Calculating Mechanical Transfer Functions with COMSOL Multiphysics

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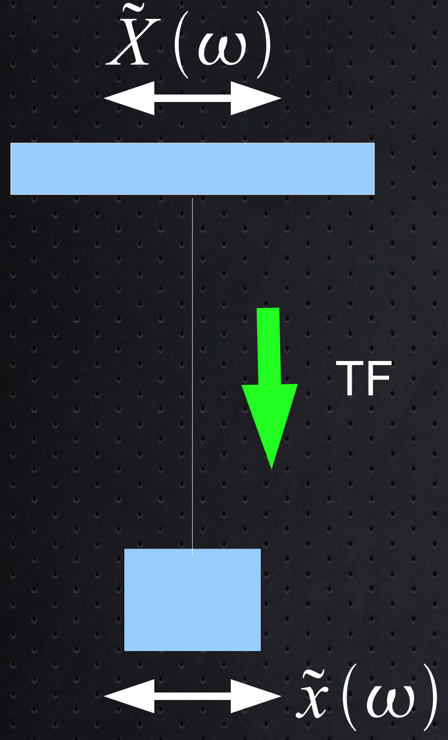
Objective

- You have a pendulum like the one shown on the right.
- You want to know the transfer function from the displacement of the suspension point to the displacement of the mass.
- The pendulum's restoring forces are the elastic force by the bending of the wire as well as the gravitational force acting on the mass.

- Elastic force can be easily calculated by COMSOL
- It is not trivial how to incorporate the gravitational force with COMSOL in the frequency domain analysis
- This presentation is intended to explain how to do it.

Frequency Domain Analysis



1D example

<u>1st Order Linear Differential Equation</u>

Linear Differential Operator

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial t}\right) \cdot u(x, t) = K\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial t}\right) \cdot f(x, t)$$

A function to be solved for (displacement for mechanics)

Source Term (force for mechanics)

We assume the source is oscillating harmonically. $f(x, t) = f_0(x) e^{i\omega t}$

Then the solution is also harmonic: $u(x, t) = u_0(x) e^{i\omega t}$

The time derivative can be substituted with iω in the above equation.

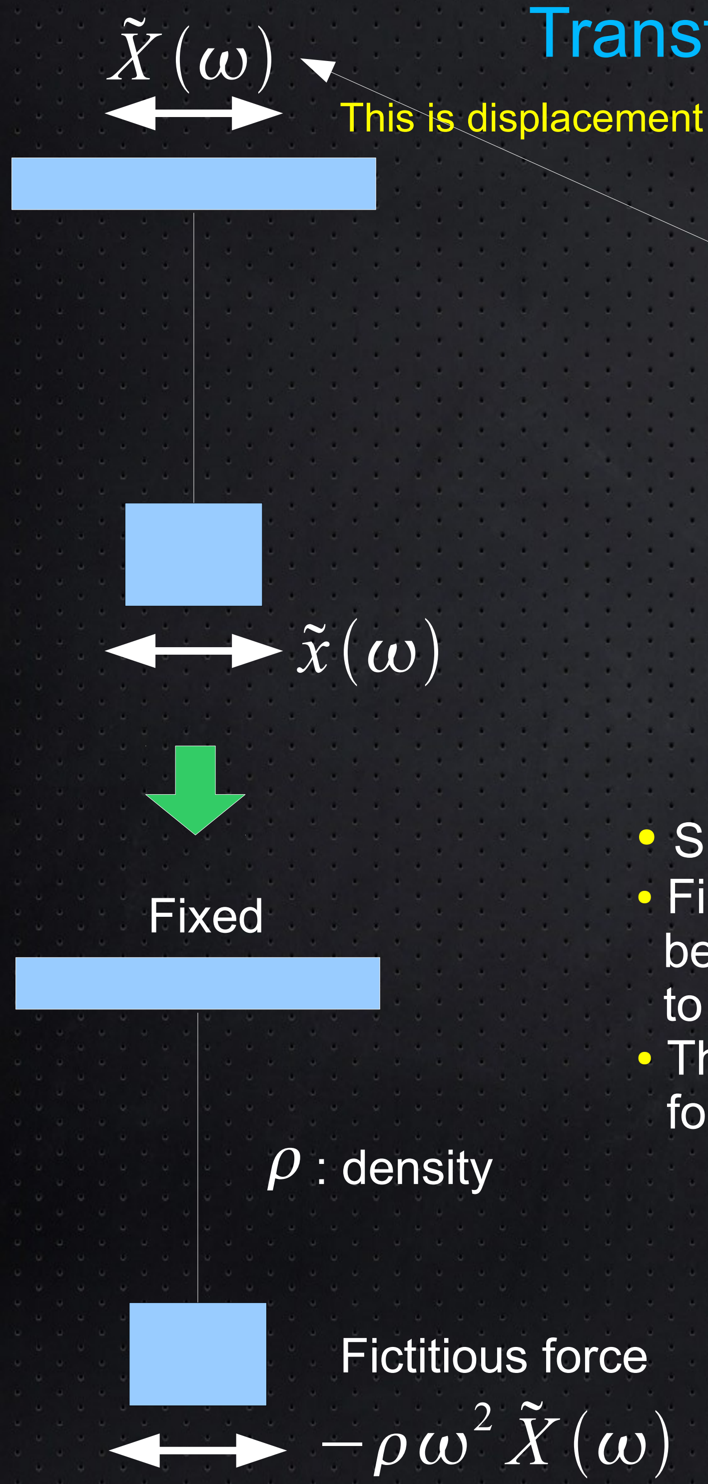
$$\frac{\partial}{\partial t} \longrightarrow i\omega \quad \Rightarrow \quad L\left(\frac{\partial}{\partial x}, \omega\right) \cdot u_0(x) = K\left(\frac{\partial}{\partial x}, \omega\right) \cdot f_0(x)$$

Non-time-dependent differential equation

→ Static solver can solve this

The solution is: $u_0(x)$ Oscillation profile at frequency ω

Transfer Function Calculation



Problem

- In the structural mechanics the source term is force
- We cannot use displacement as input

Trick

- Change the reference frame from the local inertial frame to the one co-moving with the excitation point (suspension point).

After the coordinate change

- Suspension point is fixed in the new frame
- Fictitious forces applied to all the body in the model because of the acceleration of the new frame with respect to the inertial frame.
- The input is converted from the displacement to the fictitious force, which appears as the source term in the equation.

$$L\left(\frac{\partial}{\partial x},\frac{\partial}{\partial t}\right)\cdot u(x,t)=K\left(\frac{\partial}{\partial x},\frac{\partial}{\partial t}\right)\cdot f(x,t)$$

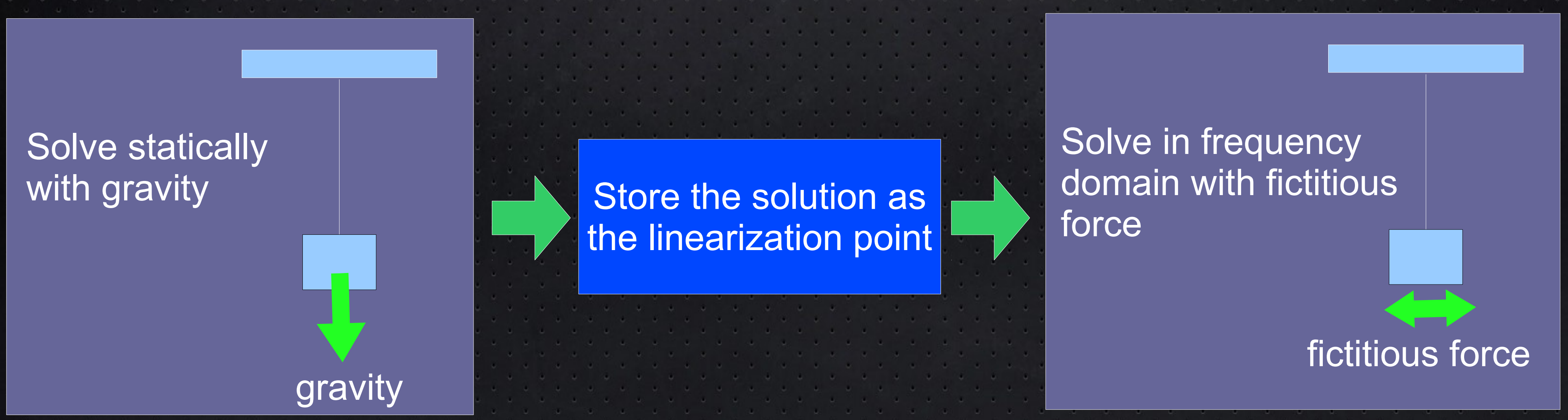
Now we can use the frequency domain analysis

How to handle the gravity ?

- Gravity is another force acting on the bodies in the model, other than the fictitious force.
- In the frequency domain analysis, all the forces are assumed to oscillate at the same frequency. Otherwise, the time-derivative substitution won't work.
- However, we don't want the gravity to oscillate.

Pre-solve the model for linearization

- A solution to this problem is to solve the model with gravity, but without the fictitious force, as a static problem first.
- Then use this solution as the linearization point for the frequency domain analysis.
- Enable the geometric nonlinearity for the static solver.
- Once the linearization point is calculated with the gravity, for the frequency domain analysis, the gravity is removed.
- This method also allows us to include the stress stiffening of the bodies by the initial deformations.



Simulation Steps

- Add “Solid Mechanics” as a Physics
- Add a “Stationary” study.
- Define variables shown on the next slide.
- Make the geometry of your model and add materials.
- Under the “Solid Mechanics” section, in the “Linear Elastic Material Model”, check “include geometric nonlinearity”.
- If you want, add “Damping” to the “Linear Elastic Material Model”. “Isotropic loss factor” is equivalent to the loss angle in the gravitational wave terminology.
- Add whatever constraints or boundary conditions you want to “Solid Mechanics”
- Add a body load of `-g*solid.rho` to all domains in the direction of gravity. `solid.rho` is the density of the material.
- Add a body load of `ax*solid.rho` to all domains in the direction of excitation.
- Disable the excitation body load for the moment.
- Solve the stationary study and store the solution (add “Store Solution” under the solver).
- Add a Frequency Domain study.
- Enable the excitation body load and disable the gravity body load.
- In the dependent variables, specify the stored solution as the source of the initial values.
- In the stationary solver, specify the stored solution as the linearization point.
- In the “Parametric” under the solver, specify the frequency points where you want to calculate the transfer function.
- Compute the frequency domain study.
- In the result section, add a 1D plot and then a Point Graph.
- Select a point in your geometry where you want to measure the TF.

Continued to the next slide

- Select the solution from the frequency domain study and put `abs(tfx)` in the expression.
- Set the axes scaling to log. You will see a TF.
- You can also plot `arg(tfx)` to see the phase of the TF.

Variables and parameters to be defined

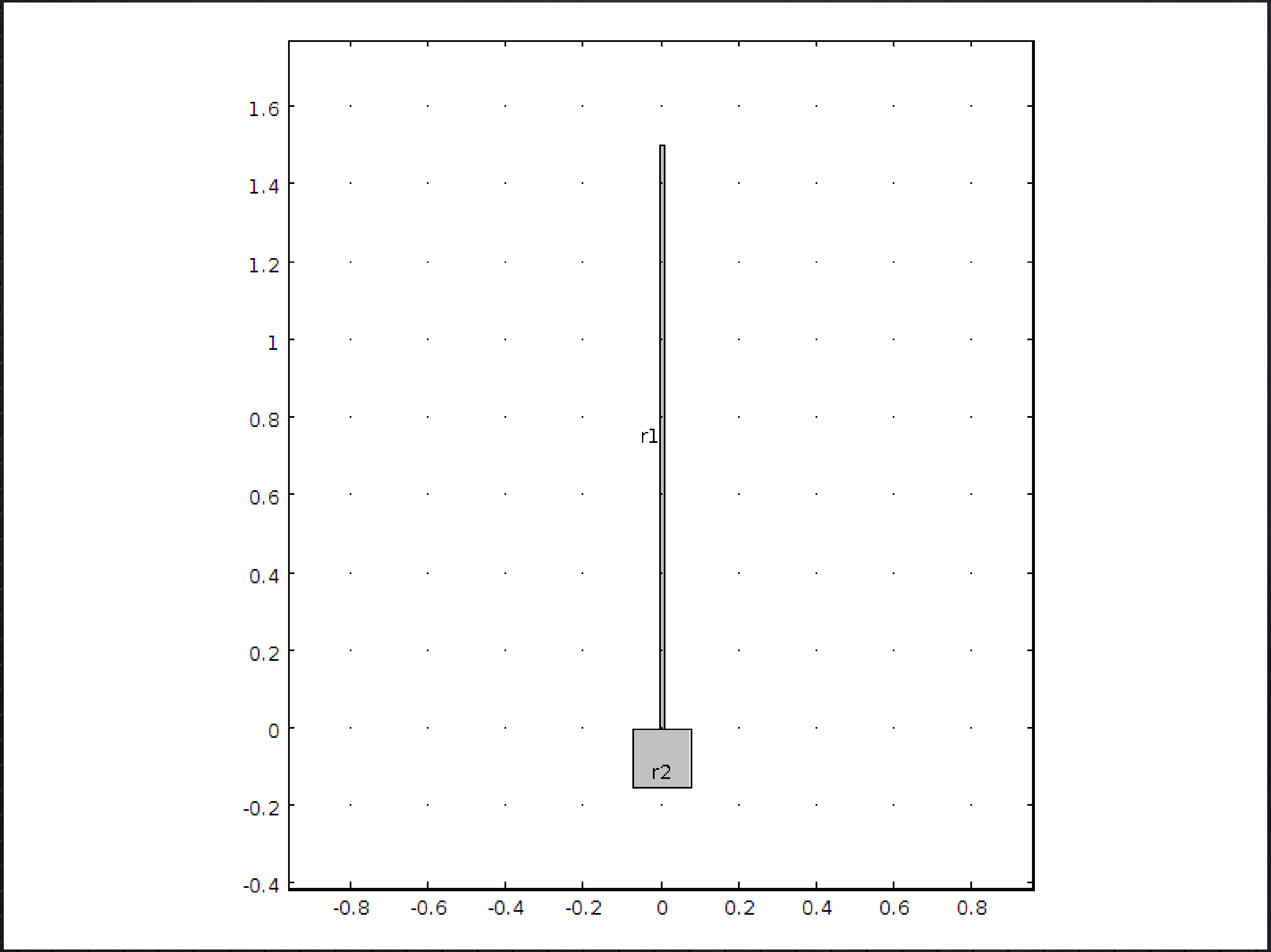
Name	Expression	Comment
excAmp	0.1[mm]	Excitation amplitude (this is a parameter)
ax	(2*pi*mod1.solid.freq)^2*excAmp	Fictitious acceleration by the excitation.
u_in	mod1.u - (-excAmp)	Displacement in the inertial frame.
tfx	u_in/excAmp	Transfer function

Note: the above variables are for x-direction transfer functions. For y- or z- directions, use v or w instead of u.

A Pendulum Example

In the next few slides, I will show some plots from a simple pendulum example.

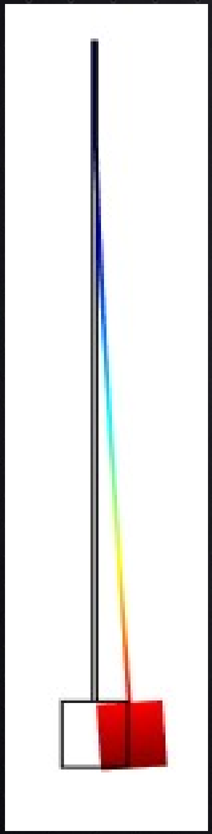
2D Pendulum



Time Series Analysis

First, we check the behavior of the pendulum with the time domain analysis.

Initial state (bent)



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