QUANTUM OPTICS

松本　伸之

1　Quantum theory of radiation

- 光の粒子性 (黒体放射 [Plank] → 光電効果 [Einstein])
- Dirac → combine the wave- and particle- like aspects of light
- **quantization of the radiation field** → **vacuum fluctuations**,
  spontaneous emission, **the Lamb shift**,
  the laser linewidth, the Casimir effect etc
- **quantum beat** → fully quantized calculation と semiclassical theory の違い
- two-photon interferometry, entangled states → chapter 21

1.1　Quantization of the free electricmagneetic field

Maxwell's equations

1.1.1　Mode expansion of the field

1. 電場 E を cavity mode(1 次元) で展開 ⇔ (1.1.5)
2. 磁場も展開 ⇔ (1.1.7)
3. the Hamiltonian of the radiation field is a sum of independent oscillator energys ⇔ (1.1.9)

1.1.2 Quantization

1 Canonical quantization ⇔ (1.1.10) $\sim$ (1.1.14)

2 電場, 磁場が演算子で表される ⇔ (1.1.15), (1.1.16)

3 a finite one-dimensional cavity $\rightarrow$ unbounced free space

3.1 電場, 磁場を平面波展開

3.2 波数ベクトル $\boldsymbol{k}$ が離散化 $\rightarrow$ さらに $\triangle\, \boldsymbol{k} \rightarrow d\, \boldsymbol{k}$ ⇔ (1.1.21), (1.1.23)

3.3 mode density ⇔ (1.1.26)

4 unbounced free space における電場, 磁場を量子化

4.1 $\pm\nu$ の成分が $a, a^\dagger$ に対応 ⇔ (1.1.29)〜(1.1.31)

1.1.3 Commutation relations between electric and magnetic field components

the parallel components of $\boldsymbol{E}$ and $\boldsymbol{H}$ may be measured simultaneously whereas the perpendicular components cannot

1.2 Fock or number states

there are nonzero fluctuations even for a vacuum state ⇔ (1.2.21)

1.3 Lamb shift

- radiative corrections due to the interaction between the atomic electron and the vacuum, shift the $2S_{1/2}$ level higher in energy by around 1057 MHz relative to the $2P_{1/2}$ level
- Dirac theory $\rightarrow$ the $2S_{1/2}$ and $2P_{1/2}$ levels should have equal energys
- the excellent agreement between the full quantum theory of radiation and matter, and experiment, e.g., the Lamb shift

1 the effect of the fluctuations in the electric and magnetic fields associated with the vacuum is a perturbation of the electron in a hydrogen atom from the standard orbits of the Coulomb potential $-e^2/4\pi\epsilon_0 r$ due to the proton ⇔ (1.3.1)

2 $\Delta V = 0$ for P-states, $\Delta V \neq 0$ for S-states ⇔ (1.3.4)

3 $\langle(\delta \boldsymbol{r})^2\rangle$ の寄与 ⇔ (1.3.7)〜(1.3.13)

3.1 全モードの和が発散してしまうように見える ⇔ (1.3.12)

3.1.1 $\nu > \pi c/a_0 \sim 1.78 \times 10^{19}$ [Hz] ⇔ $k > \pi/a_0 \sim 5.9 \times 10^{10}$ [m]
$k < mc/\hbar \sim 7.5\ \times 10^{14}$[m]
の下での積分なので発散しない。(コンプトン波長より大きくボーア半径より小さい程度の波長領域での積分)

1.4 Quantum beats

- Quantum beat phenomena provide us with a simple example of a case in witch the results of a self-consistent fully quantized calculation differ substantially from those obtained via a semiclassical theory (SCT) even when argumented by the notion of vacuum fluctuations

- SCT → matter is treated quantum mechanically while radiation is described according to Maxwell's equations, to which one adds vacuum fluctuations

1 We consider two different types of three-level atoms in the so-callde V and Λ type configrations ⇔ Fig. 1.3

2 dipole matrix elements ⇔ (1.4.2)

3 from a semiclassical perspective, the radiated field is (1.4.4)

4 for atoms of both types V and Λ, an interference term is predicted by SCT

5 from a quantum electrodynamics (QED), the transition probability of the detector is (1.4.6) ← 4.2

6 for only the V type atoms, an interference term is predicted by QED ⇔ (1.4.11)

7.1 V type → since both transitions lead to the same final atomic state, one cannot determine along which path, ν_1, ν_2 the atom decayed

7.2 Λ type → after the emission is long pastt, an observation of the atom would tell us which decay channel (1 or 2) was taken

1.5 What is light? - The photon concept

- with such examples as **quantum beat phenomena** , **the quantum eraser**, and certain **two-photon interference phenomena** , as discussed later, it becomes necessary to think of the photon as a quantum mechanical entity whose basic physics is much deeper than the semiclassical theory plus vacuum fluctuation logic
- **negative probability concepts** with indefinite-metric physics

1.5.1 Vacuum fluctuations and the photon concept

- the photoelectric effect, stimulated emission, resonance fluorescence, and other many effects do not require the full machinery of the quantum theory of radiation for their explanation ; they can rather be explained by a semiclassical analysis
- the Michelson-Morley experiment and the Rayleigh-Jeans catastrophe associated with black-body radiation involved the concept of a photon
- there are three issues associated with the photoelectric effect

First (1.5.1)

Second the rate of electron ejection is proportional to the square of the electric field of the incident light

Third there is no time delay between the time in which the field begins falling on the photoactive surface and the instance of photoelectron emission

- back-action is contained by forcing the theory to be self-consistent as shown in Fig. 1.4
- the question of where SCT breaks down

1.5.2 Vacuum fluctuations

- perhaps the most important example of a situation which is not covered by the SCT of Fig. 1.4 is the **spontaneous emission** of light
- if there are no fluctuations to get things started, the atom remains in the excited state for a long, potentially infinite, time
- Lamb shift, Plank distribution of black-body radiation, linewidth of the laser can be understood by SCT plus vacuum fluctuation arguments
- we soon realize that this concept of a ’photon’, while useful, is incomplete

1.5.3 Quantum beats, the quantum eraser, Bell's theorem, and more

- the quantum beat argument provides an example of the insufficiency of semiclassical theory plus vacuum fluctuations to, understand the physics of the phenomena
- quantum eraser → 光子が２重スリットの左側を通る場合と，右側を通る場合の重ね合わせとなり，スクリーン上で干渉する．このとき，干渉する条件は光子が２重スリットのどちらを通ったかが分からないことである．２重スリットのところでどちらが分かったか分かるようにしていても、その後その量子的な情報を消すことで干渉を復活させることができる。これを、Quantum Eraser と呼ぶ。(http://www-lab15.kuee.kyoto-u.ac.jp/index.php?id=22)

1.5.4 'Wave function for photons'

- the fact that there is, strictly speaking, no such a thing as a 'photon wave function'
- How far and how exactly can one consistently compare the radiation field with an ensemble of independent particles?
- one cannot speak of particles in a radiation field in the same sense as in the (non-relativistic) quantum mechanics of systems of point particles
- the reason is that the wave equation solutions of Schrodinger's time-dependent wave function corresponding to an energy E_λ have a circular freqency $\omega_\lambda = E_\lambda/\hbar$, while the monochromatic solutions of the wave equation have both $\pm\omega_\lambda$
- the logic of semiclassical and fully second quantized treatments of the radiation-matter system is summarized in Fig. 1.5

1 second quantization ⇔ (1.5.2)～(1.5.6)

2 the probability that a single-photon state of the radiation field ⇔ (1.5.11)

3 'electric field' associated with the single photon state is (1.5.16), (1.5.18)

3.1 'electric field' associated with the state is prepared by atomic is (1.5.17) , (sharply peaked about the atomic transition frequency ω)

4 the photodetection probability amplitude is (1.5.20)

4.1 'electric field' is (1.5.21)

5 'magnetic field' is (1.5.22)～(1.5.24)

6 Maxwell's equations in terms of the photodetection probability amplitude ⇔ (1.5.25)

7. matrix form ⇔ (1.5.26), (1.5.27)

8. the Dirac equations for the neutrino is (1.5.28)∼(1.5.33)

9. the comparison between photon and neutrino is summarized in Fig. 1.6

10. important and basic differences between the photon and the neutrino eqquations of motion ⇔ (1.5.34)∼(1.5.38)& (1.5.39)∼(1.5.44)

11. the photon 'wave function' is different from that of a nonrelativistic massiv particles, and a 'photon-as-a-particle' picture can be misleading

1.6 Problems

1.6.1

1.6.2

$f(x) = e^{xA}e^{xB}$ とすると, (5) から

$$\begin{aligned}\frac{df(x)}{dx} &= Ae^{xA}e^{xB} + e^{xA}Be^{xB} \\ &= (A + e^{xA}Be^{-xA})e^{xA}e^{xB} \\ &= (A + B + x[A,B])f(x) \end{aligned} \tag{1}$$

$[[A,B],A] = 0$, $[[A,B],B] = 0$, $f(0) = 1$ だから

$$f(x) = \exp\big((A+B)x + [A,B]x^2/2\big) \tag{2}$$

1.6.3

$g(x) = e^{-xA}Be^{xA}$ とすると

$$g(0) = B \tag{3}$$

$$\frac{dg(x)}{dx} = -[A, g(x)] \tag{4}$$

ここで $\mathrm{ad}(A)B \equiv [A,B]$ とすると, (4) は

$$\begin{aligned} g(x) &= \sum_{x=0}^{\infty} \frac{(-x)^n}{n!}(\mathrm{ad}(A))^n B \\ &\left(= B - x[A,B] + \frac{x^2}{2!}[A,[A,B]] + \ldots + \frac{(-x)^n}{n!}[A, g^{\{n\}}(x)] + \ldots\right) \\ &= e^{-x\mathrm{ad}A}B \end{aligned} \tag{5}$$

1.6.4

(a) まず

$$[a, (a^\dagger)^n] = n(a^\dagger)^{n-1} \tag{6}$$

を仮定すると

$$\begin{aligned}[a, (a^\dagger)^{n+1}] &= a^\dagger[a,(a^\dagger)^n] - [a,(a^\dagger)^n]a^\dagger \\ &= (n+1)(a^\dagger)^n \end{aligned} \tag{7}$$

次に $f(a,a^\dagger)=\sum_n \alpha_n(a)(a^\dagger)^n$ と展開できるので

$$
\begin{aligned}
[a,f(a,a^\dagger)] &= \sum_n \alpha_n [a,(a^\dagger)^n] \\
&= \sum_n \alpha_n \frac{(\partial a^\dagger)^n}{\partial a^\dagger} \\
&= \frac{\partial}{\partial a^\dagger}\sum_n \alpha_n (a^\dagger)^n \\
&= \frac{\partial f}{\partial a^\dagger} \qquad (8)
\end{aligned}
$$

(b) $f(a,a^\dagger)=\sum_n \beta_n(a^\dagger)a^n$ と展開

(c)

1.6.5

$n\equiv a^\dagger a$ とすると,

$$[a,n]=a \qquad (9)$$

$$an^m=(n+1)^m a \qquad (10)$$

であり,

$$[a,n^m]=((n+1)^m-n^m)a \qquad (11)$$

を仮定すると,

$$
\begin{aligned}
[a,n^{m+1}] &= n[a,n^m]+[a,n]n^m \\
&= n((n+1)^m-n^m)a+an^m \\
&= (n+1)^m na-n^{m+1}a+(n+1)^m a \\
&= (n+1)^{m+1}a-n^{m+1}a \qquad (12)
\end{aligned}
$$

である。従って

$$
\begin{aligned}
[a,e^{-\alpha a^\dagger a}] &= \sum_{m=0}^{\infty}\frac{(-\alpha)^m}{m!}[a,(a^\dagger a)^m] \\
&= \sum_{m=0}^{\infty}\frac{(-\alpha)^m}{m!}[a,(a^\dagger a)^m] \\
&= \sum_{m=0}^{\infty}\frac{(-\alpha)^m}{m!}\left[(n+1)^m a-n^m a\right]
\end{aligned}
\qquad (13)
$$

1.6.6

$$
\begin{aligned}
\mathscr{H} \sum_n |n\rangle\langle n| &= \hbar\nu \left(a^\dagger a + \frac{1}{2} \right) \sum_n |n\rangle\langle n| \\
&= \sum_n \hbar\nu \left(n + \frac{1}{2} \right) |n\rangle\langle n| \\
&= \sum_n E_n |n\rangle\langle n| \qquad (14)
\end{aligned}
$$

1.6.7

$$
\begin{aligned}
(\boldsymbol{s} \cdot \nabla)\boldsymbol{V} &= (s_i \nabla_i)\boldsymbol{V} \\
&= \begin{pmatrix} 0 & -\nabla_z & \nabla_y \\ \nabla_z & 0 & -\nabla_x \\ -\nabla_y & \nabla_x & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} \\
&= \nabla \times \boldsymbol{V} \qquad (15)
\end{aligned}
$$

1.6.8

$$
\begin{aligned}
\frac{\partial}{\partial t} \boldsymbol{\Psi}_\gamma^\dagger \boldsymbol{\Psi}_\gamma &= \dot{\boldsymbol{\Psi}}_\gamma^\dagger \boldsymbol{\Psi}_\gamma + \boldsymbol{\Psi}_\gamma^\dagger \dot{\boldsymbol{\Psi}}_\gamma \\
&= \dot{\boldsymbol{\varphi}}_\gamma^\dagger \boldsymbol{\varphi}_\gamma + \dot{\boldsymbol{\chi}}_\gamma^\dagger \boldsymbol{\chi}_\gamma + \boldsymbol{\varphi}_\gamma^\dagger \dot{\boldsymbol{\varphi}}_\gamma + \boldsymbol{\chi}_\gamma^\dagger \dot{\boldsymbol{\chi}}_\gamma \\
&= -(c\nabla \boldsymbol{\chi}_\gamma^\dagger \cdot \boldsymbol{s}^t)\boldsymbol{\varphi}_\gamma + (c\nabla \boldsymbol{\varphi}_\gamma^\dagger \cdot \boldsymbol{s}^t)\boldsymbol{\chi}_\gamma + \boldsymbol{\varphi}_\gamma^\dagger (c\boldsymbol{s} \cdot \nabla \boldsymbol{\chi}_\gamma) - \boldsymbol{\chi}_\gamma^\dagger (c\boldsymbol{s} \cdot \nabla \boldsymbol{\varphi}_\gamma) \\
&= -c\nabla \cdot (\boldsymbol{\chi}_\gamma^\dagger \boldsymbol{s} \boldsymbol{\varphi}_\gamma - \boldsymbol{\varphi}_\gamma^\dagger \boldsymbol{s} \boldsymbol{\chi}_\gamma) \\
&= -\nabla \cdot \boldsymbol{j} \qquad (16)
\end{aligned}
$$

1.6.9