論文紹介
Exploring Macroscopic Quantum Mechanics in Optomechanical Devices
written by Haixing Miao

Nobuyuki Matsumoto

2011/11/9 坪野研輪講

Decrease your frequency by expanding your horizon. Increase your Q by purifying your mind. Eventually, you will achieve inner peace and view the internal harmony of our world.

-A lesson from a harmonic oscillator

Abstract

- For surpassing the SQL

I. Modifying the input and output optics

II. Modifying the dynamics

III. Measuring a conserved quantity

- For an MQM experiment with optomechanical devices

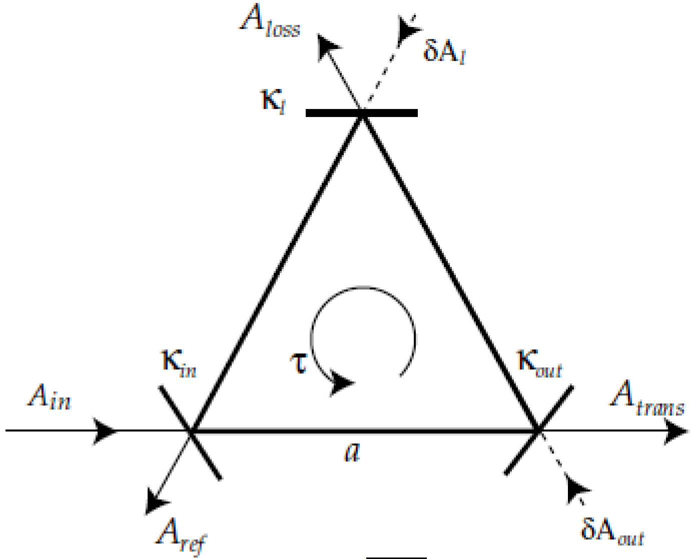
I. State preparation

II. State verification

Contents

I. Introduction

- Fabry-Perot cavity



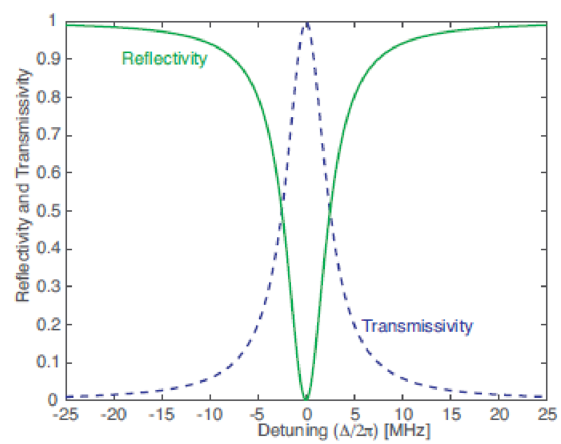
$$\bar{a} = \frac{\sqrt{2\kappa_{in}}}{\kappa + i\Delta}\bar{A}_{in}.$$

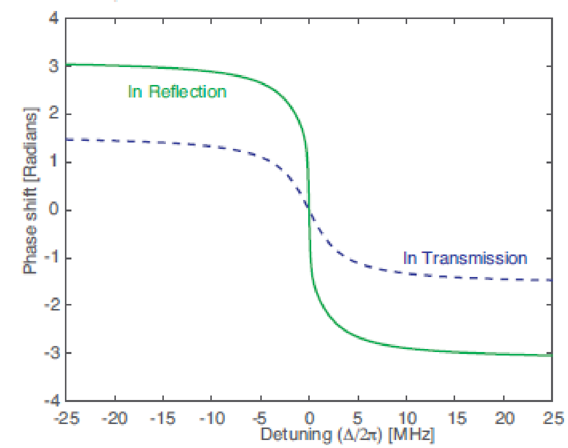
$$\nu_{fsr} = \frac{c}{p} \qquad \mathcal{F} = \frac{\pi(R_{in}R_{out}R_l)^{\frac{1}{4}}}{1-\sqrt{R_{in}R_{out}R_l}} \approx \frac{2\pi}{T_{in}+T_{out}+T_l}.$$

$$\delta\nu = \frac{\nu_{fsr}}{\mathcal{F}}. \qquad P_{circ} = \frac{\hbar\omega a^\dagger a}{\tau} = \frac{4T_{in}}{(T_{in}+T_{out}+T_l)^2}P_{in}$$

$$\mathcal{T}(\Delta) = \frac{\bar{A}_{trans}}{\bar{A}_{in}} = \frac{2\sqrt{\kappa_{in}\kappa_{out}}}{\kappa + i\Delta},$$

$$\mathcal{R}(\Delta) = \frac{\bar{A}_{ref}}{\bar{A}_{in}} = \frac{(2\kappa_{in}-\kappa-i\Delta)}{\kappa+i\Delta}$$



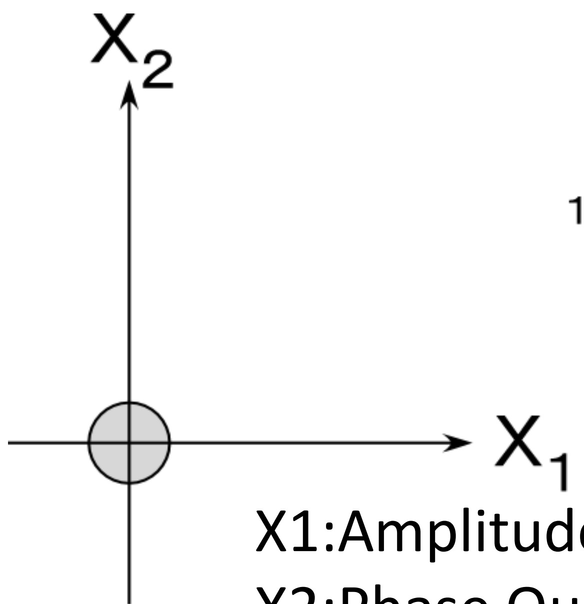


I. Introduction

真空場

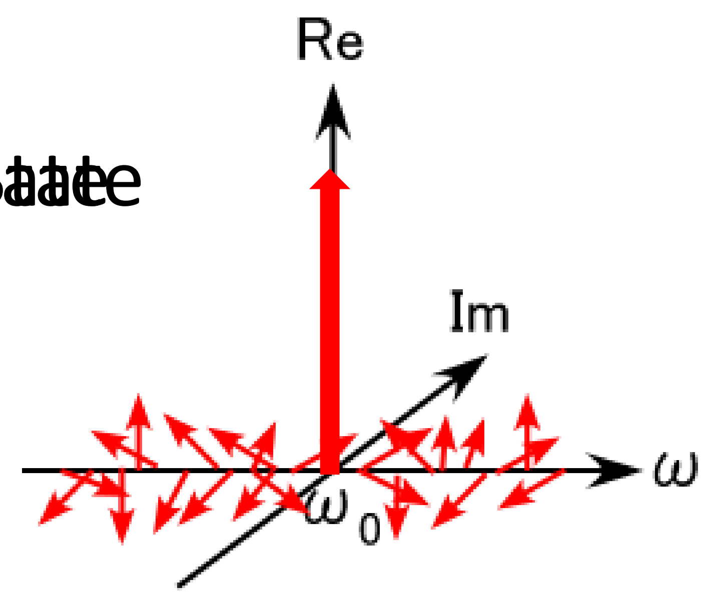
- 光子数が0の状態　$|0\rangle = |n=0\rangle = |\alpha=0\rangle$
- 揺らぎを伴う　$\langle 0|\,\Delta X\,|0\rangle \neq 0$

The Ball-on-Stick Picture



Vacuum State
Coherent State

The Sideband Picture



X1:Amplitude Quadrature

X2:Phase Quadrature

I. Introduction

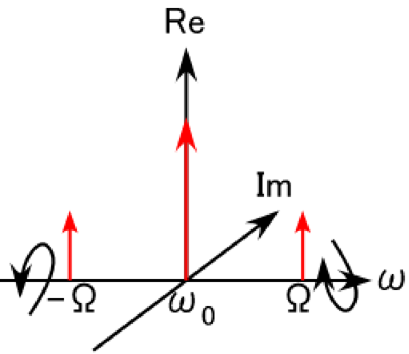
- Squeezed state

i. 零点振動のサイドバンド成分が相関を持った状態

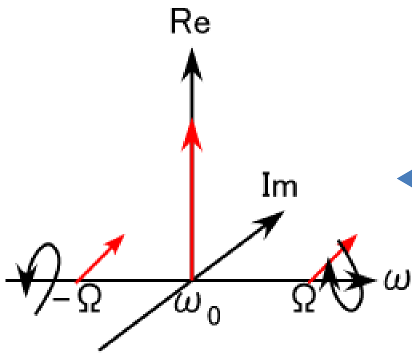
ii. 古典的な場合の変調がかかった状態と類似

$$
\begin{aligned}
a_{\mathrm{am}} e^{\mathrm{i}\omega t} &= a_0 \left(1 + m\cos(\Omega t)\right) e^{\mathrm{i}\omega t} \\
&= a_0 \left(e^{\mathrm{i}\omega t} + \frac{m}{2} e^{\mathrm{i}(\omega+\Omega)t} + \frac{m}{2} e^{\mathrm{i}(\omega-\Omega)t}\right)
\end{aligned}
$$

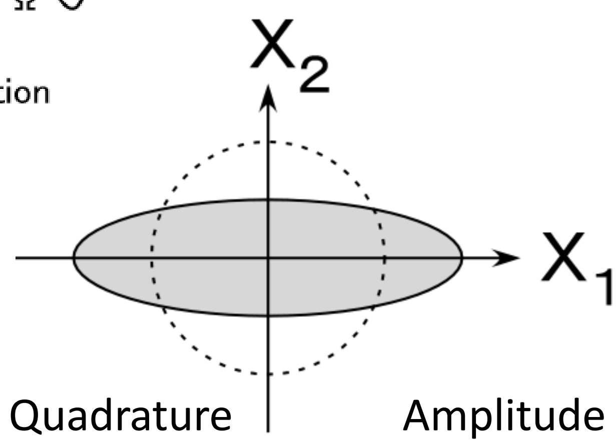
$$
\begin{aligned}
a_{\mathrm{pm}} e^{\mathrm{i}\omega t} &= a_0 e^{\mathrm{i}(\omega t + m\cos(\Omega t))} \\
&= a_0 e^{\mathrm{i}\omega t} \left(\mathrm{J}_0(m) + \sum_{l=1}^{\infty} \mathrm{i}^l \mathrm{J}_l(m) (e^{\mathrm{i}l\Omega t} + e^{-\mathrm{i}l\Omega t})\right) \\
&= a_0 e^{\mathrm{i}\omega t} \left(\mathrm{J}_0(m) + \mathrm{i}\mathrm{J}_1(m)(e^{\mathrm{i}\Omega t} + e^{-\mathrm{i}\Omega t}) + \mathcal{O}(l \geq 2)\right) \\
&\simeq a_0 \left(e^{\mathrm{i}\omega t} + \mathrm{i}\frac{m}{2} e^{\mathrm{i}(\omega+\Omega)t} + \mathrm{i}\frac{m}{2} e^{\mathrm{i}(\omega-\Omega)t}\right)
\end{aligned}
$$



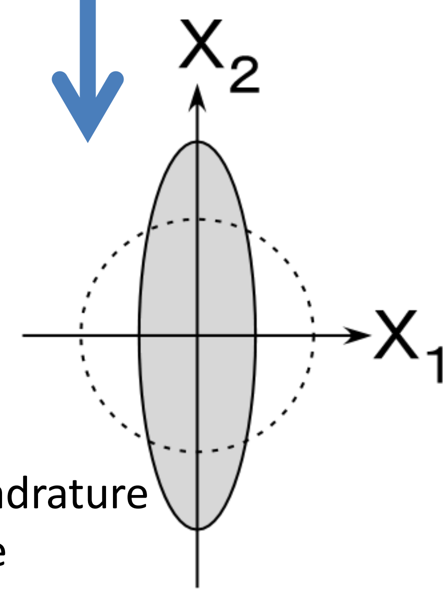
Amplitude modulation



Phase modulation



Phase Quadrature
Squeezed State



Amplitude Quadrature
Squeezed State

I. Introduction

- SQL (Standard Quantum Limit) $S_h^{\rm SQL}(\Omega) = \dfrac{2\hbar}{m\Omega^2 L^2}$

-origin-

the dynamics of the optical field, and of the test-mass

shot and radiation pressure noise

$[\hat{x}(t), \hat{x}(t')] \neq 0 \, (t \neq t')$

I. Introduction

$$h_{shot}(\Omega) = \frac{1}{L}\sqrt{\frac{\hbar c\lambda}{2\pi P}},$$

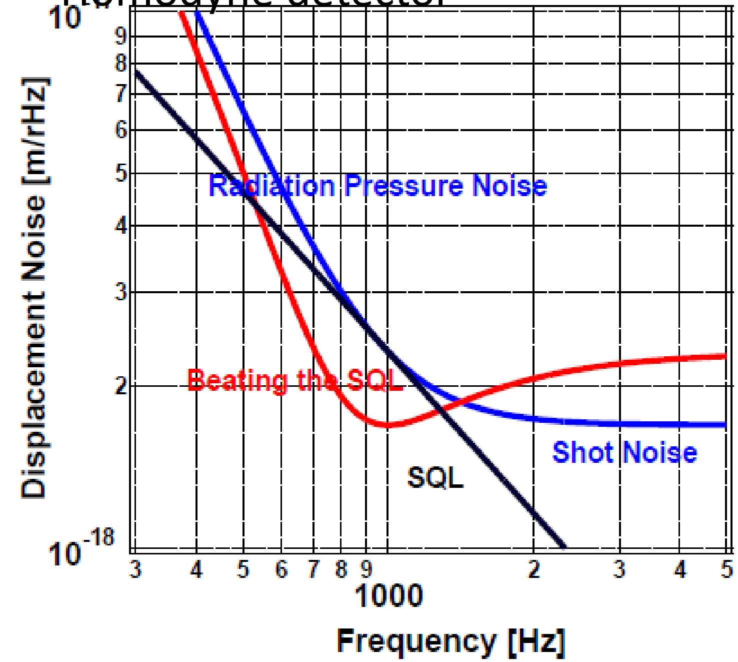
$$h_{rad}(\Omega) = \frac{1}{2m\Omega^2 L}\sqrt{\frac{8\pi\hbar P}{c\lambda}},$$

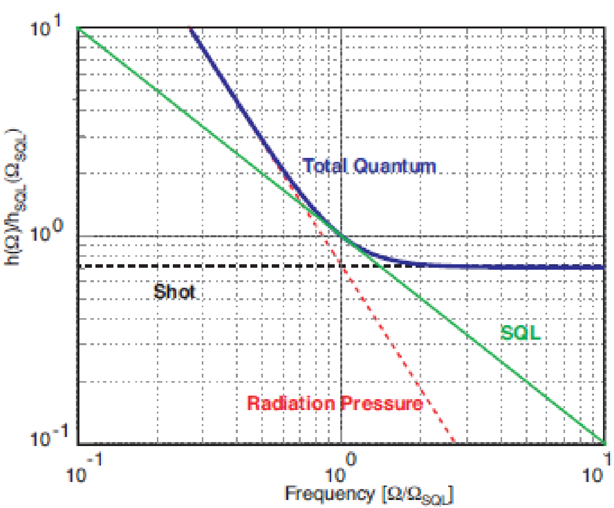
$$h_{total} = \sqrt{h_{shot}^2(\Omega) + h_{rad}^2(\Omega)}.$$

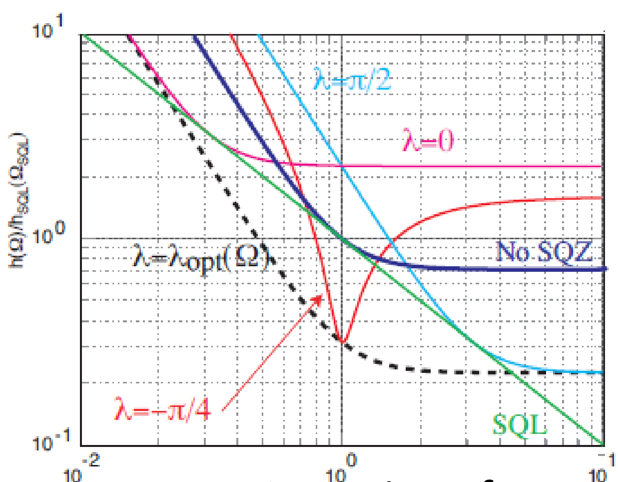
$$h_{SQL} = \sqrt{\frac{4\hbar}{m\Omega^2 L^2}},$$

$$S_h = \frac{h_{SQL}^2}{2}\left(\frac{1}{\mathcal{K}} + \mathcal{K}\right)$$

Homodyne detector







Inputting of FD squeezed vacuum

II-i. Modifying Input and Output Optics

- 量子雑音を低減させるためにinput opticsとoutput opticsを操作する。

i. Shot noise => input squeezing

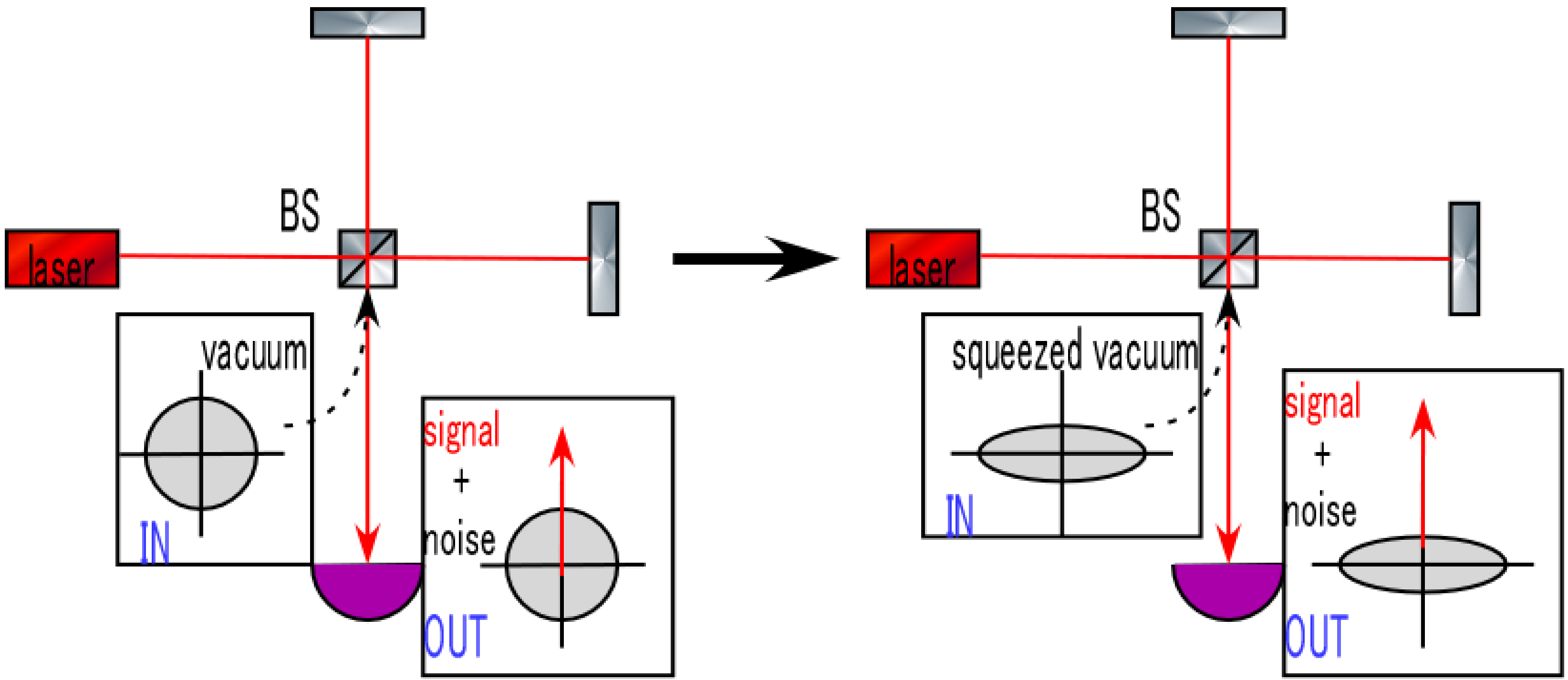
ii. Radiation pressure noise => (FD) homodyne measurement (ponderomotive squeezing), input squeezing

- Beat the SQL => FD squeezing, (FD) homodyne

  => filter cavity

II-i. Modifying Input and Output Optics

- Input squeezing (高周波領域)



$$b_1 = a_1 e^{2i\beta}$$

$$b_2 = (a_2 - \mathcal{K} a_1) e^{2i\beta} + \sqrt{2\mathcal{K}} \frac{h}{h_{SQL}} e^{i\beta}$$

$$\mathcal{K} = \frac{4 I_0 \omega_0}{m c^2 \Omega^2}$$

II-i. Modifying Input and Output Optics

- homodyne measurement(中間周波数帯以下)

PHYSICAL REVIEW D, **65**, 022002, (2001)

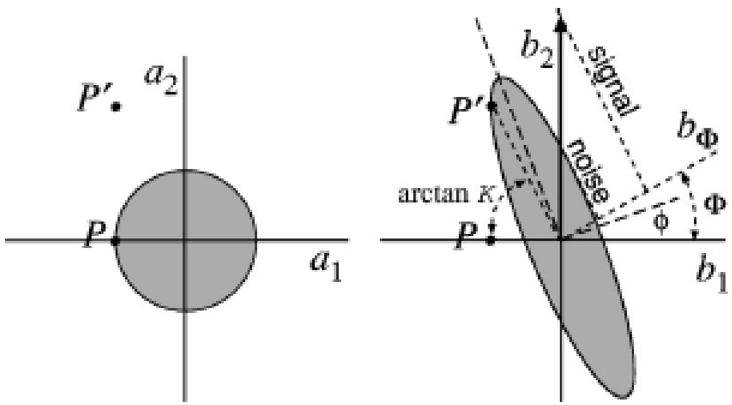


FIG. 7. Noise ellipses for a variational-output interferometer. *Left:* Noise for the ordinary vacuum that enters the interferometer's dark port. *Right:* Noise for the field that exits at the dark port along with the gravitational-wave signal. These noise ellipses are the same as for a conventional interferometer, Fig. 5, but here the quantity measured is the quadrature amplitude b_Φ with frequency dependent phase $\Phi \equiv \mathrm{arccot}\,\mathcal{K}$. It is informative to compare the measured phase Φ with the angle of ponderomotive squeeze $\phi = \frac{1}{2}\mathrm{arccot}(\mathcal{K}/2)$. They are related by $\tan\Phi = \frac{1}{2}\tan 2\phi = \tan\phi/(1-\tan^2\phi)$, so Φ is always larger than ϕ; but for large $\mathcal{K}$ (strong beating of the SQL), they become small and nearly equal.

輻射圧雑音によるバックアクションが相関を持ってマイケルソン干渉計の両腕のミラーを揺らす。
⇒ポンデロモーティブスクイージング;揺らぎが相関を持つ。テストマスの周波数依存性に応じた依存性を伴う。

II-ii. Variations of Filter Cavity

- Kimble

$$S_h = \frac{h_{\mathrm{SQL}}^2}{2}\left(\frac{1}{\mathcal{K}} + \mathcal{K}\right)(\cosh 2R - \cos[2(\lambda + \Phi)]\sinh 2R).$$

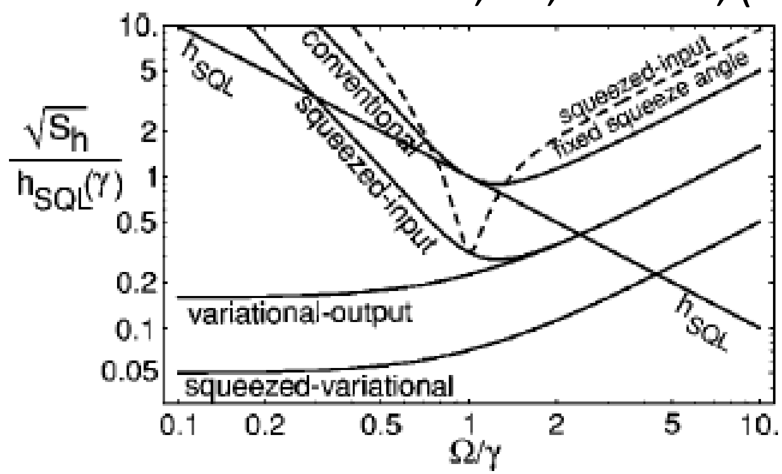
$$\lambda(\Omega) = -\Phi(\Omega) \equiv -\mathrm{arccot}\,\mathcal{K}(\Omega).$$

$$S_h = \frac{h_{\mathrm{SQL}}^2}{2}\left(\frac{1}{\mathcal{K}} + \mathcal{K}\right)e^{-2R}. \qquad \mathcal{K} \equiv \frac{(I_o/I_{\mathrm{SQL}})2\gamma^4}{\Omega^2(\gamma^2+\Omega^2)}$$

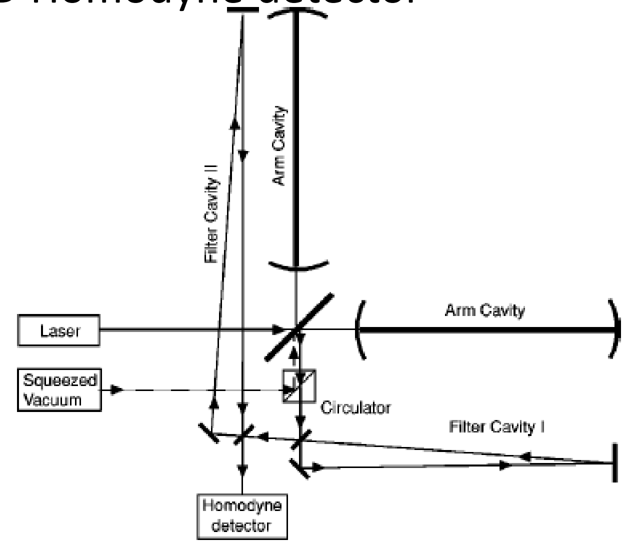
キャビティ長が長い～ km

性能は良い

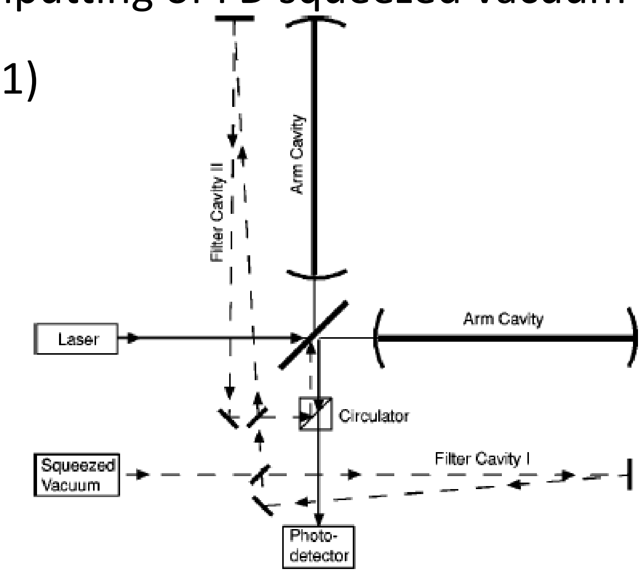
PHYSICAL REVIEW D, **65**, 022002, (2001)



FD Homodyne detector



Inputting of FD squeezed vacuum



II-ii. Variations of Filter Cavity

- Corbitt

i. 中間帯はダメ

ii. キャビティ短い

iii. ロスに強い

PHYSICAL REVIEW D, **70**, 022002, (2004)

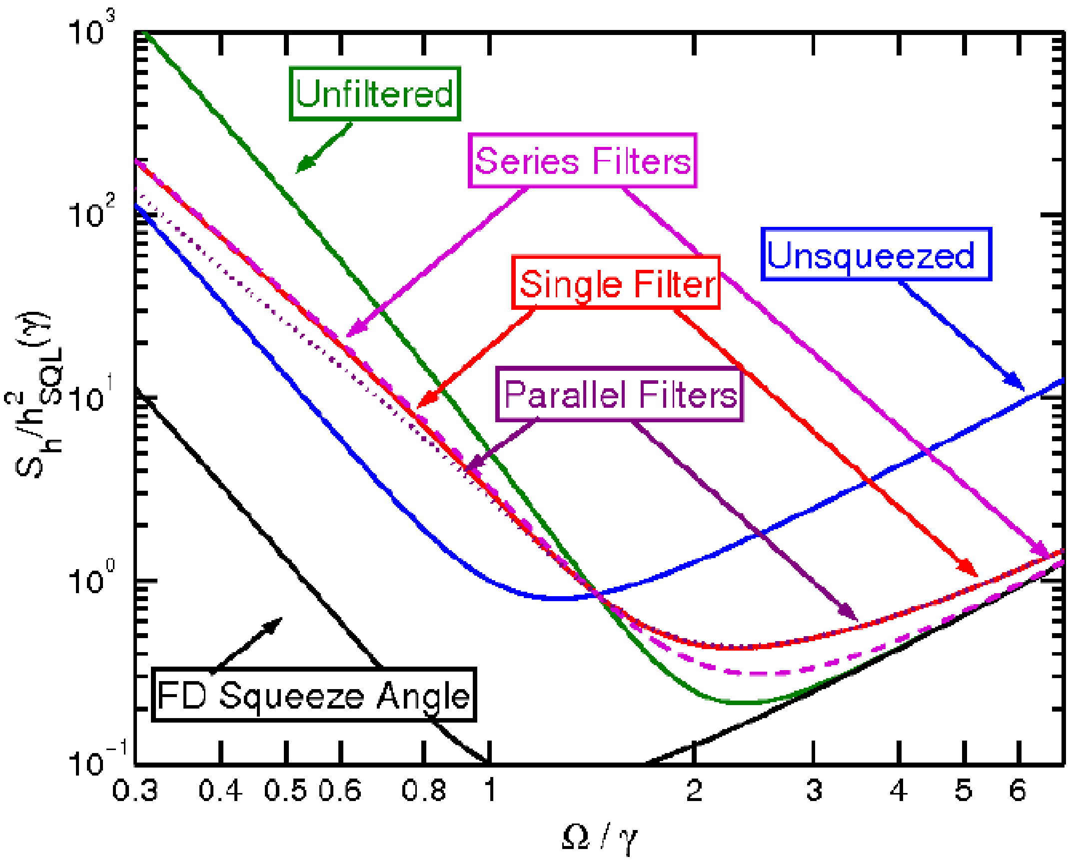


Figure 2-6: The square root of the noise spectral density is shown for a conventional interferometer with (i) no squeezed input ("Unsqueezed"), (ii) Squeezed vacuum injected ("Unfiltered"); Squeezed input filtered by (iii) a single filter cavity ("Single Filter"), (iv) series filter cavities ("Series Filter"), (v) parallel filter cavities ("Parallel Filter"); and (vi) frequency-dependent squeeze angle ("FD Squeeze Angle").

II-ii. Variations of Filter Cavity

- Khalili　　i. キャビティ短い(～30m)

  ii. ロスに強い

  iii. 中間周波数帯でも感度向上

PHYSICAL REVIEW D, **77**, 062003 (2008)

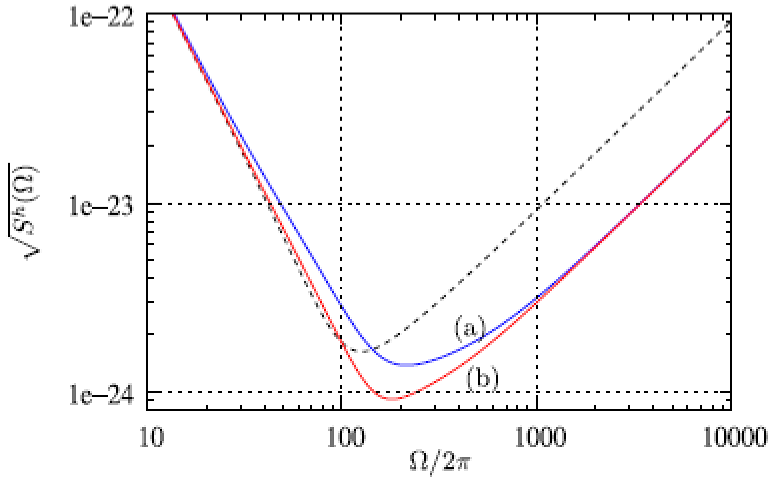
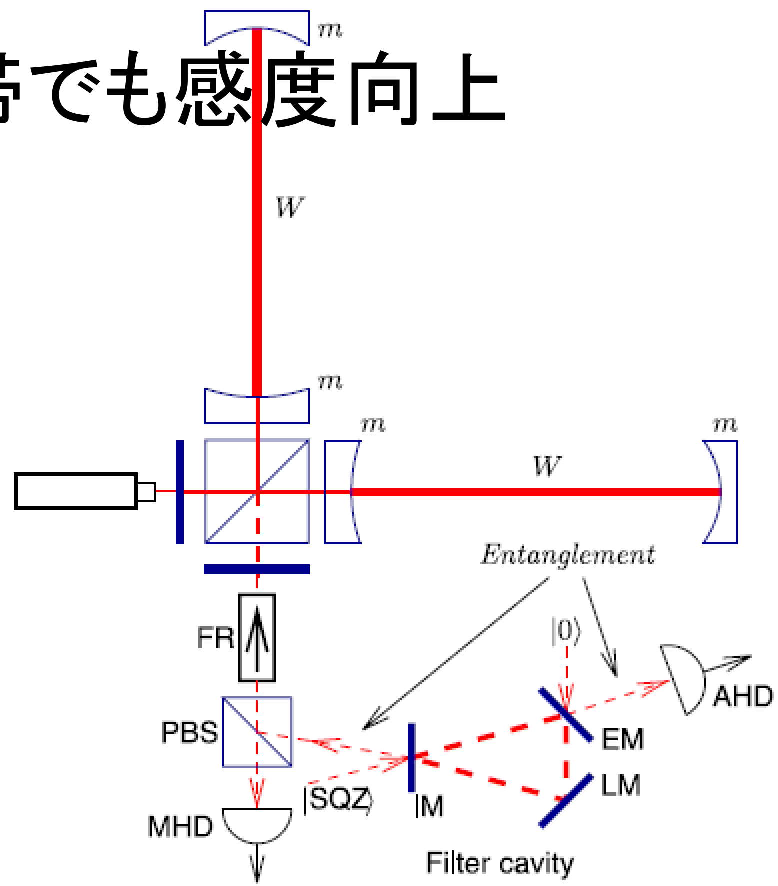


FIG. 3 (color online). Square root of the sum quantum noise spectral density of the conventional unsqueezed interferometer (dashed line); the original CMW scheme (a); the CMW scheme with the additional homodyne detector (b). In all three cases, $W = 840$ kW, $\gamma = 2\pi \times 100$ s^{-1}, and $\eta = 1$.



II-ii. Variations of Filter Cavity

- Haixing

i. 観測帯域全域に渡り量子雑音を低減できる

ii. キャビティ長が短い（～30 m）

iii. スクイーザーが二台必要

Haixing Miao, Doctor thesis

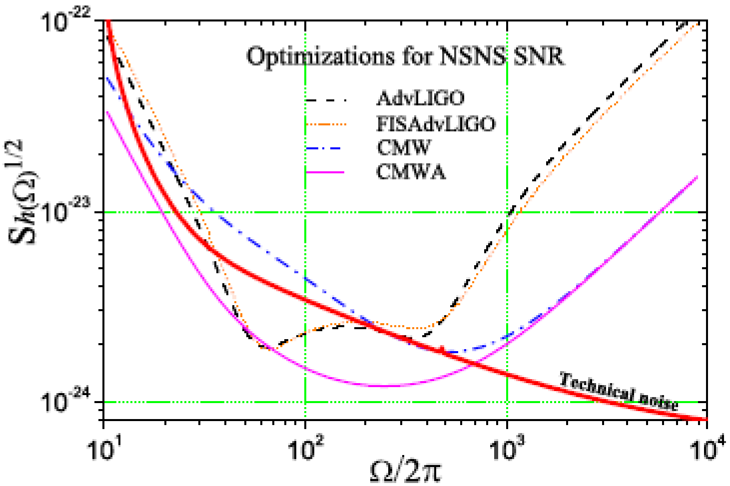
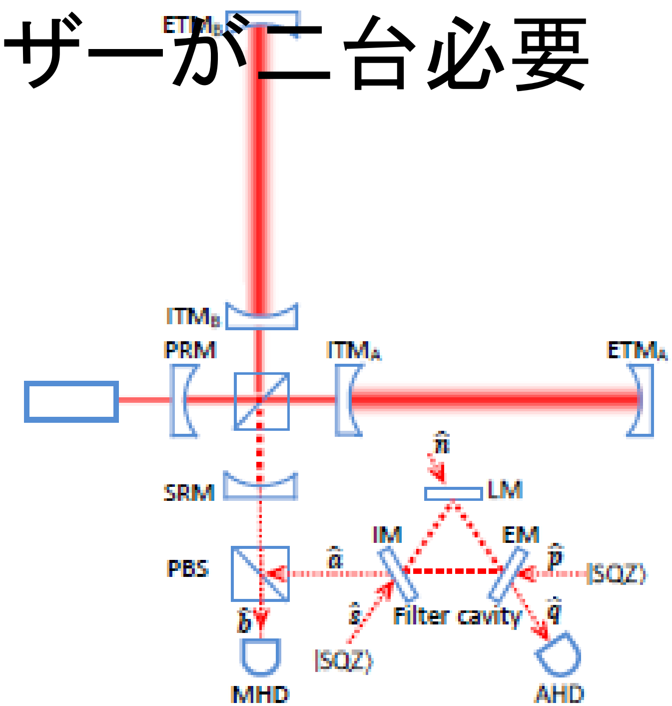


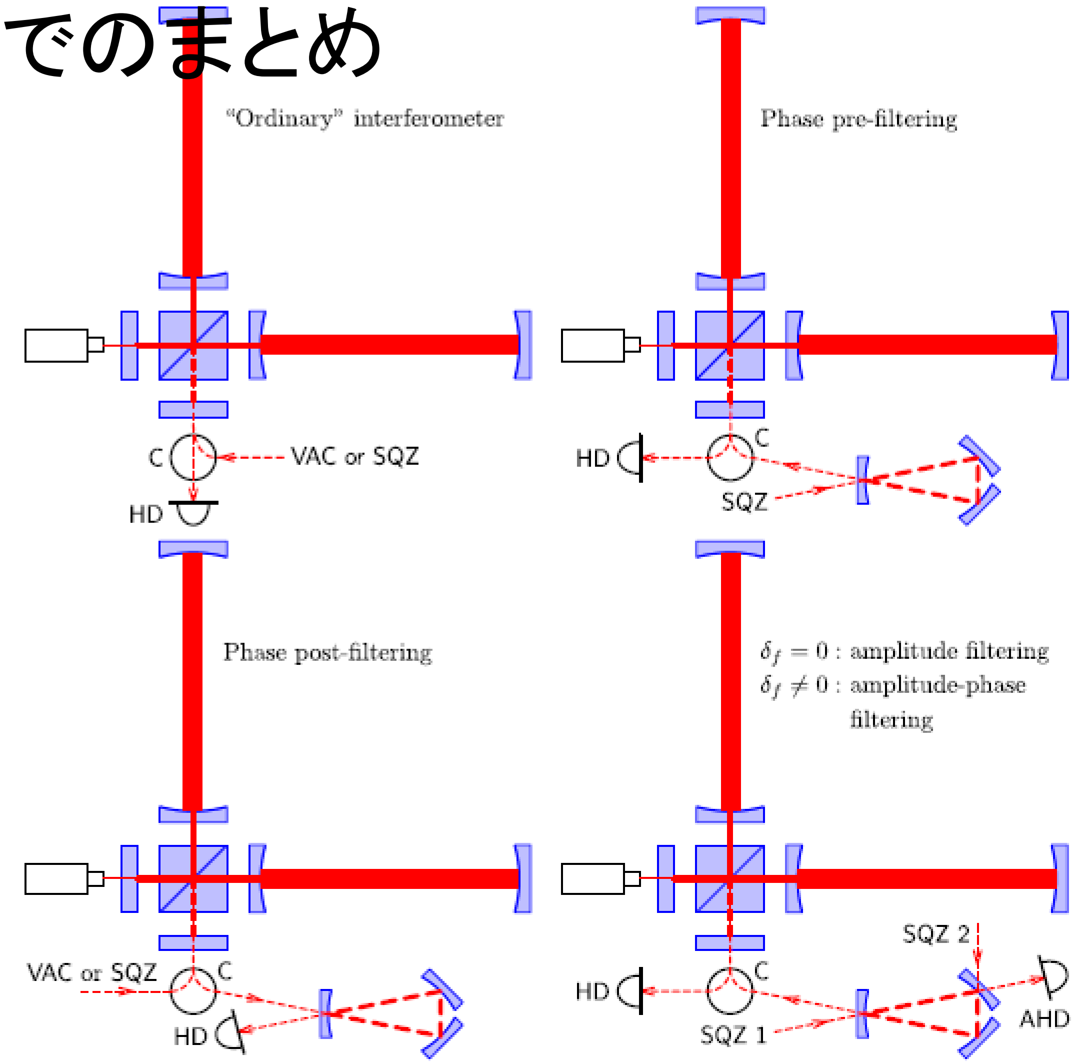
Figure 3.2 – Quantum-noise spectra of different schemes with optimized parameters for detecting gravitational waves from NS-NS binaries. The optimal values for the parameters are listed in Table. 3.2.



II-ii. Variations of Filter Cavity

PHYSICAL REVIEW D, **81**, 122002, (2010)

- ここまでのまとめ



II-iii. Comparing Performances

PHYSICAL REVIEW D, **81**, 122002, (2010)

SNRs for

the burst sources(left)

the NSNS binary(right)

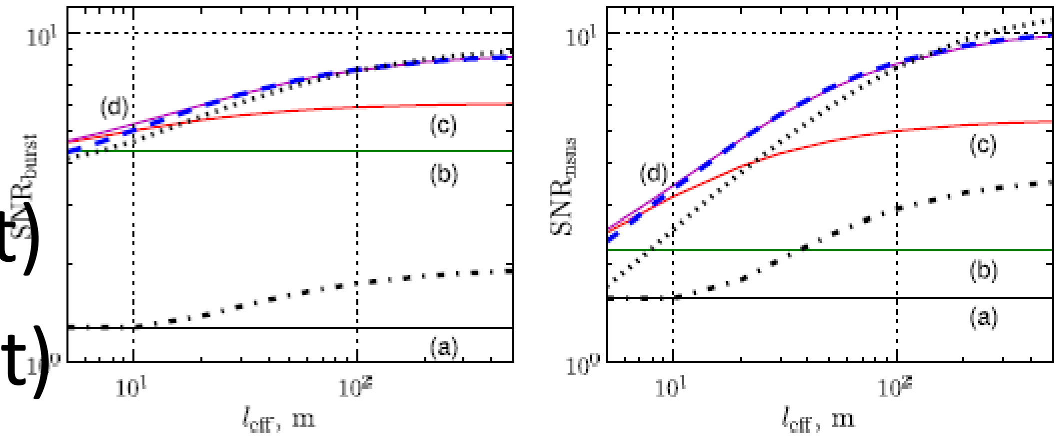


FIG. 2 (color online). Normalized signal-to-noise ratios as functions of the filter cavity effective length. Solid lines: (a) —ordinary interferometer, vacuum input; (b) —frequency-independent squeezing; (c) —amplitude filtering, (d) —combined amplitude-phase filtering; Dashed line: phase prefiltering; dashed-dotted line: phase post-filtering, vacuum input; dotted line: phase post-filtering, squeezed input. Left: optimization for bursts; Right: optimization for neutron star-neutron star events.

$$\mathrm{SNR}_{\mathrm{burst}} = \frac{1}{N_{\mathrm{burst}}} \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}^{\mathrm{burst}}} \frac{df}{f S^h(2\pi f)},$$

$$\mathrm{SNR}_{\mathrm{nsns}} = \frac{1}{N_{\mathrm{nsns}}} \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}^{\mathrm{nsns}}} \frac{df}{f^{7/3} S^h(2\pi f)},$$

Khalili のフィルターは
そんなに良くない？？

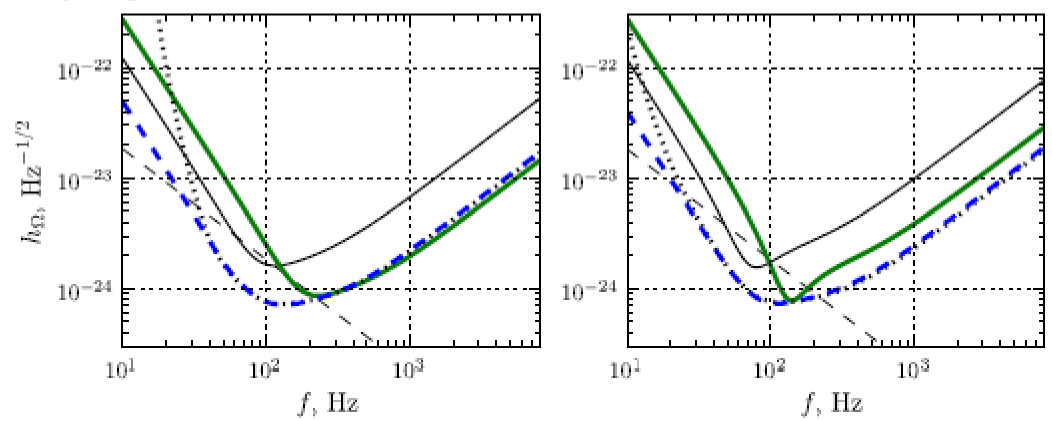


FIG. 4 (color online). Optimal quantum noises spectral densities. Thin dashed lines: SQL; thin solid lines: ordinary interferometer, vacuum input; thick solid lines: ordinary interferometer, frequency-independent squeezing; thick dashed lines: phase prefiltering; dotted line: phase post-filtering. Left: optimization for bursts; Right: optimization for neutron star-neutron star events.

III. Modifying the Dynamics

- スクイーズを用いた方法ではロスに弱い

$$\xi = \frac{h}{h_{\rm SQL}^{\rm fm}} = \sqrt[4]{\frac{1-\eta}{\eta}}\,. \qquad \xi \gtrsim 0.5 - 0.3\,.$$

- Optical spring effect を利用し信号を増幅

(reducing the effective inertia of the probe mass)

$$\eta \equiv \frac{S^F_{\rm SQL}|_{\rm modified}}{S^F_{\rm SQL}|_{\rm free\ mass}} = \frac{\Omega^2}{|(\Omega^2 - \omega_m^2) + 2i\gamma_m\Omega|}.$$

⇒ロスに強い

⇒i. single optical spring

anti-damping force による不安定性.狭帯域

ii. double optical spring

不安定性を解消し、かつ広帯域に

III. Modifying the Dynamics

- Optical spring effect (FD restoring force)

PHYSICAL REVIEW A, **69**, 051801(R), (2004)

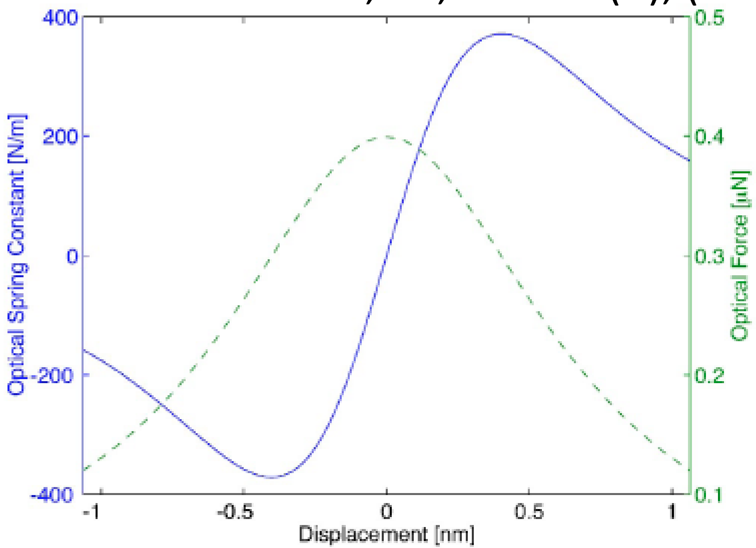


FIG. 1. The dashed curve shows the radiation pressure on the cavity mirror as a function of the detuning from resonance. The solid curve shows the derivative of the optical force, i.e., the optical spring constant. Positive displacement corresponds to increasing the cavity length. The parameters for calculating this curve are the ones from the experiment described here. The circulating power on resonance is 60 W, corresponding to the maximum input power used–400 mW.

古典的なフィードバック
を用いる場合と異なり
熱雑音をもたらさない
（hν/k_B～10000 k）

detuning の正負によって

Δ>0 => K>0 (restore), γ<0 (anti-damp)

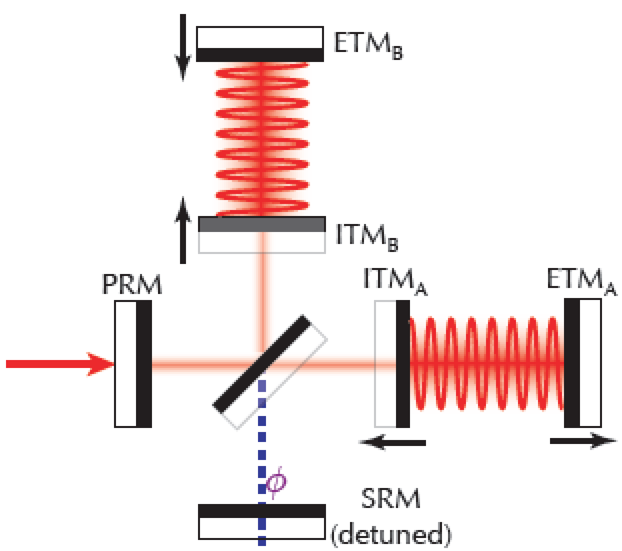
Δ<0 => K<0 (anti-restore), γ>0 (damp)

III. Modifying the Dynamics

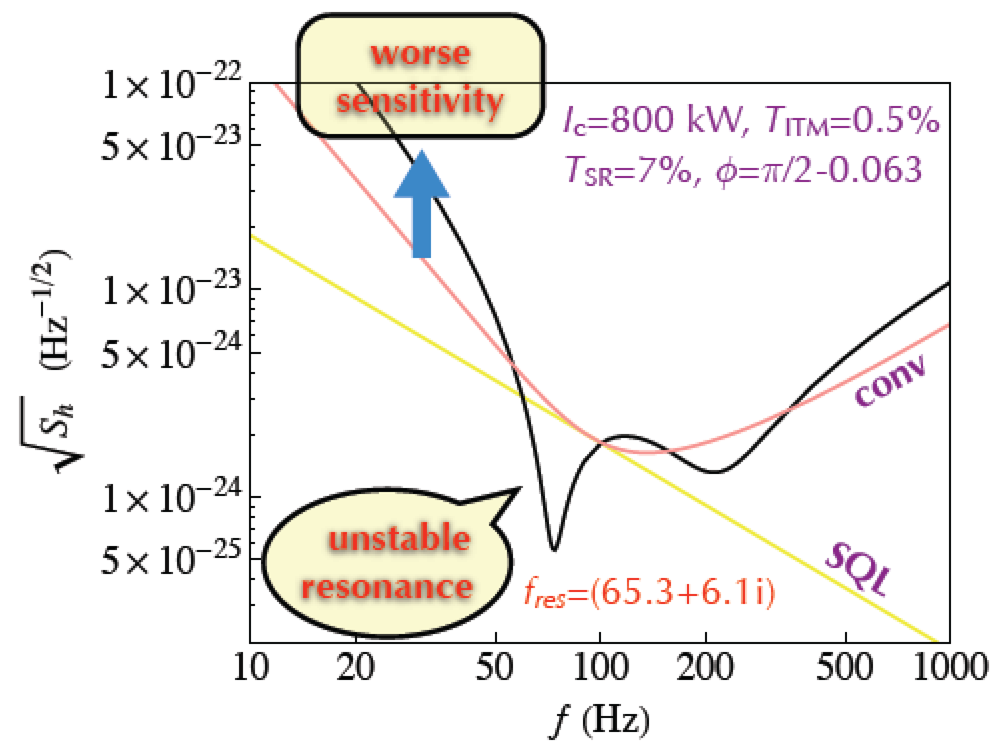
Yanbei Che
Parametric Instability Workshop, July 17, 2007

Optical Spring: introduction and issues

- **Optical springs** in detuned signal recycling interferometers [Buonanno & Chen 00 & 01] differential mode can be viewed as single detuned cavity [Buonanno & Chen, 03].
- **Sensitivity:** *enhanced* around optical and mechanical resonances (beating SQL), but **suppressed** in other, especially low frequencies.
- **Instability:** optical spring resonance is unstable; in-band control does not impose fundamental noise [Buonanno & Chen, 00 & 01]



optical spring connects ITM and ETM

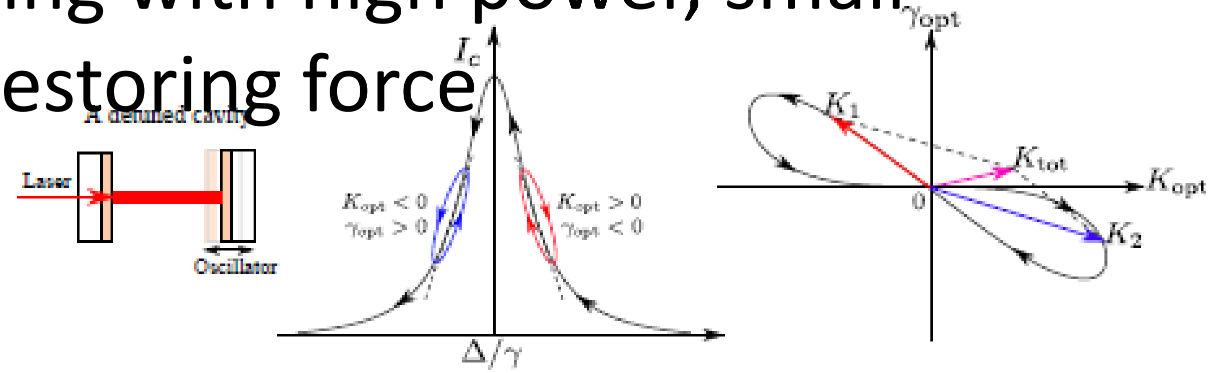


III. Modifying the Dynamics

- 異なる2つの周波数のレーザーを用い、それぞれのdetuningを調整(double optical spring)

⇒i. small “-” detuning, large “+” damping

ii. large “+” detuning with high power, small “-” damping, strong restoring force



- K>0, γ>0 を実現

Haixing Miao, Doctor thesis

Figure 1.5 – Plot showing the optical spring effect in a detuned optical cavity. The radiation pressure is proportional to the intra-cavity power which depends on the position of the test mass. The non-zero delay in the cavity response gives rise to an (anti-)damping force. By injecting two laser beams at different frequencies, this creates a double optical spring and the system can be stabilized (right panel).

III. Modifying the Dynamics

- Optical spring に課せられる条件

$$K_1(0) + K_2(0) = 0\,, \quad \frac{1}{2}\frac{\partial^2[K_1(s) + K_2(s)]}{\partial s^2}\Big|_{s=0} + m = 0\,.$$

i. 静的にはバネ定数はキャンセル

ii. Zero inertia の実現

$$\Rightarrow \quad \chi^{-1}(s) \approx -2\gamma s + \frac{4\gamma s^3}{\Delta^2} + \frac{s^4}{\Delta^4} + O(s^5)\,.$$

- Friction term が低周波での感度向上を妨げる

$$\Rightarrow \quad \frac{\partial[K_1(s) + K_2(s)]}{\partial s}\Big|_{s=0} = 0.$$

III. Modifying the Dynamics

- Detuned signal-recycling cavity を用いた場合のテストマスの感受率（∝S_(SQL)^-1）

test mass $m = 0.8\,\mathrm{kg}$ and a cavity length $L = 80\,\mathrm{m}$,

$\Delta_1/2\pi = 200\,\mathrm{Hz}$, $\Delta_2/2\pi = -500\,\mathrm{Hz}$, $\gamma_1/2\pi = 36\,\mathrm{Hz}$, $\gamma_2/2\pi = 400\,\mathrm{Hz}$.

$I_1 = 3\,\mathrm{kW}$, $I_2 = 10\,\mathrm{kW}$.

i. 広帯域で感度向上
ii. 制御は？

Haixing Miao, Doctor thesis

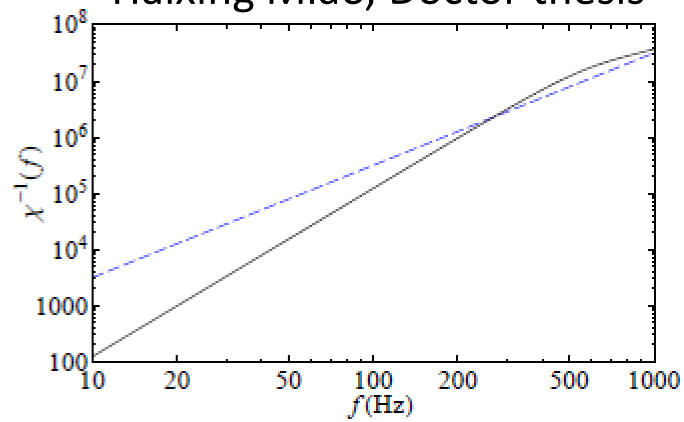


Figure 4.1 – The resulting response function with a double optical spring (solid), compared with that of a free mass (dashed).

IV. Measuring a Conserved Quantity

- [a(t),a(t')]=0(測定による反作用を回避)

⇒位置の測定ではなくmechanical quadratureの測定を！

$$\hat{x}(t) \equiv \hat{X}_1(t)\cos\omega_m t + \frac{\hat{X}_2(t)}{m\omega_m}\sin\omega_m t.$$

$$[\hat{X}_1(t),\, \hat{X}_1(t')] = [\hat{X}_2(t),\, \hat{X}_2(t')] = 0, \quad [\hat{X}_1(t),\, \hat{X}_2(t')] = i\hbar\delta(t-t').$$

$$\hat{H}_{\rm int} = \hbar G(t)\hat{x}(t)\hat{a}_1(t) = \hbar G_0\cos\omega_m t\left[\hat{X}_1(t)\cos\omega_m t + \frac{\hat{X}_2(t)}{m\omega_m}\sin\omega_m t\right]\hat{a}_1(t)$$

$$= \frac{1}{2}\hbar G_0\left[\hat{X}_1(t) + \hat{X}_1(t)\cos 2\omega_m t + \frac{\hat{X}_2(t)}{m\omega_m}\sin 2\omega_m t\right],$$

- γ<<ω_m　ならば簡単に測定可能(high finesse cavity)

⇒このような制限の無い測定方法とは？

IV. Measuring a Conserved Quantity

- Dynamics

test-mass にcavityがアシストされている

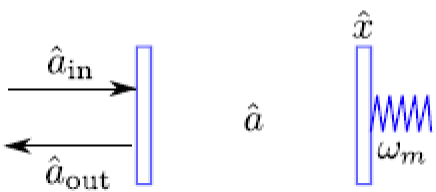


Figure 5.1 – A schematic plot showing an optomechanical system. A mechanical oscillator with eigenfrequency ω_m interacts with a cavity mode $\hat{a}$, which is also coupled to the input ($\hat{a}_{\rm in}$) and output ($\hat{a}_{\rm out}$) optical modes.

$$\dot{\hat{x}}(t) = \hat{p}(t)/m,$$
$$\dot{\hat{p}}(t) = -m\,\omega_m^2 \hat{x}(t) + \alpha\,\hat{a}_1(t) + \hat{\xi}_{\rm th}(t).$$

$$\hat{x}_q(t) = \delta x_q[\hat{X}_0 \cos\omega_m t + \hat{P}_0 \sin\omega_m t],$$

$$\delta\hat{x}(t) = \int_0^t dt' G_x(t-t')[\alpha\,\hat{a}_1(t') + \xi_{\rm th}(t') + F_{\rm ext}(t')],$$

$$\hat{b}_1(t) = \hat{a}_1(t),$$
$$\hat{b}_2(t) = \hat{a}_2(t) + (\alpha/\hbar)\hat{x}(t).$$
$$\alpha \equiv 8\sqrt{2}(\mathcal{F}/\lambda)\sqrt{\hbar I_0/\omega_0}$$

キャビティのdecay γ は

γ>>ω_m　とする

IV. Measuring a Conserved Quantity

- Read out

Cavity の線幅に影響の無い測定

測定結果をN個に分け解析(それぞれの quadrature は同じ)

$$\hat{Y}_i = \int_{\tau_i}^{\tau_{i+1}} dt \left[g_1^{(i)}(t)\hat{b}_1(t) + g_2^{(i)}(t)\hat{b}_2(t) \right], \quad (\tau_i = i\,\Delta\tau,\ i = 0, 1, \cdots, N-1).$$

$$g_1^{(i)}(t) + (\alpha^2/\hbar)\int_t^{\tau_{i+1}} dt'\, G_x(t'-t)\, g_2^{(i)}(t') = 0. \qquad \int_{\tau_i}^{\tau_{i+1}} dt\, g_2^{(i)}(t)\hat{x}_q(t) = 0.$$

$$g_2(t) = \frac{1}{\tau_m} \sum_{n=-\infty}^{+\infty} \tilde{g}_{2,n}(n\,\omega_m)\, e^{-i n \omega_m t}, \quad \tilde{g}_{2,n}(n\,\omega_m) = \int_0^{\tau_m} dt\, g_2(t)\, e^{i n \omega_m t}.$$

$$\Rightarrow \; S_{ZZ}(\Omega) = \sum_{k=-\infty}^{\infty} |\tilde{g}_k(k\,\omega_m)|^2 \left[\frac{1}{2} + \frac{\alpha^2}{\hbar^2} \frac{\sin^2(\omega\,\tau_m/2)}{[(\omega\,\tau_m/2) + k\,\pi]^2} R_{xx}^2(\Omega) S_F(\Omega) \right]$$

Conclusions

- Kimbleのfilter cavityが最適？
- Khaliliのfilterなら実験室でできそう（面白い実験ではあるが意義はないかも･･･）
- Double optical spring使えば広帯域でSQLを超える事が出来て、かつ制御なしで安定
- Mechanical quadrature の測定でQND測定