

Status report and future plans

Hinata Takidera (M1)
Department of Physics, University of Tokyo

Ando Lab Seminar March 3rd, 2023

- Abstract about Axion
- Cavity
 - Design
 - Mode matching ratio
- Result
 - Finesse, mode matching ratio
 - Phase difference between s-pol. and p-pol.
- Incident angle
 - Calculated result about range of angle
 - How to measure the incident angle accurately
- Future Plans

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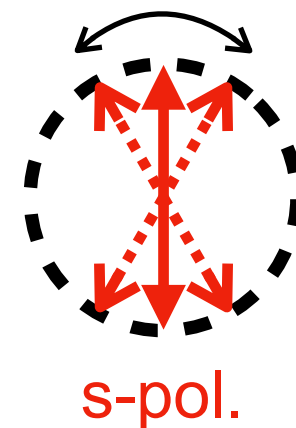
- Axion behaves as a coherent field $a(t)$ with its mass m_a and a phase factor $\delta_\tau(t)$

$$a(t) = a_0 \cos (m_a t + \delta_\tau(t))$$

- The axion-photon interaction gives a phase velocity difference between left-handed and right-handed circularly polarized light.

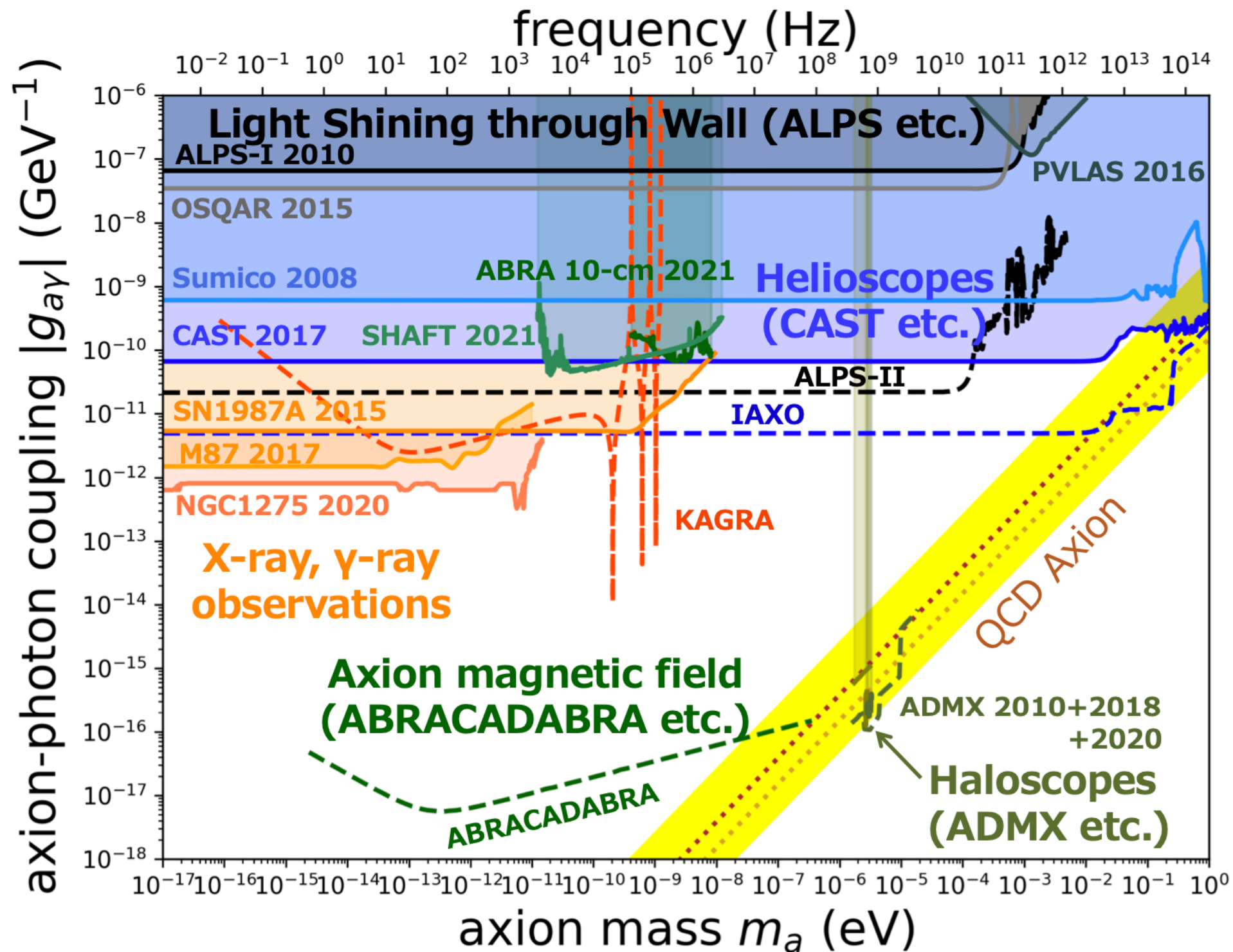
$$c_{L/R}(t) \simeq 1 \mp \frac{g_{a\gamma} \dot{a}(t)}{2k}$$

- The modulated phase velocity difference gives the modulation of the polarization angle of linearly polarized light.

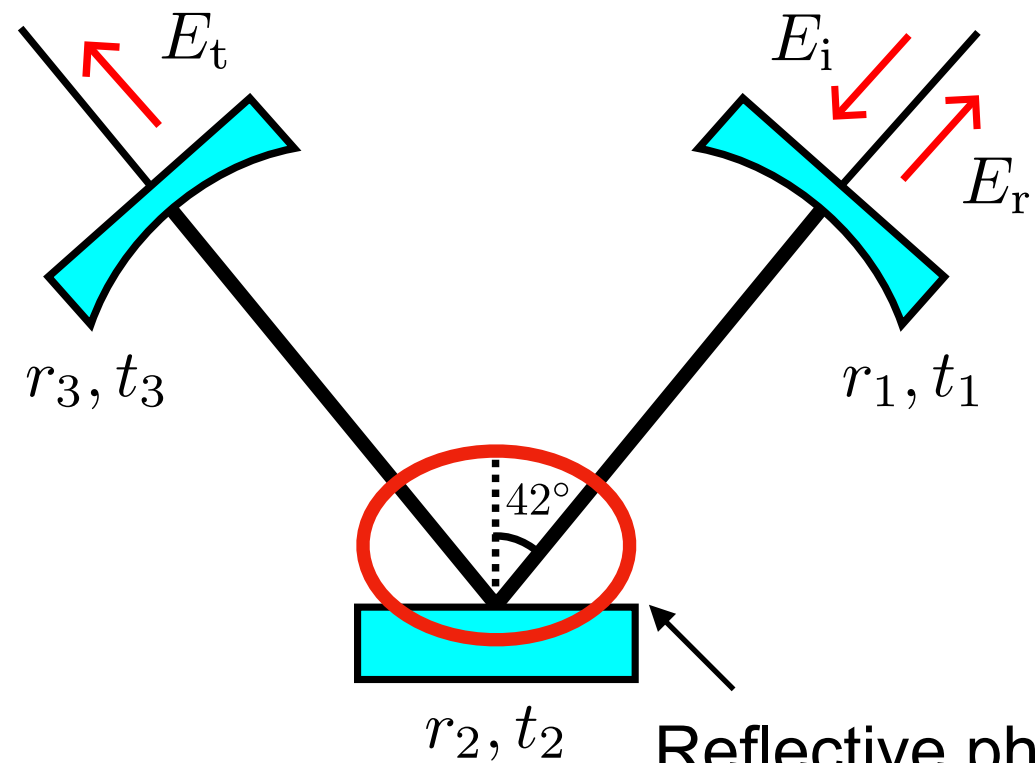


Abstract about Axion

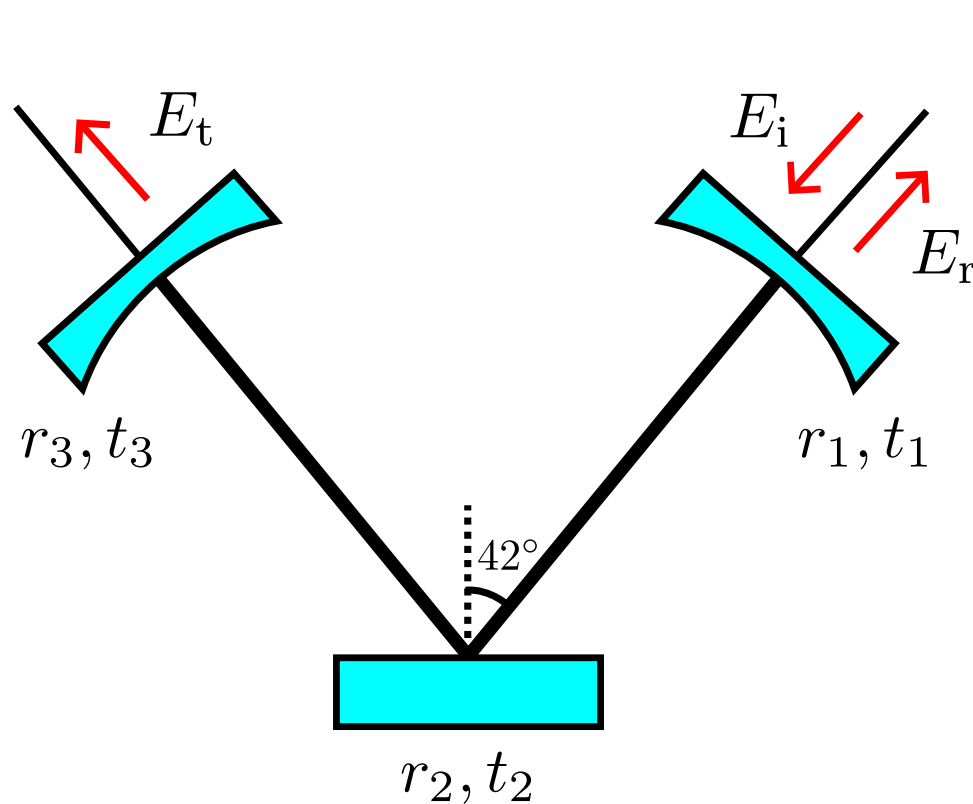
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- Need for calibration
- Obtain time dependence data to investigate whether phase difference between s-pol. and p-pol. is drifting



phase shift: $\phi = \frac{\omega L}{c}$

$$E_r = E_i(-r_1) + E_i t_1^2 r_2^2 r_3 e^{-i2\phi} + E_i t_1^2 r_1 r_2^4 r_3 e^{-i4\phi} + E_i t_1^2 r_1^2 r_2^6 r_3^3 e^{-i6\phi} + \dots$$

$$= E_i \left(-r_1 + \frac{t_1^2 r_2^2 r_3 e^{-i2\phi}}{1 - r_1 r_2^2 r_3 e^{-i2\phi}} \right)$$

$$E_t = E_i t_1 r_2 t_3 e^{-i\phi} + E_i t_1 r_1 r_2^3 t_3 r_3 e^{-i3\phi} + E_i t_1 r_1^2 r_2^5 r_3^2 t_3 e^{-i5\phi} + \dots$$

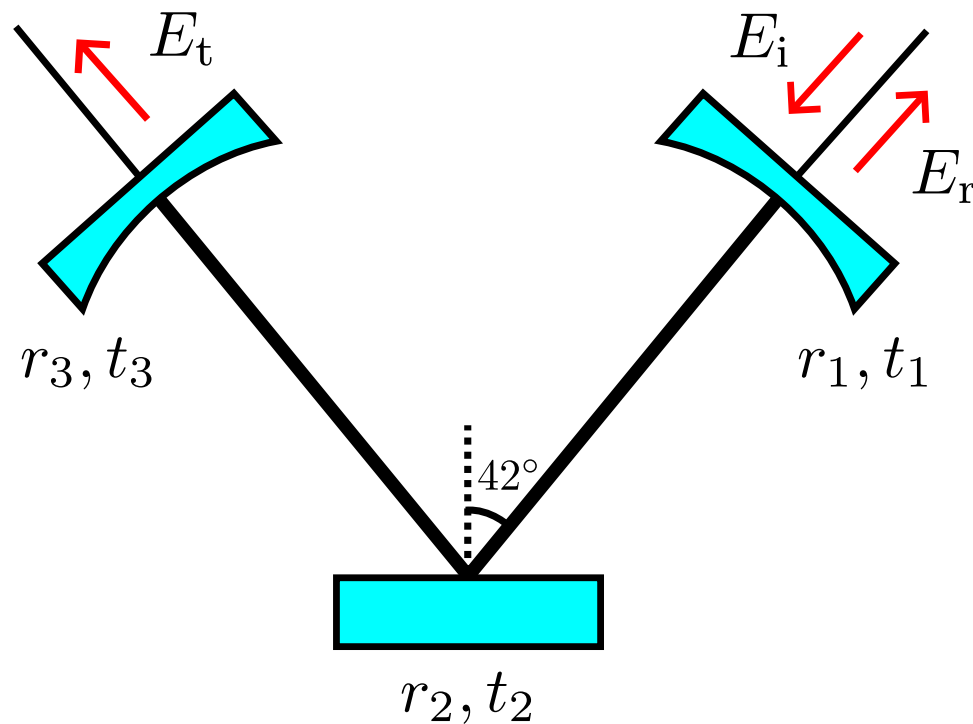
$$= E_i \frac{t_1 r_2 t_3 e^{-i\phi}}{1 - r_1 r_2^2 r_3 e^{-i2\phi}}$$

Amplitude reflectance: $r(\phi) = -r_1 + \frac{t_1^2 r_2^2 r_3 e^{-i2\phi}}{1 - r_1 r_2^2 r_3 e^{-i2\phi}}$

Amplitude transmittance: $t(\phi) = \frac{t_1 r_2 t_3 e^{-i\phi}}{1 - r_1 r_2^2 r_3 e^{-i2\phi}}$

Reflected light intensity: $P_r = |E_r|^2 = \frac{[(t_1^2 + r_1^2) r_2^2 r_3 - r_1]^2 + 4 r_1 r_2^2 r_3 (t_1^2 + r_1^2) \sin^2 \phi}{(1 - r_1 r_2^2 r_3)^2 + 4 r_1 r_2^2 r_3 \sin^2 \phi} |E_i|^2$

Transmitted light intensity: $P_t = |E_t|^2 = \frac{(t_1 r_2 t_3)^2}{(1 - r_1 r_2^2 r_3)^2 + 4 r_1 r_2^2 r_3 \sin^2 \phi} |E_i|^2$



Resonance condition

$$\phi = \pi m \rightarrow 2L = m\lambda (m \in \mathbb{N})$$

$$\text{Free spectral range: } \nu_{FSR} = \frac{c}{2L}$$

By Reflected light intensity P_r ,

$$\frac{1}{1 + \frac{4r_1r_2^2r_3}{(1 - r_1r_2^2r_3)^2} \sin^2\left(\frac{\pi\nu_{FWHM}L}{c}\right)} = \frac{1}{2}$$

$$\text{Assume } \frac{\pi\nu_{FWHM}L}{c} = \frac{\pi\nu_{FWHM}}{2\nu_{FSR}} \ll 1,$$

Full Width at Half Maximum:

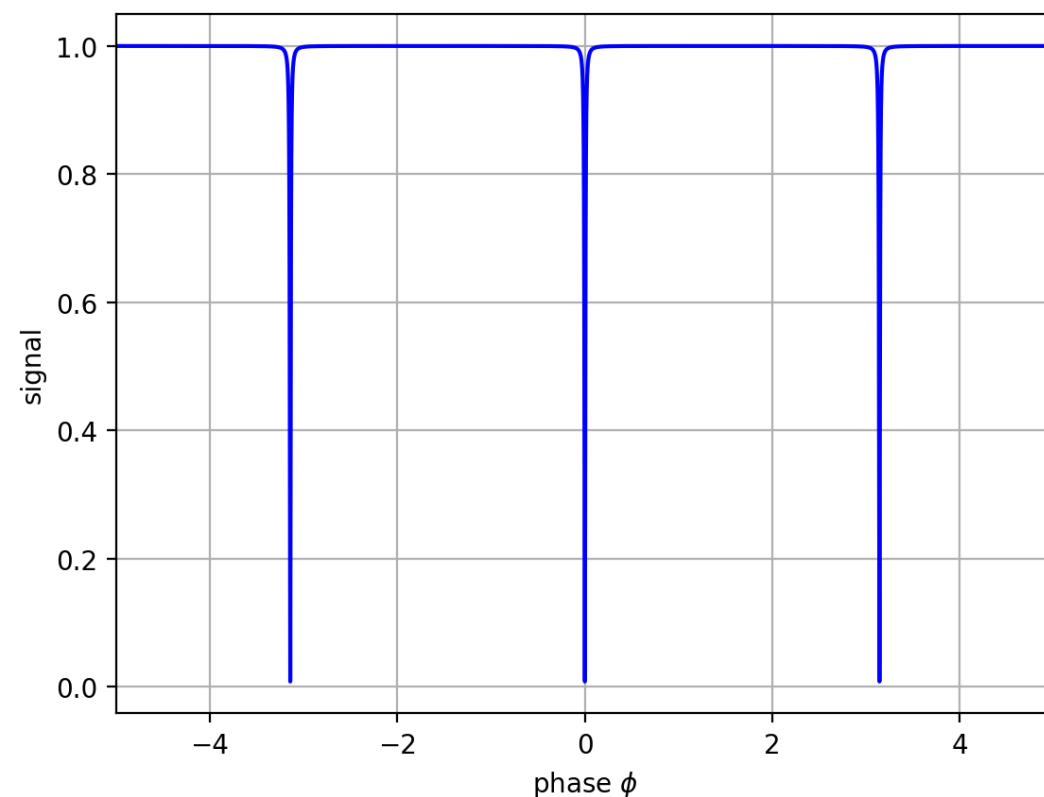
$$\nu_{FWHM} = \frac{c(1 - r_1r_2^2r_3)}{2\pi L\sqrt{r_1r_2^2r_3}}$$

Finesse:

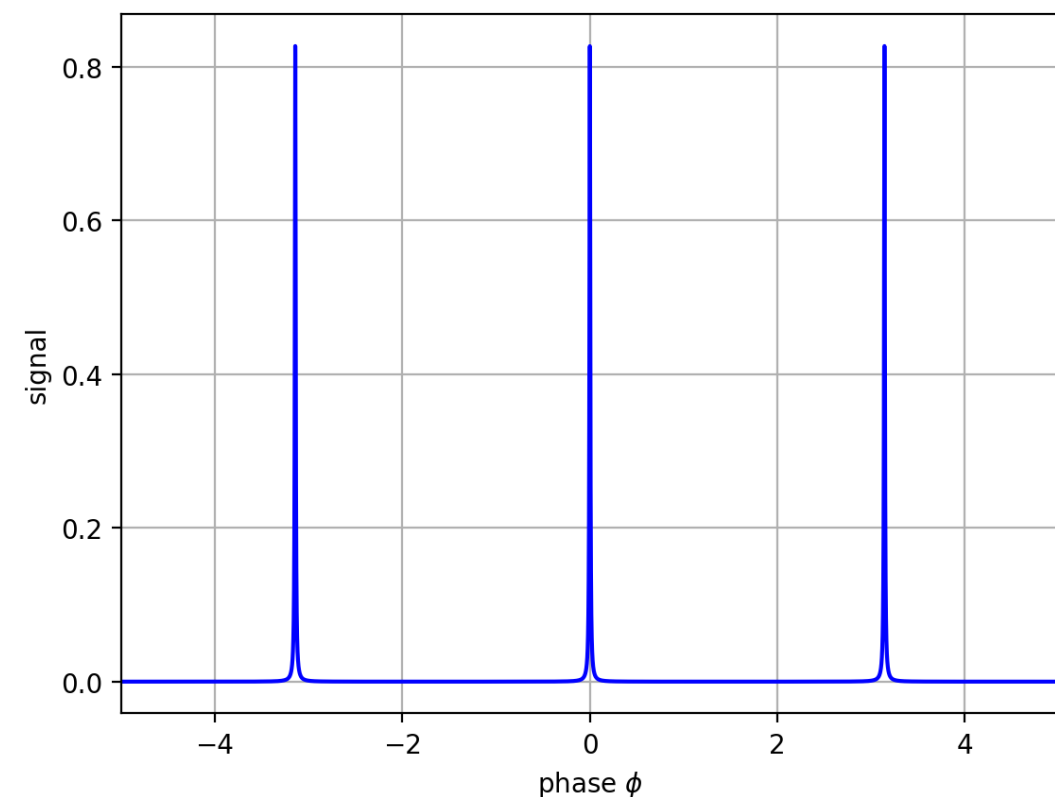
$$\mathcal{F} = \frac{\nu_{FSR}}{\nu_{FWHM}} = \frac{\pi\sqrt{r_1r_2^2r_3}}{1 - r_1r_2^2r_3}$$

$$r_1 = \sqrt{0.99}, r_2 = \sqrt{0.999}, r_3 = \sqrt{0.99}, t_1 = \sqrt{1 - r_1^2}, t_2 = \sqrt{1 - r_2^2}, t_3 = \sqrt{1 - r_3^2}, |E_i|^2 = 1, L = 60mm$$

$$P_r = \frac{[(t_1^2 + r_1^2) r_2^2 r_3 - r_1]^2 + 4r_1 r_2^2 r_3 (t_1^2 + r_1^2) \sin^2 \phi}{(1 - r_1 r_2^2 r_3)^2 + 4r_1 r_2^2 r_3 \sin^2 \phi} |E_i|^2 \quad P_t = \frac{(t_1 r_2 t_3)^2}{(1 - r_1 r_2^2 r_3)^2 + 4r_1 r_2^2 r_3 \sin^2 \phi} |E_i|^2$$



Reflected light



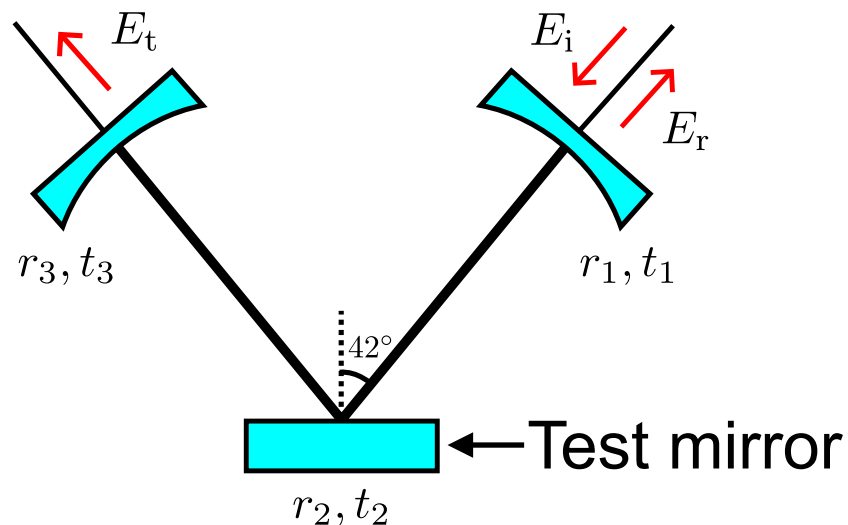
Transmitted light

Calculated value:



$$\nu_{FSR} = 2.5\text{GHz}, \nu_{FWHM} = 8.8\text{MHz}, \mathcal{F} = 284.2$$

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Mode matching ratio M

$$M = 4 \left[\frac{z_{R1}^2 z_{Rx} z_{Ry}}{\{(z_{R1} + z_{Rx})^2 + (d_1 - d_x)^2\} \{(z_{R1} + z_{Ry})^2 + (d_1 - d_y)^2\}} \right]^{\frac{1}{2}}$$

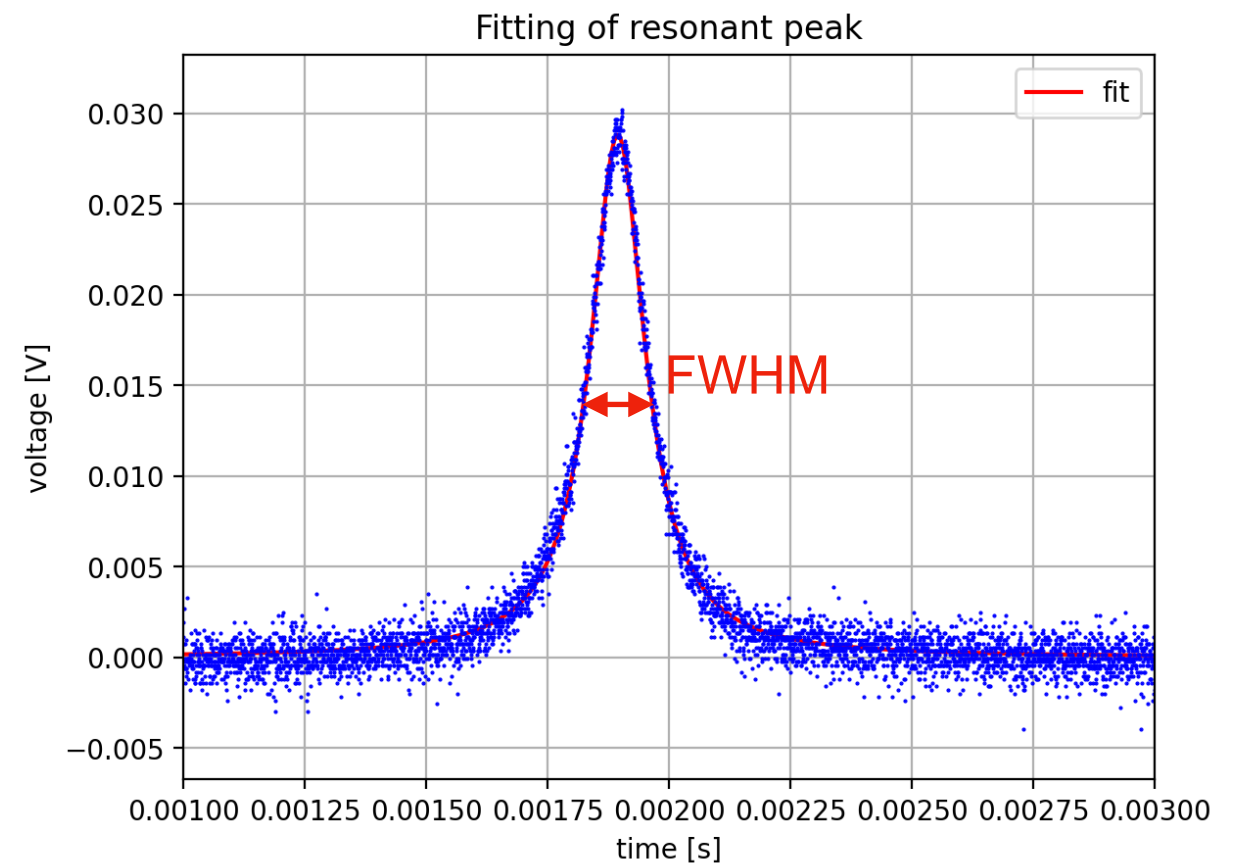
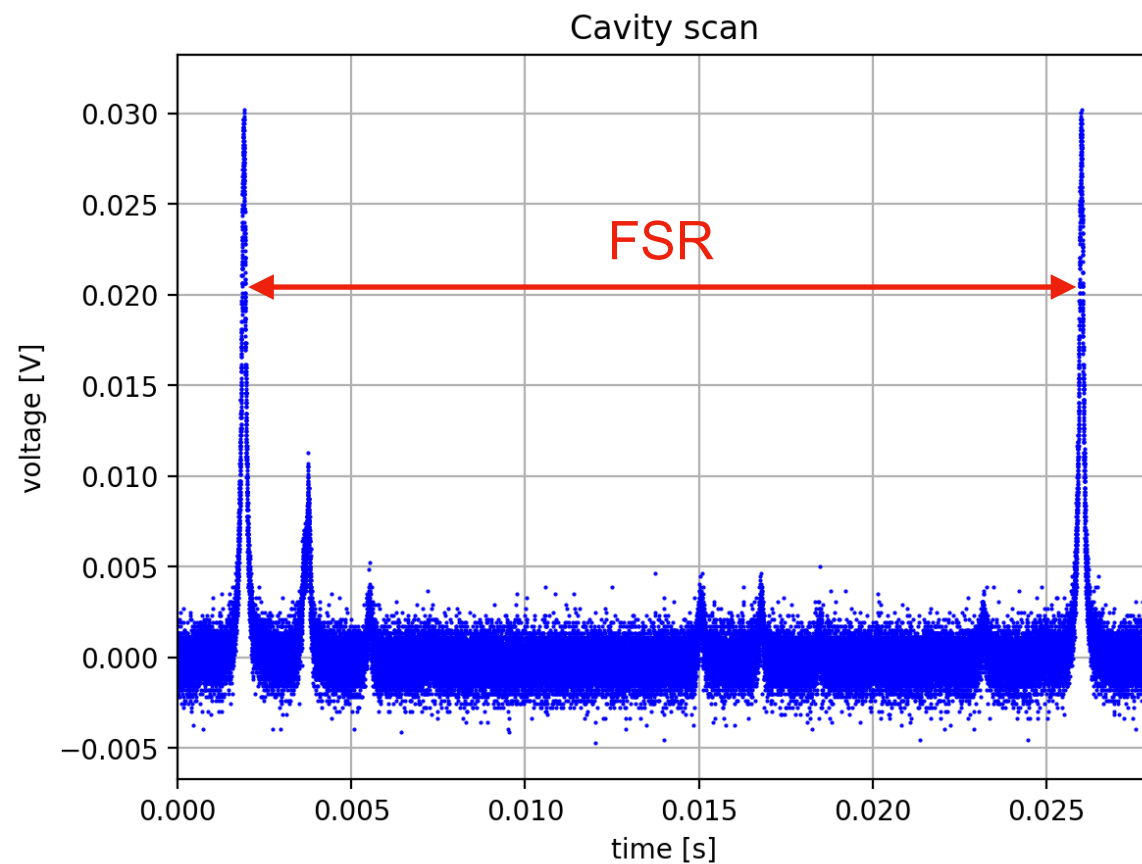
Raab, F.J. and S.E. Whitcomb (1992) Estimation of Special Optical Properties of a Triangular Ring Cavity. *LIGO Tech. Rep.*, No.108: 1-8.

Assume the beam is circle $z_{Rx} = z_{Ry} = z_{R2}$, $d_x = d_y = d_2$, M is given by

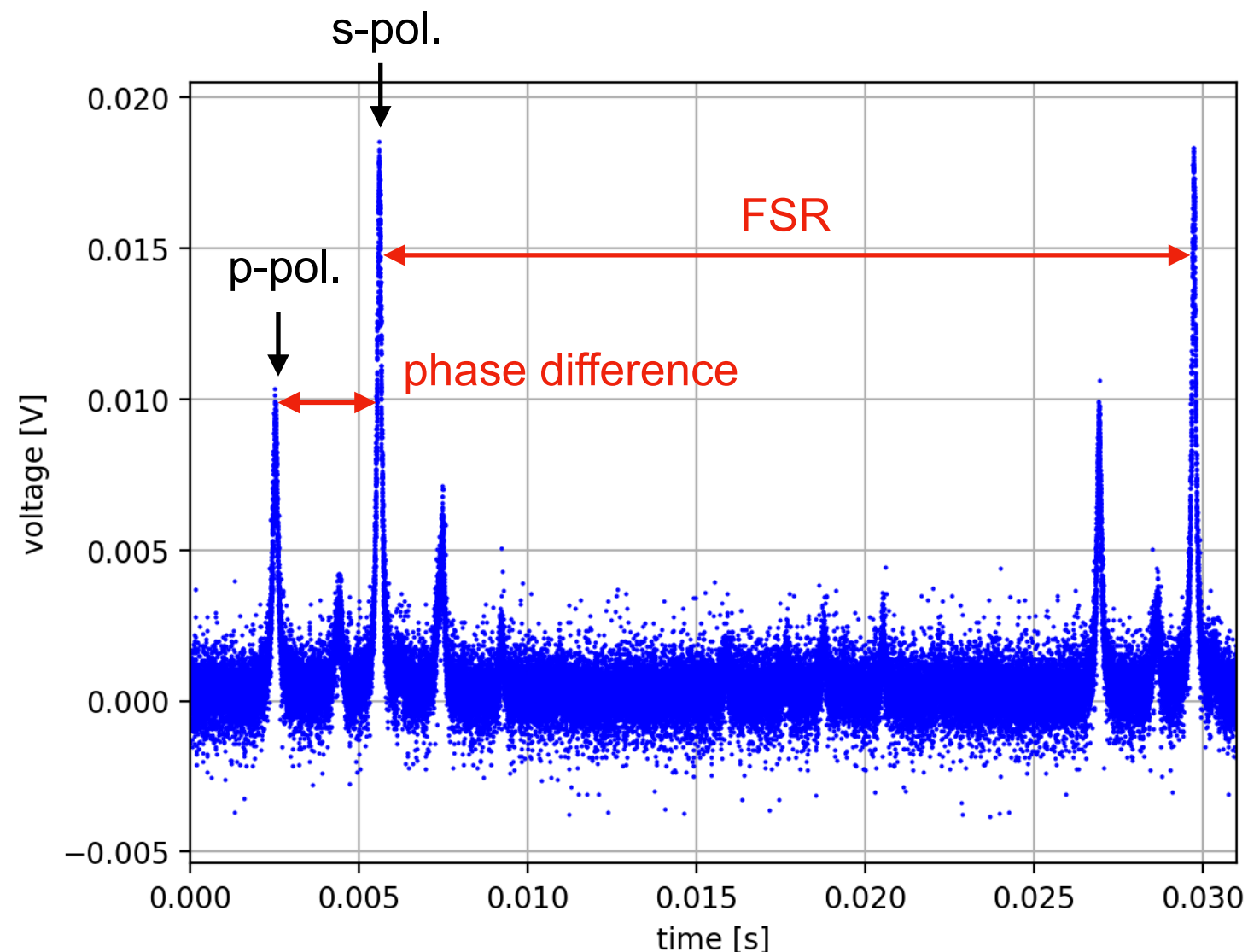
$$M = \frac{4z_{R1}z_{R2}}{(z_{R1} + z_{R2})^2 + (d_1 - d_2)^2}$$

- Test mirror with flat: 100%
- Test mirror with CC(center of curvature)1000mm
Front and end mirror with CC 50mm: 99.94%
Front and end mirror with CC1000mm: 97.17%
→ The larger CC, mode matching ratio gets worse

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➔ Finesse: $\mathcal{F} = 166.6 \pm 8.9$
Mode matching ratio: $M = 65.6 \%$

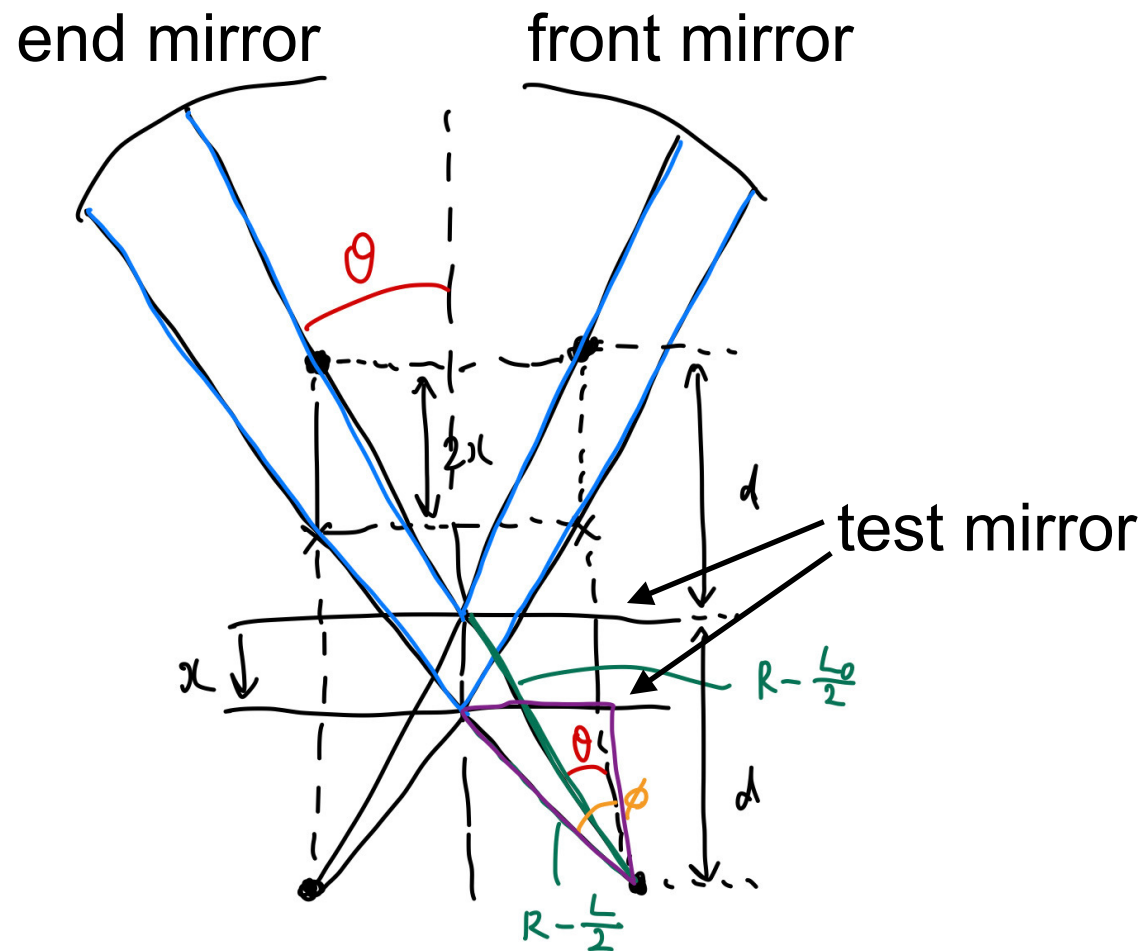


Phase difference between s-pol. and p-pol.

$$\Delta\phi = 23.27 \pm 0.46[\text{deg}]$$

➡ Compared with [previous result](#), this result matches!
(Fujimoto-san's)

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x : motion of test mirror

d : distance between test mirror and center of curvature

R : curvature of front and end mirror

L_0 : cavity length (initial)

θ : incident angle (initial)

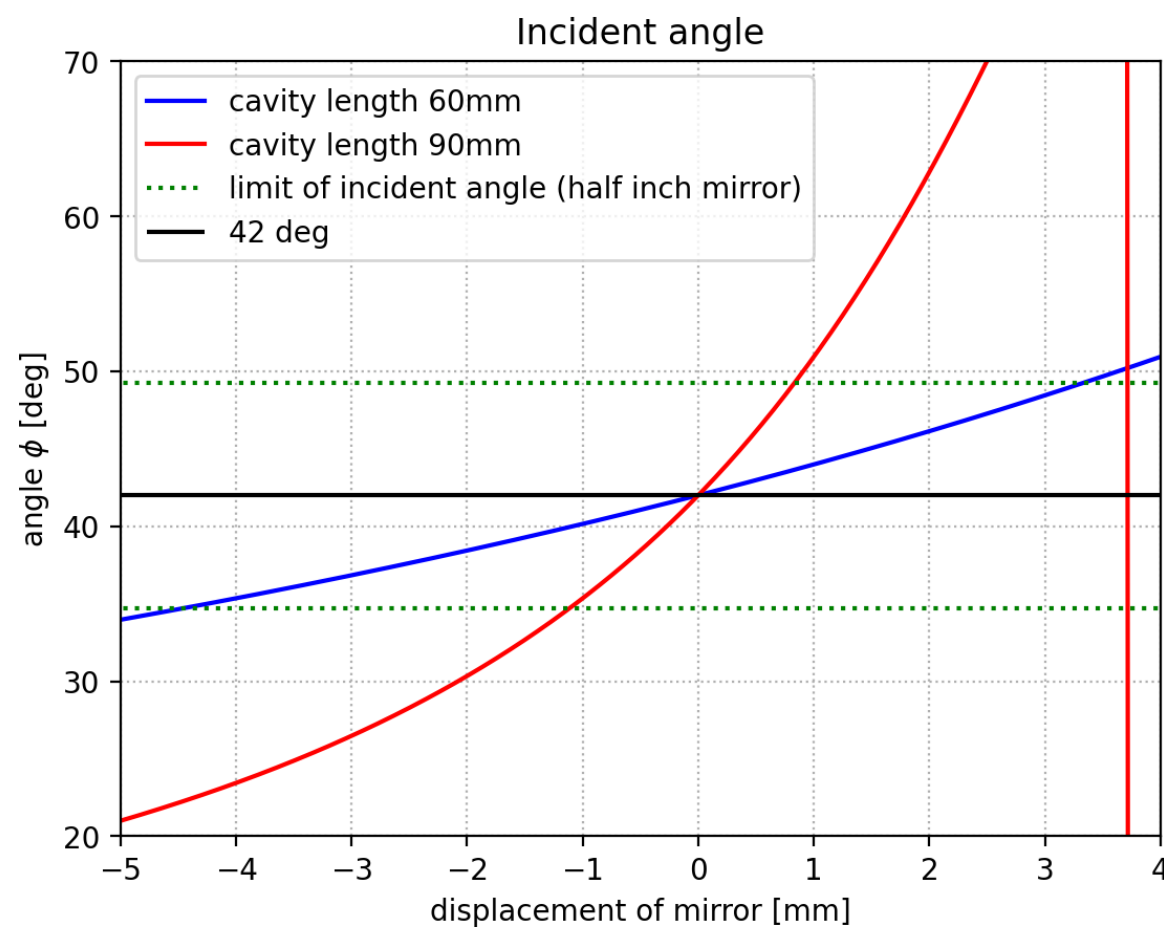
Focus on purple triangle, incident angle ϕ is given by

$$\phi = \tan^{-1} \left[\frac{1}{d - x} \left(R - \frac{L_0}{2} \right) \sin \theta \right] \quad \left(d = \left(R - \frac{L_0}{2} \right) \cos \theta \right)$$

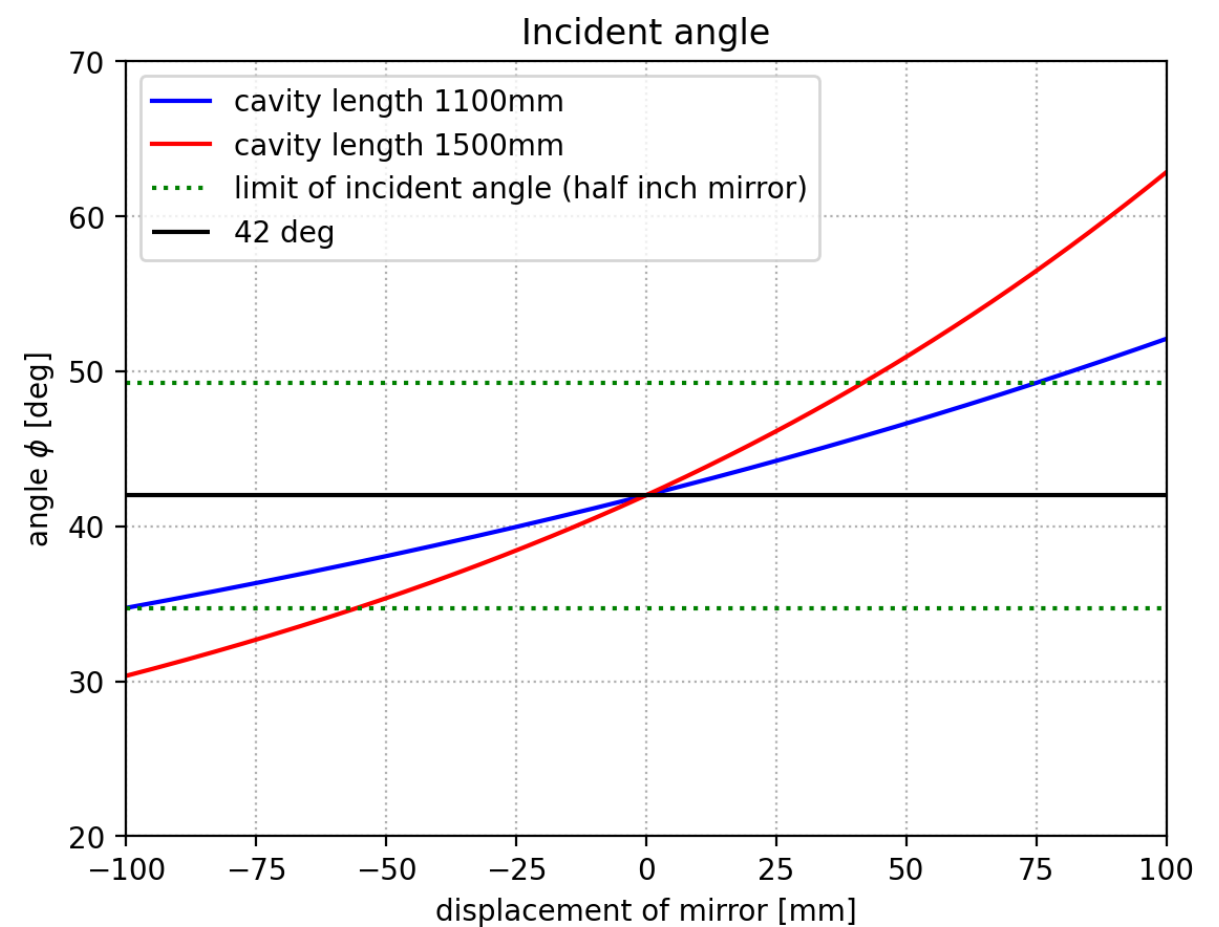
Calculated result about range of angle

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$$\phi = \tan^{-1} \left[\frac{1}{d-x} \left(R - \frac{L_0}{2} \right) \sin \theta \right]$$



Front and end mirror with $R = 50\text{mm}$



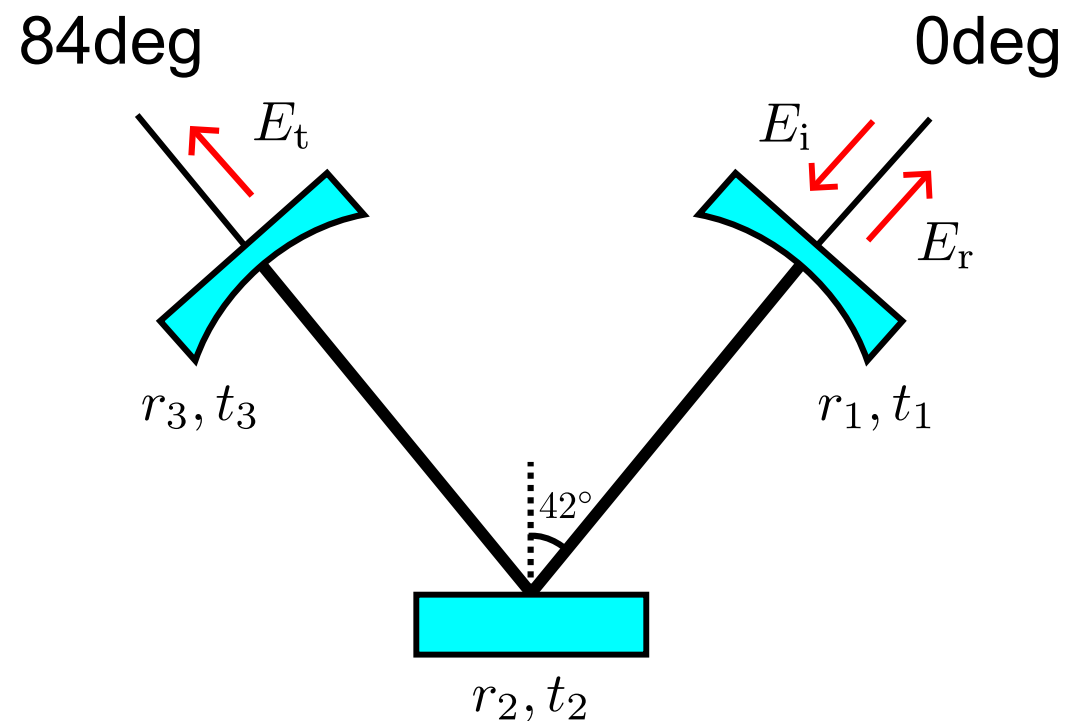
Front and end mirror with $R = 1000\text{mm}$

➡ When the cavity length is long, Incident angle is sensitive to displacement of mirror

How to measure the angle accurately

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1. Reflect the beam to the laser source (0deg)
2. Using the rotation mount to tune 42deg



example(rotation mount)



<https://www.thorlabs.co.jp/thorproduct.cfm?partnumber=PR01/M>

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1. Measure the time dependence of phase difference between s-pol. and p-pol.
2. Confirm whether the phase difference depends on incident angle
3. Prepare the wavelength sensitive phase-shifting mirror
4. Introduce wavelength tunable laser to realize simultaneous resonance