Parameter estimation with inspiral waveforms of compact binary coalescences including nontensorial gravitational waves polarizations

Hiroki Takeda, Atsushi Nishizawa, Yuta Michimura, Koji Nagano, Kentaro Komori, Masaki Ando, Kazuhiro Hayama
Abstract

• The observation of gravitational waves (GW) from compact binary coalescences (CBC) enabled some experimental studies to probe into the nature of gravity.
• The separability of the polarization modes for the inspiral GW from the CBC systematically.

  • The three polarization modes of the GW would be separable even with the global network of three detectors to some extent.
  • With four detectors, even the four polarization modes would be separable.
1. Polarization of Gravitational Wave
Polarization mode of GW

The information of GW polarization modes is expected to bring more knowledge about gravity.

\[ h_{ab}(t, \hat{\Omega}) = h_A(t) e^A_{ab}(\hat{\Omega}) \]

\[ A = +, \times \]

Tensor

Plus

\[ e^+_{ab} = \hat{e}_x \otimes \hat{e}_x - \hat{e}_y \otimes \hat{e}_y \]

Cross

\[ e^\times_{ab} = \hat{e}_x \otimes \hat{e}_y + \hat{e}_y \otimes \hat{e}_x \]

In General Relativity (GR), GW has only two tensor mode (plus & cross)

Polarization mode of GW

\[ h_{ab}(t, \hat{\Omega}) = h_A(t) e_{ab}^A(\hat{\Omega}) \]

\[ A = +, \times, x, y, b, l \]

Tensor
- Plus
  \[ e_{ab}^+ = \hat{e}_x \otimes \hat{e}_x - \hat{e}_y \otimes \hat{e}_y \]
- Cross
  \[ e_{ab}^\times = \hat{e}_x \otimes \hat{e}_y + \hat{e}_y \otimes \hat{e}_x \]

Vector
- Vector x
  \[ e_{ab}^x = \hat{e}_x \otimes \hat{e}_z + \hat{e}_z \otimes \hat{e}_x \]
- Vector y
  \[ e_{ab}^y = \hat{e}_y \otimes \hat{e}_z - \hat{e}_z \otimes \hat{e}_y \]

Scalar
- Breathing
  \[ e_{ab}^b = \hat{e}_x \otimes \hat{e}_x + \hat{e}_y \otimes \hat{e}_y \]
- Longitudinal
  \[ e_{ab}^l = \sqrt{2} \hat{e}_z \otimes \hat{e}_z \]

Test of GR and Alternative theories with GW polarization modes allowed in a theoretical model

<table>
<thead>
<tr>
<th>Theory</th>
<th>plus</th>
<th>cross</th>
<th>vector x</th>
<th>vector y</th>
<th>breathing</th>
<th>longitudinal</th>
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<tbody>
<tr>
<td>GR</td>
<td>☑</td>
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<td>Kaluza-Klein theory</td>
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<td>f(R) theory</td>
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<td>Bimetric theory</td>
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</tbody>
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Test of Polarization modes of GW is a powerful tool for pursuing the nature of space-time.

Separate and Reconstruct polarization modes of GW model independently from detector signal.
Polarization test with GW from CBC

In principle, \((\text{The number of polarization modes}) = (\text{The number of detectors})\)

More polarization modes can be probed with the larger number of detectors

However

Few studies have focused on the reconstruction of the polarization of GW from CBC

Because…

Waveforms of GW from CBC have the source model parameters, which determine the frequency evolution in time and are correlated each other.

It is necessary to gain a better understanding of the correlations and degeneracies among the parameters in a realistic waveform of CBC including nontensorial modes.
2. Parameter Estimation
Parameter estimation including nontensorial GW polarizations

the separability of the polarization modes for the inspiral GW from the CBC systematically.

Fisher Information Matrix

\[ \Gamma_{ij} = 4 \text{Re} \int_{f_{\text{min}}}^{f_{\text{max}}} df \sum_I \frac{1}{S_{n,I}(f)} \left( \frac{\partial h_I^*(f)}{\partial \chi^i} \right) \left( \frac{\partial h_I(f)}{\partial \chi^j} \right) \]

Antenna pattern functions

Detector Signal

\[ h_I(t, \hat{\Omega}) = F_I^A(\hat{\Omega}) h_A(t) \]

GW amplitude for polarization mode “A”
Inclination angle

GW amplitude depends on the inclination angle

Geometrical factor $\leftarrow$ Inclination-angle dependence + antenna pattern

e.g. Tensor mode

$$h_I = \frac{2}{5} G_{T,I} h_{GR}$$

$$G_{T,I} := \frac{5}{2} \{ (1 + \cos^2 \iota) F_{+,I}(\theta_s, \theta_e) + 2 \cos \iota F_{\times,I}(\theta_s, \theta_e) \} e^{i \Phi_{D,I}(\theta_2, \phi_2, \theta_e, \phi_e)}$$

Observer

$\iota$: Inclination angle

Orbital plane
Geometrical factor for nontensorial modes

\[ G_{V_x, I} := \sqrt{\frac{525}{56}} \sin 2\nu F_{V_x, I}(\theta_s, \theta_e) e^{i\phi_D, I(\theta_s, \phi_s, \theta_e, \phi_e)} \]

\[ G_{V_y, I} := \sqrt{\frac{15}{2}} \sin \nu F_{V_y, I}(\theta_s, \theta_e) e^{i\phi_D, I(\theta_s, \phi_s, \theta_e, \phi_e)} \]

\[ G_{S_2, I} := \sqrt{\frac{225}{8}} \sin^2 \nu F_{b, I}(\theta_s, \theta_e) e^{i\phi_D, I(\theta_s, \phi_s, \theta_e, \phi_e)} \]

\[ G_{S_1, I} := \sqrt{\frac{45}{2}} \sin \nu F_{b, I}(\theta_s, \theta_e) e^{i\phi_D, I(\theta_s, \phi_s, \theta_e, \phi_e)} \]
<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model T:</td>
<td>GR model</td>
<td>[ h_I = G_{T,I} h_{GR} ]</td>
</tr>
<tr>
<td>Model TS1:</td>
<td>Tensor (+, ×) Scalar (dipole)</td>
<td>[ h_I = { G_{T,I} + A_{S_1} G_{S_1,I} } h_{GR} ]</td>
</tr>
<tr>
<td>Model TS2:</td>
<td>Tensor (+, ×) Scalar (quadrupole)</td>
<td>[ h_I = { G_{T,I} + A_{S_2} G_{S_2,I} } h_{GR} ]</td>
</tr>
<tr>
<td>Model TVxS2:</td>
<td>Tensor (+, ×) Scalar (quadrupole)</td>
<td>[ h_I = { G_{T,I} + A_{S_2} G_{S_2,I} + A_{V_x} G_{V_x,I} } h_{GR} ]</td>
</tr>
<tr>
<td>Model TVyS1:</td>
<td>Tensor (+, ×) Scalar (dipole)</td>
<td>[ h_I = { G_{T,I} + A_{S_1} G_{S_1,I} + A_{V_y} G_{V_y,I} } h_{GR} ]</td>
</tr>
<tr>
<td>Model TV:</td>
<td>Tensor (+, ×) Vector-x, Vector-y</td>
<td>[ h_I = { G_{T,I} + A_{V_x} G_{V_x,I} + A_{V_y} G_{V_y,I} } h_{GR} ]</td>
</tr>
</tbody>
</table>
Parameter estimation including nontensorial GW polarizations

**Setup**
- the inspiral waveform up to 3.0 PN in amplitude & 3.5 PN in phase
  $$h_{GR} = A_{ins} e^{-i\phi_{ins}},$$
  $$A_{ins} = \frac{1}{\sqrt{6\pi^{2/3}d_L}} M^{5/6} f^{-7/6} \sum_{i=0}^{6} (\pi Mf)^i/3,$$
  $$\phi_{ins} = 2\pi ft_c - \phi_c - \frac{\pi}{4} + \frac{3}{128} (\pi Mf)^{-5/3} \sum_{i=0}^{7} \phi_i (\pi Mf)^i/3$$
- 11 model parameters in GR + additional polarization parameters, \((A_{S1}, A_{V1}, \ldots)\) fiducial value unity
- Source (500)
  - binary black holes (BBH) with equal mass 10M⊙ at \(z = 0.05\)
  - binary neutron stars (BNS) with equal mass 1.4M⊙ at \(z = 0.01\)
- Detector Network
  - Two aLIGO detectors at Hanford- Livingston-AdV(HLV)
  - Two aLIGO detectors at Hanford-Livingston-AdV-KAGRA(HLVK)
  all design sensitivity

uniformly random

(log \(M, \log \eta, t_c, \phi_c, \log d_L, \chi_s, \chi_a, \theta_s, \phi_s, \cos \iota, \psi_p\) \(\theta_s, \phi_s, \cos \iota, \psi_p\) \(\theta_s, \phi_s, \cos \iota, \psi_p\) \(\theta_s, \phi_s, \cos \iota, \psi_p\))
Results

Model TS1

red: BBH-HLV

green: BBH-HLVK

blue: BNS-HLV

magenta: BNS-HLVK

can break a degeneracy among amplitude parameters.
<table>
<thead>
<tr>
<th></th>
<th>BBH(HLV)</th>
<th>BBH(HLVK)</th>
<th>Improvement Factor</th>
<th>BNS(HLV)</th>
<th>BNS(HLVK)</th>
<th>Improvement Factor</th>
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<tbody>
<tr>
<td>ModelT SNR</td>
<td>33.3</td>
<td>40.2</td>
<td></td>
<td>36.4</td>
<td>44.3</td>
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<tr>
<td>ModelT $\Delta \ln d_L$</td>
<td>0.269</td>
<td>0.137</td>
<td>1.96</td>
<td>0.183</td>
<td>0.107</td>
<td>1.71</td>
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<tr>
<td>ModelT $\Delta \Omega_s [\text{deg}^2]$</td>
<td>5.91</td>
<td>1.77</td>
<td>3.34</td>
<td>1.39</td>
<td>0.517</td>
<td>2.69</td>
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<tr>
<td>ModelTS1 $\Delta \ln d_L$</td>
<td>0.678</td>
<td>0.179</td>
<td>3.79</td>
<td>0.359</td>
<td>0.134</td>
<td>2.68</td>
</tr>
<tr>
<td>ModelTS1 $\Delta \Omega_s [\text{deg}^2]$</td>
<td>4.74</td>
<td>0.912</td>
<td>5.20</td>
<td>0.919</td>
<td>0.250</td>
<td>3.68</td>
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<tr>
<td>ModelTS1 $\Delta A_s$</td>
<td>1.16</td>
<td>0.284</td>
<td>4.08</td>
<td>0.606</td>
<td>0.197</td>
<td>3.08</td>
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<tr>
<td>ModelTS1 $C(A_{S1}, \log d_L)$</td>
<td>0.998</td>
<td>0.989</td>
<td></td>
<td>0.996</td>
<td>0.984</td>
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<tr>
<td>ModelTS1 $C(A_{S1}, \cos \psi)$</td>
<td>-0.553</td>
<td>-0.500</td>
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<td>-0.231</td>
<td>-0.159</td>
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<tr>
<td>ModelTS2 $\Delta \ln d_L$</td>
<td>0.676</td>
<td>0.182</td>
<td>3.71</td>
<td>0.358</td>
<td>0.134</td>
<td>2.67</td>
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<td>ModelTS2 $\Delta \Omega_s [\text{deg}^2]$</td>
<td>4.74</td>
<td>0.913</td>
<td>5.09</td>
<td>0.862</td>
<td>0.246</td>
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<td>ModelTS2 $\Delta A_{S2}$</td>
<td>1.51</td>
<td>0.385</td>
<td>3.92</td>
<td>0.765</td>
<td>0.256</td>
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<tr>
<td>ModelTS2 $C(A_{S2}, \log d_L)$</td>
<td>0.997</td>
<td>0.989</td>
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<td>0.996</td>
<td>0.984</td>
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<tr>
<td>ModelTS2 $C(A_{S2}, \cos \psi)$</td>
<td>-0.609</td>
<td>-0.564</td>
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<td>-0.246</td>
<td>-0.189</td>
<td></td>
</tr>
</tbody>
</table>

ModelTS1, TS2

- Improved by factor 4 in $\Delta A_s$(BBH), about 3 in $\Delta A_s$(BNS)
- $A_s$ is correlated with $\ln d_L$ and $\cos \psi$ (for all models)
- BBH estimation is worse than BNS with HLV due to the short signal of BBH.
ModelTVxS2, TVyS1, TV more improved than in Model TS1, TS2.
<table>
<thead>
<tr>
<th>Model TVxS2, TVyS1, TV</th>
<th>BBH(HLV)</th>
<th>BBH(HLVK)</th>
<th>Improvement Factor</th>
<th>BNS(HLV)</th>
<th>BNS(HLVK)</th>
<th>Improvement Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ ln d_L</td>
<td>1.58</td>
<td>0.258</td>
<td>6.12</td>
<td>1.05</td>
<td>0.190</td>
<td>5.53</td>
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<tr>
<td>ΔΩ_s [deg^2]</td>
<td>6.13</td>
<td>0.885</td>
<td>6.92</td>
<td>0.783</td>
<td>0.179</td>
<td>4.37</td>
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<tr>
<td>Δ A_{S2}</td>
<td>4.15</td>
<td>0.486</td>
<td>8.54</td>
<td>2.48</td>
<td>0.340</td>
<td>7.29</td>
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<tr>
<td>Δ A_{Vx}</td>
<td>2.23</td>
<td>0.399</td>
<td>5.59</td>
<td>1.24</td>
<td>0.228</td>
<td>5.44</td>
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<tr>
<td>Δ ln d_L</td>
<td>1.69</td>
<td>0.253</td>
<td>6.68</td>
<td>1.05</td>
<td>0.183</td>
<td>5.74</td>
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<tr>
<td>ΔΩ_s [deg^2]</td>
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<td>0.879</td>
<td>7.69</td>
<td>0.831</td>
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<td>Δ A_{S1}</td>
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<td>Δ A_{Vx}</td>
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<td>Δ ln d_L</td>
<td>1.98</td>
<td>0.310</td>
<td>6.39</td>
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<td>Δ A_{Vy}</td>
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<td>7.62</td>
<td>2.12</td>
<td>0.298</td>
<td>7.11</td>
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</table>

- Model TVxS2, TVyS1, TV more improved than in Model TS1,TS2.
- The error is more than 5 times improved for both BBH and BNS.
- Even the four polarization modes would be separable with four detectors.
· Even if the number of detectors is equal to the number of the polarization modes, it is difficult to separate the modes depending on the correlation among the amplitude parameters.

· The three polarization modes would be more separable by breaking a degeneracy of the polarization modes and even the four polarization modes would be separable with the global four detectors network.

· The participation of KAGRA in the network of the GW detectors will make it possible to extract the polarization information.
ありがとうとうございました！