Paper Review:

Measurement of Gravitational Coupling between Millimeter-Sized Masses

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Overview

Measurement of Gravitational Coupling between Millimeter-Sized Masses

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We demonstrate gravitational coupling between two gold spheres of radius $r \approx 1\,\mathrm{mm}$ and mass $m \approx 90\,\mathrm{mg}$. By periodically modulating the source mass position at a frequency $f_{mod} = 12.7\,\mathrm{mHz}$ we generate a time-dependent gravitational acceleration at the location of the test mass, which is measured off resonance in a miniature torsional balance configuration. Over an integration time of 350 hours the test mass oscillator enables measurements with a systematic accuracy of $4 \times 10^{-11}\,\mathrm{m/s^2}$ and a statistical precision of $4 \times 10^{-12}\,\mathrm{m/s^2}$. This is sufficient to resolve the gravitational signal at a minimal surface distance of 400 µm between the two masses. We observe both linear and quadratic coupling, consistent in signal strength with a time-varying $1/r$ gravitational potential. Contributions of non-gravitational forces could be kept to less than 10 % of the observed signal. We expect further improvements to enable the isolation of gravity as a coupling force for objects well below the Planck mass. This opens the way for precision tests of gravity in a new regime of isolated microscopic source masses.

arxiv:2009.09546

- Measurement of gravitational force between two tiny spheres using a torsion pendulum
- Modulating a source mass off resonance
- Almost limited by thermal noise of the torsion mode
- Statistical error: 1 %, systematic error: 10 %

Contents

- Introduction
- Experiment
- Results
- Discussion & Outlook

- Supplement Material
  - Torsion pendulum
  - Calibration
  - etc…

Introduction

- Testing gravity is one important experimental endeavor:
  - Unification within standard model
  - Consistency with quantum theory
- Example:
  - Astronomically
    - Direct detection of GW
    - Direct imaging of SMBH
  - Earth-bound
    - Test of equivalent principle
    - Precision measurement of G (Newton’s constant)
    - Test of inverse square law

Motivation

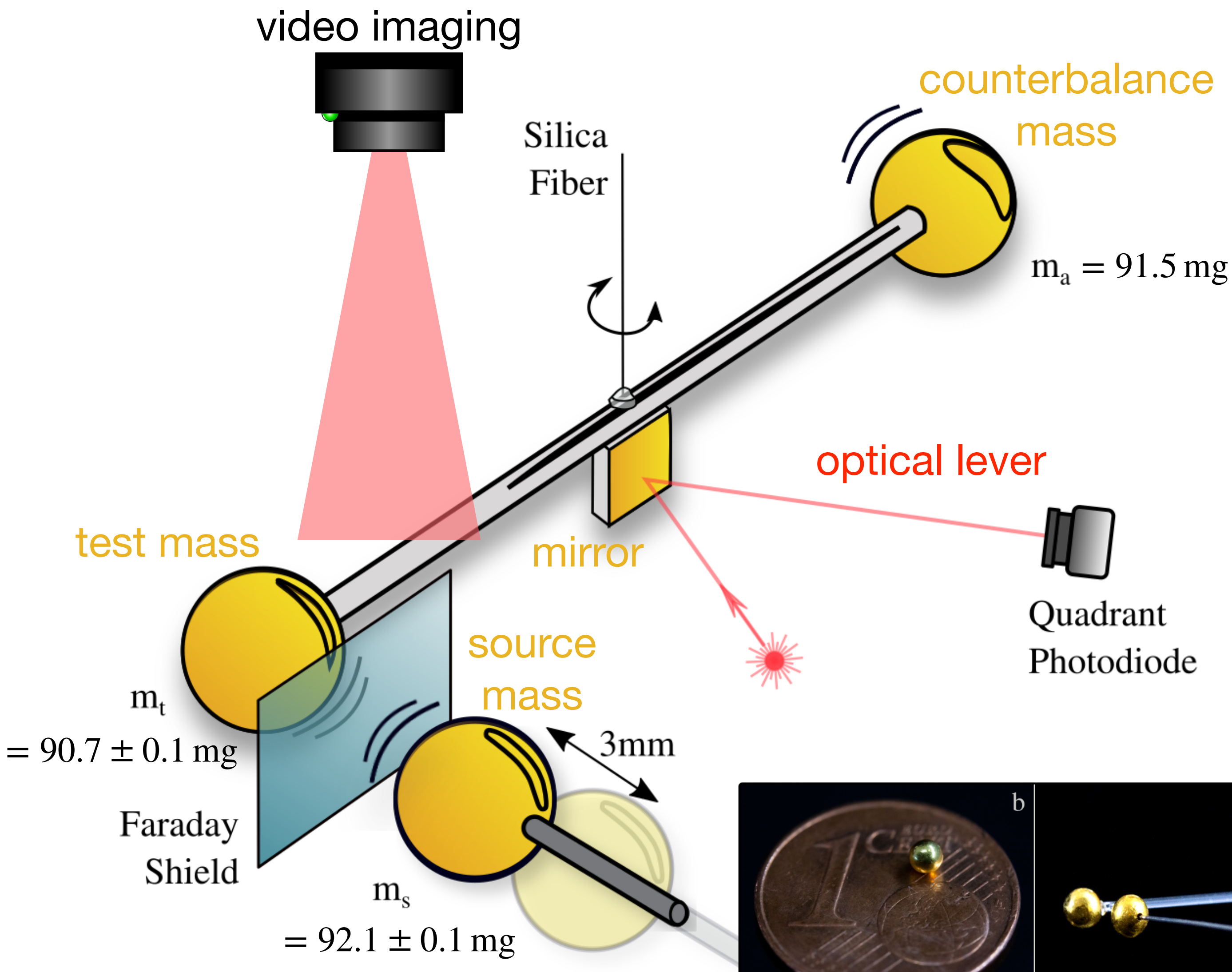
For such experiments,

- Test mass: from quantum system to macroscopic scale
- Source mass: kg-scale and beyond
  - Microscopic source mass is a frontier!
- In this paper they show gravitational coupling in sub-100mg scale

Goal:

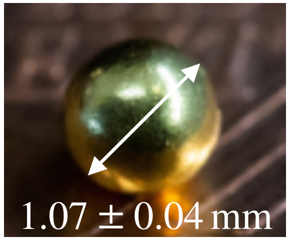
- Show a prospects to Planck mass scale (~ 22 μg) experiment

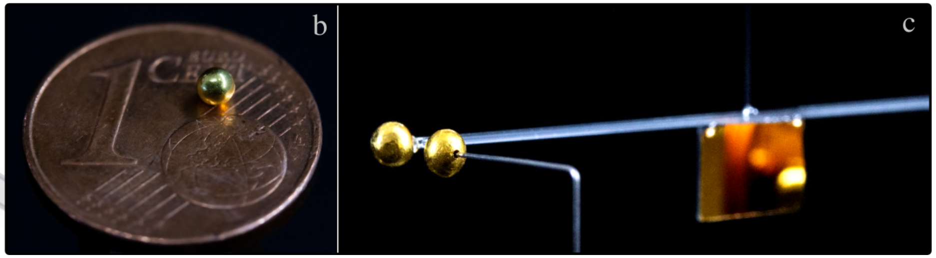
Experiment Setup: Torsion Pendulum



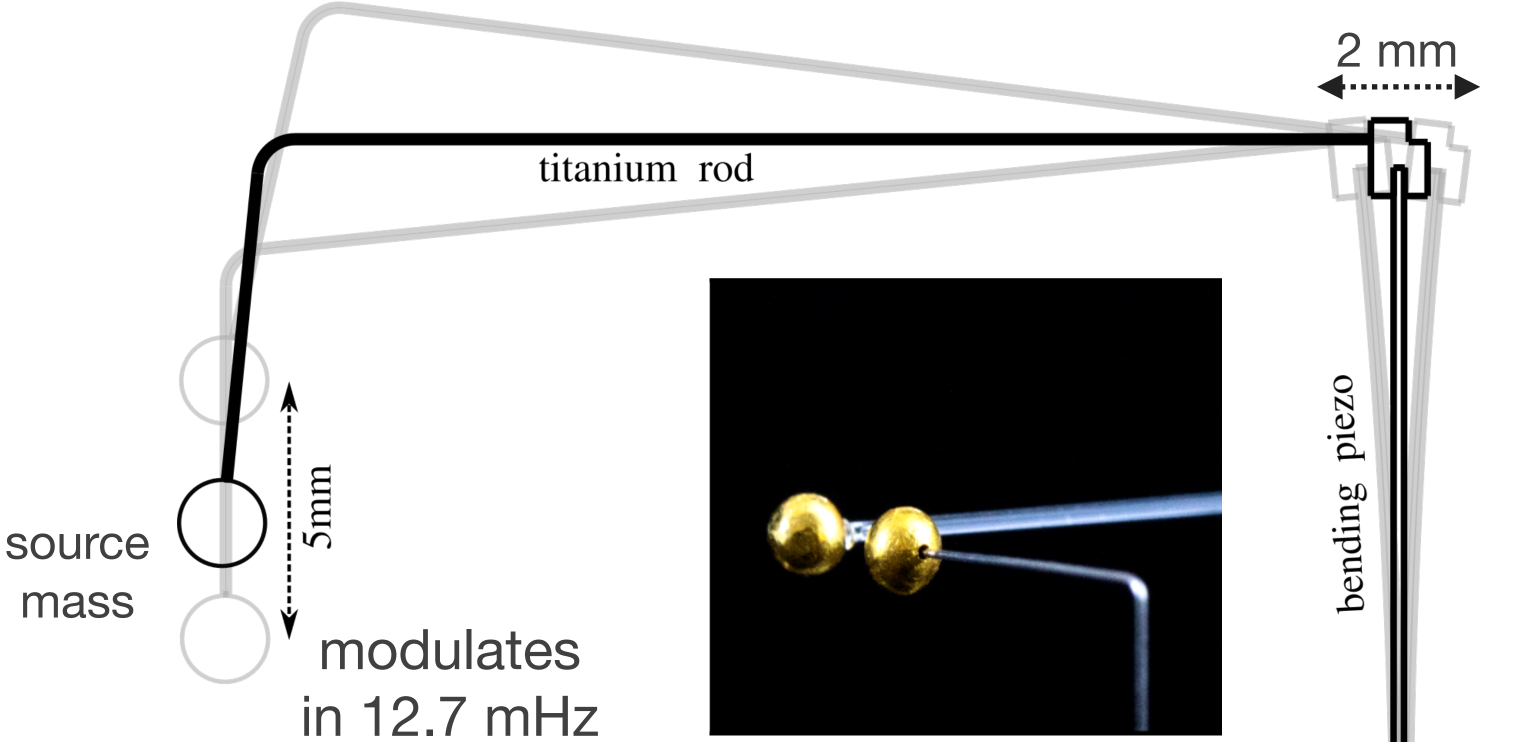
Torsion pendulum

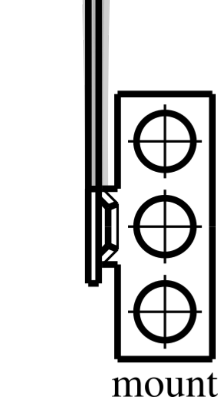
- f_0 = 3.59 mHz
- Q = 4.9





Experiment Setup: Source Mass Drive





Source mass position x_s:

$$x_s = d_0 + d_m \cos(2\pi f_{mod} t)$$

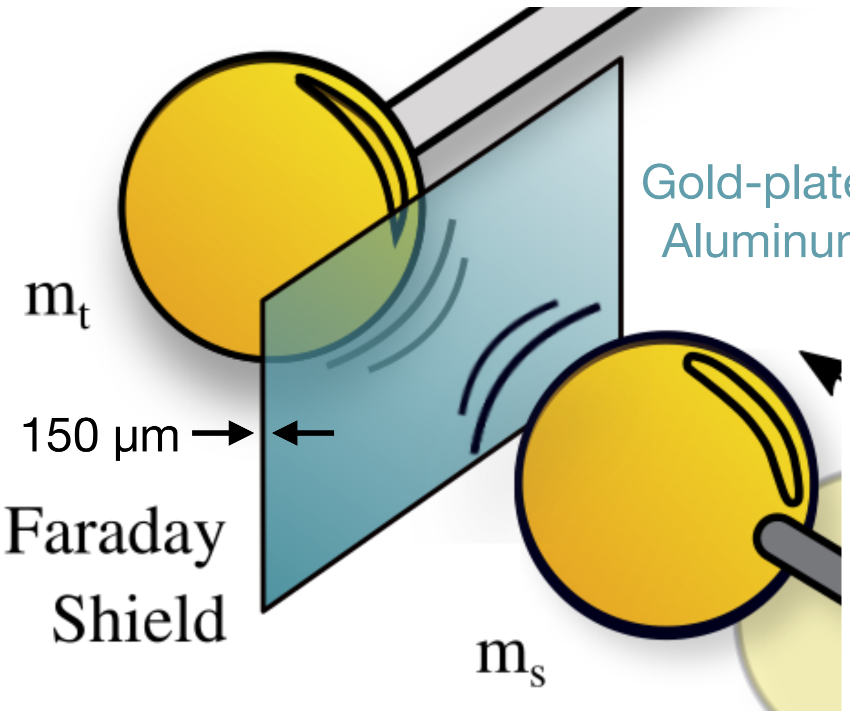
where $d_0 \approx 3.5\,\mathrm{mm}$, $d_m \approx 1.6\,\mathrm{mm}$,

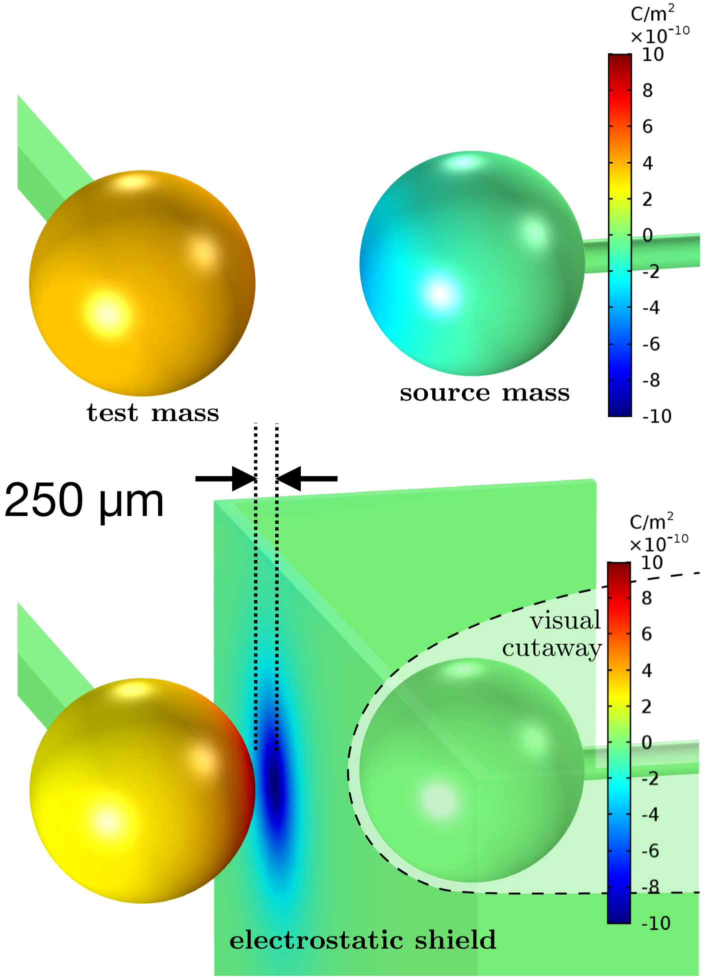
$f_{mod} = 12.7\,\mathrm{mHz}$

Experimental Setup: Faraday Shield

Faraday shield to reduce electrostatic coupling

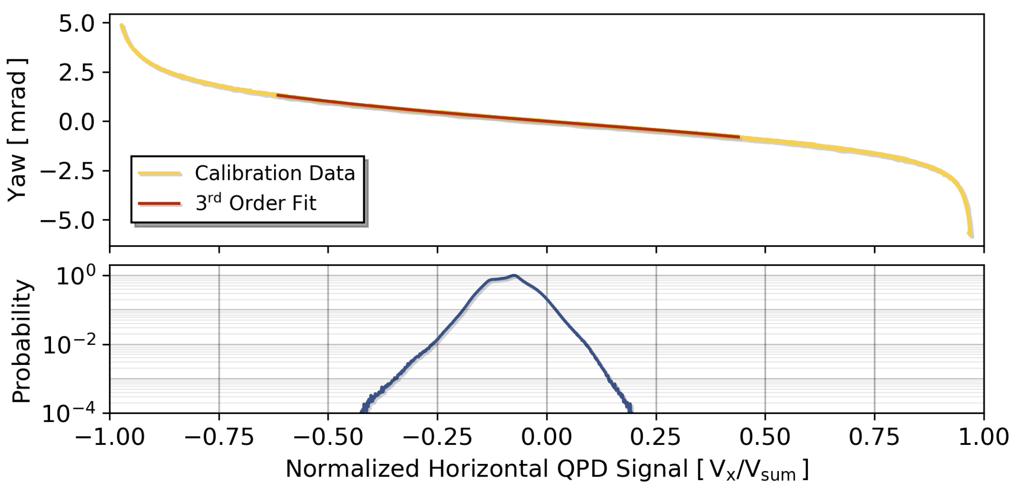
- w/o: Electrostatic ~ 3 x Gravity
- w/: Electrostatic < 0.03 x Gravity

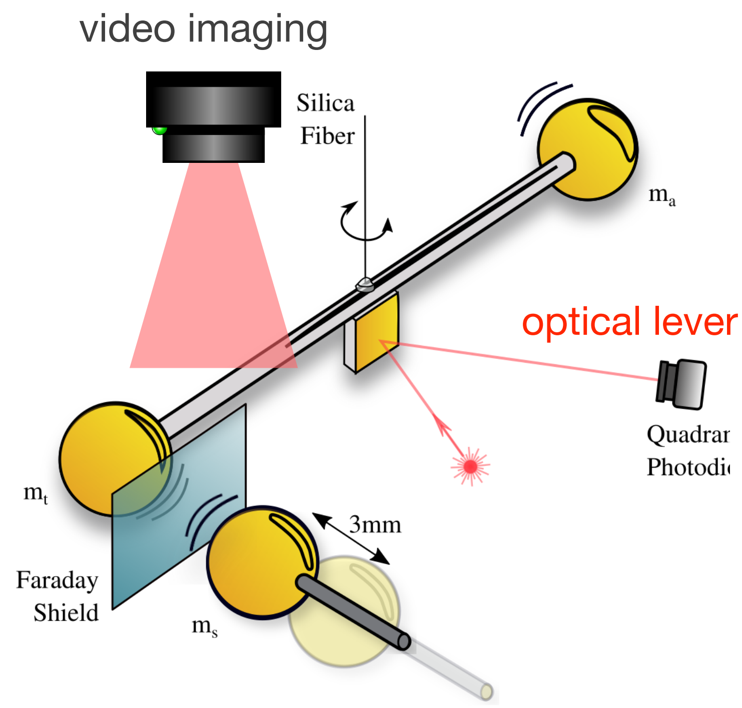




Calibration: Test Mass Position

- Calibration of TM position is done by comparing these two signal:
- Optica lever
  - ◉ Good resolution (2e-9 m/√Hz)
  - ◉ Narrow linear region (~ 0.5 mrad)
- Video tracking: tracking by pixel image
  - ◉ Wide range
  - ◉ Worse resolution (24μm/px)



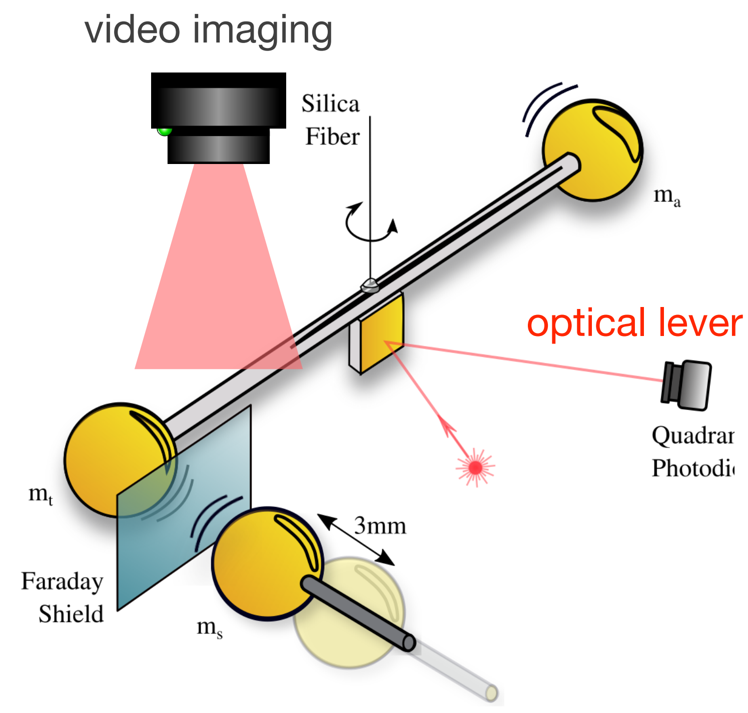


Calibration: Piezo Drive

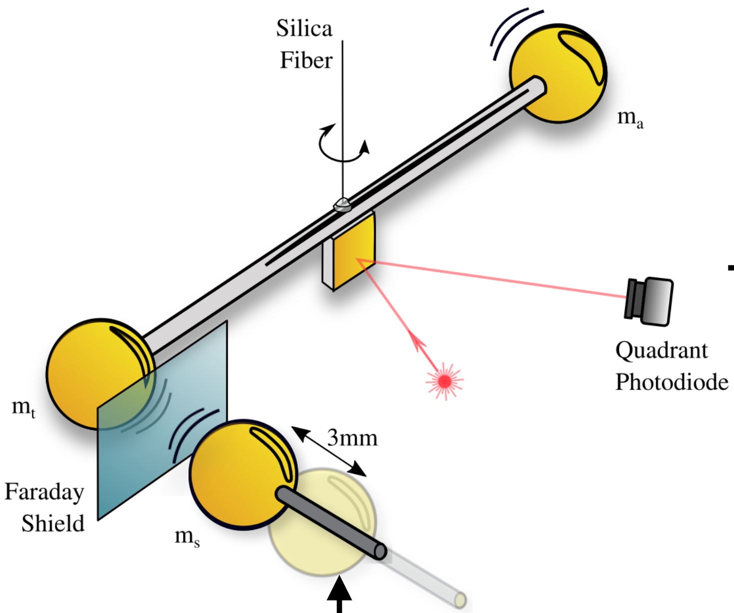
Piezo drive motion is calibrated by video tracking

- Time lag, long term drift exist
  - Modeled as a transducer with an additional low-pass filter
  - Still unexplainable drift exists

- In observation run, seperation between TM and SM is measured at the beginning and the end of the measurement
  - Not monitored continuously during measurement
  - Induced shift of mean position (55 μm) and change of modulation depth (13 μm)
  - They are supposed to be linear over the measurement run



Data Processing



$x_t \xrightarrow[\text{deconvolution}]{\chi^{-1}} F_t$

calibration → x_t position of TM

deconvolution (χ^{-1}) → F_t force on TM

harmonics of f_{mod} → F_G modulated force

remove known singal → F_{res} residual force

modulation

f_{mod} → calibration → x_s position of SM

combination

fit

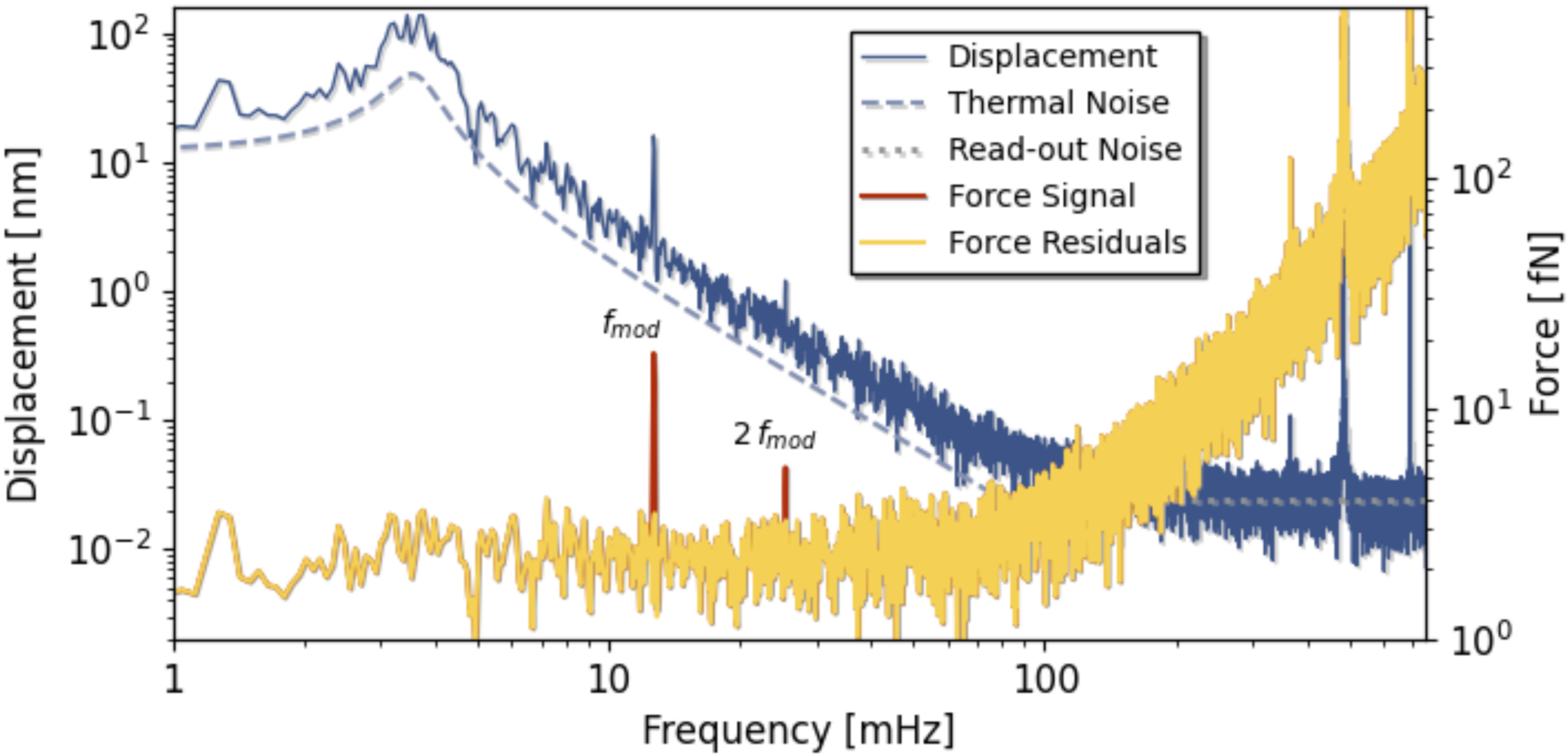
$$F_G = G\frac{m_t m_s}{x_s(t)^2}$$

G_{fit} fitted value of G

moving average $1/f_{mod}$ ~ 79 s → force variance

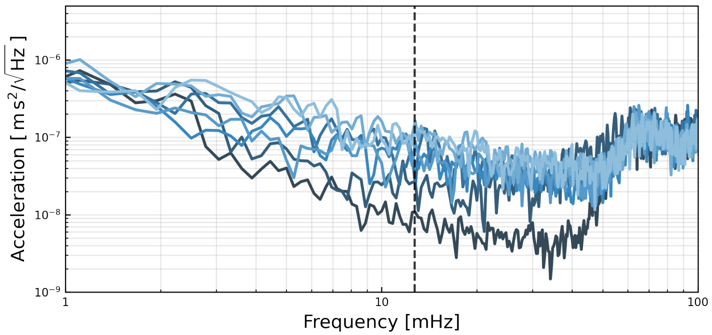
Displacement Sensitivity

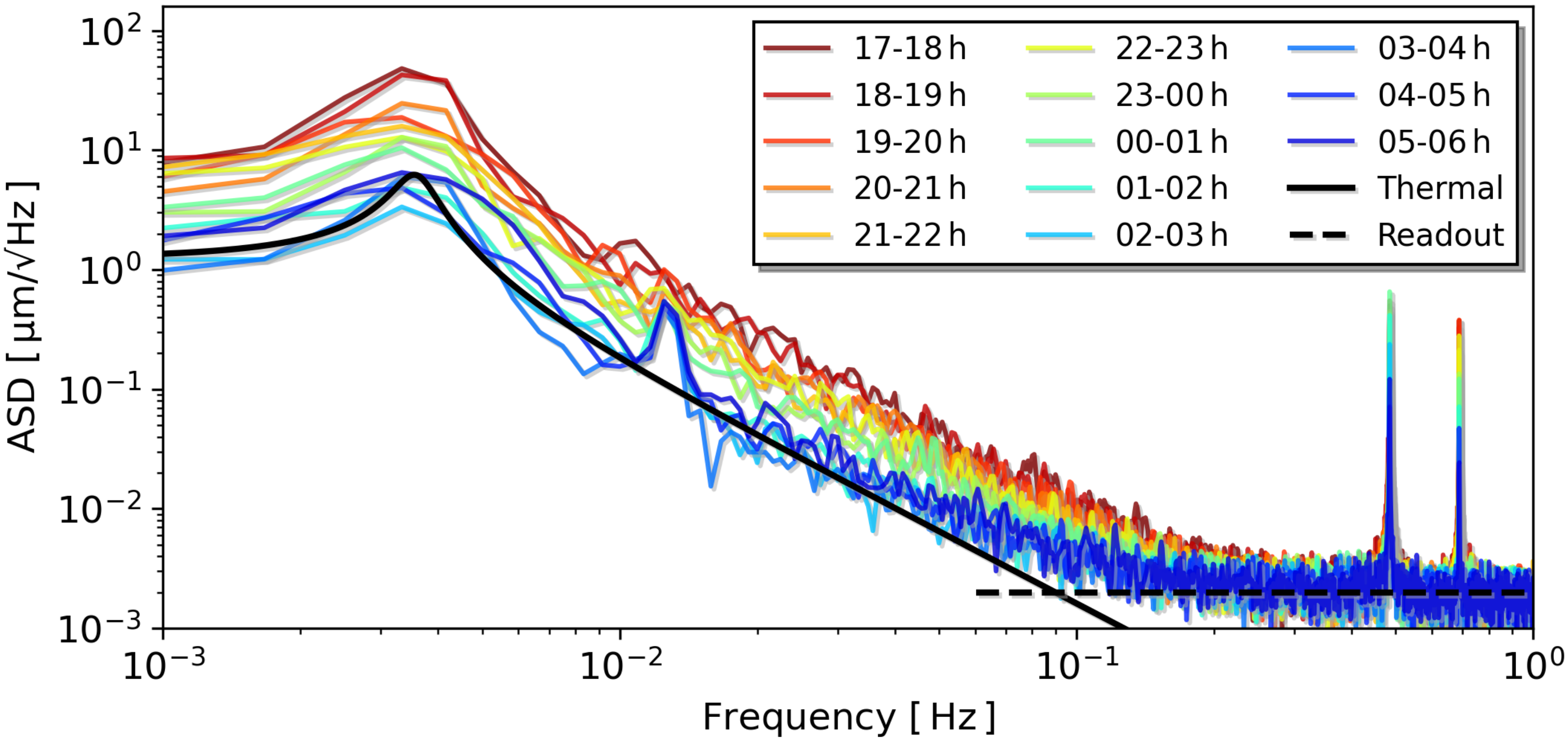
Thermal-noise-limited sensitivity



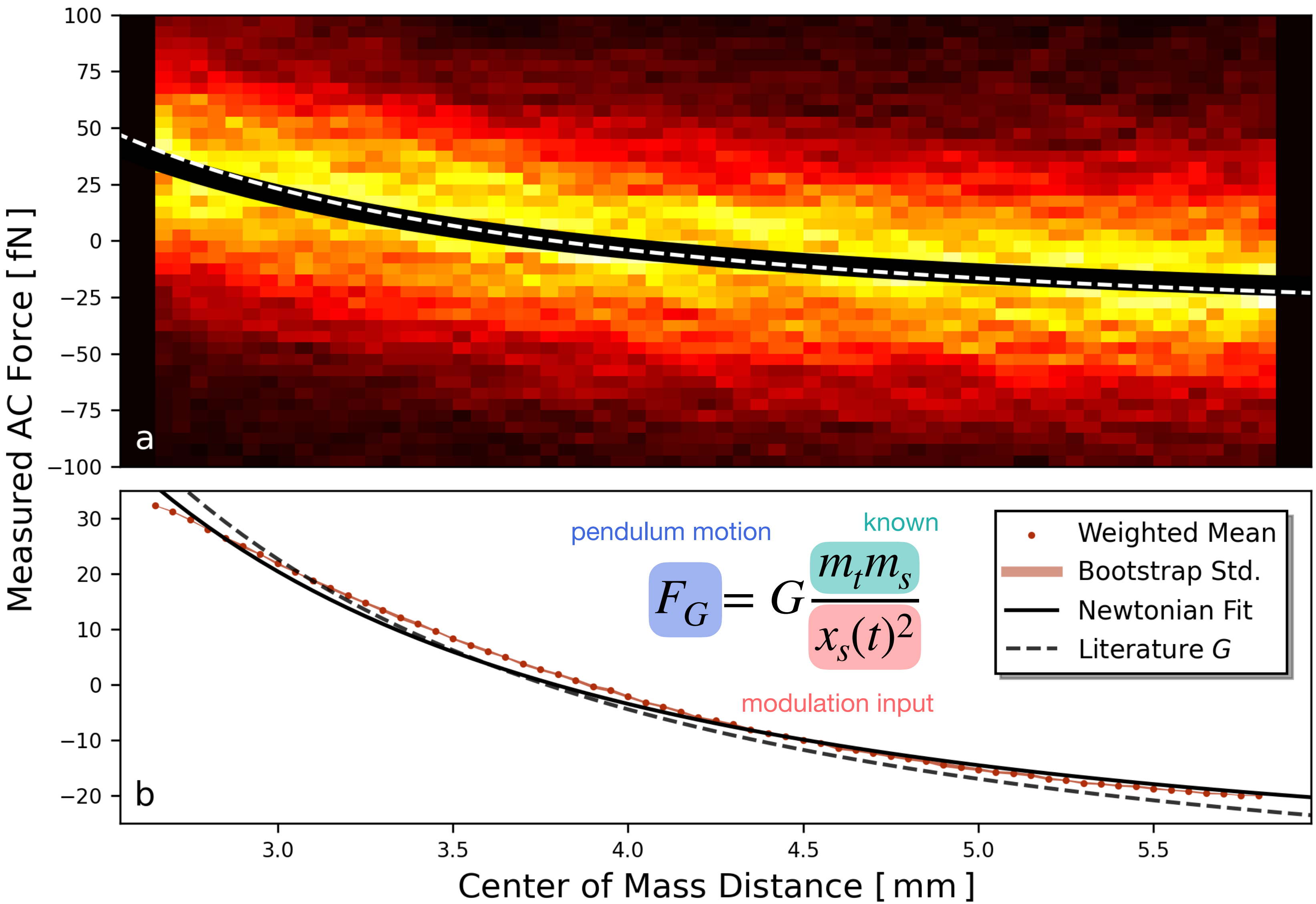
Non-Stationary Noise

- Vary in time
  - Correlation with seismic noise
  - But not linear response





Force vs Separation



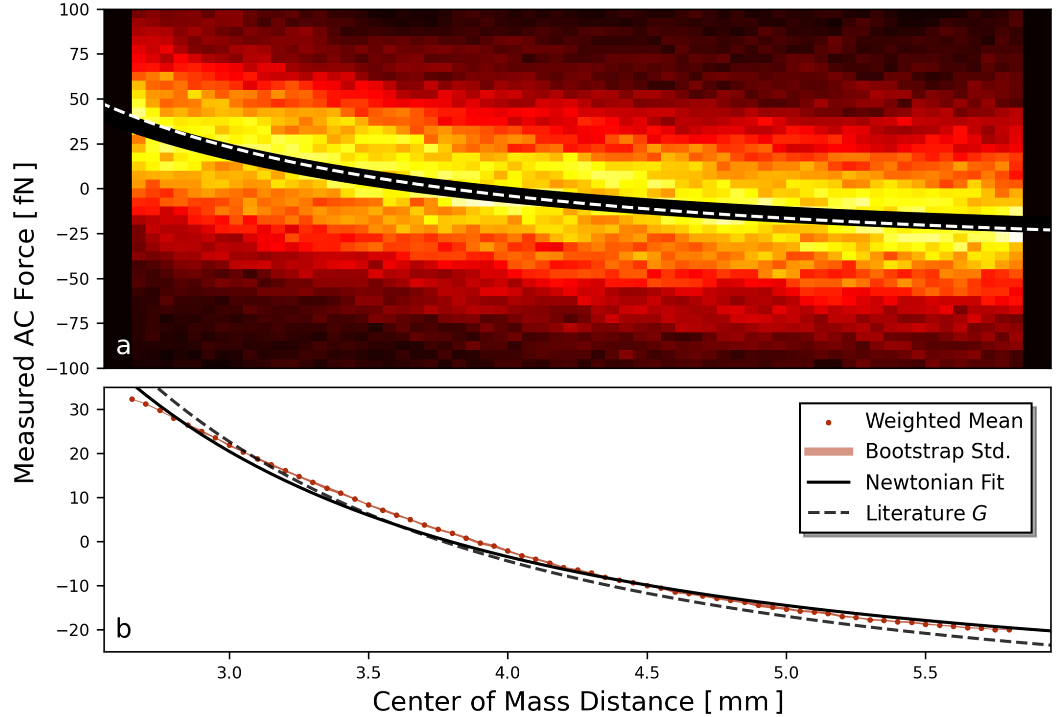
Force vs Separation

pendulum motion

known

$$F_G = G\frac{m_t m_s}{x_s(t)^2}$$

modulation input



One data set in 13.5 hours

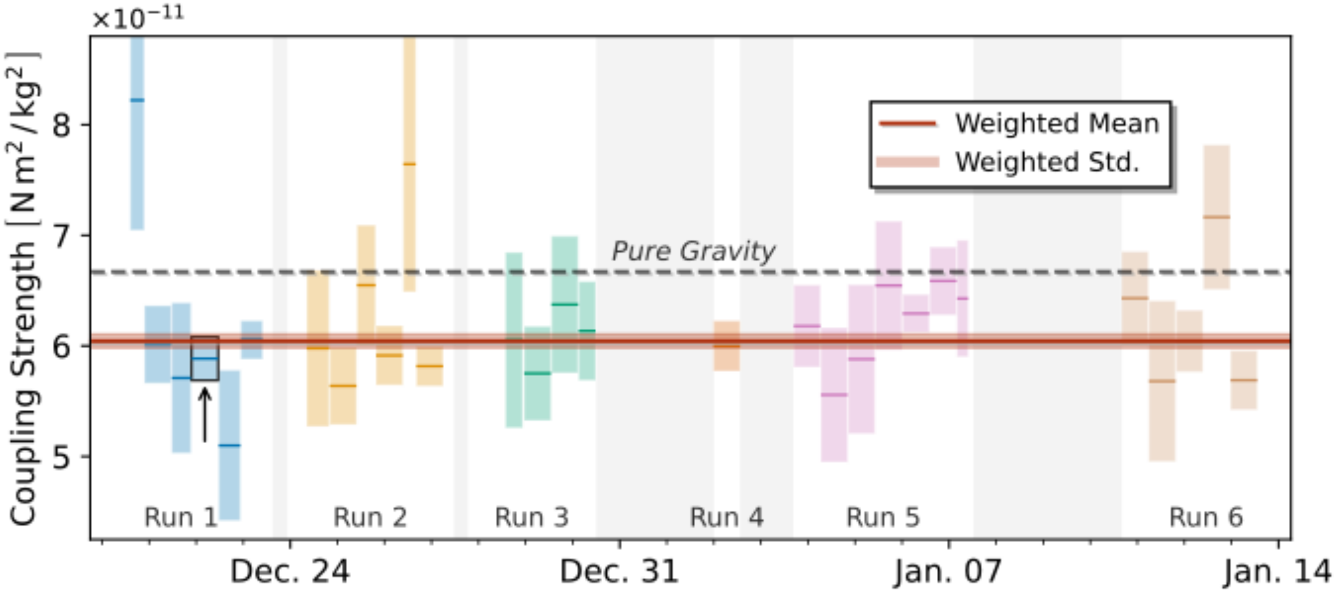
- Using this data, fitted G is

$$G_{fit} = 5.89 \pm 0.2 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$$

- Literature value:

$$G_{lit} = 6.67 \times 10^{-11}\mathrm{N\,m/kg^2}$$

Combination of All Measurement Run



- Measurement over 3 weeks during 2019 Christmas season (because seismic noise due to human activity was quiet)
- Total measurement time: 350 hours
- Combining measurement runs, estimated G is

$$G_{comb} = 6.04 \pm 0.06 \times 10^{-11} \mathrm{N\,m\,/kg^2}$$

- 1 % statistical error, 9 % systematic deviation below G_{lit}

Systematic Influences

- The quantified systematic influences is not fully cover the deviation of the estimated G 9 % below

- Video-tracking related items
  - calibration
  - drive calibration
  - height offset
    - In the future they will improve by
      - continuous tracking
      - 3D monitoring

- Most items relate with scale of the source mass
  - They put a limit on the accuracy of the estimation of G, but not limit the measurement of smaller source masses

effect	**influence** $[F_G]$
act. mass gravity	-1.1E-4
capillary gravity	$+4.5$E-3
glue gravity	$+3$E-3
titanium gravity	$+7$E-3
electrostatic	± 3E-2
magnetic	± 1E-4
calibration	± 6E-3
drive calib.	± 1.6E-2
mass separation	± 1E-3
m_s accuracy	± 1.1E-3
m_t accuracy	± 1.1E-3
m_{glue} accuracy	$+3$E-3
height offset	± 1.5E-2
m_s roundness	unknown
m_t roundness	unknown
Q accuracy	± 5E-3
bandpass $1 f_m$	$+1.6$E-2
bandpass $2 f_m$	-7.6E-2
downsamplig	± 1E-3
compensation pole	± 1E-5
anti aliasing filter	± 2E-6
upper limit	$+11.0$E-2
lower limit	-7.6E-2

Table II: identified systematic deviations

Summary of the Results

- Over an integration time of 350 hour,
  - Resolution $\approx 4 \times 10^{-10}\,\mathrm{m/s^2}$
  - Systematic precision $\approx 4 \times 10^{-11}\,\mathrm{m/s^2}$
  - Statistical accuracy $\approx 4 \times 10^{-12}\,\mathrm{m/s^2}$
- Contribution of non-gravitational forces is kept below 10 %
- Current sensitivity is limited by thermal noise when ambient condition is optimal (quiet seismic motion)

Toward Smaller Masses

Consider possibilities of realizing the system with Planck mass ($m_P \approx$ 22 μg) objects

- Reduction of thermal noise

- Current Q factor is Q ~ 5 for small amplitude
- When driving large amplitude (π rad), Q = 20,000 and different rest position
  - Potentially Q = 20,000 is achievable for this pendulum
  - The sensitivity will improve by √20000/5 ~ 65 times
- Current SNR is 6.06/0.04 ~ 100 with 100 mg object
  - For Q=20,000 SNR will improve to ~ 6.04/0.04 x 65 ~ 9800
  - For Planck mass objects, SNR will 9800*2.2e-4 ~ 2
  - Measurable!
- Optical levitation is one way to reduce thermal noise

Other Challenges to Smaller Mass

- Casimir force between a plate and a sphere

$$F_{Casimir} = \frac{r_t k_B T \zeta(3)}{8 d_s^2}$$

- In this experiment ($r_t = 1\,\mathrm{mm}$, $d_s \approx 250\,\mu\mathrm{m}$) Casimir force is more than four orders of magnitude smaller than gravitational force
- For a Planck mass sized sphere with separation bellow 100 μm Casimir force will dominates the Newtonian gravity
  - We can avoid this by modulating gravitational force
- Casimir force might introduce instability of the torsion pendulum
- W/o Faraday shields Casimir force btw TM and SM exists
  - For Planck mass sized spheres the lower limit of the separation is ~ 20 μm

Future Application

- Application of Planck mass sized gravitational force source

◉ Determine value of G in a different way

- CQG **33**, 125031 (2016)

◉ Search for new scalar field

- Science **349**, 849-851 (2015)

◉ Test of gravitation theory beyond Newtonian dynamics

- Canadian Journal of Physics **93**, 892-895 (2015)

◉ Quantum nature of gravity

- PRL **124**, 013603 (2020)

Summary

- Gravitational force test in the regime of microscopic source mass is unexplored frontier
- They measured gravitational force between two tiny spheres (~ 100 mg) using a torsion pendulum
- Almost limited by thermal noise of the torsional mode
- Estimated G: $G_{comb} = 6.04 \pm 0.06 \times 10^{-11} \mathrm{N\,m/kg^2}$

◉ Statistical error: 1 %, systematic error: 10 %

◉ Not fully covered by the quantified systematic errors

- Planck mass scale measurement will be realizable if Q is improved by factor of 4000 (or introducing optical levitation)

Appendix

125031 (2016).

[35] Shimoda, T., Aritomi, N., Shoda, A., Michimura, Y., and Ando, M., Seismic cross-coupling noise in torsion pendulums. *Physical Review D* **97**(10), 104003 (2018).

[36] Shimoda, T. and Ando, M., Nonlinear vibration transfer in torsion pendulums. *Classical and Quantum Gravity* **36**(12), 125001 (2019).

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[41] Komori, K. *et al.*, Attonewton-meter torque sensing with a macroscopic optomechanical torsion pendulum. *Physical Review A* **101**(1), 011802 (2020).

[42] Ranjit, G., Cunningham, M., Casey, K., and Geraci,