Seminar @ Ando lab, University of Tokyo

Enhancement of quantum gravity signal in an optomechanical experiment

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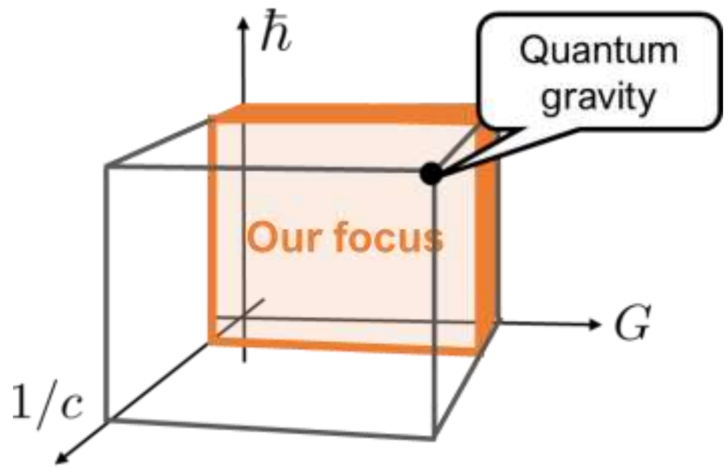
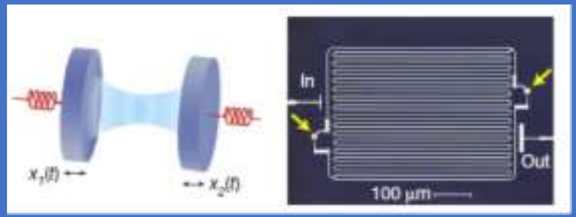
Introduction

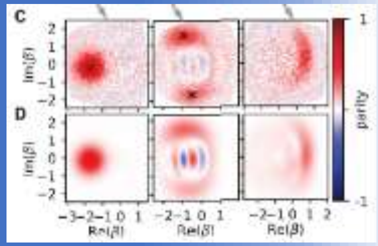
Introduction

- We need experimental evidence to explore quantum gravity
- Quantum gravity theory ∋ non-relativistic gravity in QM regime
  Our focus
- Regarding the recent progress of quantum experiment in mesoscopic scale, it is expected to be realized in near future.



Quantum superposition





Planck mass scale

Measuring gravitational const

$7 \times 10^{-11}[g]$ $2 \times 10^{-5}[g]$ $9 \times 10^{-2}[g]$ Probe mass

Let's consider how to test the quantum nature of non-relativistic gravity targeting on mesoscopic object!

Quantum gravity witness

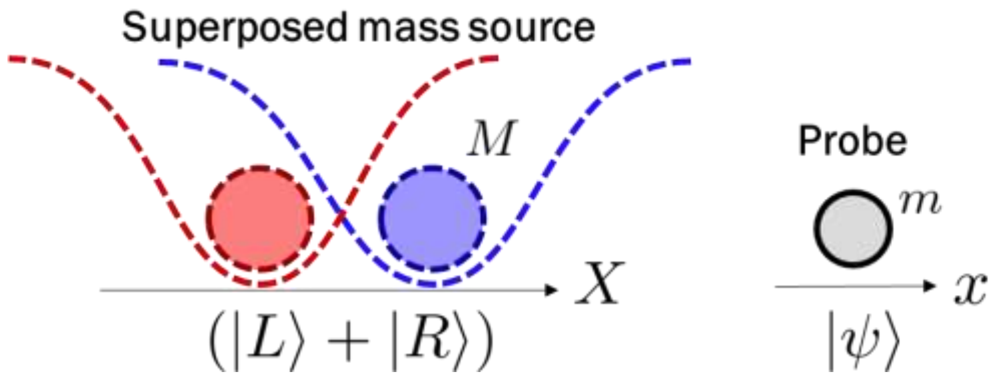
Feynman (1957), Zeh (2008)
Bose et al. (2017),
Marletto, Vederal (2017)

We want to clarify if superposition of gravity is realized or not.

- Quantized Newtonian potential

Potential depends on mass source position.

$$\Phi(\hat{x}, \hat{X}) = -\frac{GM}{|\hat{x} - \hat{X}|}$$



Time evolution of probe & mass source systems

$$e^{-im\Phi t/\hbar}|\psi\rangle \otimes (|L\rangle + |R\rangle)$$: Separable state

$$= e^{\phi_L}|\psi\rangle \otimes |L\rangle + e^{\phi_R}|\psi\rangle \otimes |R\rangle$$: Entangled state

$$(\phi_j = -im\Phi(\hat{x}, j)t/\hbar)$$

If Newtonian potential is quantized, probe feels different gravitational potential depending on mass source position and evolves with different phase.

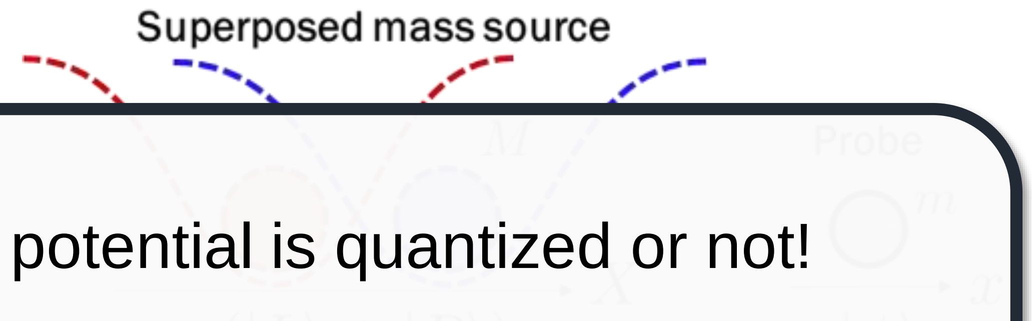
Quantum gravity witness

Feynman (1957), Zeh (2008)
Bose et al. (2017),
Marletto, Vederal (2017)

We want to clarify if superposition of gravity is realized or not.

- Quantized Newtonian potential

  Potential depends on mass source position.



We have not tested yet if Newtonian potential is quantized or not!

What kind of experiment should we consider to see quantum nature of gravity?

⇒ ”Quantum gravity witness in optomechanical system” by Balushi, Cong, Mann

If Newtonian potential is quantized, probe feels different gravitational potential depending on mass source position and evolves to quantum entangled state.

[Previous work]

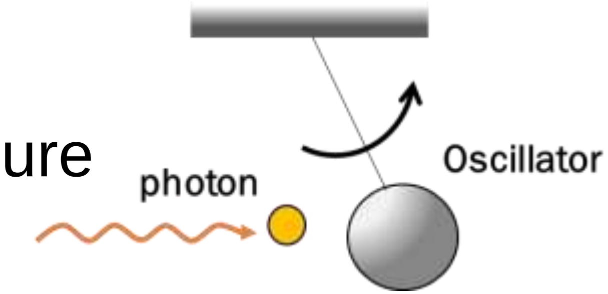
Quantum Gravity witness in optomechanical system

Balushi, Cong, Mann (2018)

Optomechanical system

Optics Mechanics

- Moving macro. system (e.g. oscillator) with photon rad. pressure

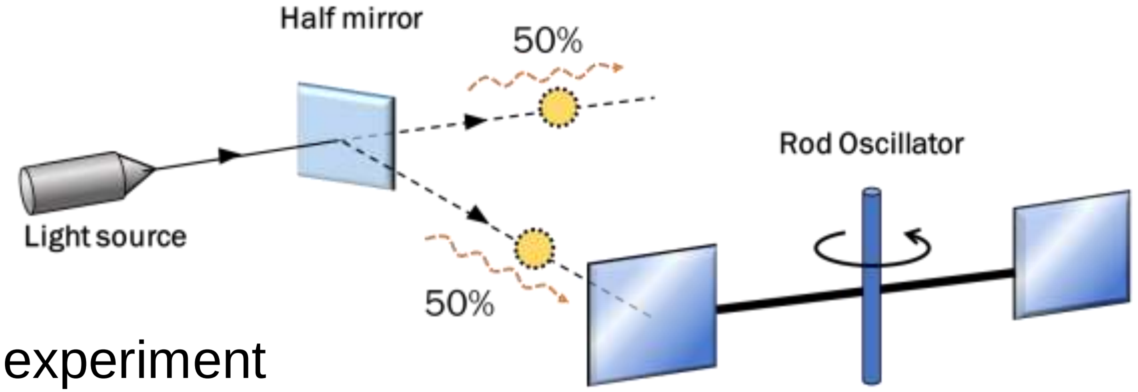


<u>Make osc. state superposed using superposed photon state</u>

– Photon state is superposed by half mirror

→ 50% chance of photon hitting the rod.

→ 50% chance of the rod oscillation



– Expected to be useful for the future quantum experiment

Observation of entanglement btw. two oscillators in optomecha. systems!

[Ockeloen-Korppi + (2018)]

– Proposal by Balushi et al.

"Let's prepare superposed mass source system in optomechanical system,
and test superposition of Newtonian gravity in photon interference experiment! "

Setup

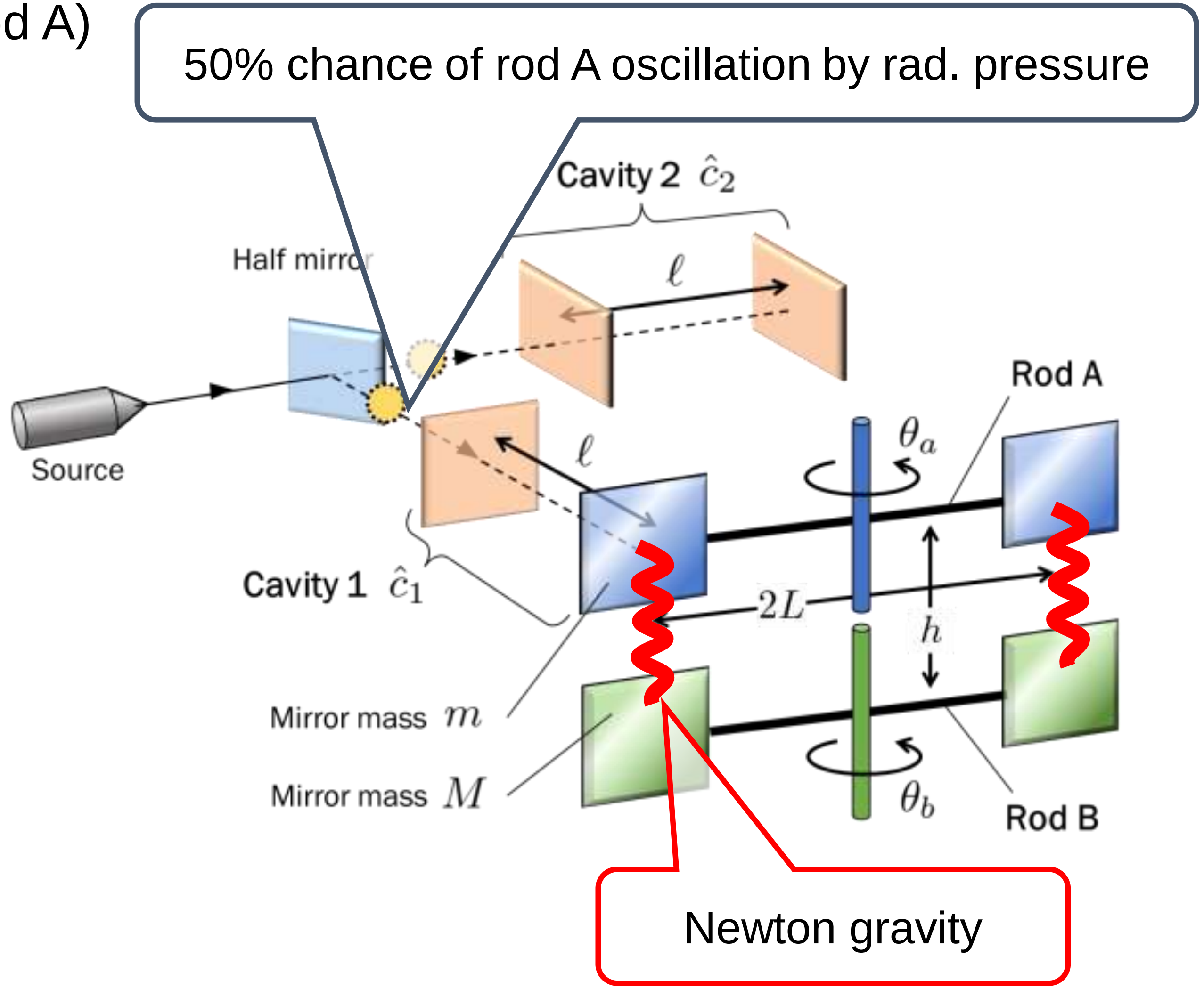
- Optomechanical system with Photon cavity × 2 + Oscillator × 2
  - One of the mirrors of cavity1 oscillates (Rod A)
  - 50% chance of photon entering cavity 1

$$\frac{1}{\sqrt{2}}\left(|0\rangle_{c_1}|1\rangle_{c_2} + |1\rangle_{c_1}|0\rangle_{c_2}\right)$$

Enter cavity 2　　Enter cavity 1

  - Rod A and B are gravitationally coupled

Finally, we observe interference of photon states in cavity 1 and cavity 2.



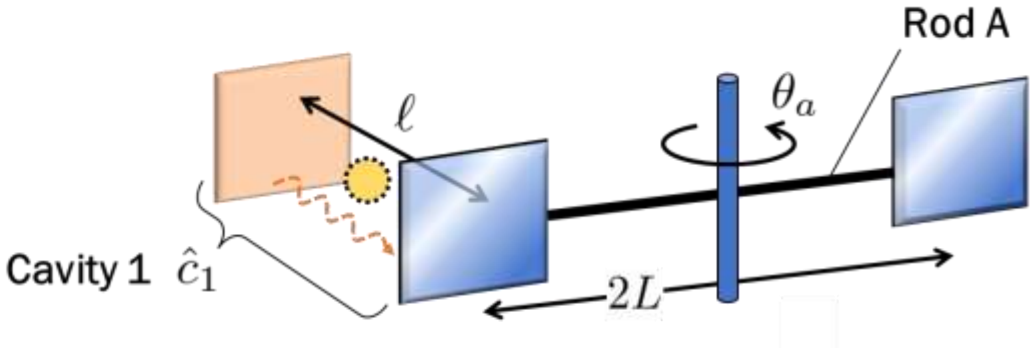
Optomechanical coupling

- Hamiltonian of cavity1 and rod A

If rod A oscillates, cavity length changes and the effective photon frequency shifts.

$$\omega_c = \frac{\pi c \mathfrak{n}}{\ell}$$

$$\longrightarrow \quad \omega_c' = \frac{\pi c \mathfrak{n}}{\ell + L \sin\theta_a} \approx \omega_c \left(1 - \frac{L}{\ell}\theta_a\right) + \mathcal{O}\left[\theta_a^2\right]$$

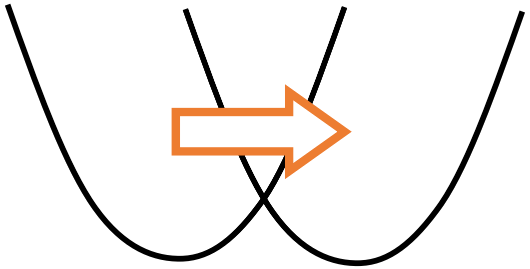


Hamiltonian of cavity 1 and rod A

$$\hat{H}_{c_1} + \hat{H}_A = \hbar\omega_c' \hat{c}_1^\dagger \hat{c}_1 + \frac{1}{2I_a}\hat{p}_a^2 + \frac{1}{2} I_a \Omega_a^2 \hat{\theta}_a^2$$

$$\approx \hbar\omega_c \hat{c}_1^\dagger \hat{c}_1 + \frac{1}{2I_a}\hat{p}_a^2 + \frac{I_a}{2}\Omega_a^2\hat{\theta}_a^2 \boxed{- \frac{\hbar\omega_c L}{\ell}\hat{c}_1^\dagger \hat{c}_1 \hat{\theta}_a}$$

Optomechanical coupling

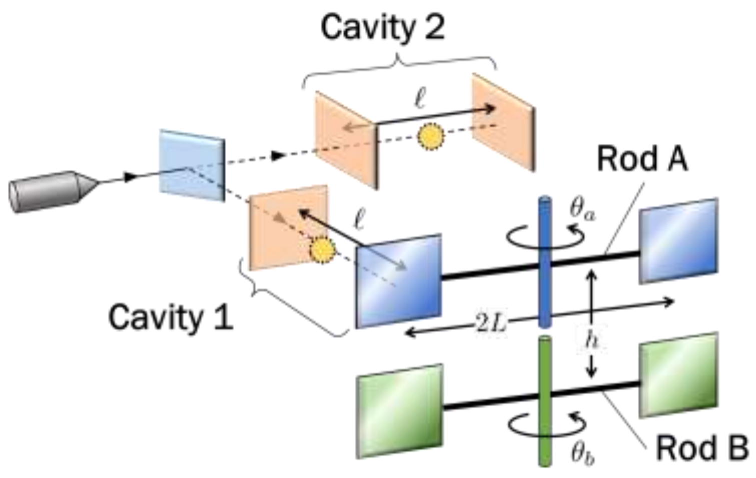


If photon hits the rod, the effective rod potential is shifted

$I_a = 2mL^2$: Moment of rod A

Ω_a: frequency of rod A

Time evolution



- Total Hamiltonian

photon; Oscillator A; Optomecha

$$\hat{H} = \hbar\omega_c \hat{c}_1^\dagger \hat{c}_1 + \hbar\omega_c \hat{c}_2^\dagger \hat{c}_2 + \frac{1}{2I_a}\hat{p}_a^2 + \frac{I_a}{2}\Omega_a^2\hat{\theta}_a^2 - \frac{\hbar\omega_c L}{\ell}\hat{c}_1^\dagger \hat{c}_1 \hat{\theta}_a$$

$$+ \frac{1}{2I_b}\hat{p}_b^2 + \frac{1}{2}I_b\Omega_b^2\hat{\theta}_b^2 - \frac{2GmM}{\sqrt{h^2 + \left(2L\sin[(\hat{\theta}_b - \hat{\theta}_a)/2]\right)^2}}$$

Oscillator B

Gravity between A and B

$$= \hbar\omega_c \hat{c}_1^\dagger \hat{c}_1 + \hbar\omega_c \hat{c}_2^\dagger \hat{c}_2 + \sum_{n=0,1} \hat{H}_{a,n} |n\rangle_{c1\,c1}\langle n| + \hat{H}_b + \hat{H}_g$$

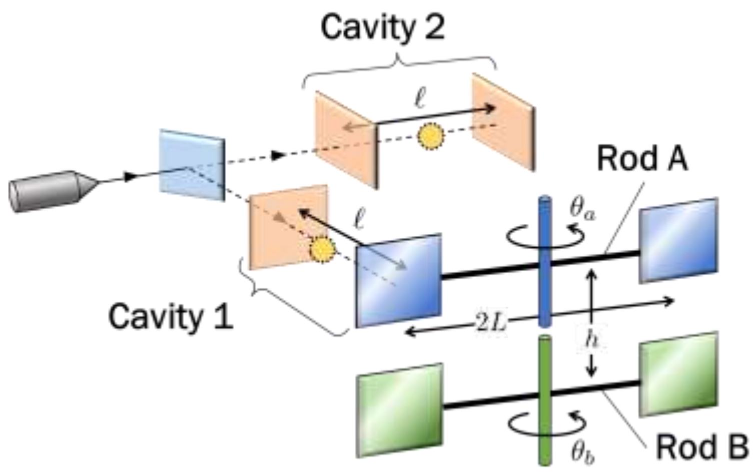
where

$$\hat{H}_{a,n} = \frac{1}{2I_a}\hat{p}_a^2 + \frac{I_a}{2}\omega_a^2\hat{\theta}_a^2 - n\frac{\hbar\omega_c L}{\ell}\hat{\theta}_a, \quad \omega_a = \sqrt{\Omega_a^2 + \frac{GM}{h^3}}$$

Effective potential of rod A is displaced depending on whether photon hits the rod (n=1) or not (n=0).

$$\hat{H}_b = \frac{1}{2I_b}\hat{p}_b^2 + \frac{1}{2}I_b\omega_b^2\hat{\theta}_b^2, \quad \omega_b = \sqrt{\Omega_b^2 + \frac{GM}{h^3}} \qquad \hat{H}_g = -g\frac{\omega_a(2L)^2}{(mM\omega_a\omega_b)^{-1/2}}\hat{\theta}_a\hat{\theta}_b, \quad g = \frac{G}{2h^3\omega_a}\sqrt{\frac{mM}{\omega_a\omega_b}}$$

Time evolution



- Initial state

Rod A: coherent state

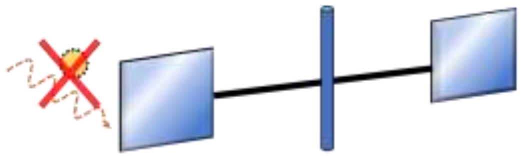
$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}\underbrace{\left(|0\rangle_{c_1}|1\rangle_{c_2} + |1\rangle_{c_1}|0\rangle_{c_2}\right)}_{\text{Superposed photon state}}|\alpha\rangle_a|\beta\rangle_b$$

Rod B: coherent state

- Time evolved state

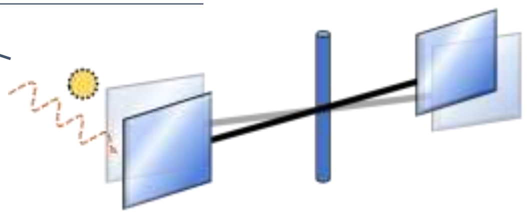
$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle$$

Direct gravitational coupling btw A and B



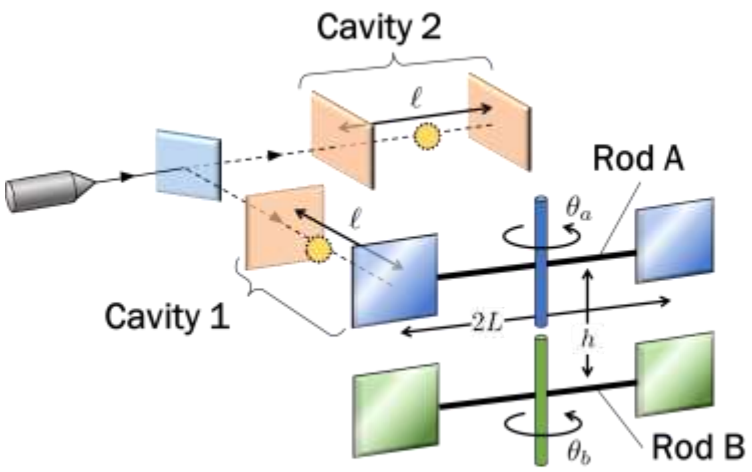
$$= \frac{e^{-i\omega_c t}}{\sqrt{2}}\Big[|0\rangle_{c_1}|1\rangle_{c_2}\left\{1+2i\,g\,\hat{\mathcal{I}}_{\omega_a}(t)\right\}e^{-i\hat{H}_{a,0}t/\hbar}|\alpha\rangle_a e^{-i\hat{H}_b t/\hbar}|\beta\rangle_b$$

$$+|1\rangle_{c_1}|0\rangle_{c_2}\left\{1+2i\,g\left(\hat{\mathcal{I}}_{\omega_a}(t)+\hat{\mathcal{J}}(t)\right)\right\}e^{-i\hat{H}_{a,1}t/\hbar}|\alpha\rangle_a e^{-i\hat{H}_b t/\hbar}|\beta\rangle_b$$



Indirect coupling of B and photon through gravity + optomecha.

Visibility



- Observe photon interference

We consider to observe interference between photon states entering in cavity1 and cavity 2, and calculate its interference term.

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle$$

$$= \frac{e^{-i\omega_c t}}{\sqrt{2}}\Big[|0\rangle_{c_1}|1\rangle_{c_2}\left\{1+2i\,g\,\hat{\mathcal{I}}_{\omega_a}(t)\right\}e^{-i\hat{H}_{a,0}t/\hbar}|\alpha\rangle_a e^{-i\hat{H}_b t/\hbar}|\beta\rangle_b$$

$$+|1\rangle_{c_1}|0\rangle_{c_2}\left\{1+2i\,g\left(\hat{\mathcal{I}}_{\omega_a}(t)+\hat{\mathcal{J}}(t)\right)\right\}e^{-i\hat{H}_{a,1}t/\hbar}|\alpha\rangle_a e^{-i\hat{H}_b t/\hbar}|\beta\rangle_b$$

Calculate innerproduct

The absolute value of interference (visibility) is

$$\mathcal{V}_c(t) := 2\left|\langle\psi(t)|1\rangle_{c1}|0\rangle_{c2}\ {}_{c1}\langle 0|_{c2}\langle 1|\psi(t)\rangle\right| \approx \mathcal{V}_c^{(0)}(t)\left|1-2ig\langle\hat{\mathcal{J}}(t)\rangle\right|$$

$$\approx \mathcal{V}_c^{(0)}(t)\left(1+4g^2\left|\langle\hat{\mathcal{J}}(t)\rangle\right|^2\right)$$

: Visibility changes with 2nd order of gravitational coupling g

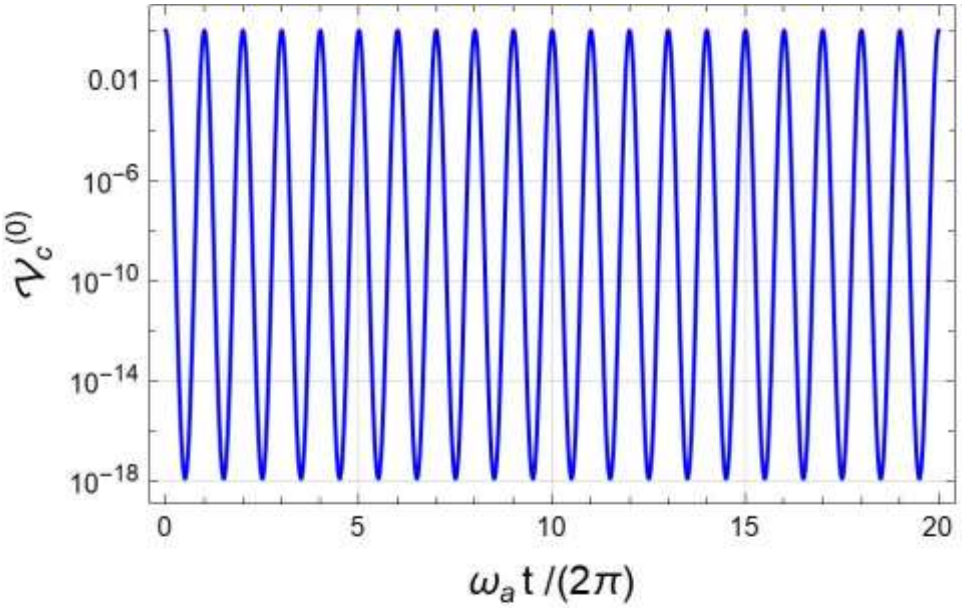
where $\mathcal{V}_c^{(0)}(t) = \left|{}_a\langle\alpha|e^{i\hat{H}_{a,0}t/\hbar}\,e^{-i\hat{H}_{a,1}t/\hbar}|\alpha\rangle_a\right|$: Visibility without gravity

Visibility

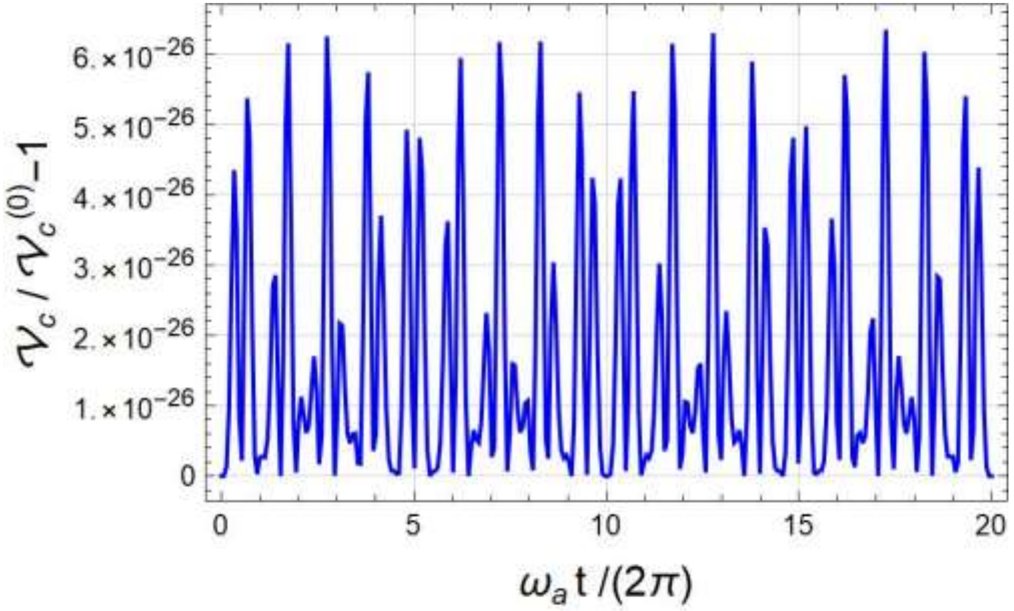
$$m = M = 10^{-13}\,[\mathrm{kg}],\ \Omega_a = 3\times 10^3\,[\mathrm{Hz}],\ \Omega_b = 0.84\times\Omega_a\,[\mathrm{Hz}],$$
$$\alpha = \beta = 1,\ \omega_c = 450\times 10^{12}\,[\mathrm{Hz}],\ \ell = 0.01\,[\mathrm{m}],\ h = 2\times 10^{-6}\,[\mathrm{m}]$$

- Measurement time dependence of visibility $\mathcal{V}_c(t) = \mathcal{V}_c^{(0)}(t)\left(1 + 4g^2\left|\langle\hat{\mathcal{J}}(t)\rangle\right|^2\right)$

No gravity case



Gravitational contribution in visibility

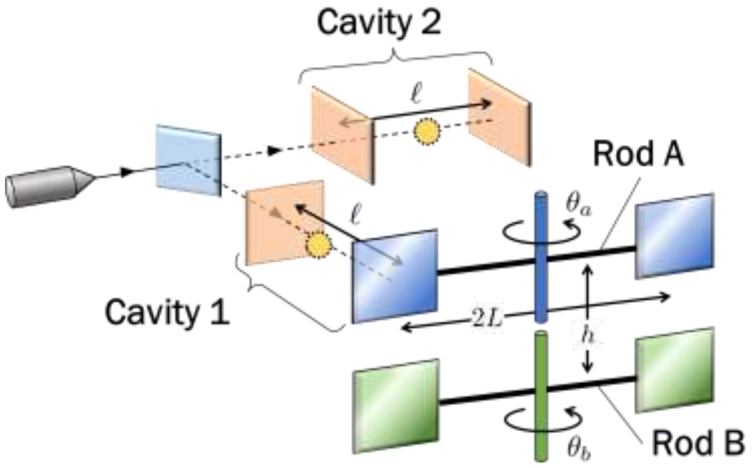


If Newtonian potential is in quantum superposition,
we would observe visibility deviation due to gravity as in the right figure!

[Our work]

Enhancement of quantum gravity signal in an optomechanical experiment

Abstract



- Consider the same setup as the previous work.
- We investigated 2 ways to enhance gravitational signal compared to previous works.

① **If we consider higher-order of optomechanical coupling, the visibility changes with the first order of gravitational coupling** $g = 5.1 \times 10^{-14}$

$$\mathcal{V}_c(t) = \mathcal{V}_c^{(0)}(t) \times \begin{cases} \left(1 + 4g^2 \left|\langle \hat{\mathcal{J}}(t) \rangle\right|^2\right) & \text{: Previous work} \\ (1 + \mathcal{O}[g]) & \text{: Higher-order of optomechanical coupling} \end{cases}$$

② **Signal enhances due to resonance of two oscillators A and B.**

$$\mathcal{V}_c(t) \propto \frac{1}{\omega_a - \omega_b}$$

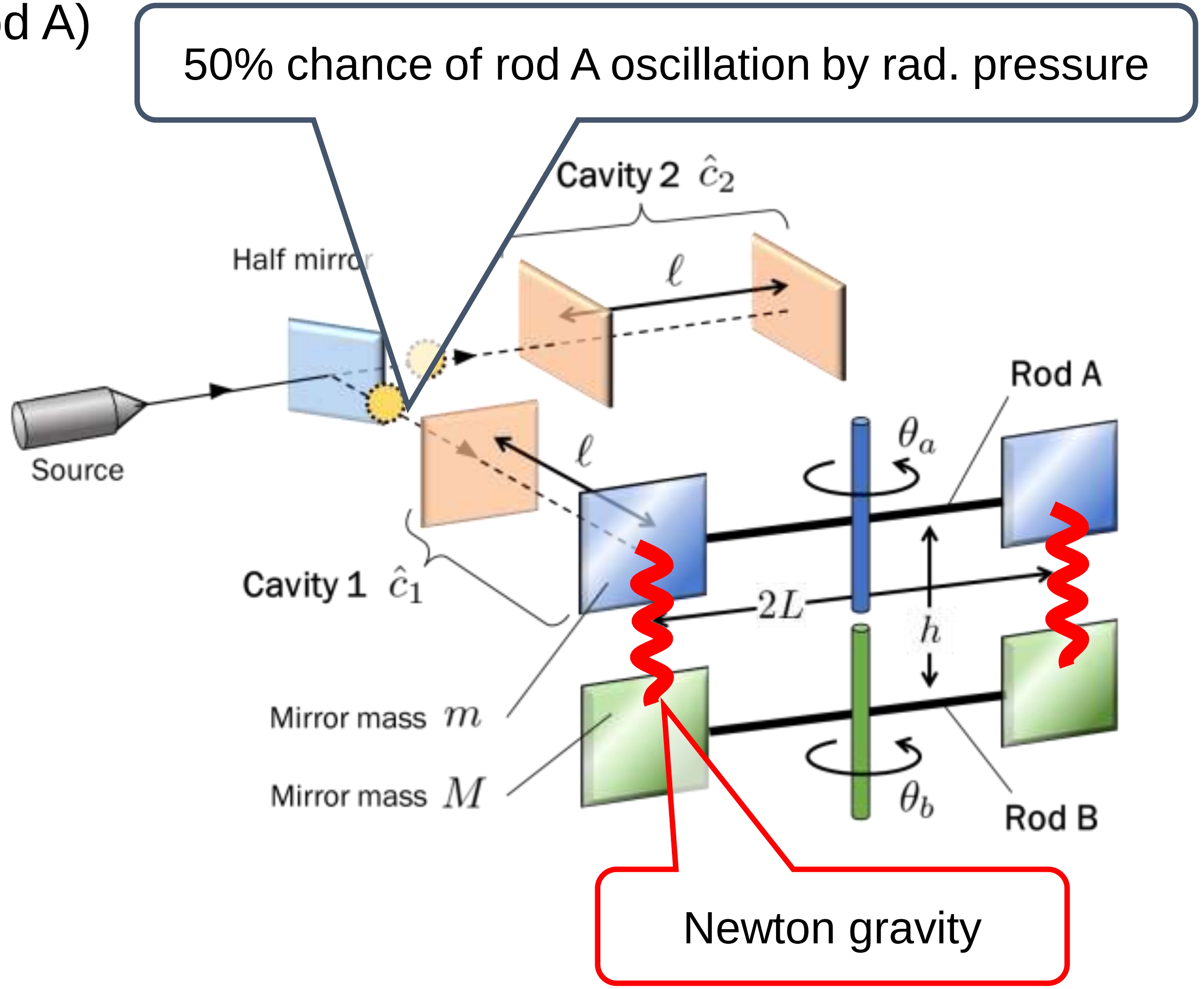
Setup

- Optomechanical system with Photon cavity × 2 + Oscillator × 2
  - One of the mirrors of cavity1 oscillates (Rod A)
  - 50% chance of photon entering cavity 1

$$\frac{1}{\sqrt{2}}\left(|0\rangle_{c_1}|1\rangle_{c_2}+|1\rangle_{c_1}|0\rangle_{c_2}\right)$$

Enter cavity 2 Enter cavity 1

  - Rod A and B are gravitationally coupled



Finally, we observe interference of photon states in cavity 1 and cavity 2.

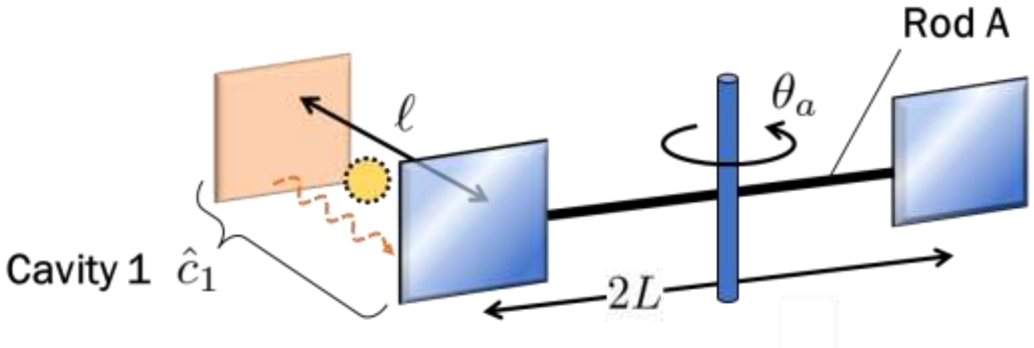
Optomechanical coupling

- Hamiltonian of cavity1 and rod A

If rod A oscillates, cavity length changes and the effective photon frequency shifts.

$$\omega_c = \frac{\pi c\,\mathfrak{n}}{\ell}$$

$$\longrightarrow \quad \omega_c' = \frac{\pi c\,\mathfrak{n}}{\ell + L\sin\theta_a} \approx \omega_c\left(1 - \frac{L}{\ell}\theta_a\right) + \mathcal{O}\left[\theta_a^2\right]$$

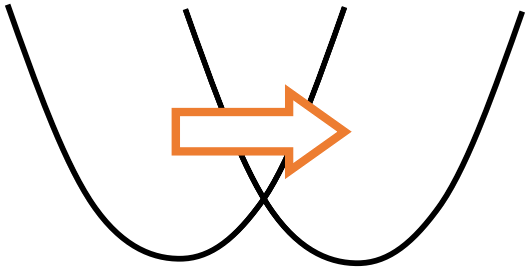


Hamiltonian of cavity 1 and rod A

$$\hat{H}_{c_1} + \hat{H}_A = \hbar\omega_c'\hat{c}_1^\dagger\hat{c}_1 + \frac{1}{2I_a}\hat{p}_a^2 + \frac{1}{2}I_a\Omega_a^2\hat{\theta}_a^2$$

$$\approx \hbar\omega_c\hat{c}_1^\dagger\hat{c}_1 + \frac{1}{2I_a}\hat{p}_a^2 + \frac{I_a}{2}\Omega_a^2\hat{\theta}_a^2 \boxed{- \frac{\hbar\omega_c L}{\ell}\hat{c}_1^\dagger\hat{c}_1\hat{\theta}_a}$$

Optomechanical coupling



If photon hits the rod, the effective rod potential is shifted

$I_a = 2mL^2$: Moment of rod A $\qquad$ Ω_a: frequency of rod A

① Higher-order of optomechanical coupling

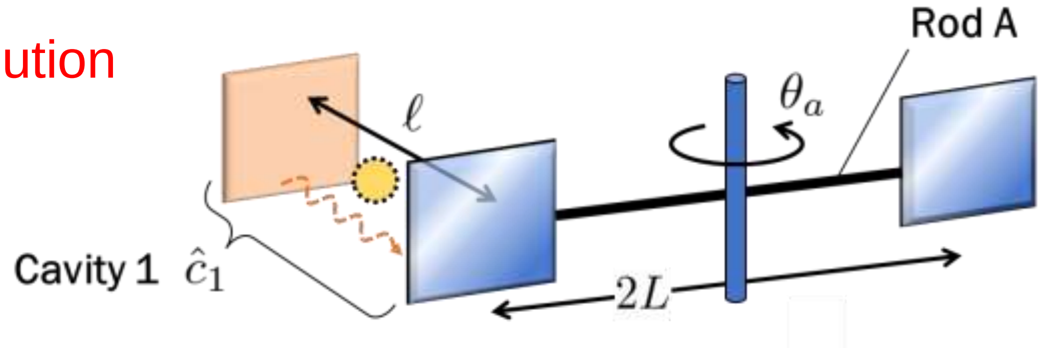
- Hamiltonian of cavity1 and rod A

If rod A oscillates, cavity length changes and the effective photon frequency shifts.

$$\omega_c = \frac{\pi c \mathfrak{n}}{\ell}$$

$$\longrightarrow \quad \omega_c' = \frac{\pi c \mathfrak{n}}{\ell + L\sin\theta_a} \approx \omega_c \left(1 - \frac{L}{\ell}\theta_a + \frac{L^2}{\ell^2}\theta_a^2\right)$$

Higher-order contribution



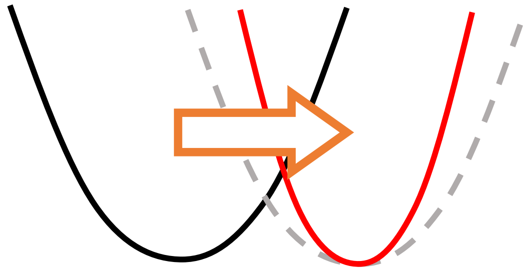
Hamiltonian of cavity 1 and rod A

$$\hat{H}_{c_1} + \hat{H}_A = \hbar\omega_c' \hat{c}_1^\dagger \hat{c}_1 + \frac{1}{2I_a}\hat{p}_a^2 + \frac{1}{2} I_a \Omega_a^2 \hat{\theta}_a^2$$

$$\approx \hbar\omega_c \hat{c}_1^\dagger \hat{c}_1 + \frac{1}{2I_a}\hat{p}_a^2 + \frac{I_a}{2}\left(\Omega_a^2 + \frac{GM}{h^3} + \frac{\hbar\omega_c}{m\ell^2}\hat{c}_1^\dagger \hat{c}_1\right)\hat{\theta}_a^2 - \frac{\hbar\omega_c L}{\ell}\hat{c}_1^\dagger \hat{c}_1 \hat{\theta}_a$$

Optomechanical coupling

Effective frequency is shifted → Potential is distorted

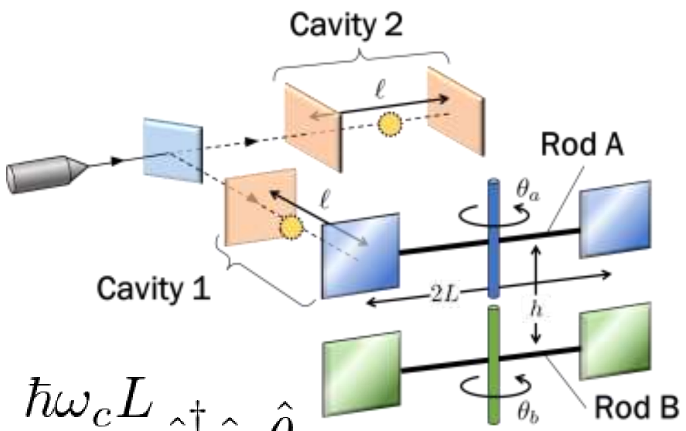


If photon hits the rod, the effective rod potential is shifted & distorted

$I_a = 2mL^2$: Moment of rod A $\quad \Omega_a$: frequency of rod A

① Higher-order optomechanical coupling

Time evolution



- Total Hamiltonian

$$\hat{H} = \hbar\omega_c \hat{c}_1^\dagger \hat{c}_1 + \hbar\omega_c \hat{c}_2^\dagger \hat{c}_2 + \frac{1}{2I_a}\hat{p}_a^2 + \frac{I_a}{2}\left(\Omega_a^2 + \frac{GM}{h^3} + \frac{\hbar\omega_c}{m\ell^2}\hat{c}_1^\dagger \hat{c}_1\right)\hat{\theta}_a^2 - \frac{\hbar\omega_c L}{\ell}\hat{c}_1^\dagger \hat{c}_1 \hat{\theta}_a$$

Optomecha

$$+ \frac{1}{2I_b}\hat{p}_b^2 + \frac{1}{2}I_b\Omega_b^2\hat{\theta}_b^2 - \frac{2GmM}{\sqrt{h^2 + \left(2L\sin[(\hat{\theta}_b - \hat{\theta}_a)/2]\right)^2}}$$

$$= \hbar\omega_c \hat{c}_1^\dagger \hat{c}_1 + \hbar\omega_c \hat{c}_2^\dagger \hat{c}_2 + \sum_{n=0,1} \hat{H}_{a,n} |n\rangle_{c1\,c1}\langle n| + \hat{H}_b + \hat{H}_g$$

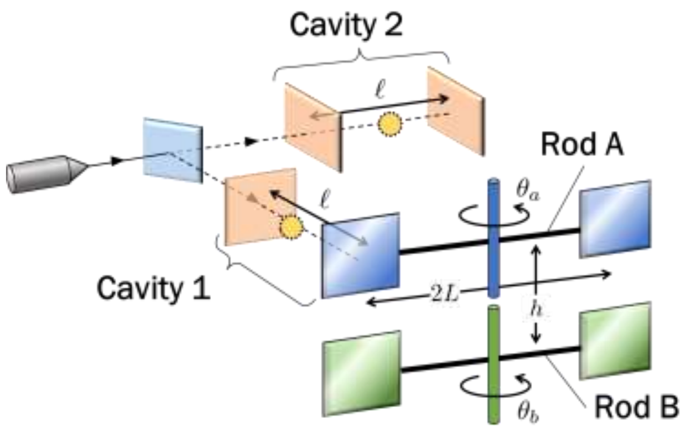
where $\hat{H}_{a,n} = \frac{1}{2I_a}\hat{p}_a^2 + \frac{I_a}{2}\omega_{a,n}^2\hat{\theta}_a^2 - n\frac{\hbar\omega_c L}{\ell}\hat{\theta}_a,\quad \omega_{a,n} = \sqrt{\Omega_a^2 + \frac{GM}{h^3}} \times \begin{cases} 1 & (n=0) \\ \sqrt{1 + \frac{2\hbar\omega_c}{I_a\omega_a^2}\frac{L^2}{\ell^2}} & (n=1) \end{cases}$

Effective potential of rod A is displaced & distorted depending on whether photon hits the rod (n=1) or not (n=0).

$$\hat{H}_b = \frac{1}{2I_b}\hat{p}_b^2 + \frac{1}{2}I_b\omega_b^2\hat{\theta}_b^2, \quad \omega_b = \sqrt{\Omega_b^2 + \frac{GM}{h^3}} \qquad \hat{H}_g = -g\frac{\omega_a(2L)^2}{(mM\omega_a\omega_b)^{-1/2}}\hat{\theta}_a\hat{\theta}_b, \quad g = \frac{G}{2h^3\omega_a}\sqrt{\frac{mM}{\omega_a\omega_b}}$$

① Higher-order optomechanical coupling

Time evolution



- Initial state

Rod A: coherent state

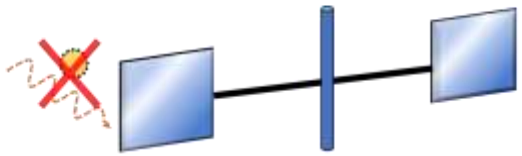
$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}} \underbrace{\left(|0\rangle_{c_1}|1\rangle_{c_2} + |1\rangle_{c_1}|0\rangle_{c_2}\right)}_{\text{Superposed photon state}} |\alpha\rangle_a |\beta\rangle_b$$

Rod B: coherent state

- Time evolved state

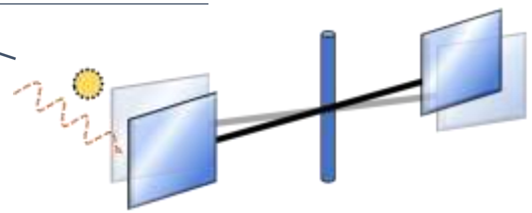
$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle$$

Direct gravitational coupling btw A and B



$$= \frac{e^{-i\omega_c t}}{\sqrt{2}} \Big[|0\rangle_{c_1}|1\rangle_{c_2} \left\{1 + 2i\, g\, \hat{\mathcal{I}}_{\omega_{a,0}}(t)\right\} e^{-i\hat{H}_{a,0}t/\hbar}|\alpha\rangle_a e^{-i\hat{H}_b t/\hbar}|\beta\rangle_b$$

$$+ |1\rangle_{c_1}|0\rangle_{c_2} \left\{1 + 2i\, g \left(\hat{\mathcal{I}}_{\omega_{a,1}}(t) + \hat{\mathcal{J}}(t)\right)\right\} e^{-i\hat{H}_{a,1}t/\hbar}|\alpha\rangle_a e^{-i\hat{H}_b t/\hbar}|\beta\rangle_b$$



Indirect coupling of B and photon through gravity + optomecha.

① Higher-order optomechanical coupling

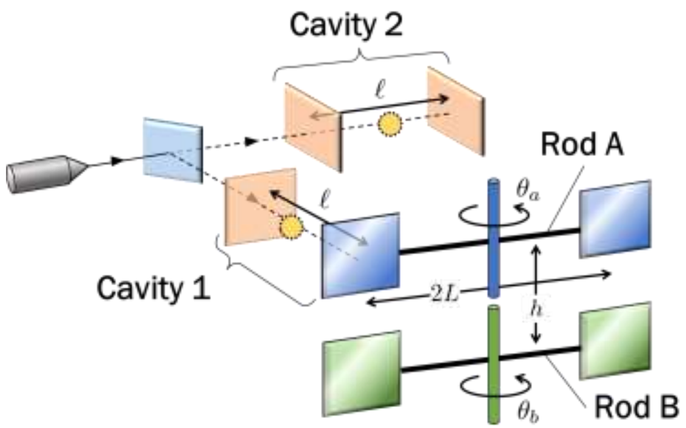
Visibility

- Observe photon interference

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle$$

$$= \frac{e^{-i\omega_c t}}{\sqrt{2}} \Big[|0\rangle_{c_1}|1\rangle_{c_2} \left\{ 1 + 2i\, g\, \hat{\mathcal{I}}_{\omega_{a,0}}(t) \right\} e^{-i\hat{H}_{a,0}t/\hbar}|\alpha\rangle_a e^{-i\hat{H}_b t/\hbar}|\beta\rangle_b$$

$$+ |1\rangle_{c_1}|0\rangle_{c_2} \left\{ 1 + 2i\, g \left(\hat{\mathcal{I}}_{\omega_{a,1}}(t) + \hat{\mathcal{J}}(t) \right) \right\} e^{-i\hat{H}_{a,1}t/\hbar}|\alpha\rangle_a e^{-i\hat{H}_b t/\hbar}|\beta\rangle_b$$



Calculate innerproduct

The absolute value of interference (visibility) is

$$\mathcal{V}_c(t) := 2\,|\langle\psi(t)|1\rangle_{c1}|0\rangle_{c2}\ {}_{c1}\langle 0|_{c2}\langle 1|\psi(t)\rangle|$$

$$= \mathcal{V}_c^{(0)}(t) \left| 1 + 2ig \left(\left\langle \hat{\mathcal{I}}^\dagger_{\omega_{a,0}}(t) \right\rangle - \left\langle \hat{\mathcal{I}}_{\omega_{a,1}}(t) \right\rangle - \langle \hat{\mathcal{J}}(t) \rangle \right) \right|$$

Not eliminated

$$\approx \mathcal{V}_c^{(0)}(t) \left(1 + 2g\ \mathrm{Im} \left[\left\langle \hat{\mathcal{I}}^\dagger_{\omega_{a,0}}(t) \right\rangle - \left\langle \hat{\mathcal{I}}_{\omega_{a,1}}(t) \right\rangle \right] + 4g^2 \left| \langle \hat{\mathcal{J}}(t) \rangle \right|^2 + \mathcal{O}[g^2] \right)$$

: Visibility changes with 1st order of gravitational coupling g

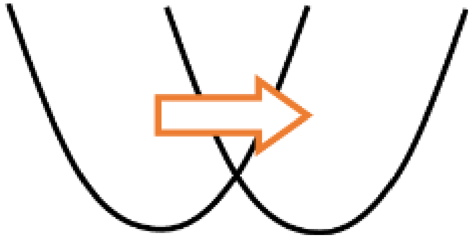
where $\mathcal{V}_c^{(0)}(t) = \left| {}_a\langle\alpha| e^{i\hat{H}_{a,0}t/\hbar} e^{-i\hat{H}_{a,1}t/\hbar} |\alpha\rangle_a \right|$: Visibility without gravity

① Higher-order optomechanical coupling

Why did we achieve $O(g)$?

- What is the physical interpretation of $O(g)$ signal in higher-order optomecha?
  - Visibility = Difference between 2 cases: photon hitting the rod A or not.
  - Higher-order optomecha. coupling makes this difference more visible.

Previous work



(i) Rod A potential is shifted by hitting photon.

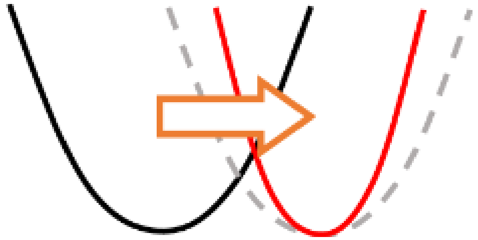
→ 2 patterns of rod A evolution w.r.t. photon state

(ii) 2 patterns of rod A oscillates in the same way under gravity from rod B.

(↔ elimination of $\hat{\mathcal{I}}_{\omega_a}$)

⇒ **Difference of whether photon hits the rod or not is less visibile.**

Our work



(i) Rod A potential is shifted & distorted by photon.

→ 2 patterns of rod A evolution w.r.t. photon state

(ii) 2 patterns of rod A oscillates in very different way under gravity from rod B.

(↔ non-elimination of $\hat{\mathcal{I}}_{\omega_{a,0}}$ and $\hat{\mathcal{I}}_{\omega_{a,1}}$)

⇒ **Difference of whether photon hits the rod or not is more visibile.**

① **Higher-order optomechanical coupling**

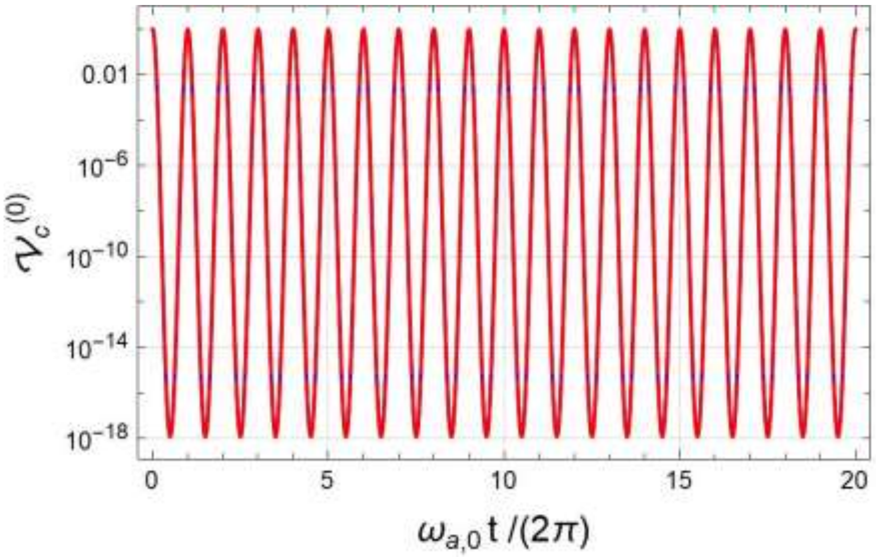
Visibility without gravity

$m = M = 10^{-13}$ [kg], $\Omega_a = 3 \times 10^3$ [Hz], $\Omega_b = 0.84 \times \Omega_a$ [Hz], $\alpha = \beta = 1$, $\omega_c = 450 \times 10^{12}$ [Hz], $\ell = 0.01$ [m], $h = 2 \times 10^{-6}$ [m]

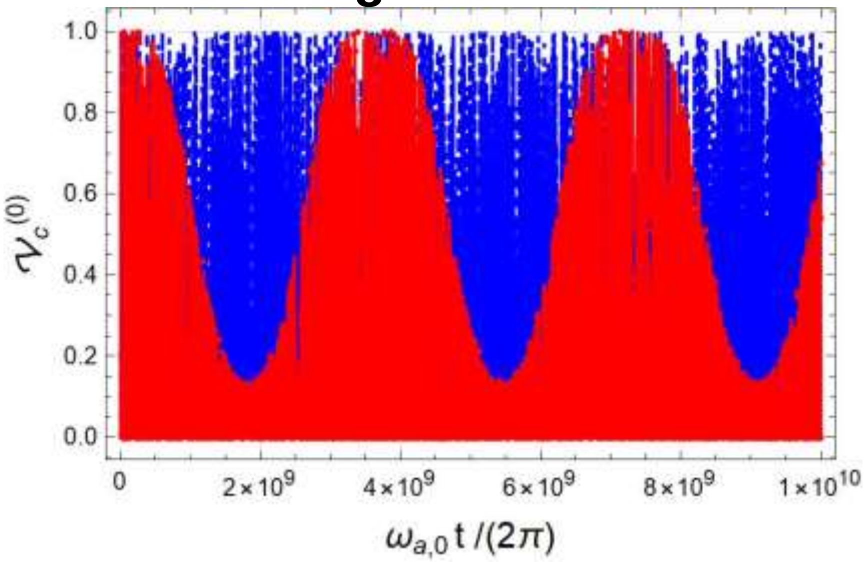
$$\mathcal{V}_c^{(0)}(t) = \left|{}_a\langle\alpha|e^{i\hat{H}_{a,0}t/\hbar}\, e^{-i\hat{H}_{a,1}t/\hbar}|\alpha\rangle_a\right|$$

Red: With higher-order optomecha contribution
Blue: Previous work

Short time scale



Long time scale



- Even without gravity, we see higher-order optomecha contribution in visibility
- At time $t \sim 2\pi(\omega_{a,1} - \omega_{a,0})^{-1}$, the periodic functions with frequency $\omega_{a,0}$ and $\omega_{a,1}$ deviates largely and we see large contribution of higher-order optomecha coupling.

① Higher-order optomechanical coupling

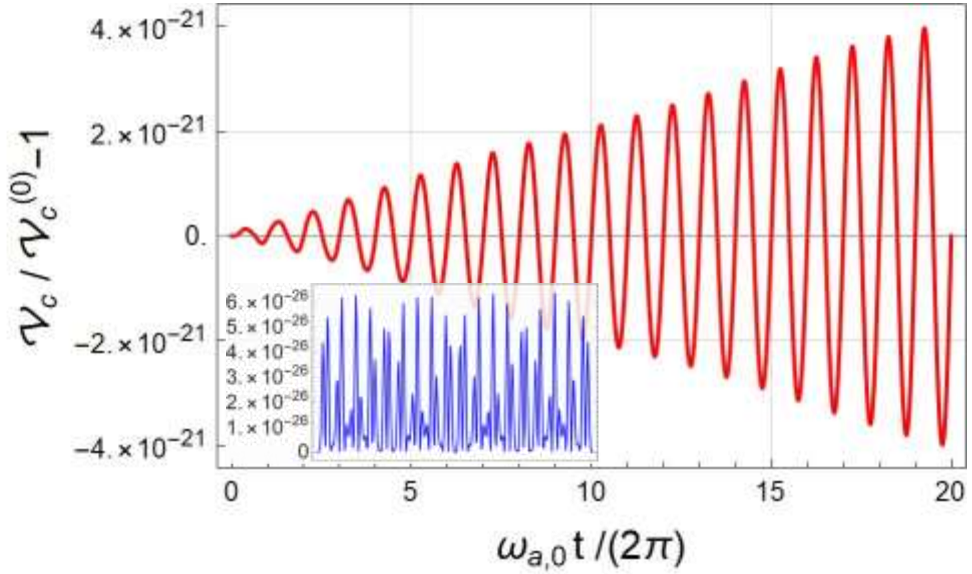
Visibility without gravity

$$m = M = 10^{-13}\,[\mathrm{kg}],\ \Omega_a = 3\times 10^3\,[\mathrm{Hz}],\ \Omega_b = 0.84\times\Omega_a\,[\mathrm{Hz}],$$
$$\alpha = \beta = 1,\ \omega_c = 450\times 10^{12}\,[\mathrm{Hz}],\ \ell = 0.01\,[\mathrm{m}],\ h = 2\times 10^{-6}\,[\mathrm{m}]$$

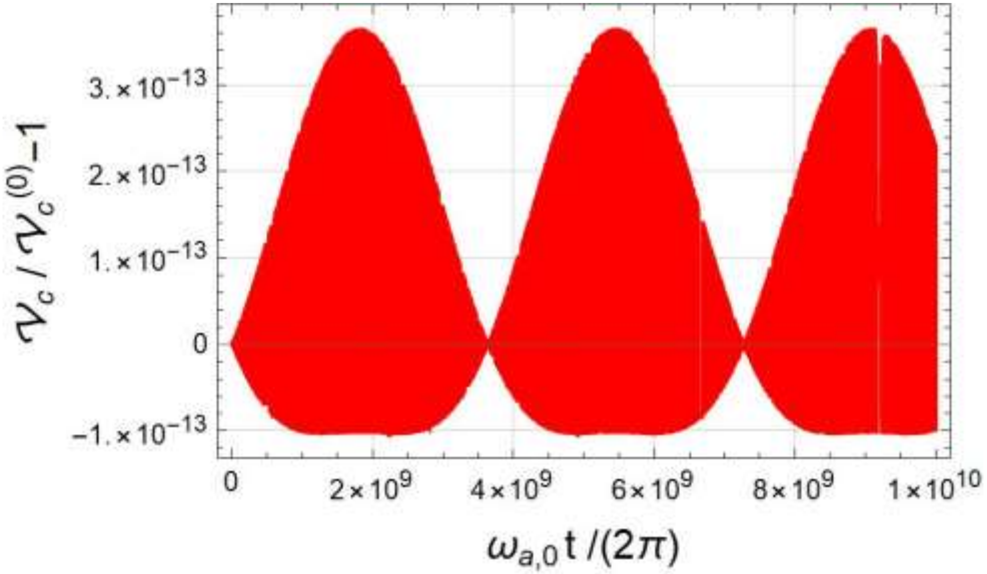
Red: With higher-order optomecha contribution
Blue: Previous work

$$\mathcal{V}_c(t) = \mathcal{V}_c^{(0)}(t)\left(1 + 2g\ \mathrm{Im}\left[\left\langle \hat{\mathcal{I}}^\dagger_{\omega_{a,0}}(t)\right\rangle - \left\langle \hat{\mathcal{I}}_{\omega_{a,1}}(t)\right\rangle\right]\right)$$

Short time scale



Long time scale



– Even in the short time scale, signal is enhanced 10^3 times due to higher-order optomecha.

– If we could do the experiment for the long time scale $t \sim 2\pi(\omega_{a,1} - \omega_{a,0})^{-1}$, we get the maximal benefit of higher-order optomecha with enhanced signal $g^{-1} = 2\times 10^{13}$.

② Resonance of 2 rods

- The case when frequency of rod A and B is close enough: $\omega_{a,0},\ \omega_{a,1} \sim \omega_b$

  By assuming $\alpha = 0,\ \beta \in \mathbb{R}$, the visibility is simplified to the following formula.

$$\mathcal{V}_c(t) = \mathcal{V}_c^{(0)}(t)\left(1 + 2g\ \mathrm{Im}\left[\left\langle \hat{\mathcal{I}}^{\dagger}_{\omega_{a,0}}(t)\right\rangle - \left\langle \hat{\mathcal{I}}_{\omega_{a,1}}(t)\right\rangle\right] + 4g^2\left|\langle \hat{\mathcal{J}}(t)\rangle\right|^2\right)$$

$$\approx \mathcal{V}_C^{(0)}(t)\left\{1 - 2g\lambda\beta\left(\frac{\sin[(\omega_{a,1}-\omega_b)t/2]}{\omega_{a,1}-\omega_b} - \frac{\sin[(\omega_{a,0}-\omega_b)t/2]}{\omega_{a,0}-\omega_b}\right)(\text{Periodic func.})\right.$$

$$\left. + g^2\frac{\sin\left[(\omega_{a,1}-\omega_b)t/2\right]^2}{\omega_{a,1}-\omega_b}(\text{Periodic func.})\right\}$$

Resonance occurs even in the previous work

If we introduce resonance parameter $\epsilon = 1 - \omega_b/\omega_{a,1} \ll 1$ and fix time at $t = \pi/(\omega_{a,1}\,\epsilon)$

Exclusive resonance: resonance occurs only if photon hits the rod

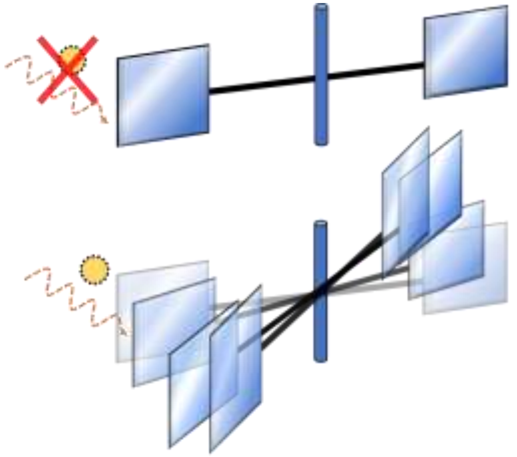
$$\frac{\mathcal{V}_C(t)}{\mathcal{V}_C^{(0)}(t)} - 1 \approx -2g\lambda\beta\ (\text{Periodic func.}) \times \begin{cases} 1/\epsilon & \left(\epsilon \ll 1 - \frac{\omega_{a,0}}{\omega_{a,1}}\right) \\ \left(1 - \frac{\omega_{a,0}}{\omega_{a,1}}\right)/\epsilon^2 & \left(\epsilon \gg 1 - \frac{\omega_{a,0}}{\omega_{a,1}}\right) \end{cases}$$

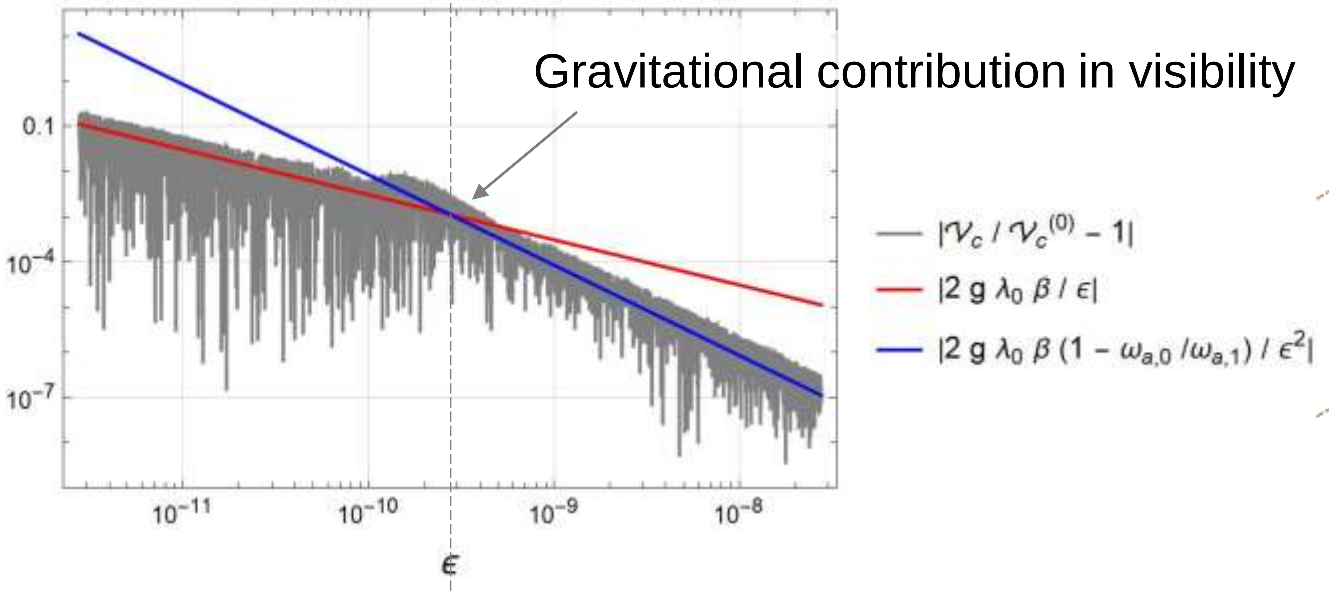
Simultaneous resonance: resonance occurs regardless of photon states

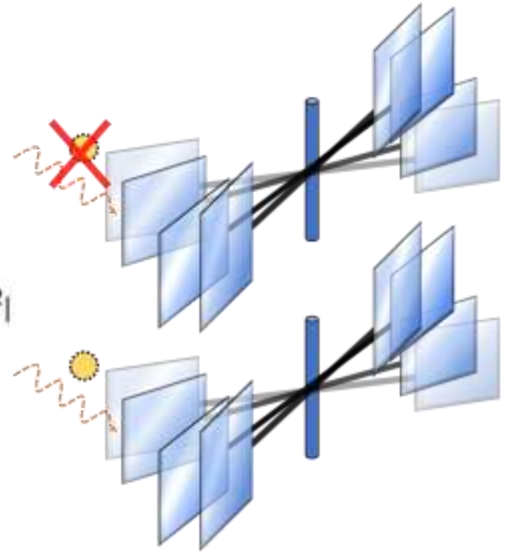
② Resonance of 2 rods

$m = M = 10^{-13}\,[\mathrm{kg}],\ \Omega_a = 3\times10^{3}\,[\mathrm{Hz}],\ \Omega_b = 0.84\times\Omega_a\,[\mathrm{Hz}],$
$\alpha=\beta=1,\ \omega_c = 450\times10^{12}\,[\mathrm{Hz}],\ \ell = 0.01\,[\mathrm{m}],\ h = 2\times10^{-6}\,[\mathrm{m}]$

- Gravitational contribution in visibility at time $t = \pi/(\omega_{a,1}\,\epsilon)$







Exclusive resonance ← → Simultaneous resonance

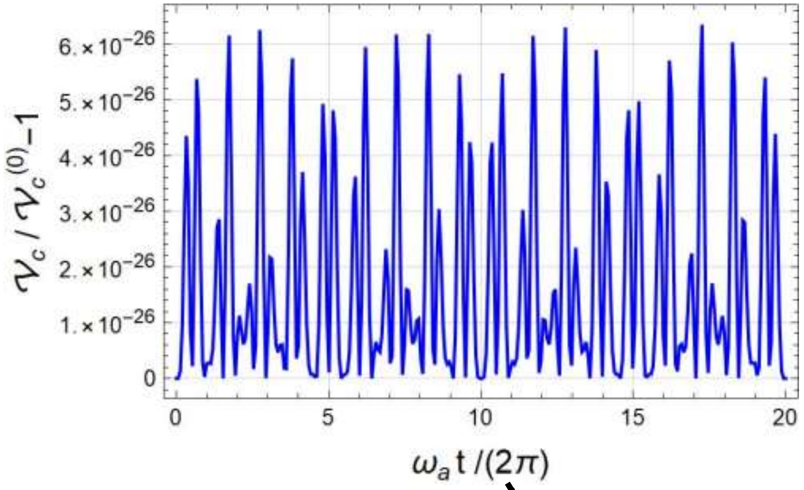
As we tune the resonance parameter ϵ to be smaller, the gravitational contribution in visibility is enhanced with its inverse ϵ.

2 ways of enhancement

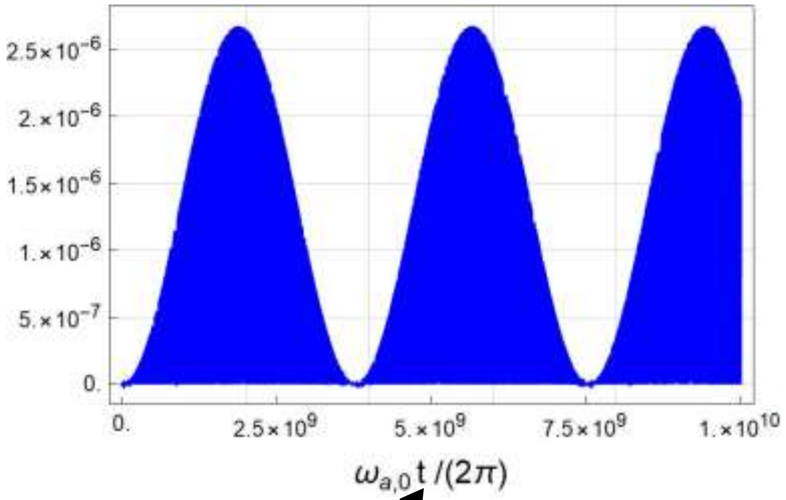
- Gravitational contribution in visibility

$g = 5.1 \times 10^{-14}$

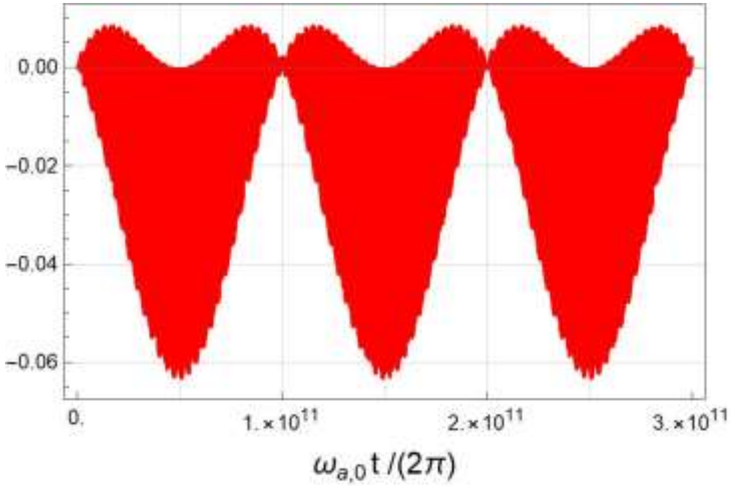
Leading-order optomecha
No resonance ($\epsilon = -0.9$)



Leading-order optomecha
Resonance ($\epsilon = 10^{-11}$)



Leading-order optomecha
Resonance ($\epsilon = 10^{-11}$)



$\langle \hat{\mathcal{J}} \rangle \sim 1/\epsilon^2 \sim 10^{22}$ times enhancement by resonance

$1/(g\epsilon) \sim 10^{25}$ times enhancement by resonance + higher-order optomecha

However, we need to sustain experiment for a sufficiently long time to achieve enough enhancement.

Conclusion

Conclusion

- We want to test quantum superposition of Newton gravity.

- Previous work: Quantum gravity witness in optomechanical system by Balushi +

  Let the oscillator superposed using superposed photon in optomechanical system.
  Then, they investigate the photon visibility deviation due to quantized Newton gravity.

- Our work

  We consider the same setup with the previous work, and study 2 ways to enhance signal.

  ① **Visibility changes with the first order of g by considering higher-order optomecha.**

$$\mathcal{V}_c(t) = \mathcal{V}_c^{(0)}(t) \times \begin{cases} \left(1 + 4g^2 \left|\langle \hat{\mathcal{J}}(t) \rangle\right|^2\right) & \text{: Previous work} \\ \left(1 + 2g \,\mathrm{Im}\left[\left\langle \hat{\mathcal{I}}^\dagger_{\omega_{a,0}}(t) \right\rangle - \left\langle \hat{\mathcal{I}}_{\omega_{a,1}}(t) \right\rangle\right]\right) & \text{: Higher-order of optomechanical coupling} \end{cases}$$

  ② **Signal enhances due to resonance of two oscillators A and B.**