w/ Y. Kaku, A. Matsumura & Y. Michimura
based on arXiv: 2308.14552
& 2501.18147

Tomohiro Fujita
(Ochanomizu & RESCEU & IPMU)
Feb 21st @ Ando Lab, UTokyo

Designing the ideal Experiment for Gravity-induced Quantum Entanglement: Instability & Resonance

Outline

1. Introduction
2. Previous Proposals
3. General Analysis
4. Our Proposal
5. Summary

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Key Question

Is Gravity Quantum?

Physics celebrities said...

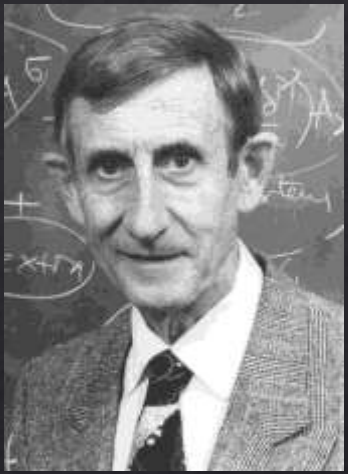
Ricard Feynman

"Maybe we should not try to quantize gravity."

Roger Penrose

"Quantum theory fits most uncomfortably with the curved space-time notion of the general relativity."

Freeman Dyson



"Should quantum mechanics and GR be unified? I don't think so. Maybe, they should not be unified..."

Key Question

Is Gravity Quantum?

We aren't sure.

Common sense?

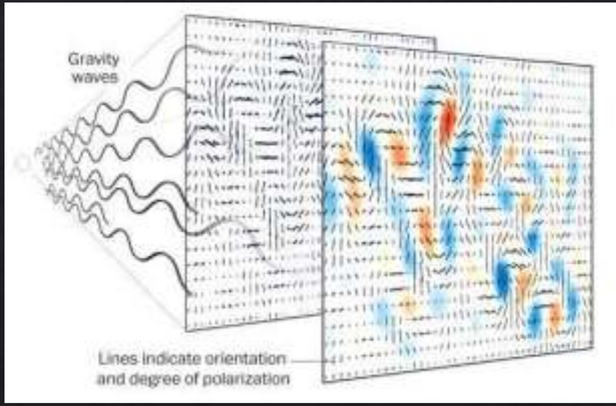
Nakayama+[0804.1827]

We often take **Quantum Gravity** for granted.

① Grav. fields can be in quantum superposition

② Graviton are quantized like QED.

(As a cosmologist, I often assume ② in my work)



Their validity has **never** been confirmed.

That's science

Let's test it with **experiments**!

Key Question

"Is Gravity Quantum?"

⇩ Testable

① Do weak gravitational fields become quantum superposition?

② graviton is (far) future step.

Key Question

Is Gravity Quantum?

We aren't sure.

Let's test it!

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2. **Previous Proposals**
3. General Analysis
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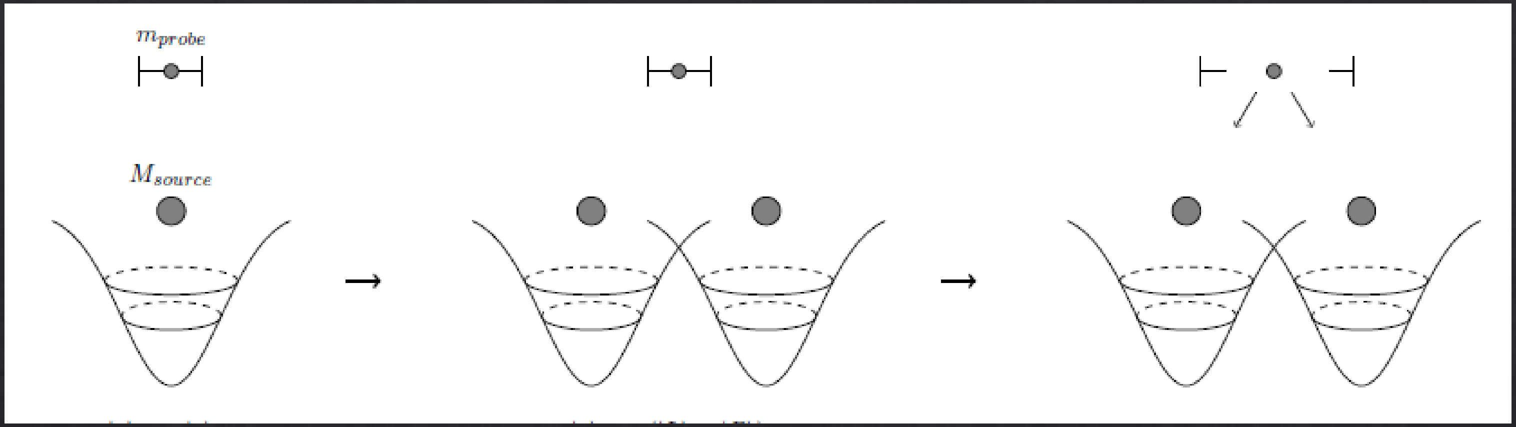
Sketch of idea

Carney+[1807.11494]

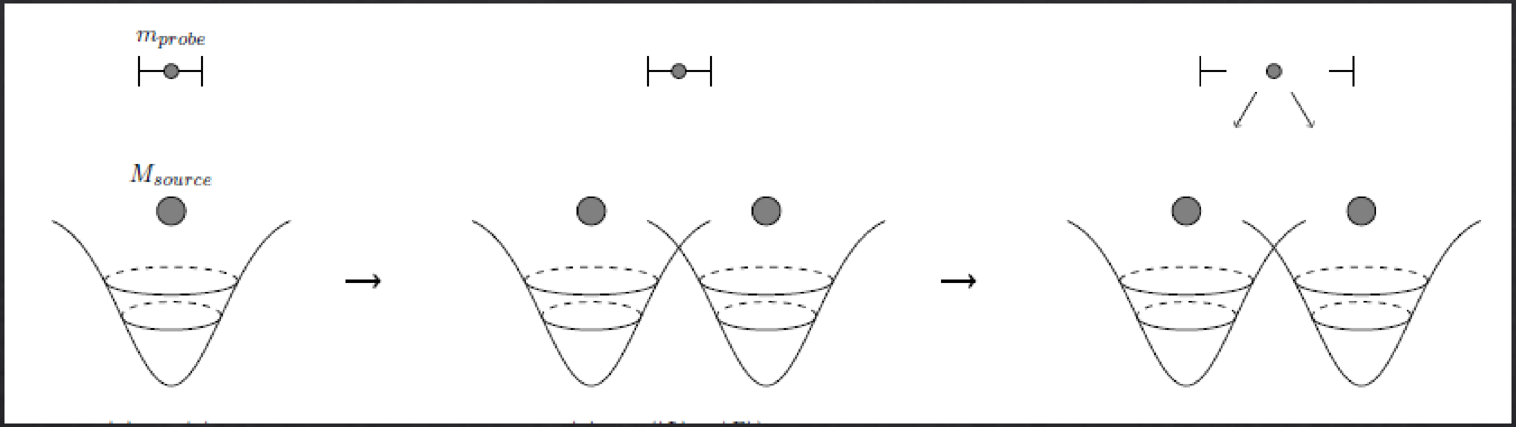


Does quantum superposition of a source mass lead to the superposition of gravitational fields?

Initial Separable: $|\psi\rangle = (|C\rangle_s \otimes |C\rangle_G) \otimes |C\rangle_{test}$

Sketch of idea

Carney+[1807.11494]



Does quantum superposition of a source mass lead to the superposition of gravitational fields?

Superpose?:

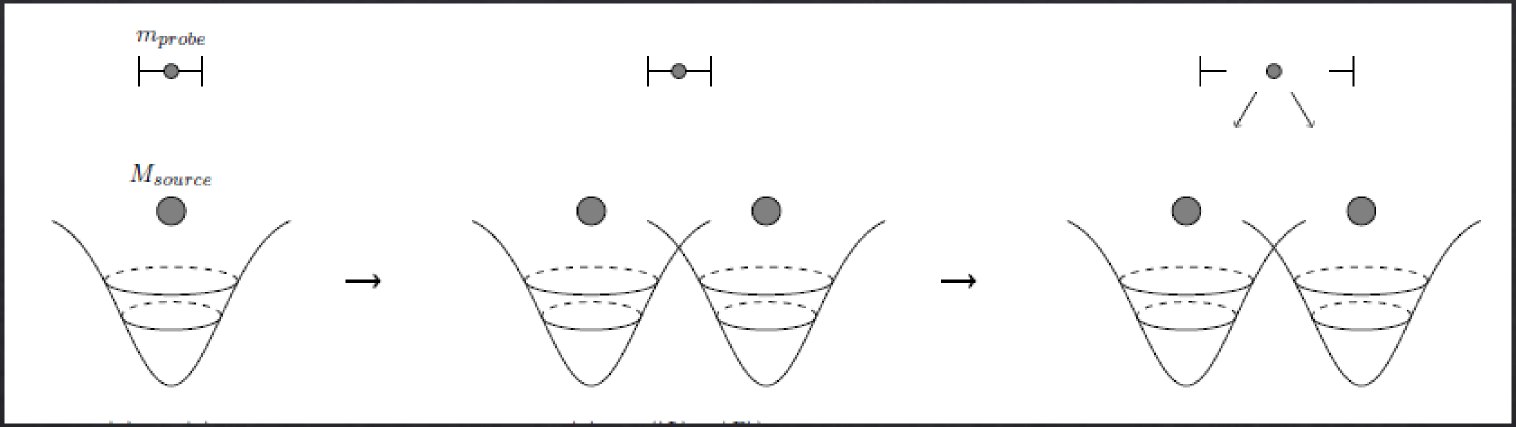
$$|\psi\rangle \stackrel{?}{=} (|L\rangle_s \otimes |L\rangle_G + |R\rangle_s \otimes |R\rangle_G) \otimes |C\rangle_{test}$$

$$|\psi\rangle \stackrel{?}{=} (|L\rangle_s + |R\rangle_s) \otimes (|L\rangle_G + |R\rangle_G) \otimes |C\rangle_{test}$$

cf. Schrodinger-Newton eq., quasi-classical approx.

Sketch of idea

Carney+[1807.11494]



Does quantum superposition of a source mass lead to the **superposition of gravitational fields**?

Entangle?:

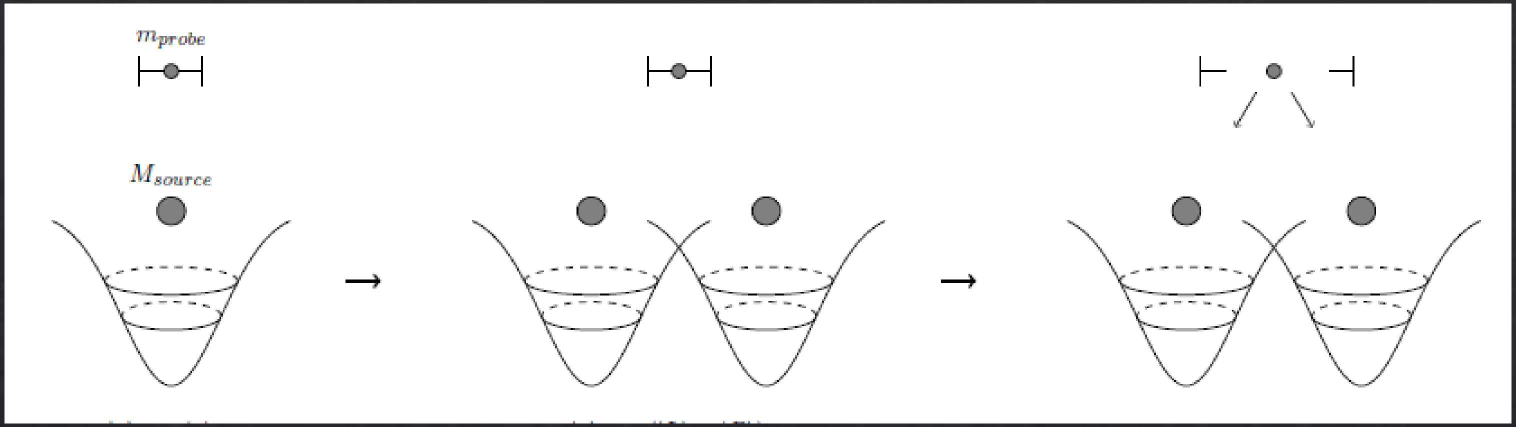
$$|\psi\rangle \overset{?}{=} (|L\rangle_s \otimes |L\rangle_G + |R\rangle_s \otimes |R\rangle_G) \otimes |C\rangle_{test}$$

⇩

$$|\psi\rangle = |L\rangle_s \otimes |L\rangle_G \otimes |L\rangle_{test} + |R\rangle_s \otimes |R\rangle_G \otimes |R\rangle_{test}$$

Sketch of idea

Carney+[1807.11494]



Does quantum superposition of a source mass lead to the **superposition of gravitational fields**?

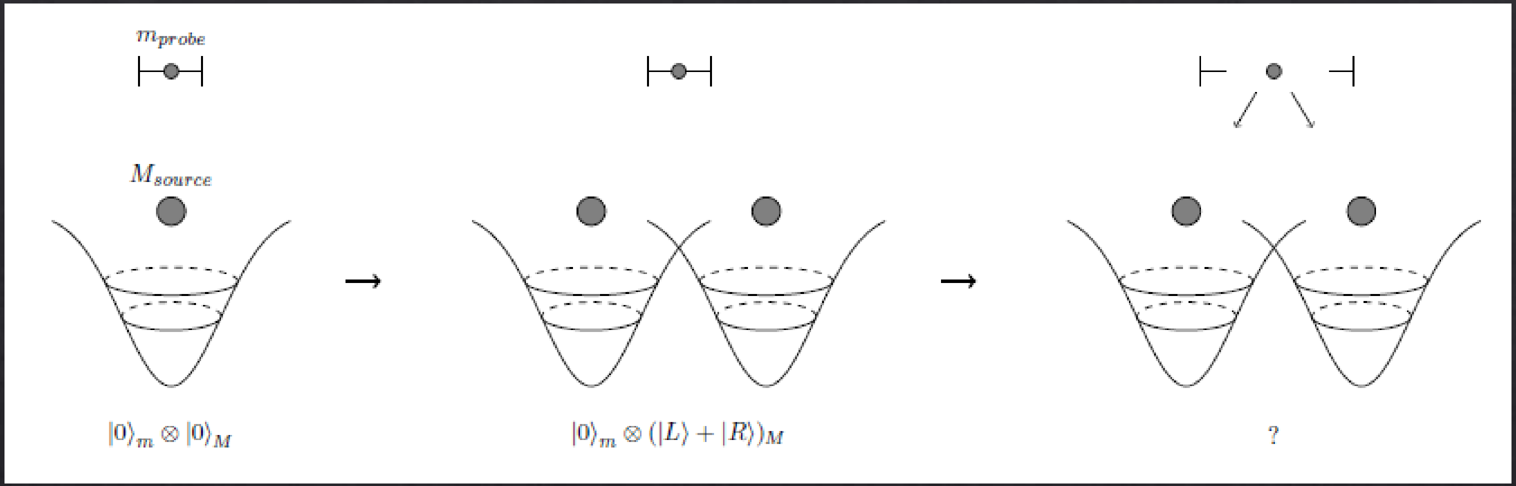
Separable: $|\psi_{\rm ini}\rangle = (|L\rangle_s + |R\rangle_s) \otimes |C\rangle_{\rm test}$

Grav. Int

Entangled: $|\psi_{\rm fin}\rangle = |L\rangle_s \otimes |L\rangle_{\rm test} + |R\rangle_s \otimes |R\rangle_{\rm test}$

Sketch of idea

Carney+[1807.11494]



Does quantum superposition of a source mass lead to the superposition of gravitational fields?

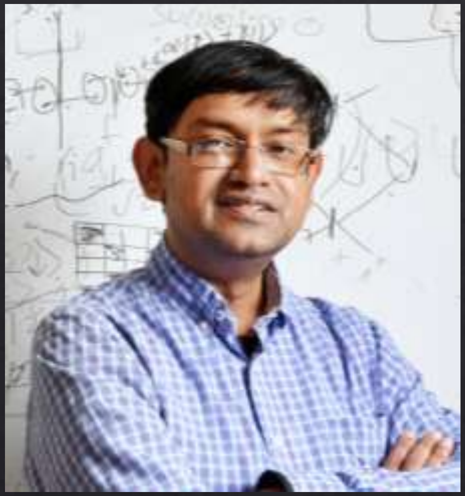
⇨ We can check it with entanglement.

Proposers

Bose+ PRL119.240401(2017)

Marletto&Vedral PRL119.240402(2017)

Sougato Bose et al.



Chiara Marletto

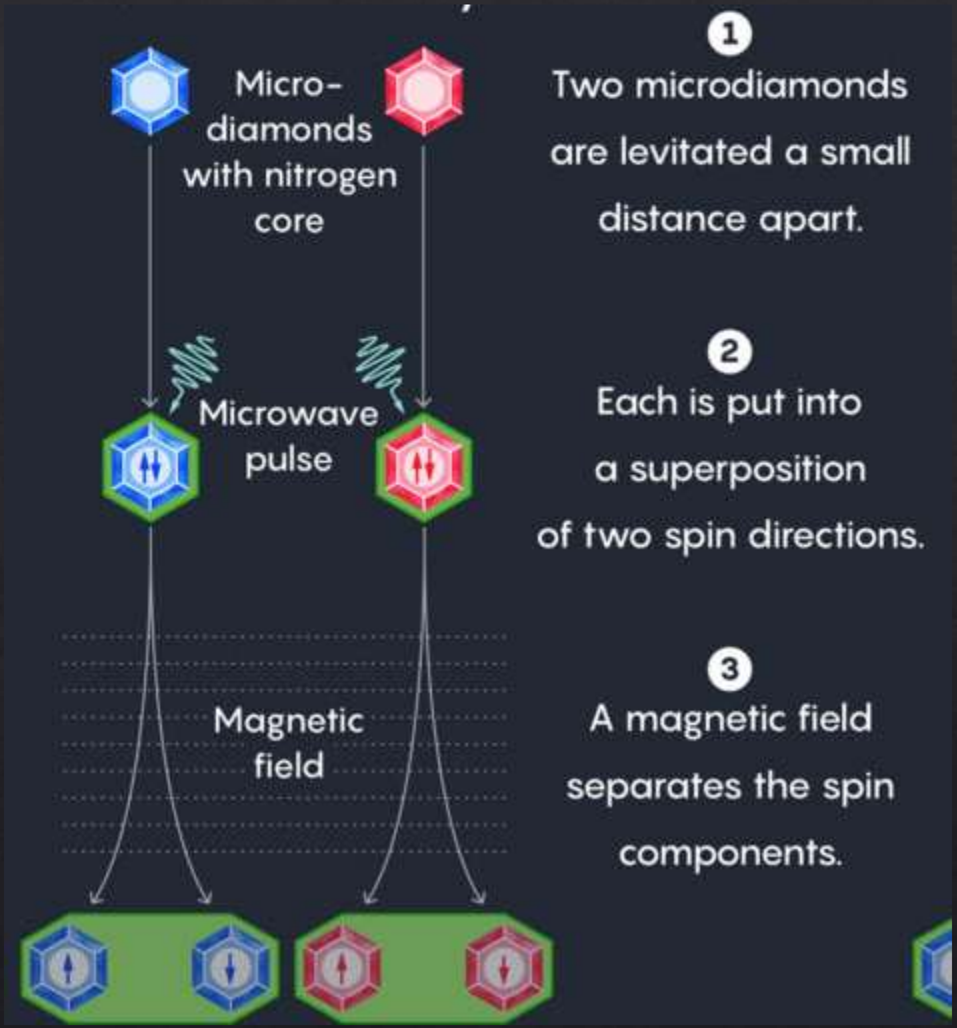
Vlatko Vedral

2 papers were published
in PRL on the same day.
= BMV proposal

BMV experiment

Bose+ PRL119.240401(2017)

Marletto&Vedral PRL119.240402(2017)



BMV experiment

Bose+ PRL119.240401(2017)

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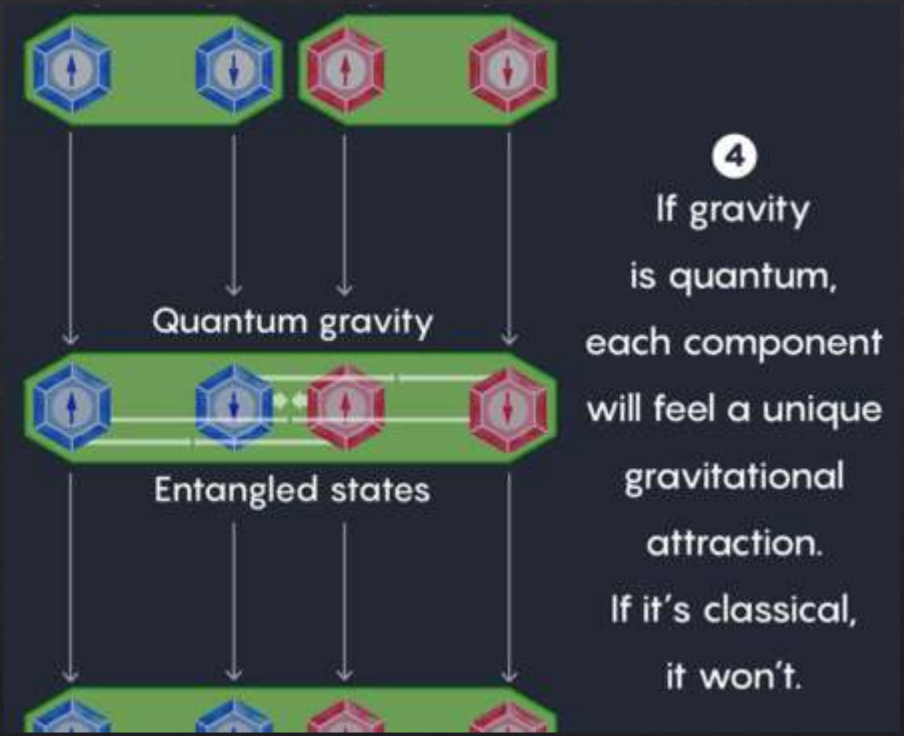

BMV experiment

Bose+ PRL119.240401(2017)

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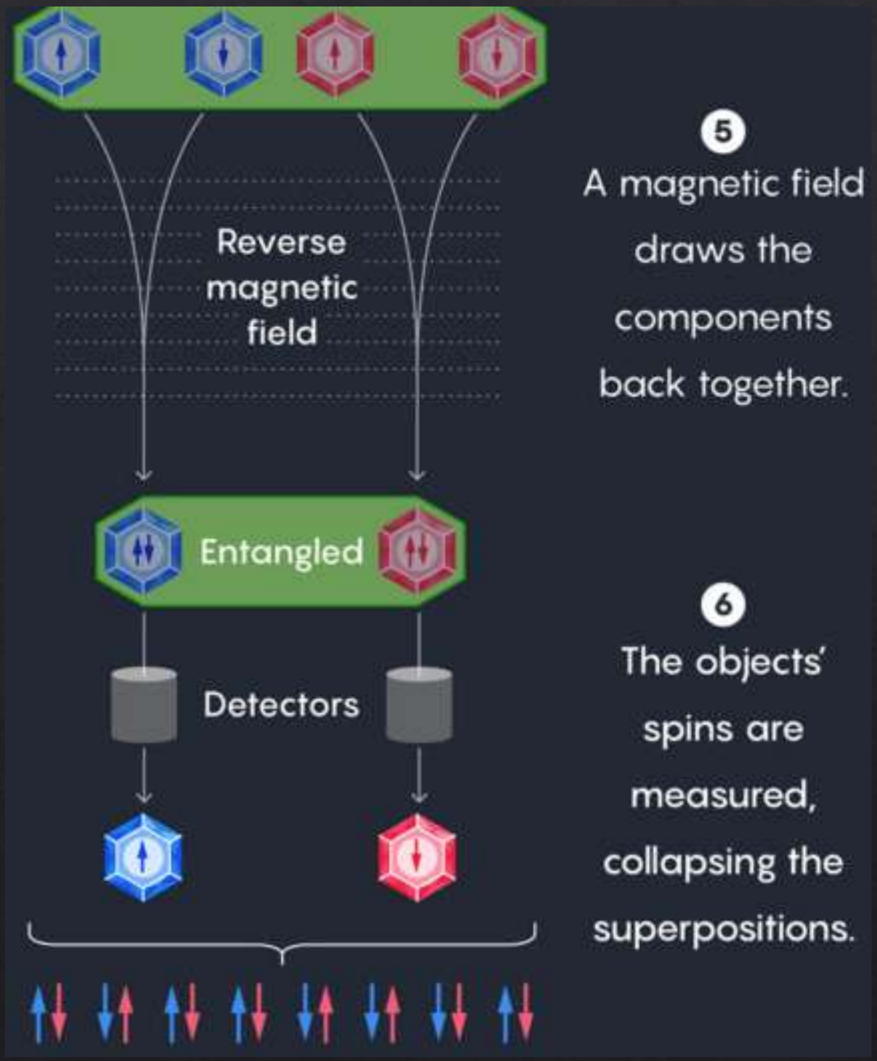
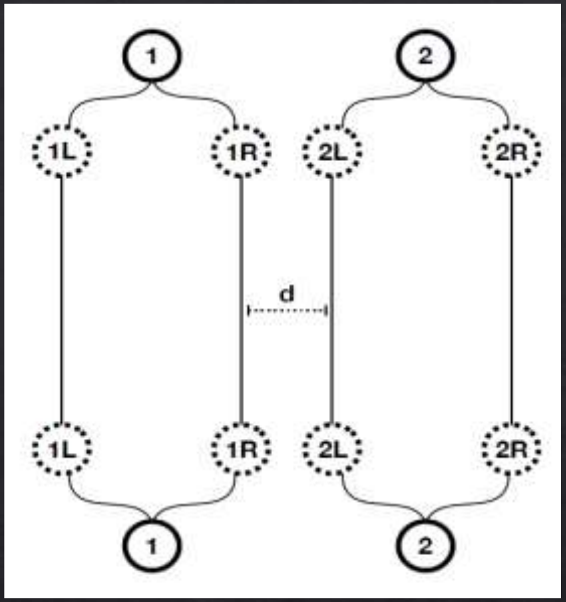


Quantum state

Christodoulou & Rovelli, PLB 792 (2019)[1808.05842]

The initial state is

$$|\Psi_1\rangle = \frac{1}{2}\Big(|\psi_1^L\rangle + |\psi_1^R\rangle\Big) \otimes \Big(|\psi_2^L\rangle + |\psi_2^R\rangle\Big) \otimes |g\rangle.$$
$$= \frac{1}{2}\Big(|LL\rangle + |RR\rangle + |LR\rangle + |RL\rangle\Big) \otimes |g\rangle.$$



if GFs can be quantum superposition

$$|\Psi_2\rangle = \frac{1}{2}\Big(|LL\rangle \otimes |g_{d_{LL}}\rangle + |RR\rangle \otimes |g_{d_{RR}}\rangle + |LR\rangle \otimes |g_{d_{LR}}\rangle + |RL\rangle \otimes |g_{d_{RL}}\rangle\Big),$$

Only the nearest pair |RL> gains a significant phase factor

$$|\Psi_3\rangle = \frac{1}{2}\Big(|LL\, g_{d_{LL}}\rangle + |RR\, g_{d_{RR}}\rangle + |LR\, g_{d_{LR}}\rangle + e^{i\frac{Gm^2 t}{\hbar d}}|RL\, g_{d_{RL}}\rangle\Big).$$

Another proposal

Krisnanda et al., npj Quant. Inf. **6**,12 (2020)

Tanjung Krisnanda

Simple Procedure:

1. Trap two masses in a harmonic potential

Another proposal

Krisnanda et al., npj Quant. Inf. 6,12 (2020)

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Simple Procedure:

1. Trap two masses in a harmonic potential
2. Release and let them grav. interact

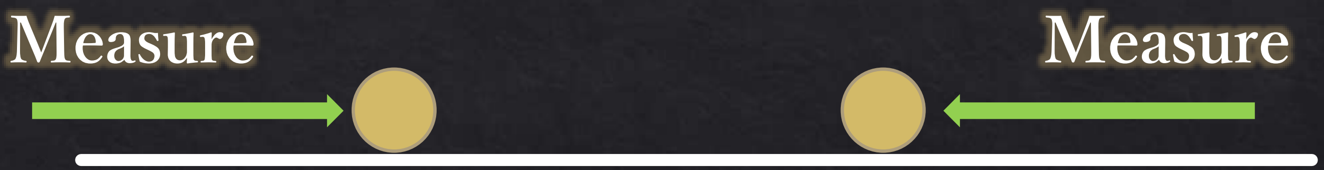
Another proposal

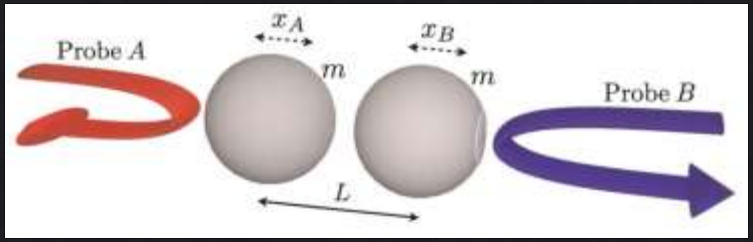
Krisnanda et al., npj Quant. Inf. **6**,12 (2020)

Tanjung Krisnanda

Simple Procedure:

1. Trap two masses in a harmonic potential
2. Release and let them grav. interact
3. Measure the positions and momenta





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Spread out

Another proposal

Krisnanda et al., npj Quant. Inf. 6,12 (2020)

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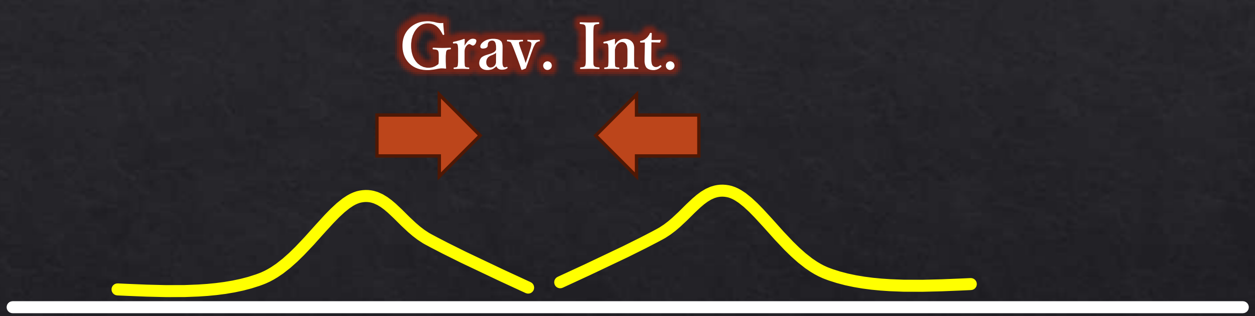


Entangled

Feasibility

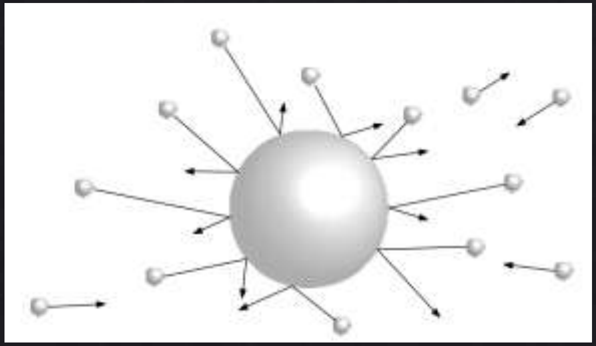
Rijavec et al., New J. Phys. **23** 043040 (2021)

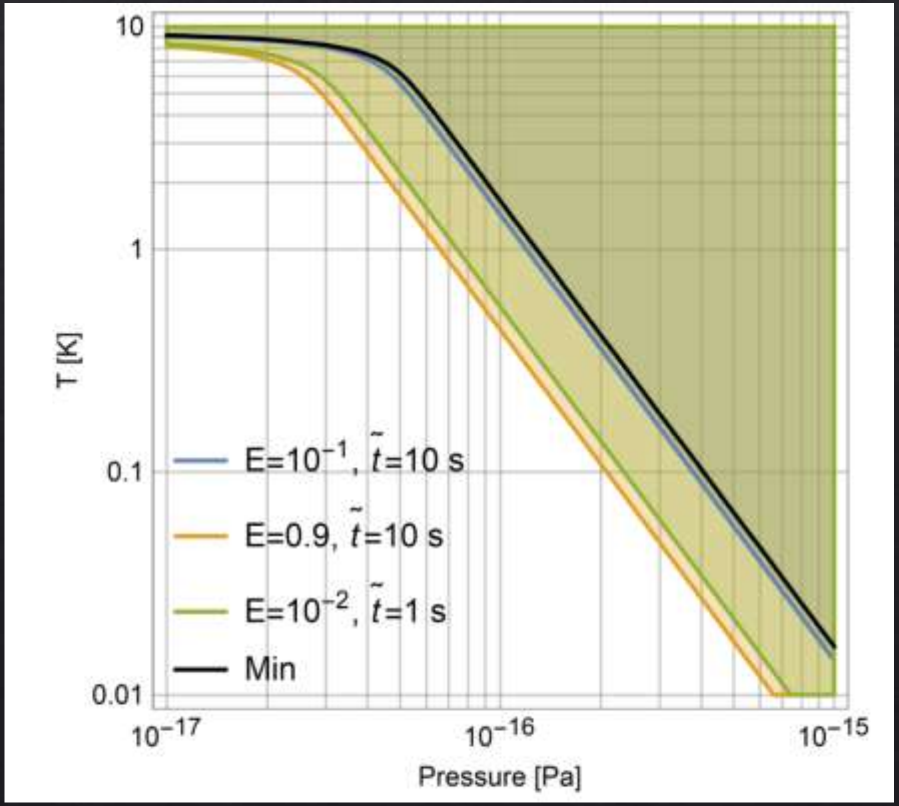
Simone Rijavec

For a real experiment,

1. Ultra-high vacuum to avoid decoherence

Air molecule Scattering





Feasibility

Rijavec et al., New J. Phys. **23** 043040 (2021)

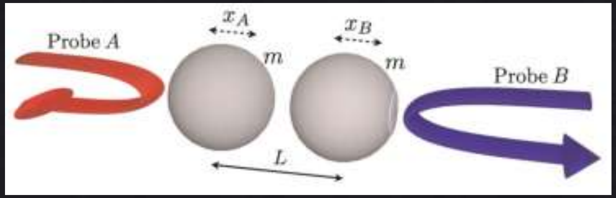
Simone Rijavec

For a real experiment,

1. Ultra-high vacuum to avoid decoherence
2. Free-fall problem

Free-fall

40m down for 3 sec



After all

Krisnanda et al., npj Quant. Inf. 6,12 (2020)

Tanjung Krisnanda

Simple Procedure:

1. Trap two masses in a harmonic potential
2. Release and let them grav. interact
3. Measure the positions and momenta

Is this the best way to produce entanglement??

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General Hamiltonian

Our quadratic Hamiltonian:

$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

oscillator1 oscillator2 Grav. Int.

$(d \gg |x_1 - x_2|)$

General Hamiltonian

Our quadratic Hamiltonian:

$$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{1}{2}k_1\hat{x}_1^2 + \frac{\hat{p}_2^2}{2m} + \frac{1}{2}k_2\hat{x}_2^2 - \frac{Gm^2}{d^3}(\hat{x}_1 - \hat{x}_2)^2,$$

oscillator1 oscillator2 Grav. Int.

$(d \gg |x_1 - x_2|)$

Should gravity be treated as quantized interaction?

⇒Entanglement test

General Hamiltonian

Our quadratic Hamiltonian:

$$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{1}{2}k_1\hat{x}_1^2 + \frac{\hat{p}_2^2}{2m} + \frac{1}{2}k_2\hat{x}_2^2 - \frac{Gm^2}{d^3}(\hat{x}_1 - \hat{x}_2)^2,$$

oscillator1 oscillator2 Grav. Int.

$(d \gg |x_1 - x_2|)$

The system is quadratic.

⇨ Exactly Solvable!

General Hamiltonian

Our quadratic Hamiltonian:

$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

Spring constant k_i

↓

Potential parameter: $\lambda_i \equiv k_i / m\omega^2$

General Hamiltonian

TF. et al. (2023) [2308.14552]

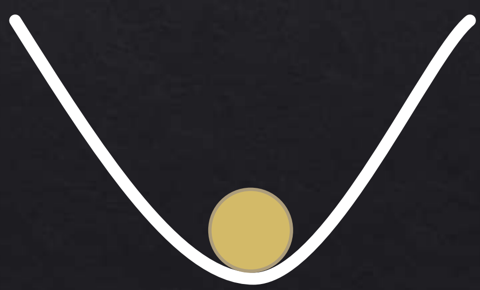
Our quadratic Hamiltonian:

$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

Spring constant k_i

Potential parameter: $\lambda_i \equiv k_i / m\omega^2$

$\lambda = 1$: Harmonic



General Hamiltonian

TF. et al. (2023) [2308.14552]

Our quadratic Hamiltonian:

$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

Potential parameter: $\lambda_i \equiv k_i / m\omega^2$

$\lambda = 1$: Harmonic

$\lambda = 0$: Free mass

General Hamiltonian

Our quadratic Hamiltonian:

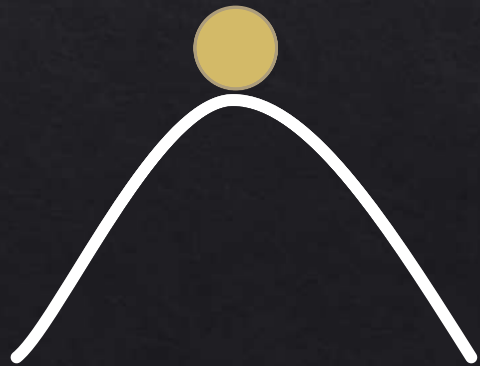
$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

Potential parameter: $\lambda_i \equiv k_i / m\omega^2$

$\lambda = 1$: Harmonic

$\lambda = 0$: Free mass

$\lambda = -1$: Inverted



Experimental Goal

Good indicator of entanglement:

Logarithmic Negativity E_N

- $E_N > 0 \quad \Leftrightarrow \quad$ Two oscillators are entangled
- Larger E_N indicates larger entanglement
- $E_N = 0.01$ is experimentally detectable.

$$E_N \equiv \max\left[0, -\log_2\left(2\tilde{\nu}_{\min}\right)\right]$$

$$\tilde{\nu}_{\min} \equiv \left[\frac{1}{2}\left(\tilde{\Sigma} - \sqrt{\tilde{\Sigma}^2 - 4\det\sigma}\right)\right]^{1/2}$$

$$u_i(t) = \Big(X_1(t), P_1(t), X_2(t), P_2(t)\Big),$$
$$\sigma_{ij}(t) = \frac{1}{2}\langle u_i(t)u_j(t) + u_j(t)u_i(t)\rangle\,.$$

$$\sigma(t) = \begin{bmatrix} \sigma_1 & \sigma_3 \\ \sigma_3^{\mathrm{T}} & \sigma_2 \end{bmatrix}$$

$$\tilde{\Sigma} \equiv \det\sigma_1 + \det\sigma_2 - 2\det\sigma_3$$

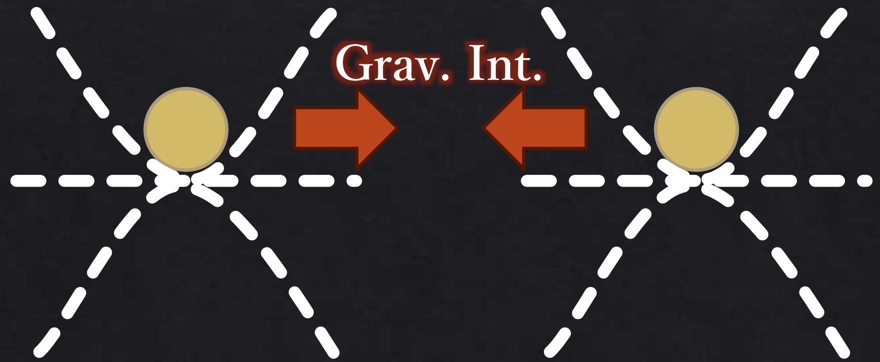
Calculation

TF. et al. (2023) [2308.14552]

We compute E_N when

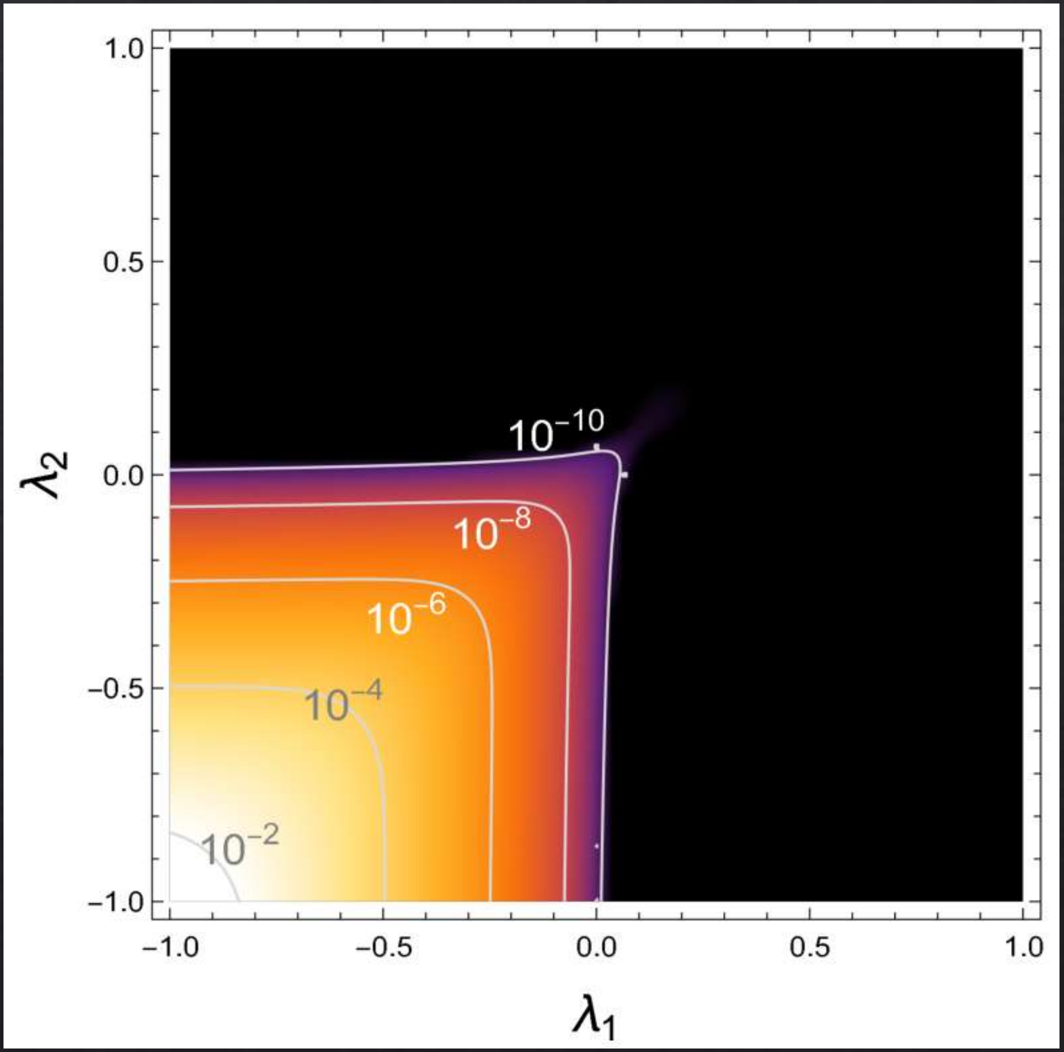
At $t = 0$,
they're in the
ground state
w/o gravity

For $t > 0$,
they evolve
in the λ_i potential
w/ gravity



Result of Entanglement

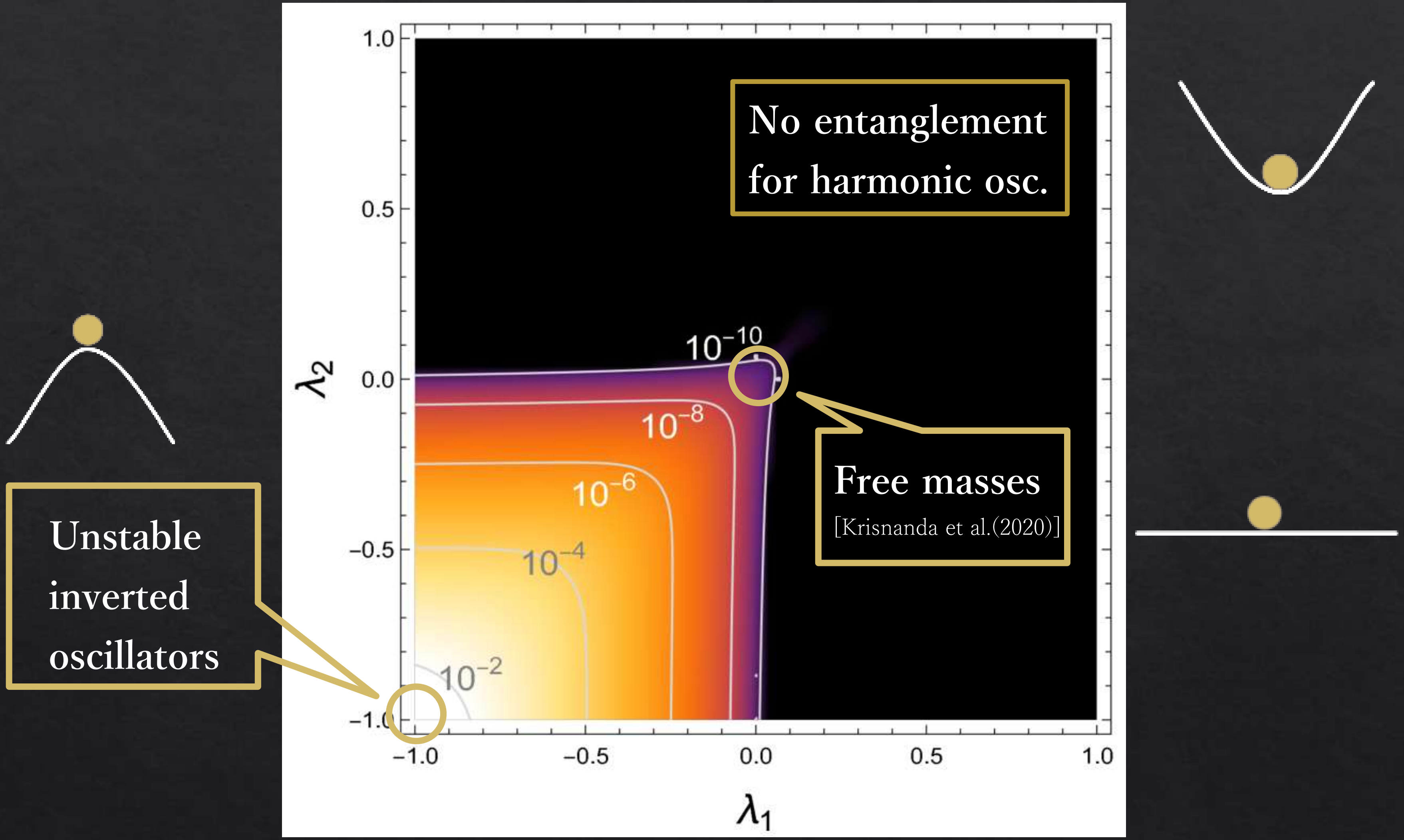
TF. et al. (2023) [2308.14552]



Contour of E_N $(\omega t = 13,\ \eta = 2\mu = 10^{-12}\ )$

Result of Entanglement

TF. et al. (2023) [2308.14552]



Contour of E_N $(\omega t = 13,\ \eta = 2\mu = 10^{-12})$

Including Decoherence

TF. et al. (2023) [2308.14552]

Heisenberg-Langevin eqs:

$$\dot{X}_i = \omega P_i, \quad \dot{P}_i = -\lambda\omega X_i + \omega\eta(X_i - X_j) + \xi_i,$$

ξ_i: random noise force ⇒ decoherence

$$\frac{1}{2}\langle \xi_i(t)\xi_j(t') + \xi_i(t')\xi_j(t) \rangle = \mu\omega\delta(t-t')\delta_{ij}\,.$$

μ: size of env. fluctuation

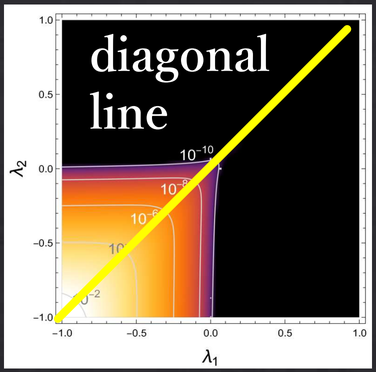
⇕

η: grav. coupling constant

$$\eta \equiv \frac{2Gm}{\omega^2 d^3} = 2.7 \times 10^{-13}\,\omega_{\mathrm{kHz}}^{-2}\left(\frac{m/d^3}{2\ \mathrm{g/cm^3}}\right)$$

Analytic Solution

TF. et al. (2023) [2308.14552]



For the identical oscillators ($\lambda_1 = \lambda_2$)

Logarithmic Negativity reads

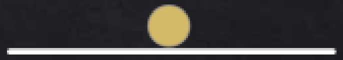
$$E_N \simeq 3(\eta - \mu) f_{\rm gra} \quad (\lambda \leq 0)$$

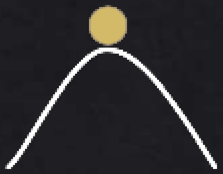
Grav. coupling constant (η)

Random noise parameter (μ)

$$f_{\rm gra} \simeq \begin{cases} \frac{1}{2}|\sin(\omega t)| & (\lambda = 1) \\ \frac{1}{6}(\omega t)^3 & (\lambda = 0) \\ \frac{1}{8}e^{2\omega t} & (\lambda = -1) \end{cases}$$

Power-law





Exponential

Time scale

TF. et al. (2023) [2308.14552]

The time required to generate observable $E_N = 0.01$

$$\tau_{\mathrm{ent}} \simeq \begin{cases} 4.2\,\omega_{\mathrm{kHz}}^{-1/3}\ \mathrm{sec} & (\lambda = 0) \\ 1.3 \times 10^{-2}\,\omega_{\mathrm{kHz}}^{-1}\ \mathrm{sec} & (\lambda = -1) \end{cases} \quad \text{w/o } \mu$$

300 times faster!

Including decoherence parameter μ

$$\tau_{\mathrm{ent}} \simeq \begin{cases} 4.2\,\omega_{\mathrm{kHz}}^{-1/3}\,[\eta/(\eta - \mu)]^{1/3}\ \mathrm{sec} & (\lambda = 0) \\ 1.3 \times 10^{-2}\,\omega_{\mathrm{kHz}}^{-1}\ \mathrm{sec} + \log[\eta/(\eta-\mu)]/(2\omega) & (\lambda = -1) \end{cases}$$

$$\simeq \mathcal{O}(10^{-3})\omega_{\mathrm{kHz}}^{-1}\mathrm{sec}$$

Time scale

TF. et al. (2023) [2308.14552]

The time required to generate observable $E_N = 0.01$

The inverted oscillators generate the gravity-induced entanglement most quickly and are most resistant to decoherence.

r!

0)

$$\simeq \mathcal{O}(10^{-3})\omega_{\mathrm{kHz}}^{-1}\mathrm{sec}$$

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Optomechanics

Rahman & Barker (2020)

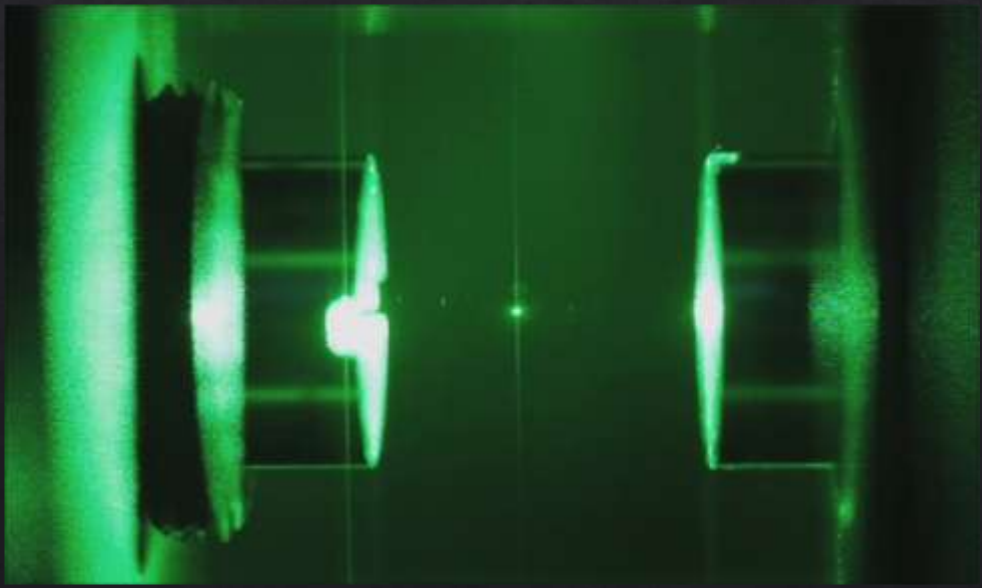
- Free fall problem

Free-fall

40m down for 3 sec

- Optical levitation

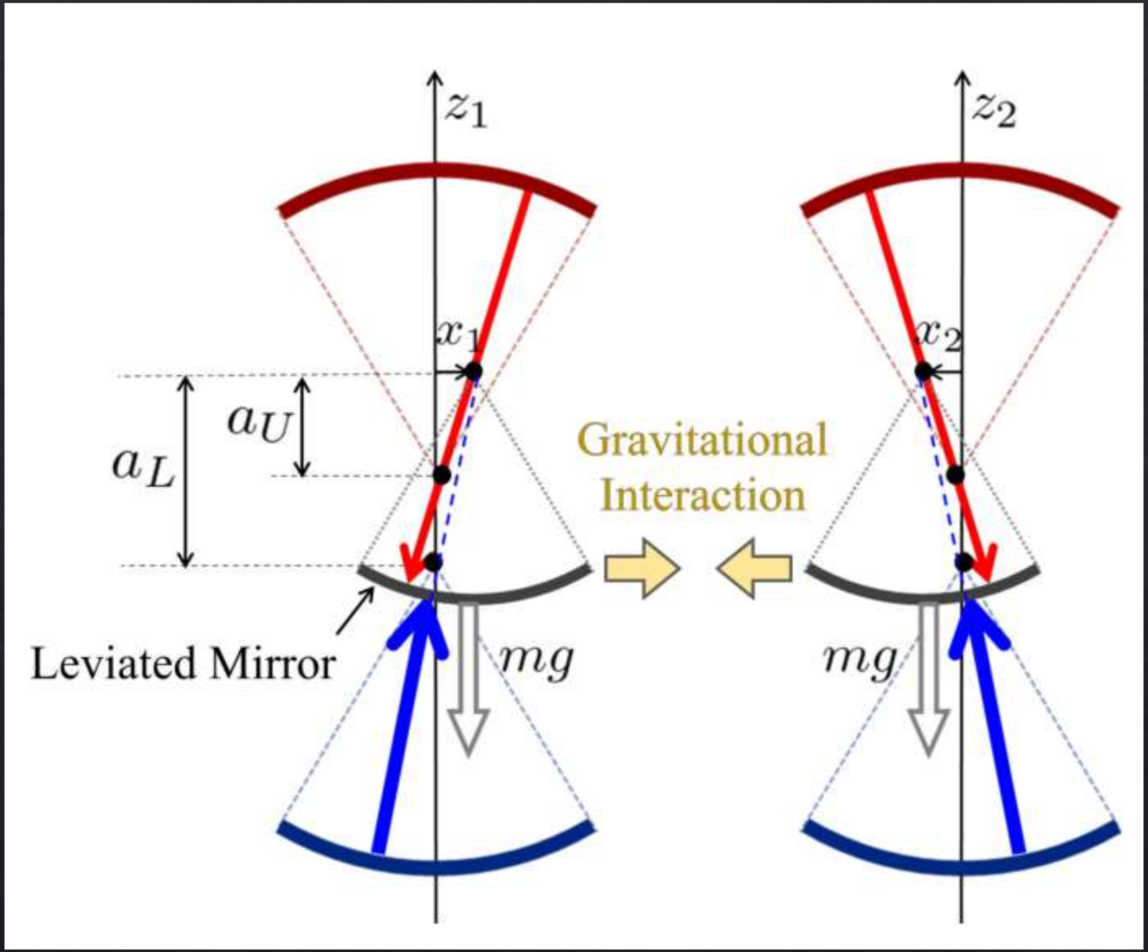
  demonstrated to levitate small particle by laser pressure w/o mechanical support



Sandwich Setup

Michimura. et al. (2017)

Inverted Oscillator ⇔ Anti-spring effect

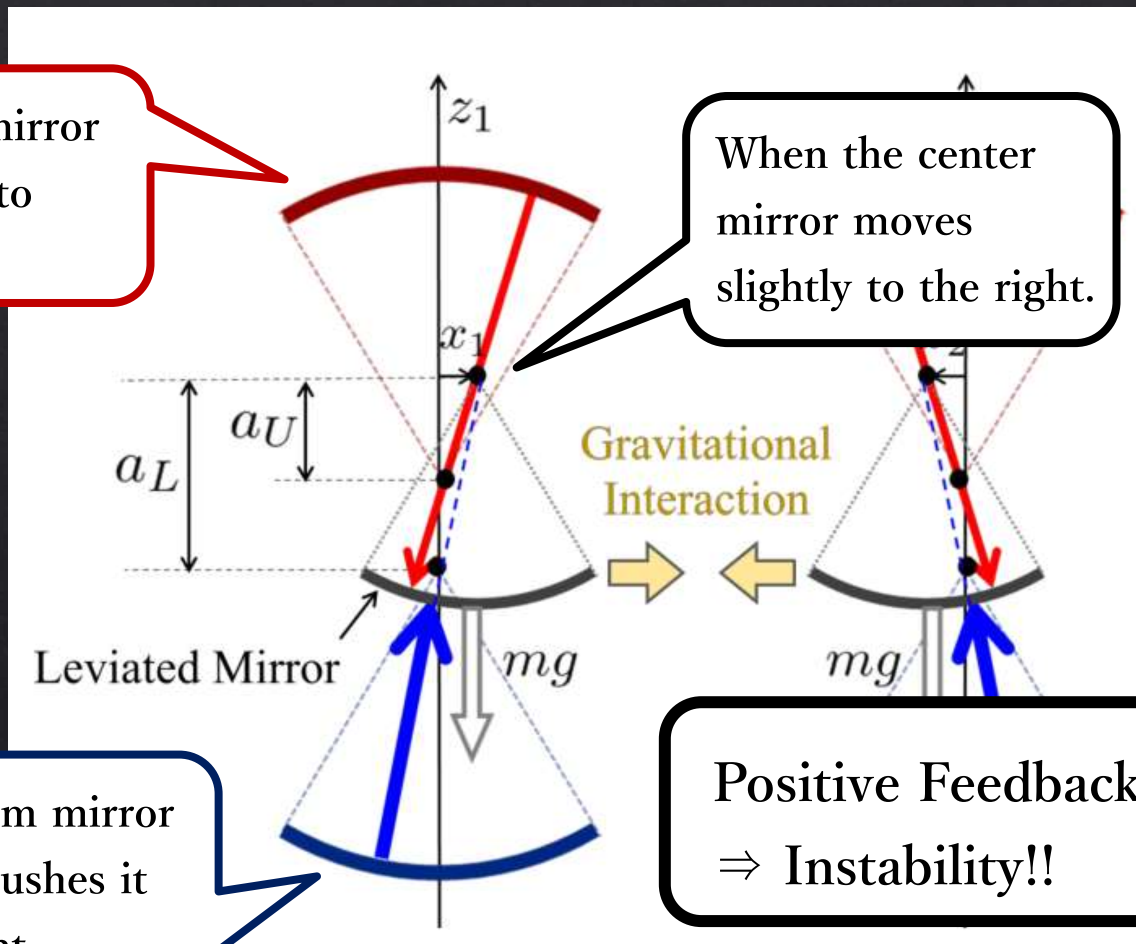


Sandwich Setup

Michimura. et al. (2017)

Inverted Oscillator ⇔ Anti-spring effect

The top mirror pushes it to the left.



When the center mirror moves slightly to the right.

The bottom mirror strongly pushes it to the right.

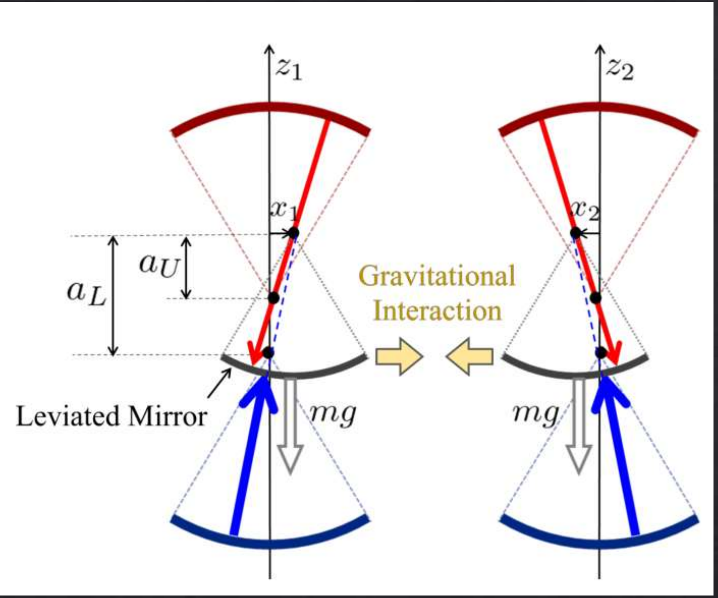
Positive Feedback
⇒ Instability!!

Sandwich Setup

Michimura. et al. (2017)

We can realize high frequency inverted oscillator

$$\omega_{\rm hor}^2 = \frac{2}{mc}\left(\frac{P_{\rm U}}{a_U} - \frac{P_{\rm L}}{a_L}\right) = \frac{2(a_L - a_U)}{mc\, a_U a_L} P_L - \frac{g}{a_U},$$
$$\simeq -(1{\rm kHz})^2 \left(\frac{m}{0.1{\rm mg}}\right)^{-1} \left(\frac{P_L}{30{\rm kW}}\right) \left(\frac{a_L}{2{\rm mm}}\right)^{-1},$$



while suppressing decoherence due to photon shot noise.

$$\mu_{\rm shot,hor} = \frac{16\omega_\ell P_L}{m\omega^2 c^2 T_{\rm in}} \left(\frac{\Delta x}{a_L}\right)^2 \simeq \frac{8\omega_\ell \Delta x^2}{c a_L T_{\rm in}},$$
$$= 2.5 \times 10^{-14}\, \omega_{\rm kHz}^3 \left(\frac{a_L}{2{\rm mm}}\right)^{-1} \left(\frac{m}{0.1{\rm mg}}\right)^{-1} \left(\frac{\omega_{\rm in}}{1{\rm MHz}}\right)^{-2}$$

Summary

- Lack of experimental verification of quantum gravity.
  Not even sure if grav. fields can be quantum superposition.

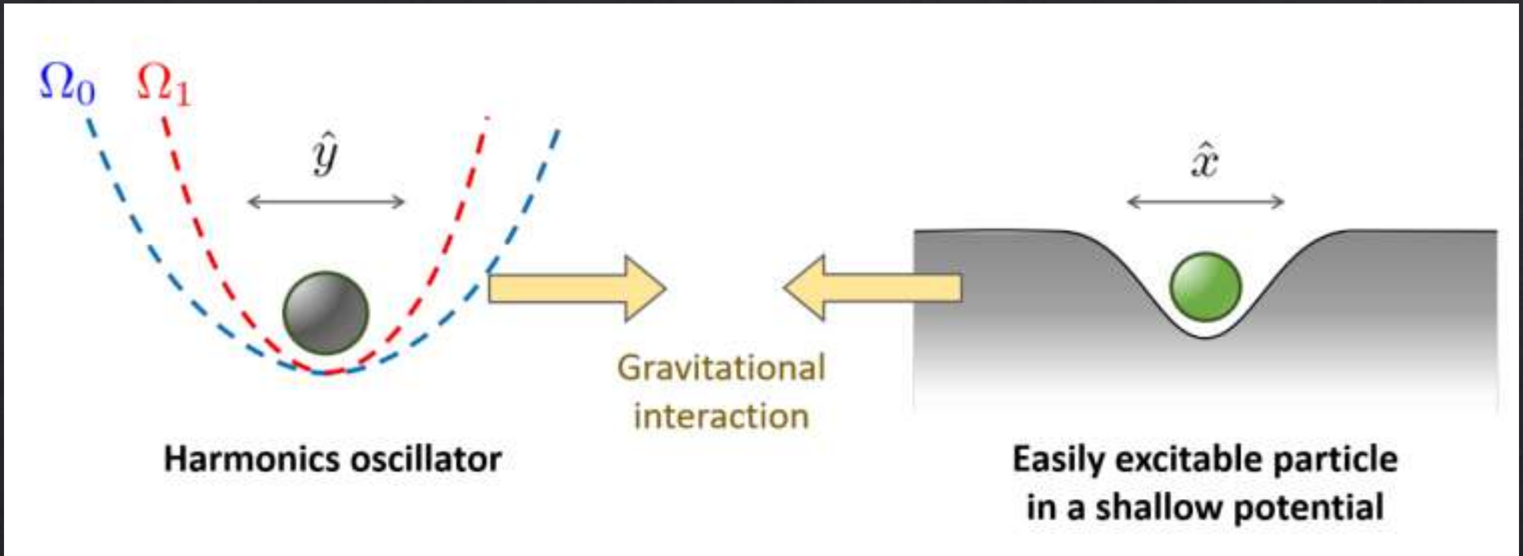
- Many proposals to test gravity-induced entanglement. "Trap & release" masses generates entanglement. (free-fall problem)

- We analyzed two general quadratic oscillators coupled by gravity and found inverted oscillators exponentially generate entanglement and resistant to decoherence.

- As an experimental implementation, we proposed levitated mirror with anti-spring effect in a sandwich configuration.

If time allows…

Resonant excitation of a particle by gravity



Qubit makes HO in a superposition of **frequency**, $|\mathrm{osc}\rangle = |\Omega_{\mathrm{high}}\rangle + |\Omega_{\mathrm{low}}\rangle$, and only $|\Omega_{\mathrm{high}}\rangle$ resonantly excites a particle $\Longrightarrow$ **Entanglement!**

Solvable systems [edit]

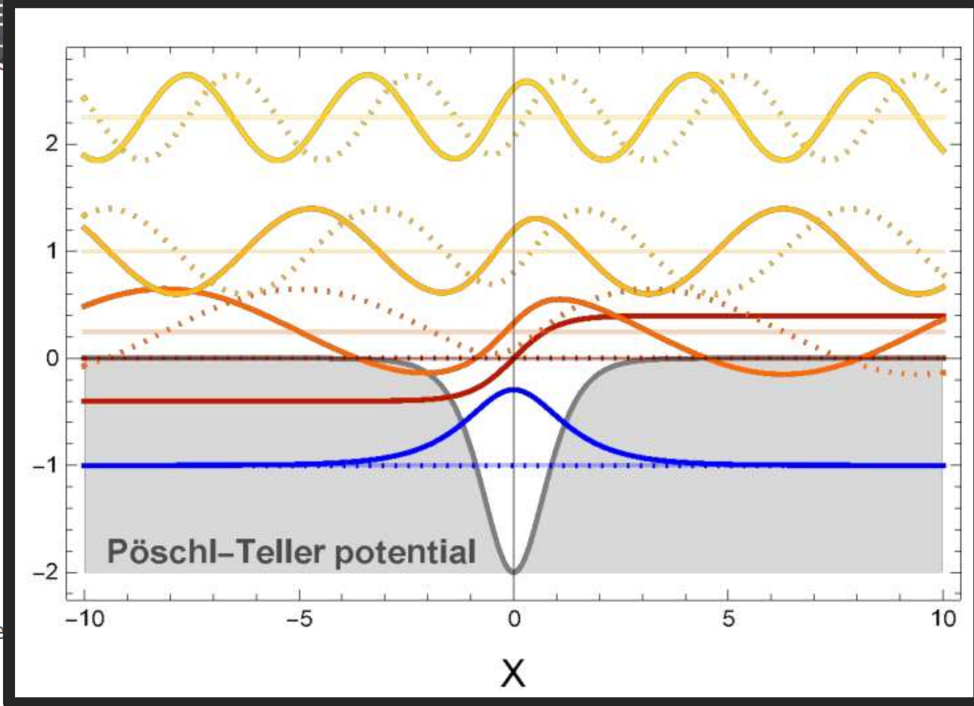
全画面表示を終了するには、E

- The two-state quantum system (the simplest possible quantum sys
- The free particle
- The one-dimensional potentials
  - The particle in a ring or ring wave guide
  - The delta potential
    - The single delta potential
    - The double-well delta potential
  - The steps potentials
    - The particle in a box / infinite potential well
    - The finite potential well
    - The step potential
    - The rectangular potential barrier
  - The triangular potential
  - The quadratic potentials
    - The quantum harmonic oscillator
    - The quantum harmonic oscillator with an applied uniform fie
  - The Inverse square root potential[2]
  - The periodic potential
    - The particle in a lattice
    - The particle in a lattice of finite length[3]
  - The Pöschl–Teller potential
  - The quantum pendulum
- The three-dimensional potentials
  - The rotating system
    - The linear rigid rotor
    - The symmetric top
  - The particle in a spherically symmetric potential
    - The hydrogen atom or hydrogen-like atom e.g. positronium
    - The hydrogen atom in a spherical cavity with Dirichlet boundary conditions[4]
    - The Mie potential[5]
    - The Hooke's atom
    - The Morse potential
    - The Spherium atom
- Zero range interaction in a harmonic trap[6]
- Multistate Landau–Zener models[7]
- The Luttinger liquid (the only exact quantum mechanical solution to a model including interparticle interactions)

Dona

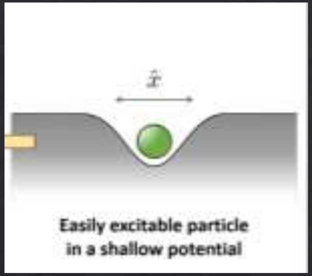
guage

Tools



これを
使おう！

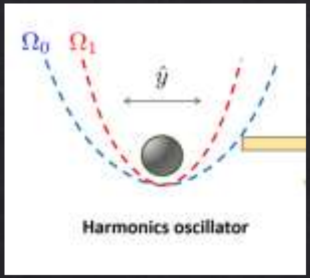
Hamiltonian of the system



Particle in a potential

Poschl-Teller potential

$$\hat{H}_{\rm PT} = \frac{1}{2m}\hat{p}_x^2 + V(\hat{x}), \quad V(\hat{x}) = -\frac{\hbar^2}{2mL^2}\frac{2}{\cosh^2(\hat{x}/L)},$$



Harmonic oscillator with superposed frequency

$$\hat{H}_{\rm osc} = \frac{1}{2M}\hat{p}_y^2 + \frac{1}{2}M\hat{\Omega}^2\hat{y}^2$$

Gravitational Interaction

$$\hat{H}_{\rm grav} = -\frac{GmM}{|d+\hat{x}-\hat{y}|} \simeq \frac{\hbar^2 g}{2mL^2}\hat{X}\hat{Y}$$

Qubit controlling HO frequency

$$\hat{\Omega} = \Omega_0|0\rangle\langle 0| + \Omega_1|1\rangle\langle 1|$$

Quantum state of Resonant exitation

$$|\Psi(t)\rangle = e^{-i\omega_b t}|b\rangle_{\rm p} \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_{\rm q}|\alpha e^{-i\Omega_0 t}\rangle_{\rm o} + |1\rangle_{\rm q}|\alpha e^{-i\Omega_1 t}\rangle_{\rm o}\right)$$

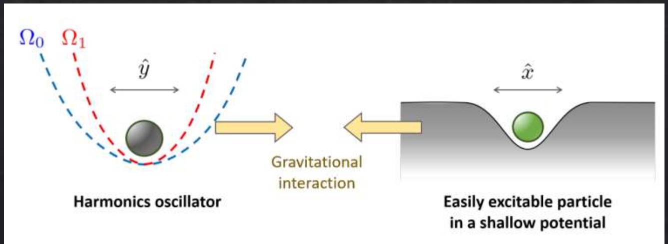
Stays in bound state
HO & qubit entangled

$$+ g\,\alpha\,|{\rm ex}(\Omega_1)\rangle_{\rm p} \otimes \frac{1}{\sqrt{2}}|1\rangle_{\rm q}|\alpha e^{-i\Omega_1 t}\rangle_{\rm o}$$

Particle excited only
by high Ω state

The excited state of the particle

$$|{\rm ex}(\Omega_1)\rangle := \int_{-\infty}^{\infty} dk e^{-i\omega_k t} c_k(\Omega_1)|k\rangle$$



$$c_k(\Omega_1) := \frac{|\omega_b|J_k}{\sqrt{2}}\,\frac{1 - e^{i(\omega_k - \omega_b - \Omega_1)t}}{\omega_k - \omega_b - \Omega_1}$$

Resonance: for $\omega_k \simeq \omega_b + \Omega_1$
the probability is enhanced!

Probability of the excitation

$$P_{\rm ex}(t) := \int_{-\infty}^{\infty} dk \, \langle k | {\rm Tr}_{\rm q,o} \big[|\Psi(t)\rangle\langle\Psi(t)| \big] | k \rangle$$

$|k\rangle$ = Particle excited state

⇓ Fermi's "**Golden rule**"-type calculation

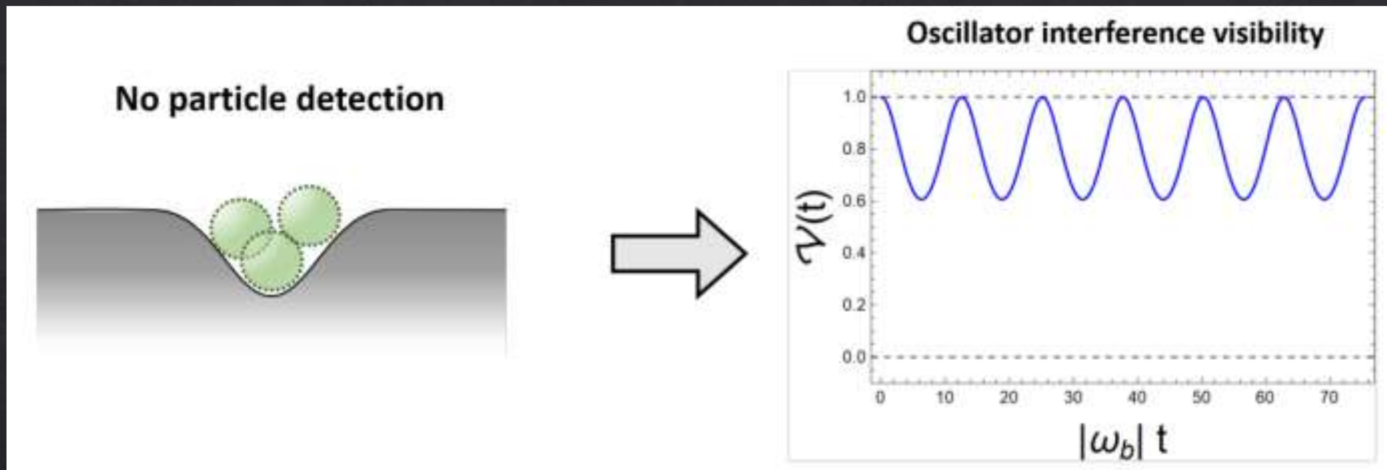
$$P_{\rm ex}(t \gtrsim t_{\rm sat}) \simeq \frac{\pi^2 g^2 |\alpha|^2}{8 k_{\rm res}} |\omega_b| t \,,$$

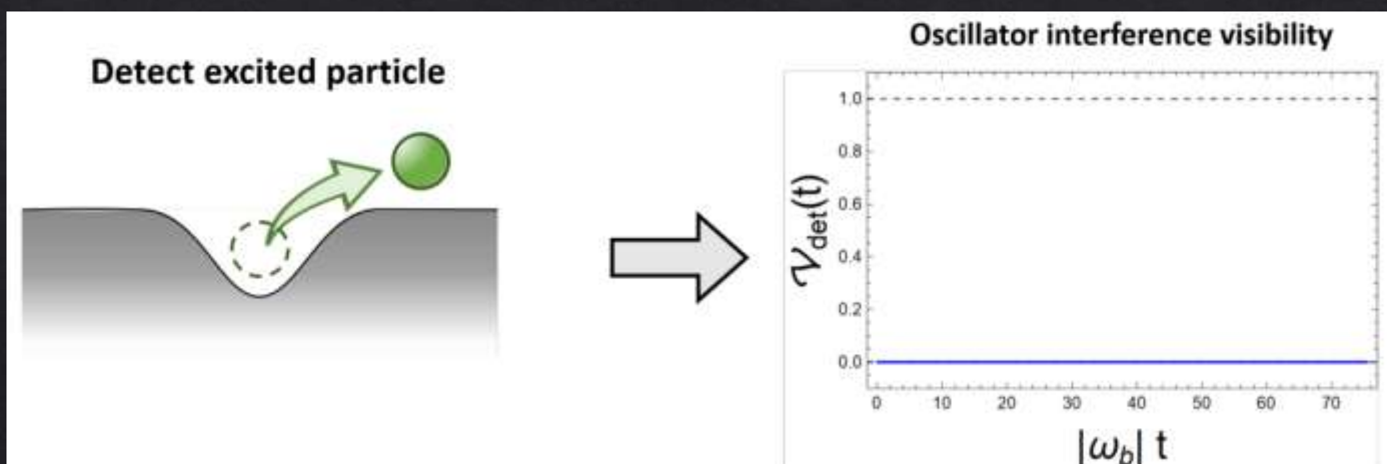
increases linearly in time!

⇓ Multiple time experiments: repeat t_{tot}/τ_1 times

$$P_{\rm ex}(t_{\rm tot}) \simeq 100\% \, \frac{m}{M} \left(\frac{M/d^3}{20\,{\rm g/cm^3}} \right)^2 \left(\frac{|\alpha|}{0.7} \right)^2 \left(\frac{|\omega_b|}{1\,{\rm Hz}} \right)^{-2} \left(\frac{\tau_1}{12\,{\rm hours}} \right) \left(\frac{t_{\rm tot}}{1\,{\rm year}} \right)$$

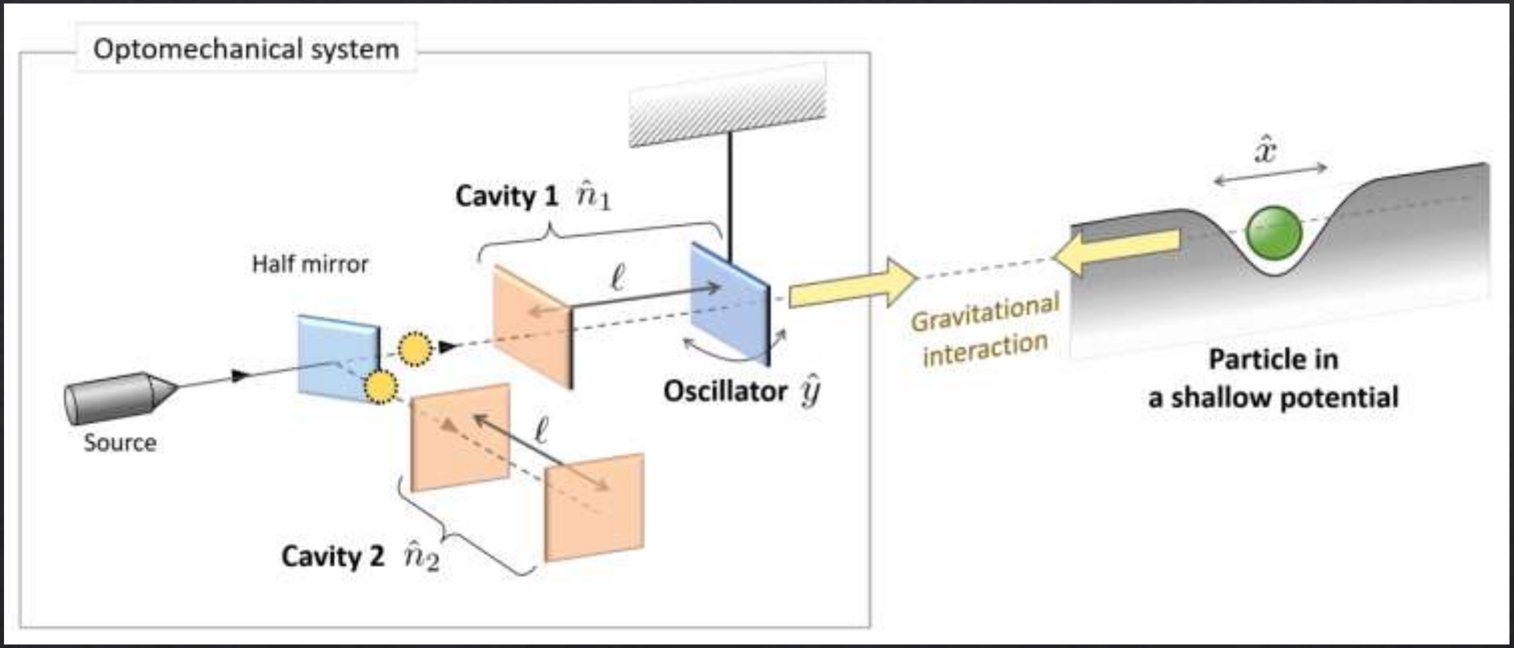
How to ensure gravitational effect?





$$|\Psi(t)\rangle = e^{-i\omega_b t}|b\rangle_{\mathrm{p}} \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathrm{q}}|\alpha e^{-i\Omega_0 t}\rangle_{\mathrm{o}} + |1\rangle_{\mathrm{q}}|\alpha e^{-i\Omega_1 t}\rangle_{\mathrm{o}}\right) + g\,\alpha\,|\mathrm{ex}(\Omega_1)\rangle_{\mathrm{p}} \otimes \frac{1}{\sqrt{2}}|1\rangle_{\mathrm{q}}|\alpha e^{-i\Omega_1 t}\rangle_{\mathrm{o}}$$

Experimental setup



$$\hat{H} = \hbar\omega_c'\hat{n}_1 + \hbar\omega_c\hat{n}_2 + \frac{1}{2M}\hat{p}_y^2 + \frac{1}{2}M\Omega_0^2\hat{y}^2 \,,$$
$$\simeq \hbar\omega_c\left(\hat{n}_1 + \hat{n}_2\right) + \frac{1}{2M}\hat{p}_y^2 + \frac{1}{2}M\hat{\Omega}^2\hat{y}^2 - \frac{\hbar\omega_c}{\ell}\hat{n}_1\hat{y}$$

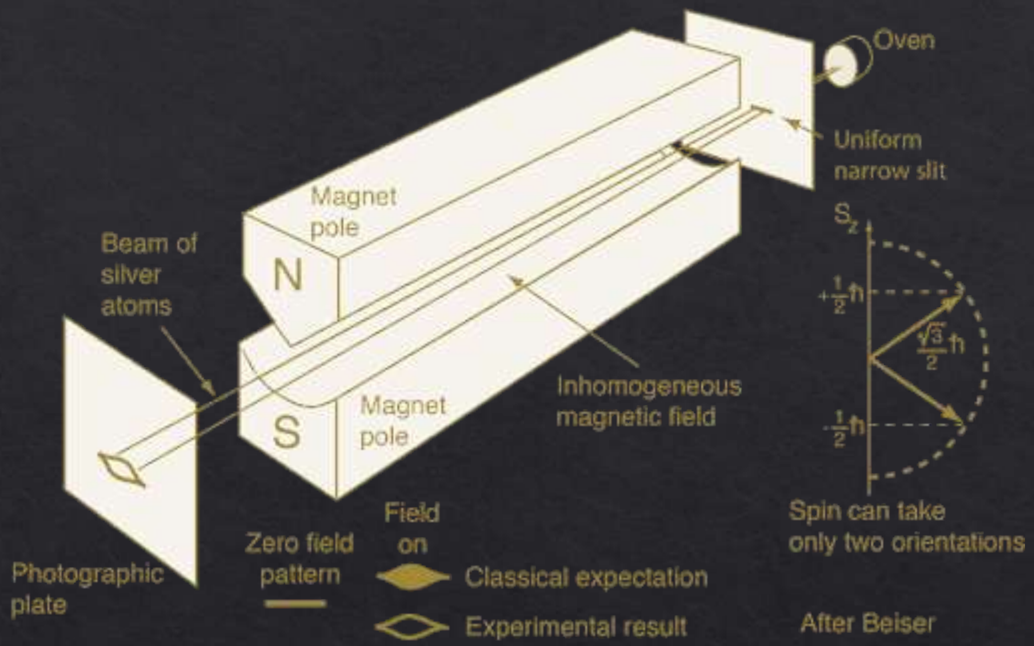
$$\Omega_1 = \sqrt{\Omega_0^2 + \frac{\hbar\omega_c}{M\ell^2}}$$

$$\omega_c' = \frac{\pi c\,\mathbf{n}}{\ell + y} \simeq \omega_c\left(1 - \frac{1}{\ell}y + \frac{1}{\ell^2}y^2\right)$$

$$\hat{\Omega} = \Omega_0|0\rangle\langle 0| + \Omega_1|1\rangle\langle 1|$$

Thank you

Sketch of idea



Stern-Gerlach experiment enables us to prepare the quantum superposition of a mass at two different locations.

Superposition

Pure state: $|\Psi\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle$

quantum superposition

It's undetermined whether $\Psi = \phi_1$ or ϕ_2 (c.f. Schrodinger's cat)

quantum

⇕

classical

It's pre-determined whether $\Psi = \phi_1$ or ϕ_2

The probabilities of each realization p_1 and p_2 are known.

Its QM description = **Mixed State**

Density matrix

$\hat{\rho}$ gives the probability and the expected value

$$p_i = \mathrm{Tr}\left[\hat{P}_i \hat{\rho}\right] \qquad \langle \hat{O} \rangle = \mathrm{Tr}\left[\hat{O} \hat{\rho}\right]$$

quantum

$$\hat{\rho}_{\mathrm{pure}} = |\Psi\rangle\langle\Psi| = \Sigma_i |c_i|^2 \, |\phi_i\rangle\langle\phi_i| + \Sigma_{i \neq j} c_i c_j^* \, |\phi_i\rangle \langle\phi_j|$$

Interference term

classical

$$\hat{\rho}_{\mathrm{mix}} = \Sigma_i p_i |\phi_i\rangle\langle\phi_i|$$

The essential difference btw quantum and classical state appears in the interference term in the density matrix.

Entanglement

A quantum system consists of subsystem A & B.

General state $|\Psi\rangle = \Sigma_{ij} c_{ij} |\psi_i\rangle_A \otimes |\phi_j\rangle_B$

If $|\psi\rangle_A = \Sigma_i a_i |\psi_i\rangle_A$ and $|\phi\rangle_B = \Sigma_j b_j |\phi_j\rangle_B$ independently,

Separable state state $|\Psi\rangle = \Sigma_{ij} a_i b_j |\psi_i\rangle_A \otimes |\phi_j\rangle_B$

Non separable = Entangled state

Interaction btw the subsystems can induce entanglement.

Trace out

Remember $\langle \hat{O} \rangle = \mathrm{Tr}[\hat{O}\hat{\rho}]$

If we only consider observables of the subsystem A, $\hat{O}_A$, we take the trace of $\hat{\rho}$ over the subsystem B,

Reduced density matrix: $\hat{\rho}_A = \mathrm{Tr}_B[\hat{\rho}]$

This operation won't change anything in A,
if A & B are separable.

Decoherence

Pure entangled state

$$|\Psi\rangle = \Sigma_i c_i |\psi_i\rangle_A \otimes |\phi_i\rangle_B$$

Density matrix

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \Sigma_{ik} c_i c_k^* |\psi_i\rangle\langle\psi_k| \otimes |\phi_i\rangle\langle\phi_k|$$

Trace out:

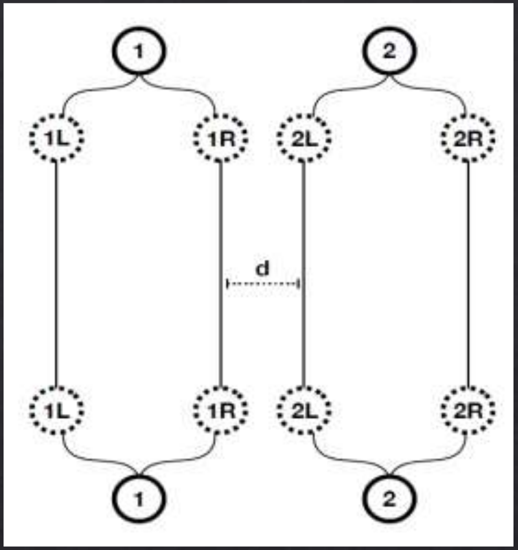
$$\mathrm{Tr}_B[\hat{\rho}] = \Sigma_l \langle\phi_l|\hat{\rho}|\phi_l\rangle = \Sigma_i |c_i|^2 |\psi_i\rangle\langle\psi_i|$$

When the original state is entangled,
tracing out it into a mixed state.

Decoherence = interference terms (quantum-ness) vanish

Phase from potential

Christodoulou & Rovelli, PLB 792 (2019)[1808.05842]



Schrodinger eq.: $i\partial_t|\psi\rangle = \hat{H}|\psi\rangle$

$\Rightarrow$Phase from mass energy $\mathrm{e}^{-imt}|\psi\rangle$

GR replaces t by the proper time s

Newtonian: $\mathrm{d}s^2 = [1+2\Phi]\mathrm{d}t^2, \qquad \Phi = -\frac{\mathrm{Gm}}{\mathrm{d}}$

c.f. $m_P \approx 2\times 10^{-5}\mathrm{g}$

$c * \mathrm{sec} \approx 3 \times 10^8 \mathrm{m}$

The relative phase that |RL> gains is

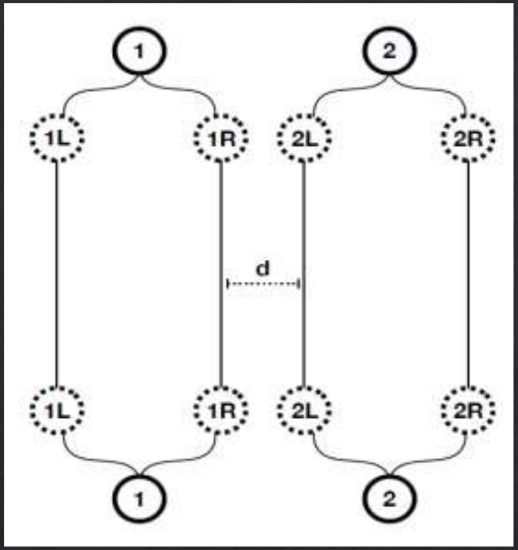
Phase: $\phi = \frac{Gm^2}{d} t \approx 2\pi \left(\frac{t}{1\mathrm{sec}}\right)\left(\frac{d}{1\mathrm{mm}}\right)^{-1}\left(\frac{m}{10^{-11}\mathrm{g}}\right)^2$

Small mass compensated by long time

Quantum state

Christodoulou & Rovelli, PLB 792 (2019)[1808.05842]

$$|\Psi_3\rangle = \frac{1}{2}\Big(|LL\, g_{d_{LL}}\rangle + |RR\, g_{d_{RR}}\rangle + |LR\, g_{d_{LR}}\rangle + e^{i\frac{Gm^2 t}{\hbar d}}|RL\, g_{d_{RL}}\rangle\Big).$$



Bring them back by inverse-SG

$$|\psi_4\rangle = \frac{1}{2}\left[|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle + |\downarrow\uparrow\rangle + e^{i\phi}|\uparrow\downarrow\rangle\right]$$

The entangled state is tested by Bell inequality

$$\mathcal{W} = |\langle\sigma_x^{(1)} \otimes \sigma_z^{(2)}\rangle - \langle\sigma_y^{(1)} \otimes \sigma_z^{(2)}\rangle|$$

If $\mathcal{W} > 1$, the state is entangled and GFs are superposed.

General Hamiltonian

TF. et al. (2023) [2308.14552]
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Our quadratic Hamiltonian:

$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

dimentionless form

$$H = \frac{\omega}{2}\left[P_1^2 + \lambda_1 X_1^2 + P_2^2 + \lambda_2 X_2^2 - \eta(X_1 - X_2)^2\right]$$

oscillator1 oscillator2 Grav. Int.

Variable: $P_i \equiv p_i/\sqrt{\hbar m\omega}$ $X_i \equiv \sqrt{m\omega/\hbar}x_i$

Coupling constant:

$$\eta \equiv \frac{2Gm}{\omega^2 d^3} = 2.7 \times 10^{-13}\,\omega_{\mathrm{kHz}}^{-2}\left(\frac{m/d^3}{2\ \mathrm{g/cm^3}}\right)$$

Gravity is weak