Modulation-type Quadrature Interferometers

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Abstract

- I will introduce three kinds of modulation-type quadrature interferometers
- In the summary slide, we can fill in all the blanks in the table

Name of interferometers	University	Design & characteristics	Sensitivity	References
Deep phase modulation interferometer				
Deep frequency modulation interferometer				
Digitally enhanced heterodyne interferometer				

Why did I choose this topic?

- I summarized sensors (most of them were quadrature interferometers) developed by Univ. of Birmingham last year
- Nagano-san said "Sensors developed by AEI might be interesting for the next seminar"

Summary

Sensors & references	Motivation	Design & characteristics	Sensitivity at 1 Hz	Discussion & status
BOSEM L.Carbone+ (2012)	Suspension control for aLIGO	Shadow sensor, coil-magnet actuator	2×10^{-10} m/√Hz	Currently used
EUCLID S. M. Aston (2011)	Suspension control for aLIGO	Quadrature homodyne interferometer, polarizing optics, cat's eye retroreflector	4×10^{-12} m/√Hz	Improved to HoQI
ILIAD F. E. P. Arellano+ (2013)	*G* measurement with torsion pendulum	Same as EUCLID, Non-planar for angular measurement	5×10^{-13} m/√Hz	Not used for *G* measurement
HoQI S. J. Cooper+ (2018)	Suspension control for aLIGO	Simpler than EUCLID	2×10^{-13} m/√Hz	Replace BOSEM in the future
QUIMETT				
HDMI Slides	AVIT for TOBA	No polarizing optics, dither	5×10^{-11} m/√Hz	Mass-produced, not installed to AVIT

Ando Lab Seminar Nov. 4, 2022 56 / 56

My seminar slides on Nov. 4, 2022

- I sent an email to Nagano-san to get references, then he modified his comment "modulation-type quadrature interferometers might be interesting"

References

- $J_1 \cdots J_4$ method
  - ★ V. S. Sudarshanam and K. Srinivasan, Optics Letters, 14, 140 (1989)
  - W. Jin+, Proc. SPIE 1267, Fiber Optic Sensors IV (1990)
  - V. S. Sudarshanam & R. O. Claus, Journal of Modern Optics, 40, 483 (1993)

- Deep phase modulation interferometer
  - ★ G. Heinzel+, Optics Express, 18, 19076 (2010)
  - T. S. Schwarze+, Optics Express, 22, 18214 (2014)
  - M. Terán+, J. Phys.: Conf. Ser., 610, 012042 (2015)

- Deep frequency modulation interferometer
  - ★ O. Gerberding, Optics Express, 23, 14753 (2015)

- Digitally enhanced heterodyne interferometer
  - ★ D. A. Shaddock, Optics Letters, 32, 3355 (2007)
  - O. P. Lay+, Optics Letters, 32, 2933 (2007)
  - ★ G. de Vine+, Optics Express, 17, 828 (2009)

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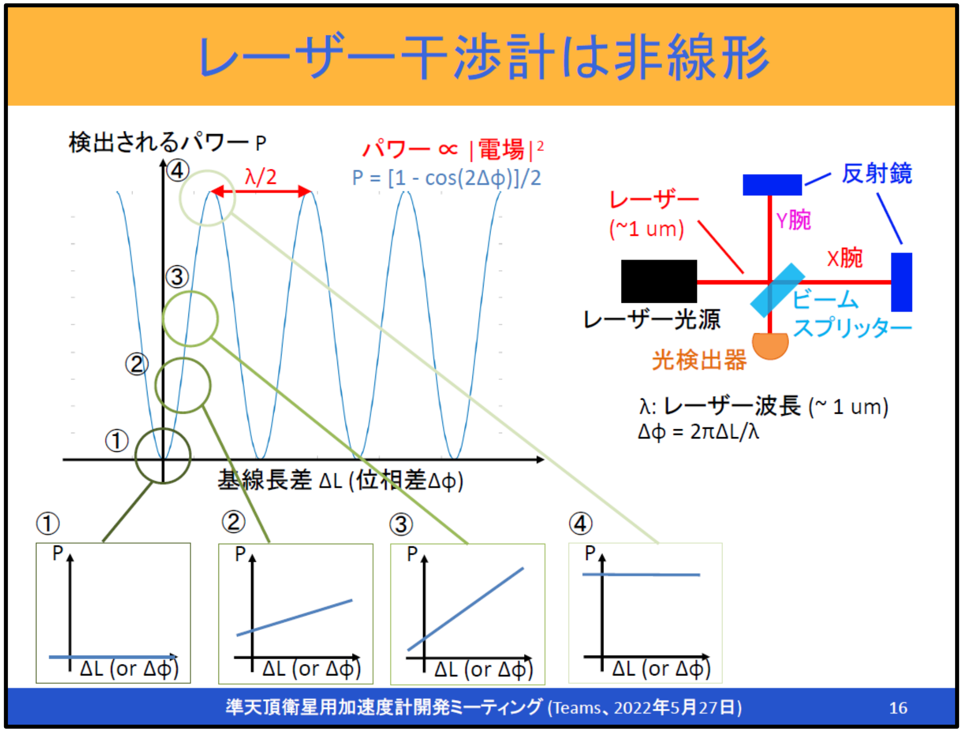
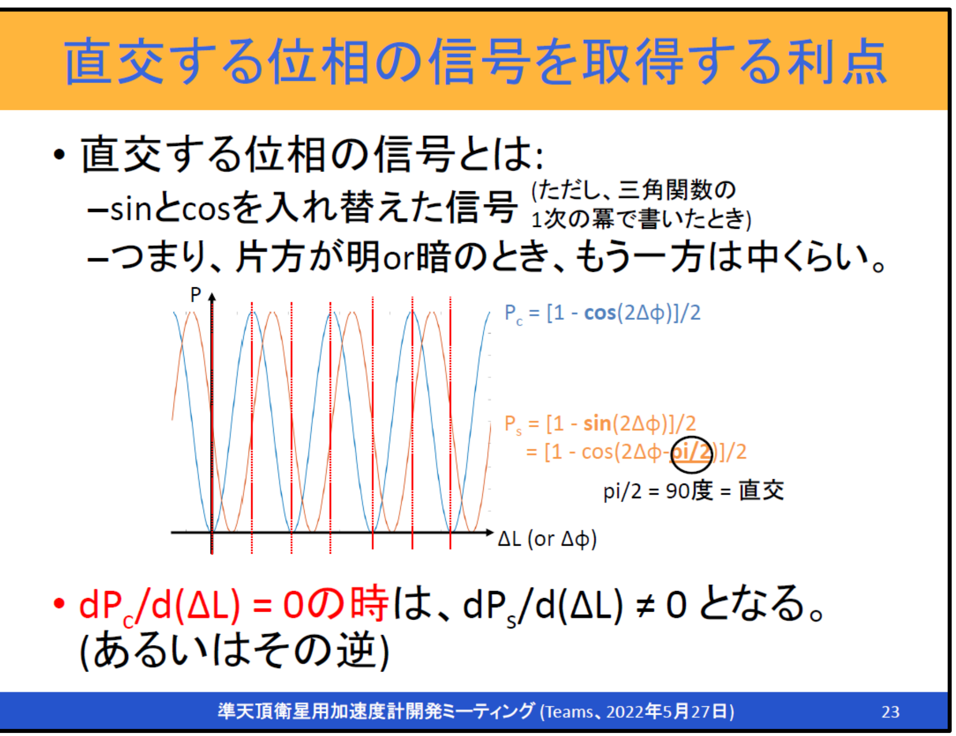
Michelson interferometer

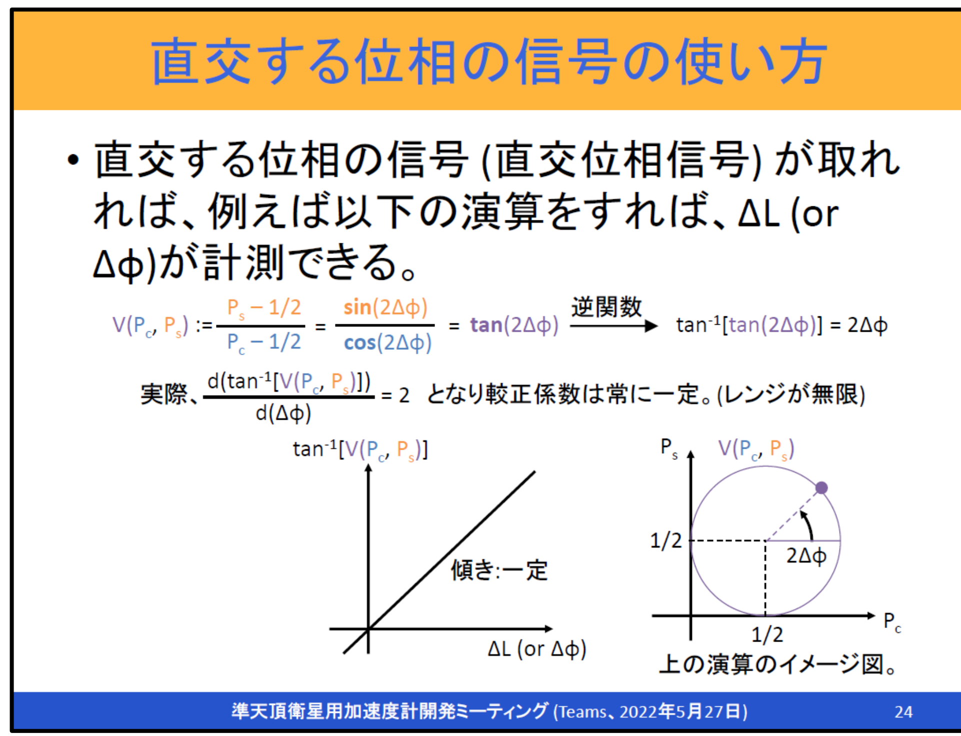
- The response of MI has non-linearity → Small range
- We usually conduct feedback control to fix the operation point



Quadrature interferometer

- When we obtain the quadrature signals (sin and cos), the information of phase can be calculated
- Range: infinity (theoretically), >10 mm (experimentally)
- No need to FB control
- Sensitivity worse than FB control method due to ADC noise

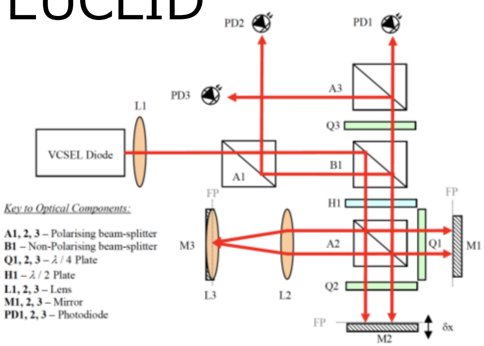




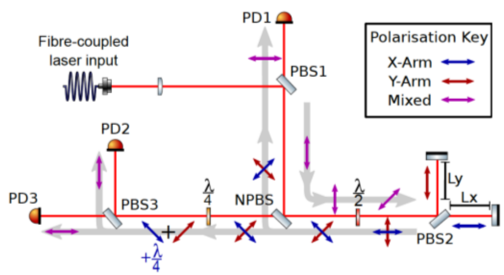
Classification of quadrature interferometers

Quadrature interferometers

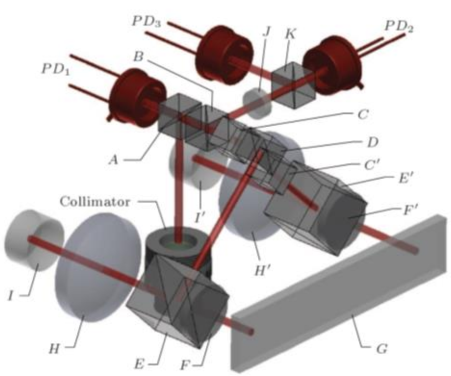
- **Polarization-type**
  - EUCLID



  - HoQI

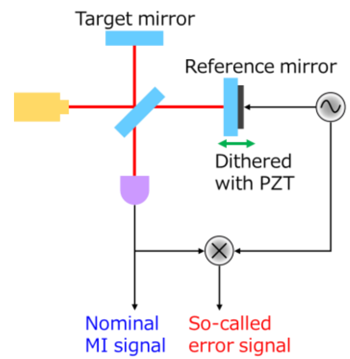


  - ILIAD



  - QUIMETT

- **Modulation-type**
  - **Mechanically**
    - HDMI



  - **Electrically**
    - Today's seminar

- ?

Contents

- Review of quadrature interferometers
- $J_1 \cdots J_4$ method
- Deep phase modulation interferometer
- Deep frequency modulation interferometer
- Digitally enhanced heterodyne interferometer

Overview

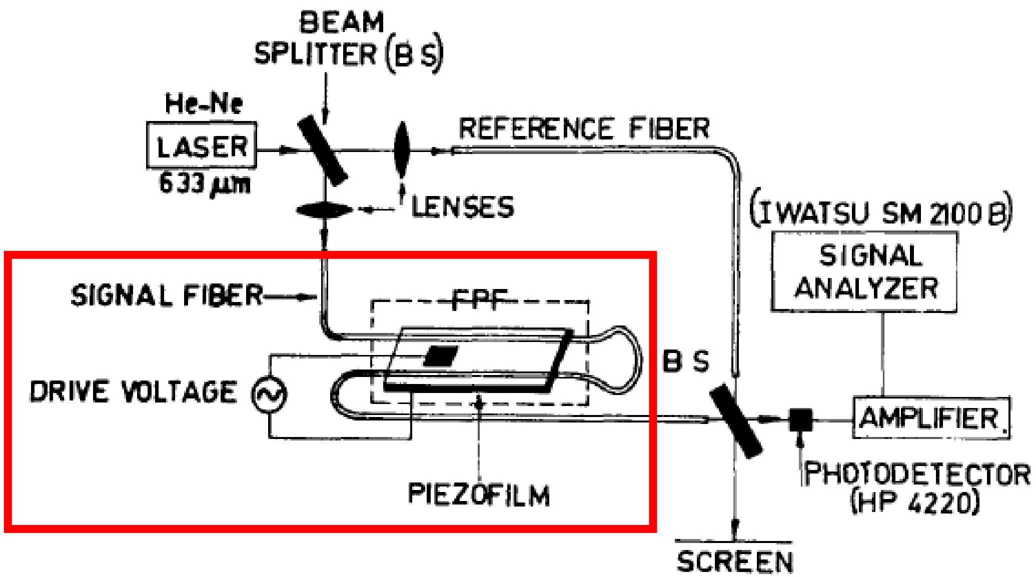
- Linear measurement over one fringe with spectral analysis and no feedback control
- Basic principle for deep phase/frequency modulation interferometer

Year	1980		1989		1993
Method	J_1 (max) J_1 (null) J_1/J_2 J_1/J_3	➡	$J_1 \cdots J_4$	➡	$J_1 \cdots J_6$ (neg) $J_1 \cdots J_6$ (pos)
Range	<1.9 rad		~ 5 rad		~ 5 rad Accuracy and minimum detectable phase improved

Setup

- A Mach-Zehnder interferometer with two fibers
- One arm fiber was stripped of its jacket and bonded onto a piezoelectric polyvinylidene fluoride film
- Piezofilm was driven by an electric signal to produce predictable phase shifts

Phase modulation (induced signal in this exp.) by piezofilm



Principle

- Photodetector output can be expressed in three ways
  - Nominal

$$I(t) = A + B\cos[\varphi_0(t) + x\sin(w_s t + \varphi_s)],$$

  - Bessel functions

φ_0: interferometer phase
x: modulation depth (signal)
w_s: modulation frequency
φ_s: modulation phase

$$I(t) = A + B\Bigg(\Bigg[J_0(x)\cos\varphi_0(t) + 2\sum_{n=1}^{\infty} J_{2n}(x)\cos\varphi_0(t)(\cos 2nw_s t\cos 2n\varphi_s - \sin 2nw_s t\sin 2n\varphi_s)\Bigg] - \Bigg\{2\sum_{n=1}^{\infty} J_{2n-1}(x)\sin\varphi_0(t)[\sin(2n-1)w_s t \times \cos(2n-1)\varphi_s + \cos(2n-1)w_s t\sin(2n-1)\varphi_s]\Bigg\}\Bigg).$$

  - Fourier series

$$I(t) = a_0 + \sum_{n=1}^{\infty}[a_n\cos(nw_s t) - b_n\sin(nw_s t)],$$

Principle

- By comparing expressions with Bessel functions and Fourier series, we can derive

$$\text{odd terms}\begin{cases} a_{2n-1} = -2BJ_{2n-1}(x)\sin\varphi_0(t)\sin(2n-1)\varphi_S, \\ b_{2n-1} = 2BJ_{2n-1}(x)\sin\varphi_0(t)\cos(2n-1)\varphi_S, \end{cases}$$

$$\text{even terms}\begin{cases} a_{2n} = 2BJ_{2n}(x)\cos\varphi_0(t)\cos 2n\varphi_S, \\ b_{2n} = 2BJ_{2n}(x)\cos\varphi_0(t)\sin 2n\varphi_S, \end{cases} \quad (n = 1, 2, 3, \ldots).$$

→ Bessel functions are extracted from Fourier transform

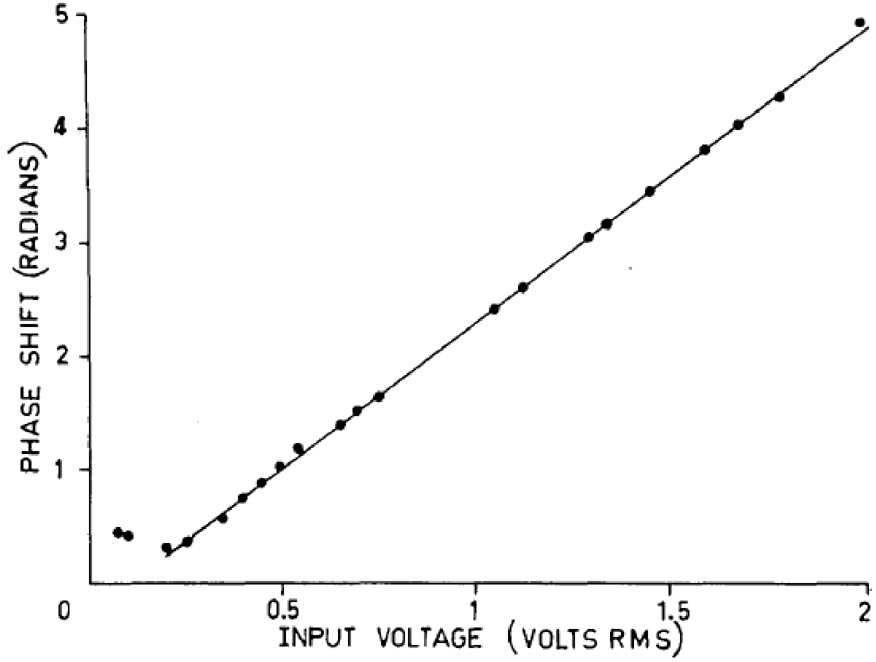
- Modulation depth is calculated from Bessel functions

$$x^2 = \frac{24J_2(x)J_3(x)}{[J_2(x) + J_4(x)][J_1(x) + J_3(x)]}.$$

- Other parameters (φ_0, $\cdots$) are also calculated (later)

Result

Measurement phase shift x [rad]



Input voltage to PZT [V]

- $J_1 \cdots J_4$ method can measured phase up to 5 rad (= over one fringe) linearly

Contents

Overview

- Proposed and developed by AEI (+ Spain)
- Extension of $J_1 \cdots J_4$ method
- Phase modulation with depth of >5 rad → "deep"

- Motivation: not clearly, but for LISA?
  - Experiment with LISA Pathfinder optical bench
  - Proceedings for LISA symposium

- Advantage: good sensitivity, large linear range, simple optics
- Disadvantage: complicated data analysis
  - Analysis for quadrature interferometers itself is already a bit complicated, but this is more complicated, I think
  - They use some algorithms (Levenberg-Marquardt, Nelder-Mead Simplex, etc.), but I will not explain today

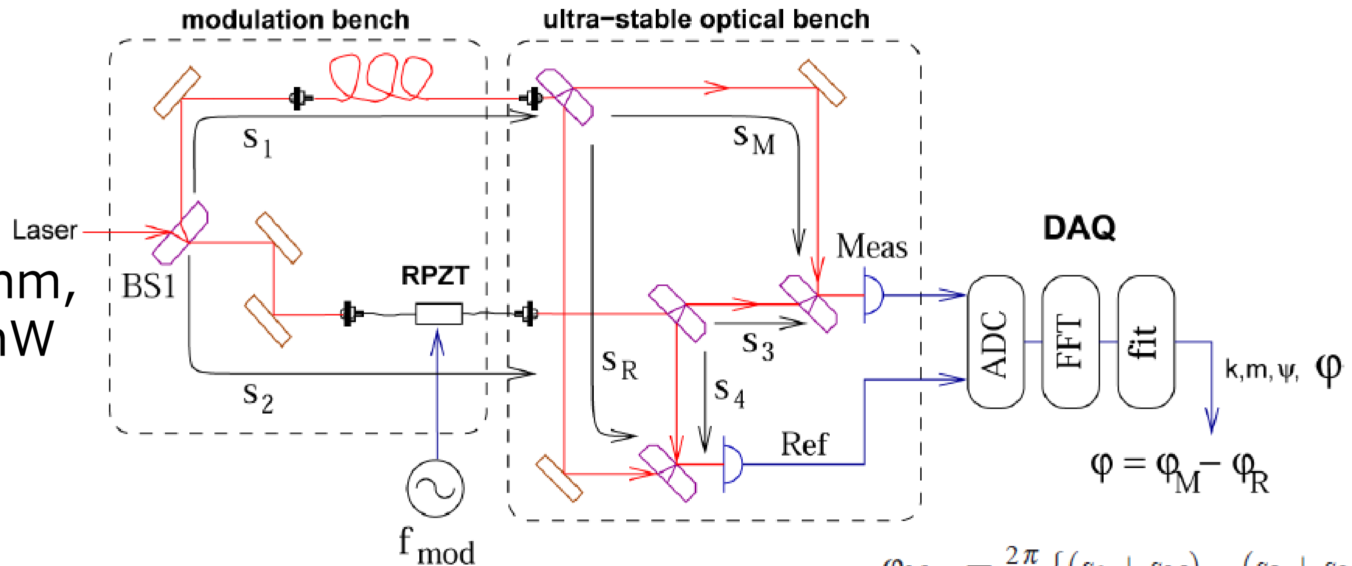
Setup

- LISA Pathfinder optical bench
- Fiber-coupled Mach-Zehnder interferometer

Standard metal optical breadboard

20 cm×20 cm Zerodur baseplate with optical components fixed by hydroxide-catalysis bonding, 5 pm/√Hz above 1 mHz

1064 nm, 300mW



A few μm, common mode rejection ↓

Fiber coiled around ring piezo-electric transducers,
$4.5\ \mathrm{V_{p-p}}, f_{\mathrm{mod}} = 280\ \mathrm{Hz}, m = 9.7$

$$\varphi_M = \frac{2\pi}{\lambda}\{(s_1+s_M)-(s_2+s_3)\} = \frac{2\pi}{\lambda}\{(s_M-s_3)+\Delta\},$$

$$\varphi_R = \frac{2\pi}{\lambda}\{(s_1+s_R)-(s_2+s_4)\} = \frac{2\pi}{\lambda}\{(s_R-s_4)+\Delta\},$$

$$\varphi = \varphi_M - \varphi_R = \frac{2\pi}{\lambda}\{s_M-(s_R+s_3-s_4)\}$$

Data analysis

- Output of PD was processed by FFT

$$V_{\mathrm{PD}}(t) = V_{\mathrm{DC}}(\varphi) + \sum_{n=1}^{\infty} a_n(m,\varphi)\cos(n(\omega_{\mathrm{m}}t + \psi))$$

$$a_n(m,\varphi) \quad = \quad kJ_n(m)\cos\left(\varphi + n\frac{\pi}{2}\right),$$

- m, φ, ψ, k was obtained by minimizing χ^2

$$\chi^2 = \sum_{n=1}^{N} (\alpha_n(m,\varphi) - \tilde{\alpha}_n(m,\varphi))^2,$$

$$n\psi \quad = \quad \arctan\left(\frac{\Im\{\alpha_n(m,\varphi)\}}{\Re\{\alpha_n(m,\varphi)\}}\right),$$

$$a_n(m,\varphi) \quad = \quad \alpha_n(m,\varphi)e^{-in\psi},$$

$$n = 1, 2, 3\ldots N,$$

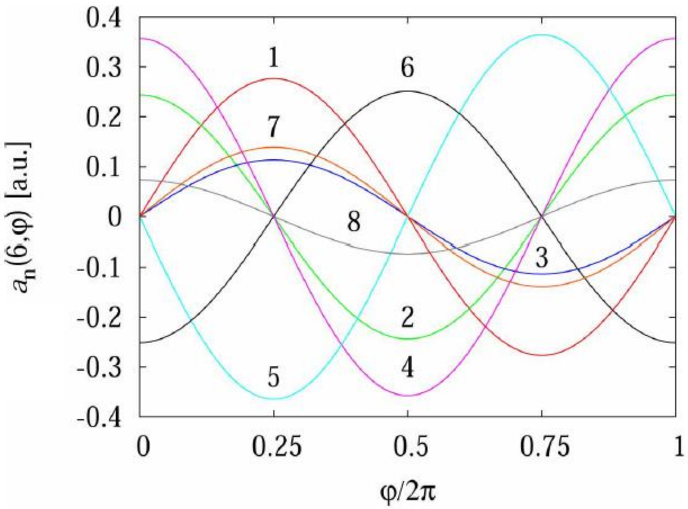


Fig. 2. Dependence of the harmonics amplitudes $a_n(m,\varphi)$ with respect to the interferometer phase φ with a modulation depth $m = 6\,\mathrm{rad}$.

Modulation index, number of bins

- To measure the phase accurately, we should choose a suitable modulation index and number of bins for FFT
- For a deeper modulation, the signal power is distributed into more and higher harmonic bins
  → Deep phase modulation is required to extract the harmonic amplitudes for processing by numerical fit
- In this experiment, $m = 9.7$ and $N = 10$ were chosen

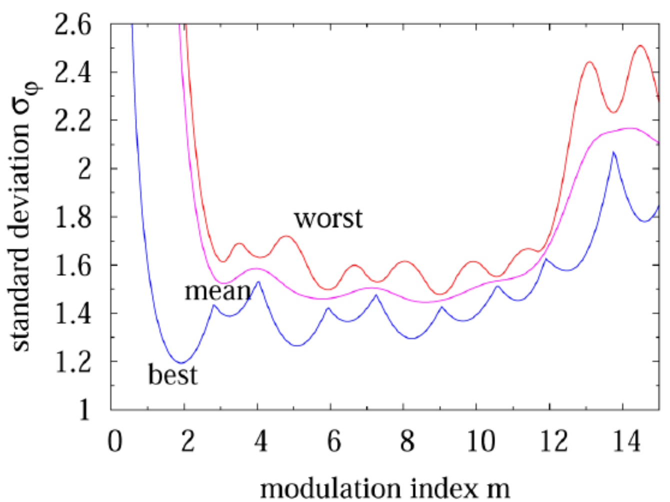


Fig. 3. Ideal resolution in φ as function of the modulation index m for $N = 10$, for the best and worst φ as well as the average for all $\varphi \in [0, 2\pi]$.

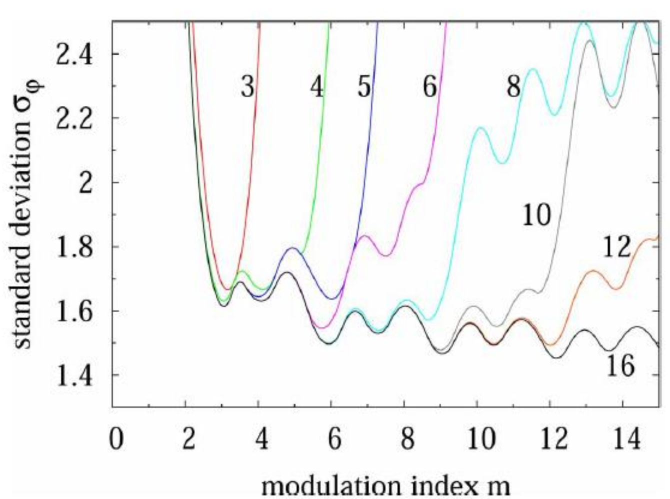


Fig. 4. Ideal resolution in φ as function of the modulation index m for different orders N, for the worst value of φ at each point of each curve.

Sensitivity

- All PDs on LPF optical bench are QPDs
  → Both length and tilt can be measured

Length

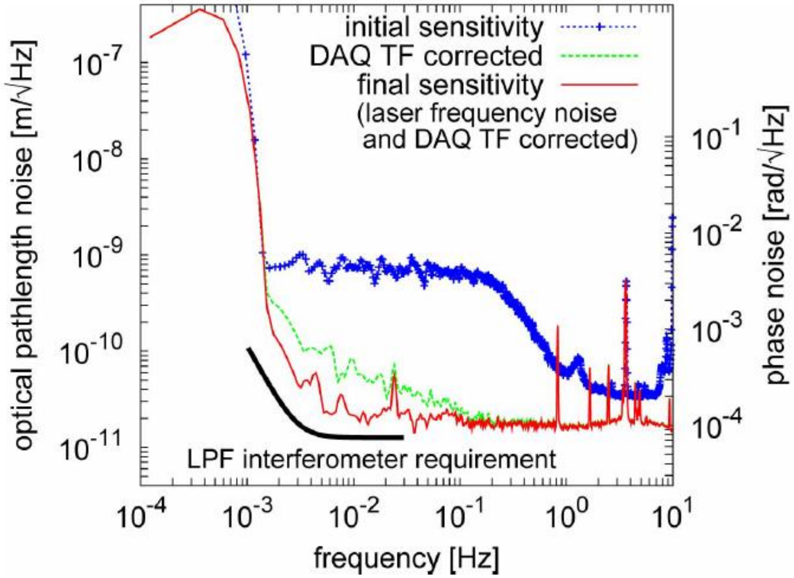


Fig. 6. Sensitivity of real optical pathlength measurements. Dashed curve with crosses: initial sensitivity prior to noise correction techniques. Dashed curve: sensitivity upon correction of DAQ frequency response. Solid curve: sensitivity reach after application of noise mitigation strategies -laser frequency noise and DAQ frequency response-.

20 pm/√Hz

Tilt

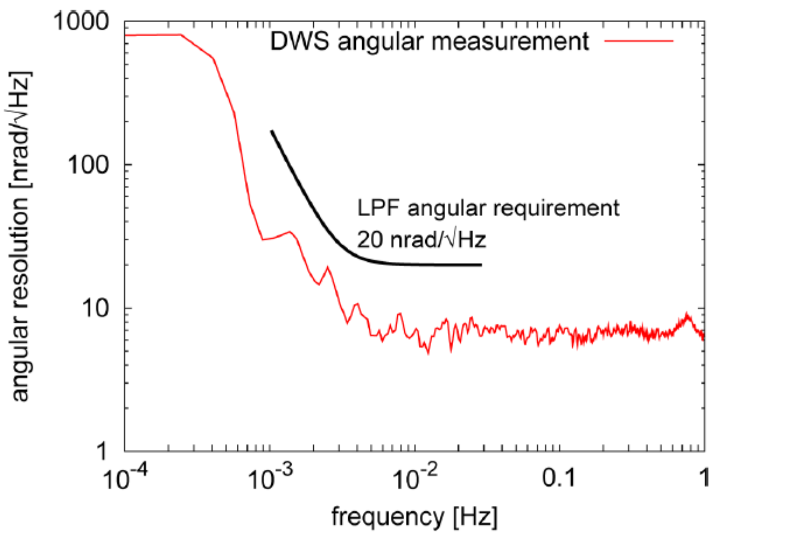


Fig. 7. Angular resolution obtained by applying a DWS algorithm to the phases extracted from individual cells of a quadrant photodetector.

10 nrad/√Hz

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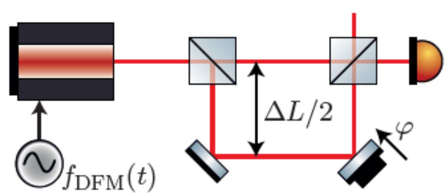
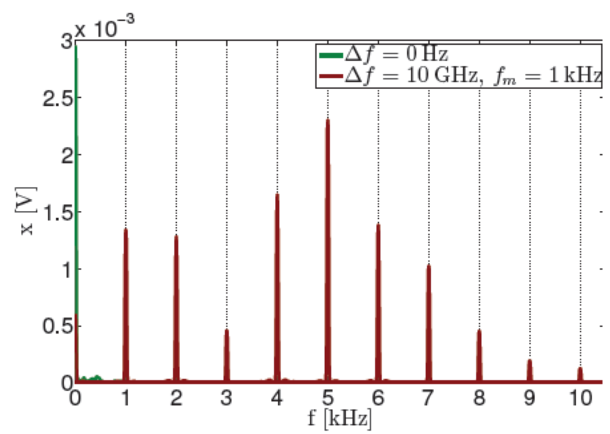
Overview

- Proposed and simulated by Univ. of Maryland
- No experimental demonstration so far
- Almost the same as deep phase modulation, but frequency modulation instead of phase modulation
- Schnapp asymmetry ΔL is needed because of modulation with a laser source (= in front of BS)
  - Of course, phase modulation also requires Schnapp asymmetry if modulating in front of BS
- Effective modulation index $m = 2\pi\Delta f \Delta L / c$
  - Larger signal with longer ΔL (when $\Delta L < \lambda_{\mathrm{mod}}$)





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Overview

- Proposed and developed by Caltech
- Currently developed by ANU

- Motivation: not clearly, but for LISA?
  - Classified as a space technology on the ANU website

ANU - Digital interferometry

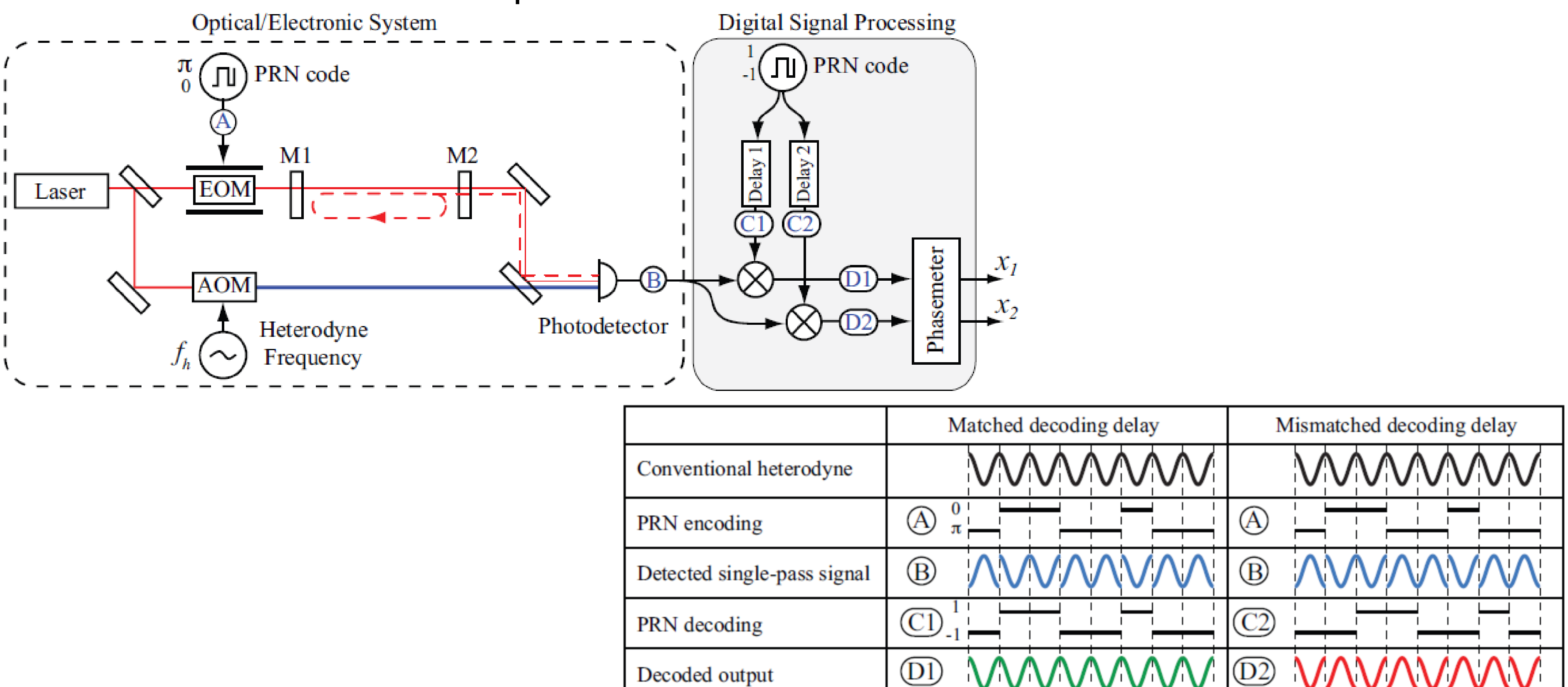
- Pseudo-random noise code → "digitally enhanced"

- Advantage: good sensitivity, large linear range, simple optics, measurement of multiple test masses with one interferometer
- Disadvantage: complicated data analysis

Digitally enhanced

Principle (intuitively)

- Pseudo-random noise (PRN) code: zero or π phase shift
- Demodulation taking into account the delay from EOM to PD
  - Single pass: EOM → M1 → M2 → PD
  - One round-trip: EOM → M1 → M2 → M1 → M2 → PD



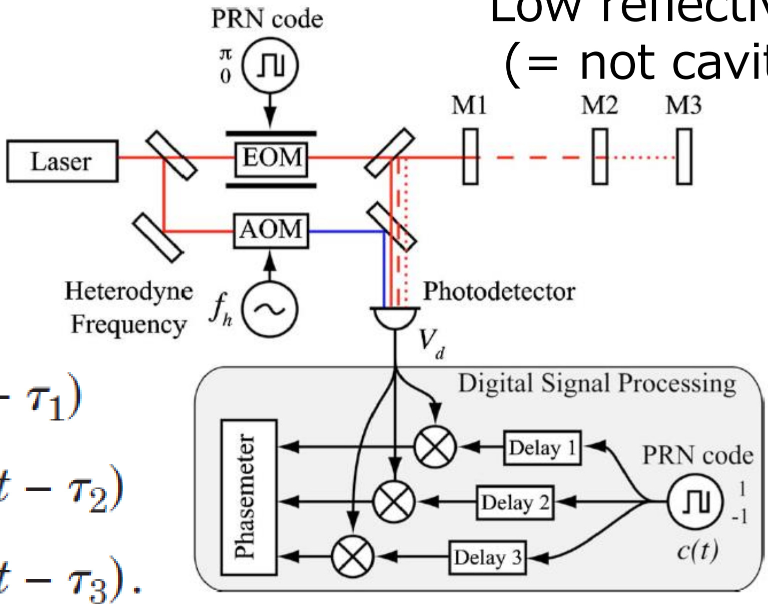
Principle (with formula)

$c(t) = \pm 1$

Probe beam at PD

$$\tilde{E}_P = E_1 e^{-i\phi_1} c(t-\tau_1) + E_2 e^{-i\phi_2} c(t-\tau_2) + E_3 e^{-i\phi_3} c(t-\tau_3),$$

Low reflectivity (= not cavity)



LO at PD

$$\tilde{E}_{LO} = e^{-i(2\pi f_h t + \phi_{LO})}.$$

AC output of PD

$$V_d(t) = E_1 \cos(\phi_1 - 2\pi f_h t - \phi_{LO}) c(t-\tau_1) + E_2 \cos(\phi_2 - 2\pi f_h t - \phi_{LO}) c(t-\tau_2) + E_3 \cos(\phi_3 - 2\pi f_h t - \phi_{LO}) c(t-\tau_3).$$

$V_{M1} = V_d \times c(t-\tau_1)$
$c(t-\tau_1) \times c(t-\tau_1)$: always +1
$c(t-\tau_1) \times c(t-\tau_{2\text{ or }3})$: random

Demodulation signal for M1

$$V_{M1}(t) = E_1 \cos(\phi_1 - 2\pi f_h t - \phi_{LO}) + E_2 \cos(\phi_2 - 2\pi f_h t - \phi_{LO}) c(t-\tau_2) c(t-\tau_1) + E_3 \cos(\phi_3 - 2\pi f_h t - \phi_{LO}) c(t-\tau_3) c(t-\tau_1).$$

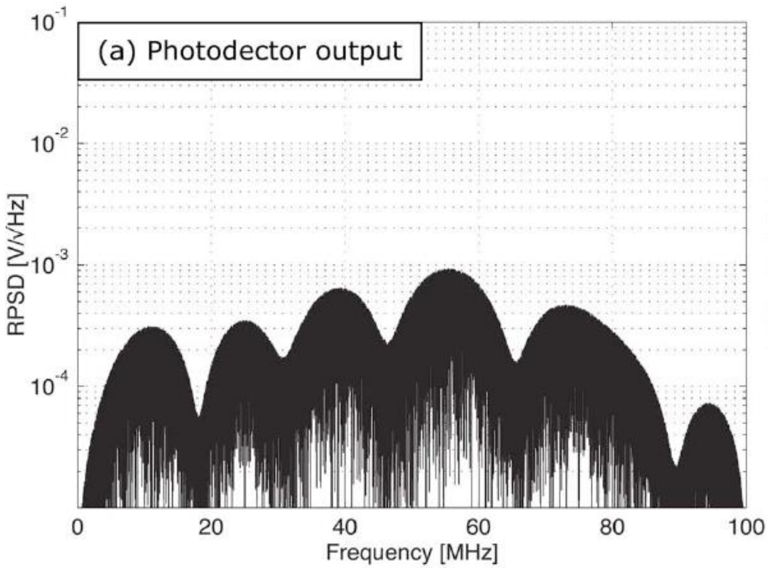
Digitally enhanced

Principle (with formula)

PSD of simulated signal

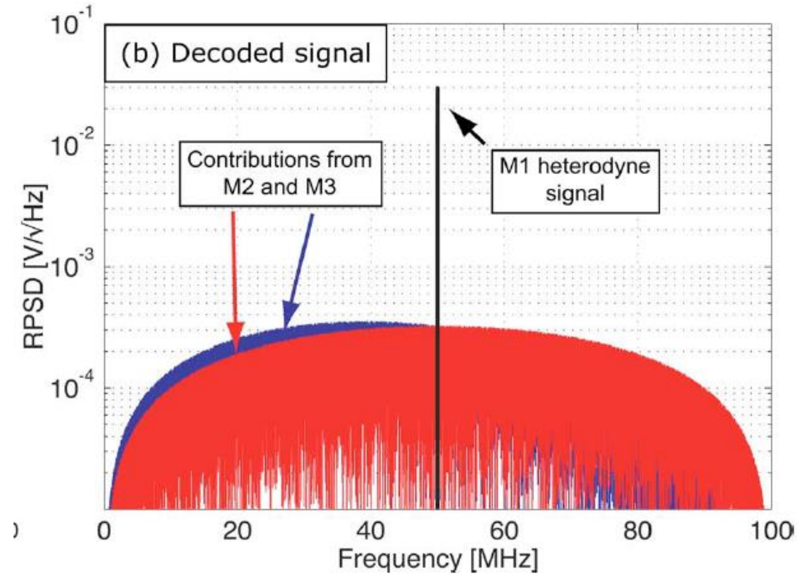
AC output of PD

$$V_d(t) = E_1 \cos(\phi_1 - 2\pi f_h t - \phi_{LO}) c(t - \tau_1) + E_2 \cos(\phi_2 - 2\pi f_h t - \phi_{LO}) c(t - \tau_2) + E_3 \cos(\phi_3 - 2\pi f_h t - \phi_{LO}) c(t - \tau_3).$$



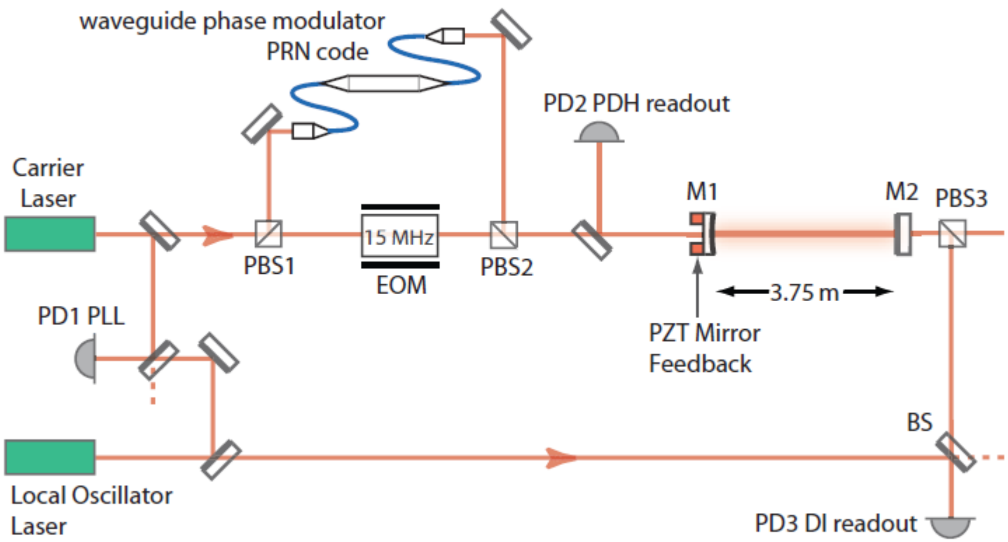
Demodulation signal for M1

$$V_{M1}(t) = E_1 \cos(\phi_1 - 2\pi f_h t - \phi_{LO}) + E_2 \cos(\phi_2 - 2\pi f_h t - \phi_{LO}) c(t - \tau_2) c(t - \tau_1) + E_3 \cos(\phi_3 - 2\pi f_h t - \phi_{LO}) c(t - \tau_3) c(t - \tau_1).$$

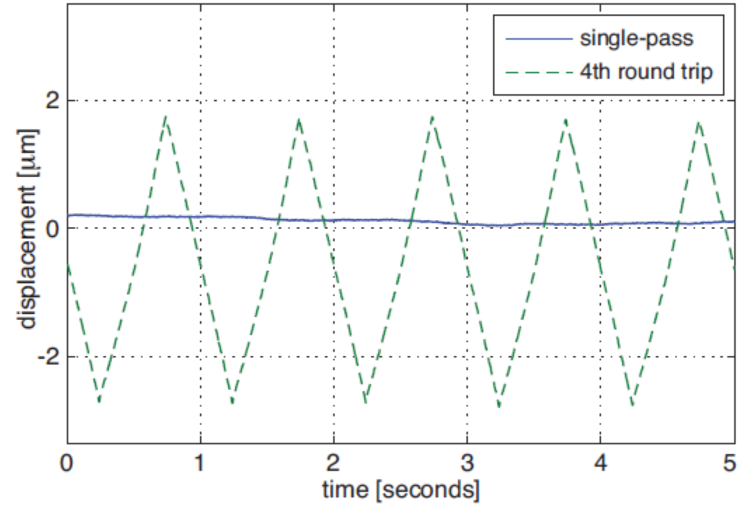


- $f_h = 50$ MHz
- $f_{\text{chip}} = 50$ Mchip/sec
- M1-M2: 6 m, M2-M3: 3 m (ΔL should be $\geq c/2f_{\text{chip}}$)

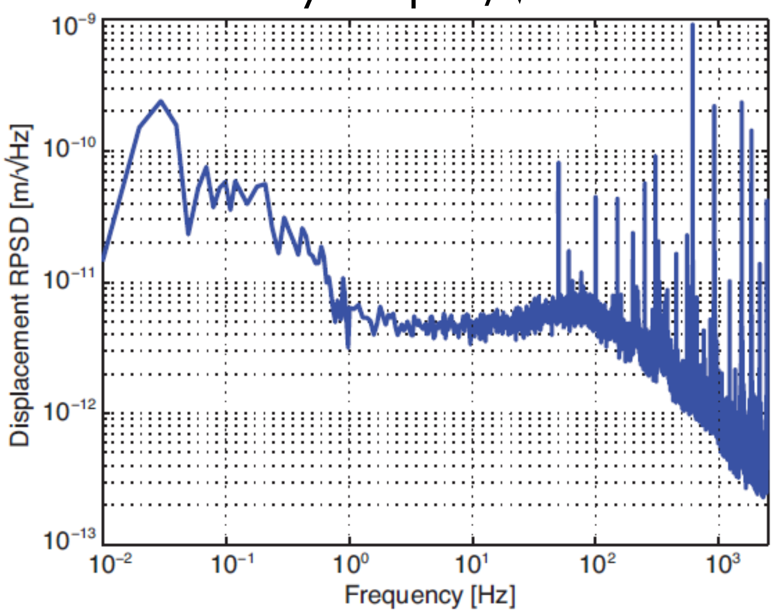
Setup and results



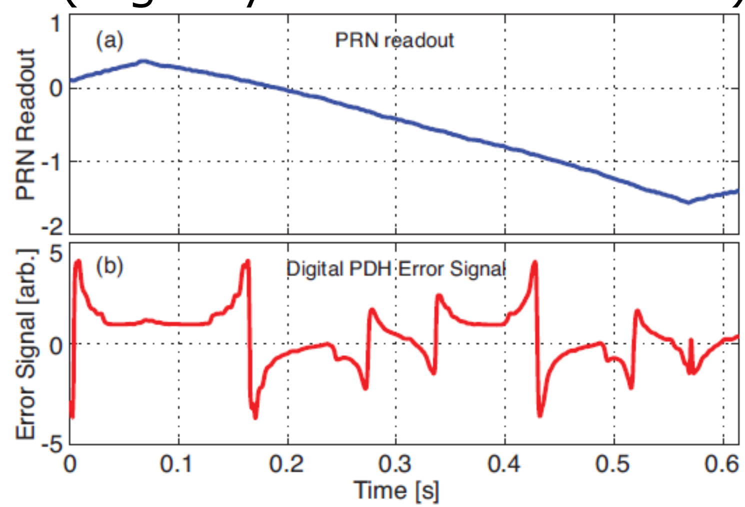
Cavity scan
(single-pass and round-trip)



Sensitivity: 5 pm/√Hz at 1 Hz



Cavity scan
(Digitally enhanced and PDH)



Summary

Name of interferometers	University	Design & characteristics	Sensitivity	References
Deep phase modulation interferometer	AEI, Spain	Homodyne, phase modulation to one arm	2×10^{-11} m/√Hz at 1 Hz	G. Heinzel+ (2010) T. S. Schwarze+ (2014) M. Terán+ (2015)
Deep frequency modulation interferometer	Univ. of Maryland	Homodyne, freq. modulation to laser source	No experiment	O. Gerberding (2015)
Digitally enhanced heterodyne interferometer	Caltech, ANU	Heterodyne, PRN code phase modulation to one arm, multiple TMs measurement	5×10^{-12} m/√Hz at 1 Hz	D. A. Shaddock (2007) O. P. Lay+ (2007) G. de Vine+ (2009)