Design of Phase-III TOBA

Yuka Oshima (D2)

Department of Physics, University of Tokyo

Overview

- I would like to discuss the configuration of Phase-III TOBA for my Ph.D. thesis
- First part: comparison of the readout optical systems
- Second part: requirement of the suspension system

- I omit the introduction of TOBA because there are no M1 students

Contents

- Comparison of the readout optical systems
- Requirement of the suspension system
- Timeline for the next two years

Contents

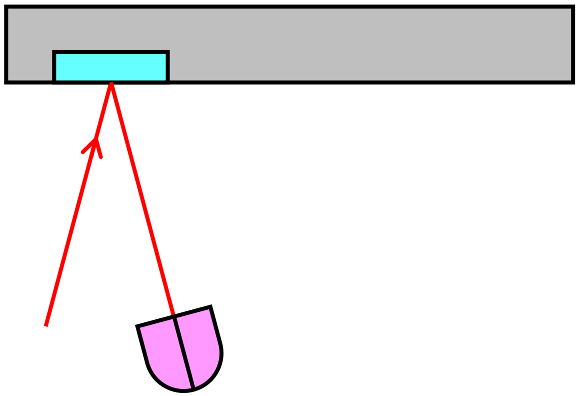
- Comparison of the readout optical systems
- Requirement of the suspension system
- Timeline for the next two years

Comparison of optical systems

- I summarized these considerations with two slides for each optical system
- We will come back to this table at the end

Unit: [rad/√Hz]	Oplev	MI	PRMI	FPMI	QI	Cavities	WFS	CWFS, FWFS
Shot noise	2×10^{-12}	2×10^{-15}	5×10^{-16}	4×10^{-18}	2×10^{-15}	4×10^{-18}	1×10^{-11}	7×10^{-14}
Thermal noise	7×10^{-17}	1×10^{-17}	1×10^{-17}	1×10^{-17}	1×10^{-17}	1×10^{-17}	7×10^{-17}	7×10^{-17}
Seismic coupling	2×10^{-12}	2×10^{-13}	2×10^{-13}	2×10^{-13}	2×10^{-13}	2×10^{-13}	No coupling	No coupling
Numbers of optics	1	6	10	11	11 polarizing	8	5	6
PD	QPD	PD	PD, RFPD	PD, RFPD	3PD	2RFPD	RFQPD	RFQPD
Experience in Ando Lab	Shimosd-san's D	Shoda-san, Shimoda-san's M	Aritomi-san's M	Nothing	Okada-san	ANU, Takano-san's D	Nothing	Nothing

Optical lever



- Shot noise:

$$\delta\theta_{\mathrm{shot}} = \frac{\pi w}{2L}\sqrt{\frac{\hbar c}{2\lambda P_0}} = 2\times 10^{-12}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

Beam radius at QPD: $w = 300\,\mu\mathrm{m}$

Arm length: $L = 30\,\mathrm{cm}$

Laser wavelength: $\lambda = 1\,\mu\mathrm{m}$

Input laser power: $P_0 = 10\,\mathrm{mW}$

- Thermal noise (mirror substrate):

$$\delta\theta_{\mathrm{thermal}} = \sqrt{\frac{2k_B T}{\pi^{3/2} f}\frac{1-\nu_s^2}{E_s}\frac{\phi_s}{w^3}} = 7\times 10^{-17}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

$f = 0.1\,\mathrm{Hz}$ $w = 100\,\mu\mathrm{m}$ $T = 4\,\mathrm{K}$

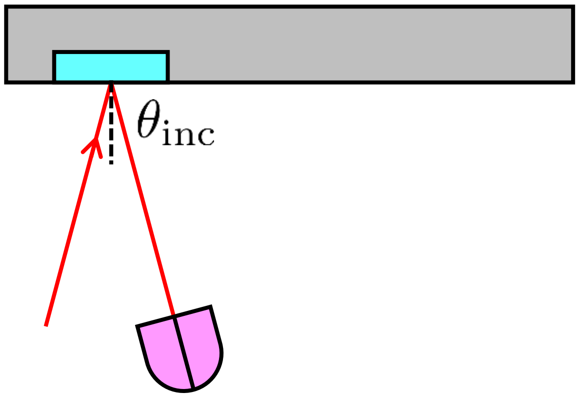
Substrate: fused silica

Young's modulus: $E_s = 7.4\,\mathrm{GPa}$

Poisson's ratio: $\nu_s = 0.17$

Loss angle: $\phi_s = 10^{-8}$

Optical lever



- Seismic coupling:

$$\delta\theta_{\rm seis} = \frac{\sin\theta_{\rm inc}}{2L}\delta x_{\rm seis}{\rm CMRR} = 2\times 10^{-12}\,{\rm rad}/\sqrt{\rm Hz}$$

Incident angle: $\theta_{\rm inc} = 1\,{\rm mrad}$

Arm length: $L = 30\,{\rm cm}$

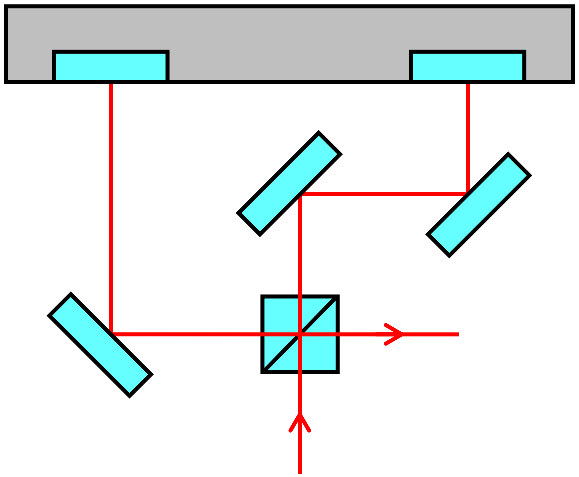
Translational seismic noise:

$$\delta x_{\rm seis} = 10^{-7}\,{\rm m}/\sqrt{\rm Hz}$$

Common mode rejection ratio between TM and OB: ${\rm CMRR} = 1/100$

- Numbers of optics: 1 mirror (only essential optics. Of course, some mirrors or BS are needed)
- PD: 1 QPD

Michelson interferometer



- Shot noise:

$$\delta\theta_{\rm shot} = \frac{1}{2L}\sqrt{\frac{\hbar c \lambda}{\pi P_0}} = 2 \times 10^{-15}\,{\rm rad}/\sqrt{\rm Hz}$$

Arm length: $L = 30\,{\rm cm}$

Laser wavelength: $\lambda = 1\,\mu{\rm m}$

Input laser power: $P_0 = 10\,{\rm mW}$

- Thermal noise (bar):

$$\delta\theta_{\rm thermal} = \frac{1}{L}\sqrt{\frac{4k_B T}{\omega}\frac{\phi_{\rm bar}}{\mu\omega_0^2}} = 1 \times 10^{-17}\,{\rm rad}/\sqrt{\rm Hz}$$

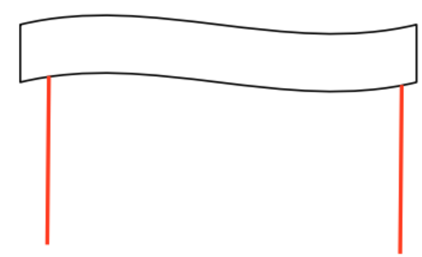


図 4.8: 試験マスの 2 次のモード

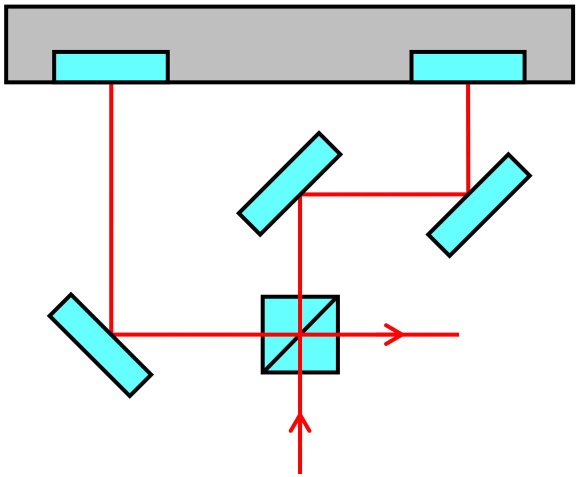
Aritomi-san’s master thesis

$T = 4\,{\rm K}$ $\omega = 2\pi \times 0.1\,{\rm Hz}$ $\phi_{\rm bar} = 10^{-6}$

Frequency of second mode: $\omega_0 = 2\pi \times 10\,{\rm kHz}$

Reduced mass of second mode: $\mu = 0.3\,{\rm kg}$

Michelson interferometer



- Seismic coupling:

$$\delta\theta_{\mathrm{seis}} = \frac{\theta_{\mathrm{mir}}}{L}\delta x_{\mathrm{seis}}\mathrm{CMRR} = 2\times 10^{-13}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

  Angle of two mirrors: $\theta_{\mathrm{mir}} = 10^{-6}\,\mathrm{rad}$

  Arm length: $L = 30\,\mathrm{cm}$

  Translational seismic noise:
  $$\delta x_{\mathrm{seis}} = 5\times 10^{-6}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$$

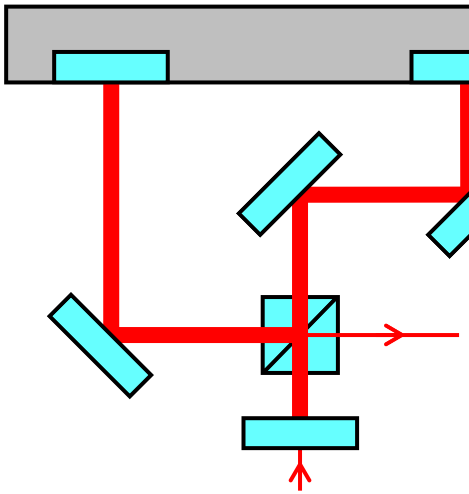
  Common mode rejection ratio between TM and OB: $\mathrm{CMRR} = 1/100$

- Cannot meet the requirement
  → discuss in the next section

- Numbers of optics: 5 mirrors, 1 BS

- PD: 1 PD

Power recycled MI



- Shot noise:

$$\delta\theta_{\mathrm{shot}} = \frac{1}{2L}\sqrt{\frac{\hbar c\lambda}{\pi G P_0}} = 5\times 10^{-16}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

Arm length: $L = 30\,\mathrm{cm}$

Laser wavelength: $\lambda = 1\,\mu\mathrm{m}$

Input laser power: $P_0 = 10\,\mathrm{mW}$

Power recycle gain: $G = 10$

- Thermal noise (bar): same as MI

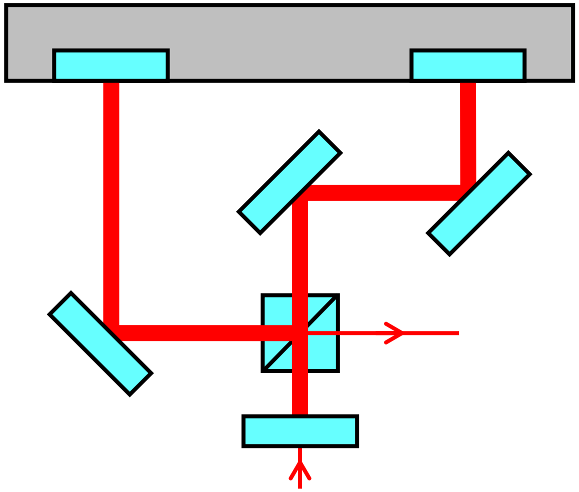
$$\delta\theta_{\mathrm{thermal}} = \frac{1}{L}\sqrt{\frac{4k_B T}{\omega}\frac{\phi_{\mathrm{bar}}}{\mu\omega_0^2}} = 1\times 10^{-17}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

$T = 4\,\mathrm{K}$ $\omega = 2\pi\times 0.1\,\mathrm{Hz}$ $\phi_{\mathrm{bar}} = 10^{-6}$

Frequency of second mode: $\omega_0 = 2\pi\times 10\,\mathrm{kHz}$

Reduced mass of second mode: $\mu = 0.3\,\mathrm{kg}$

Power recycled MI



- Seismic coupling: same as MI

$$\delta\theta_{\mathrm{seis}} = \frac{\theta_{\mathrm{mir}}}{L}\delta x_{\mathrm{seis}}\mathrm{CMRR} = 2\times 10^{-13}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

Angle of two mirrors: $\theta_{\mathrm{mir}} = 10^{-6}\,\mathrm{rad}$

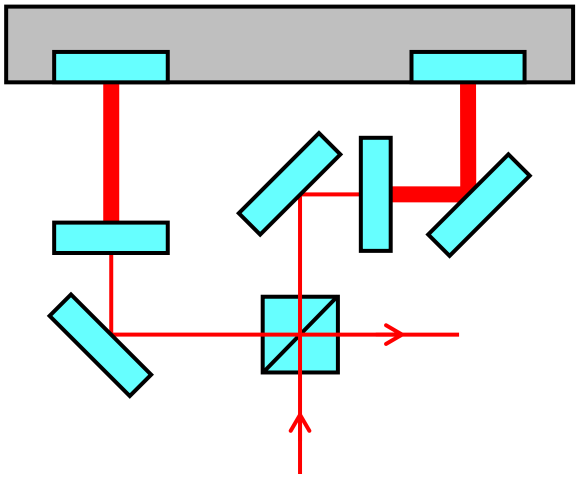
Arm length: $L = 30\,\mathrm{cm}$

Translational seismic noise:
$$\delta x_{\mathrm{seis}} = 5\times 10^{-6}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$$

Common mode rejection ratio
between TM and OB: $\mathrm{CMRR} = 1/100$

- Numbers of optics: 6 mirrors, 2 BS (1 for PDH), 2 lenses
- PD: 1 RFPD, 1 PD

Fabry–Pérot MI



- Shot noise:

$$\delta\theta_{\rm shot} = \frac{t^2}{8L}\sqrt{\frac{\hbar c\lambda}{\pi P_0}} = 4\times 10^{-18}\,{\rm rad}/\sqrt{\rm Hz}$$

Arm length: $L = 30\,{\rm cm}$

Laser wavelength: $\lambda = 1\,\mu{\rm m}$

Input laser power: $P_0 = 10\,{\rm mW}$

Transmittance of front mirrors: $t^2 = 1\%$ (End: 0%)

- Thermal noise (bar): same as MI

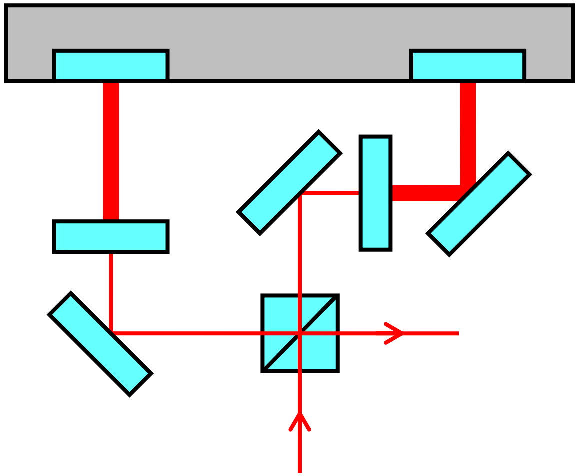
$$\delta\theta_{\rm thermal} = \frac{1}{L}\sqrt{\frac{4k_BT}{\omega}\frac{\phi_{\rm bar}}{\mu\omega_0^2}} = 1\times 10^{-17}\,{\rm rad}/\sqrt{\rm Hz}$$

$T = 4\,{\rm K}$ $\quad\omega = 2\pi\times 0.1\,{\rm Hz}$ $\quad\phi_{\rm bar} = 10^{-6}$

Frequency of second mode: $\omega_0 = 2\pi\times 10\,{\rm kHz}$

Reduced mass of second mode: $\mu = 0.3\,{\rm kg}$

Fabry–Pérot MI



- Seismic coupling: same as MI

$$\delta\theta_{\rm seis} = \frac{\theta_{\rm mir}}{L}\delta x_{\rm seis}{\rm CMRR} = 2\times10^{-13}\,{\rm rad}/\sqrt{\rm Hz}$$

Angle of two mirrors: $\theta_{\rm mir} = 10^{-6}\,{\rm rad}$

Arm length: $L = 30\,{\rm cm}$

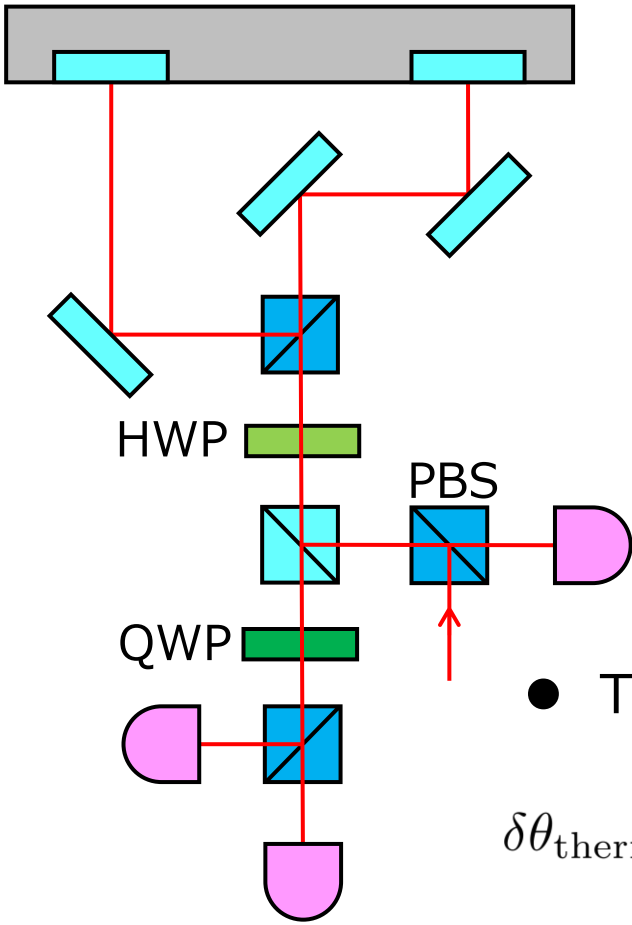
Translational seismic noise:

$$\delta x_{\rm seis} = 5\times10^{-6}\,{\rm m}/\sqrt{\rm Hz}$$

Common mode rejection ratio between TM and OB: ${\rm CMRR} = 1/100$

- Numbers of optics: 7 cavity mirrors, 2 BS, 2 lenses
- PD: 1 RFPD, 1 PD

Quadrature interferometer



- Shot noise: same as MI

$$\delta\theta_{\rm shot} = \frac{1}{2L}\sqrt{\frac{\hbar c\lambda}{\pi P_0}} = 2\times 10^{-15}\,{\rm rad}/\sqrt{\rm Hz}$$

Arm length: $L = 30\,{\rm cm}$

Laser wavelength: $\lambda = 1\,\mu{\rm m}$

Input laser power: $P_0 = 10\,{\rm mW}$

- Thermal noise (bar): same as MI

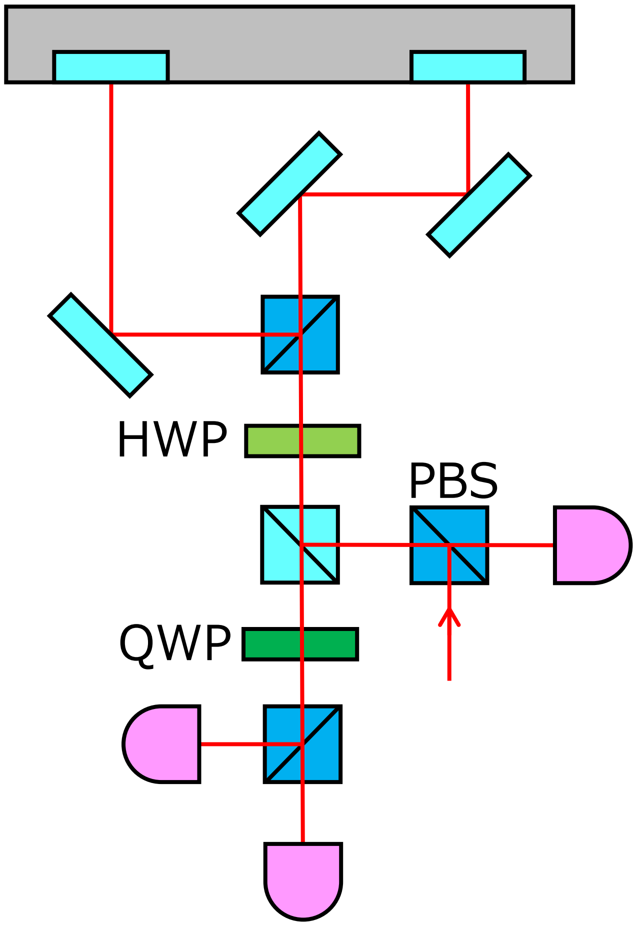
$$\delta\theta_{\rm thermal} = \frac{1}{L}\sqrt{\frac{4k_BT}{\omega}\frac{\phi_{\rm bar}}{\mu\omega_0^2}} = 1\times 10^{-17}\,{\rm rad}/\sqrt{\rm Hz}$$

$T = 4\,{\rm K}$ $\quad\omega = 2\pi\times 0.1\,{\rm Hz}$ $\quad\phi_{\rm bar} = 10^{-6}$

Frequency of second mode: $\omega_0 = 2\pi\times 10\,{\rm kHz}$

Reduced mass of second mode: $\mu = 0.3\,{\rm kg}$

Quadrature interferometer



- Seismic coupling: same as MI

$$\delta\theta_{\mathrm{seis}} = \frac{\theta_{\mathrm{mir}}}{L}\delta x_{\mathrm{seis}}\mathrm{CMRR} = 2\times 10^{-13}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

Angle of two mirrors: $\theta_{\mathrm{mir}} = 10^{-6}\,\mathrm{rad}$

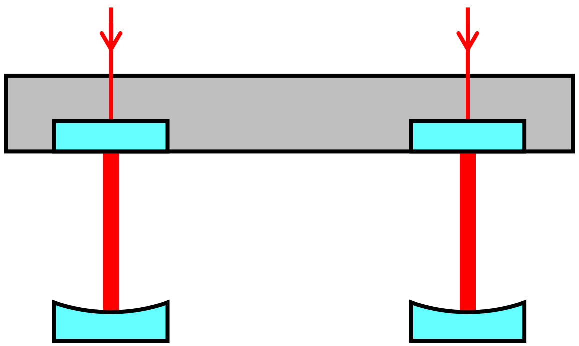
Arm length: $L = 30\,\mathrm{cm}$

Translational seismic noise:
$$\delta x_{\mathrm{seis}} = 5\times 10^{-6}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$$

Common mode rejection ratio
between TM and OB: $\mathrm{CMRR} = 1/100$

- Numbers of optics: 5 mirrors, 1 BS, 3 PBS, 1 QWP, 1 HWP
- PD: 3 PD

Differential Fabry–Pérot cavities



- Shot noise: same as FPMI

$$\delta\theta_{\mathrm{shot}} = \frac{t^2}{8L}\sqrt{\frac{\hbar c\lambda}{\pi P_0}} = 4\times 10^{-18}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

Arm length: $L = 30\,\mathrm{cm}$

Laser wavelength: $\lambda = 1\,\mu\mathrm{m}$

Input laser power: $P_0 = 10\,\mathrm{mW}$

Transmittance of front mirrors: $t^2 = 1\%$ (End: 0%)

- Thermal noise (bar): same as MI

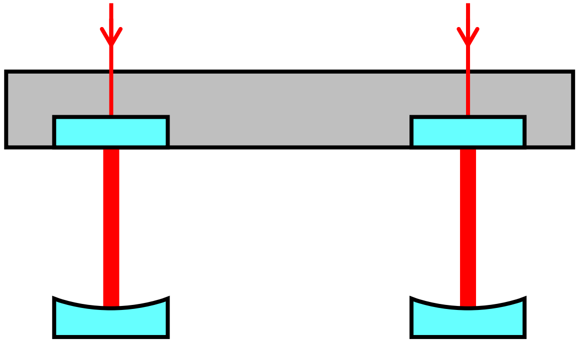
$$\delta\theta_{\mathrm{thermal}} = \frac{1}{L}\sqrt{\frac{4k_BT}{\omega}\frac{\phi_{\mathrm{bar}}}{\mu\omega_0^2}} = 1\times 10^{-17}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

$T = 4\,\mathrm{K}$ $\quad\omega = 2\pi\times 0.1\,\mathrm{Hz}$ $\quad\phi_{\mathrm{bar}} = 10^{-6}$

Frequency of second mode: $\omega_0 = 2\pi\times 10\,\mathrm{kHz}$

Reduced mass of second mode: $\mu = 0.3\,\mathrm{kg}$

Differential Fabry–Pérot cavities



- Seismic coupling: same as MI

$$\delta\theta_{\rm seis} = \frac{\theta_{\rm mir}}{L}\delta x_{\rm seis}{\rm CMRR} = 2\times 10^{-13}\,{\rm rad}/\sqrt{\rm Hz}$$

Angle of two mirrors: $\theta_{\rm mir} = 10^{-6}\,{\rm rad}$

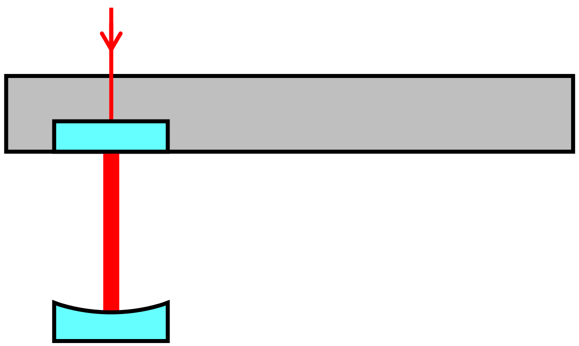
Arm length: $L = 30\,{\rm cm}$

Translational seismic noise:
$$\delta x_{\rm seis} = 5\times 10^{-6}\,{\rm m}/\sqrt{\rm Hz}$$

Common mode rejection ratio
between TM and OB: ${\rm CMRR} = 1/100$

- Numbers of optics: 4 mirrors, 2 BS, 2 lenses
- PD: 2 RFPD

Wavefront sensor



- Shot noise:

$$\delta\theta_{\mathrm{shot}} = \frac{1}{8J_0J_1w}\sqrt{\frac{2\hbar c\lambda}{P_0}} = 1\times10^{-11}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

$J_0 = 1 \qquad J_1 = 0.05$

Beam radius: $w = 300\,\mu\mathrm{m}$

Laser wavelength: $\lambda = 1\,\mu\mathrm{m}$

Input laser power: $P_0 = 10\,\mathrm{mW}$

- Thermal noise (mirror substrate): same as oplev

$$\delta\theta_{\mathrm{thermal}} = \sqrt{\frac{2k_BT}{\pi^{3/2}f}\frac{1-\nu_s^2}{E_s}\frac{\phi_s}{w^3}} = 7\times10^{-17}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

$f = 0.1\,\mathrm{Hz} \qquad w = 100\,\mu\mathrm{m} \qquad T = 4\,\mathrm{K}$

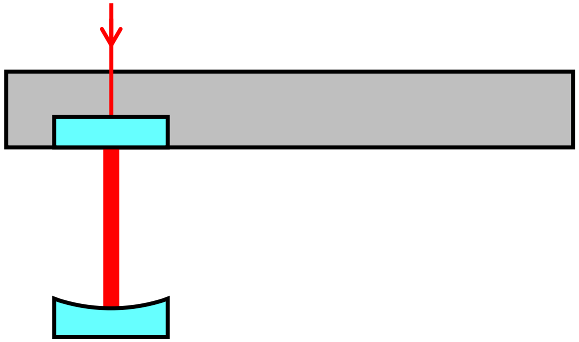
Substrate: fused silica

Young's modulus: $E_s = 7.4\,\mathrm{GPa}$

Poisson's ratio: $\nu_s = 0.17$

Loss angle: $\phi_s = 10^{-8}$

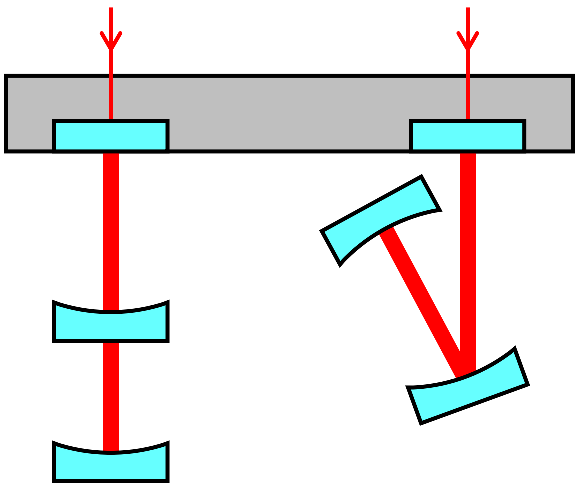
Wavefront sensor



- Seismic coupling:
  - In principle, no coupling by using a flat mirror and sufficient PDH (length) control
  - Non-linear coupling?

- Numbers of optics: 2 cavity mirrors, 1 BS, 2 lenses
- PD: 1 RFQPD

Coupled or Folded WFS



- Shot noise:

$$\delta\theta_{\rm shot} = \frac{1}{8J_0 J_1 \mathcal{F} w}\sqrt{\frac{2\hbar c\lambda}{P_0}} = 7\times 10^{-14}\,{\rm rad}/\sqrt{\rm Hz}$$

$J_0 = 1 \qquad J_1 = 0.05$

Beam radius: $w = 300\,\mu{\rm m}$

Laser wavelength: $\lambda = 1\,\mu{\rm m}$

Input laser power: $P_0 = 10\,{\rm mW}$

Finesse: $\mathcal{F} = 300$

- Thermal noise (mirror substrate): same as oplev

$$\delta\theta_{\rm thermal} = \sqrt{\frac{2k_B T}{\pi^{3/2} f}\frac{1-\nu_s^2}{E_s}\frac{\phi_s}{w^3}} = 7\times 10^{-17}\,{\rm rad}/\sqrt{\rm Hz}$$

$f = 0.1\,{\rm Hz} \qquad w = 100\,\mu{\rm m} \qquad T = 4\,{\rm K}$

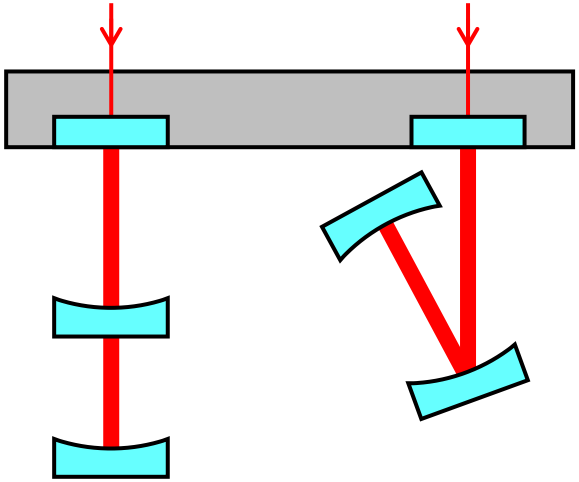
Substrate: fused silica

Young's modulus: $E_s = 7.4\,{\rm GPa}$

Poisson's ratio: $\nu_s = 0.17$

Loss angle: $\phi_s = 10^{-8}$

Coupled or Folded WFS



- Seismic coupling:
  - In principle, no coupling by using a flat mirror and sufficient PDH (length) control
  - Non-linear coupling?

- Numbers of optics: 3 cavity mirrors, 1 BS, 2 lenses
- PD: 1 RFQPD

Comparison of optical systems

- Oplev, WFS, CWFS cannot meet the shot noise requirement
- WFS cannot meet even with KAGRA parameter Aso+ (2013)
- CWFS will meet with large beam and high finesse

Unit: [rad/√Hz]	Oplev	MI	PRMI	FPMI	QI	Cavities	WFS	CWFS, FWFS
Shot noise	2×10^{-12}	2×10^{-15}	5×10^{-16}	4×10^{-18}	2×10^{-15}	4×10^{-18}	1×10^{-11}	7×10^{-14}
Thermal noise	7×10^{-17}	1×10^{-17}	1×10^{-17}	1×10^{-17}	1×10^{-17}	1×10^{-17}	7×10^{-17}	7×10^{-17}
Seismic coupling	2×10^{-12}	2×10^{-13}	2×10^{-13}	2×10^{-13}	2×10^{-13}	2×10^{-13}	No coupling	No coupling
Numbers of optics	1	6	10	11	11 polarizing	8	5	6
PD	QPD	PD	PD, RFPD	PD, RFPD	3PD	2RFPD	RFQPD	RFQPD
Experience in Ando Lab	Shimosd-san's D	Shoda-san, Shimoda-san's M	Aritomi-san's M	Nothing	Okada-san	ANU, Takano-san's D	Nothing	Nothing

Comparison of optical systems

- I give them rough scores to make the ranking
- Michelson interferometer: very simple
- Two cavities: sensitive and quite simple

Unit: [rad/√Hz]	MI	PRMI	FPMI	QI	Cavities
Shot noise	2×10^{-15} (1)	5×10^{-16} (2)	4×10^{-18} (3)	2×10^{-15} (1)	4×10^{-18} (3)
Thermal noise	1×10^{-17}	1×10^{-17}	1×10^{-17}	1×10^{-17}	1×10^{-17}
Seismic coupling	2×10^{-12}	2×10^{-13}	2×10^{-13}	2×10^{-13}	2×10^{-13}
Numbers of optics	6 (5)	10 (3)	11 (2)	11 polarizing (1)	8 (4)
PD	PD (3)	PD, RFPD (2)	PD, RFPD (2)	3PD (2)	2RFPD (1)
Experience in Ando Lab	Shoda-san, Shimoda-san's M (3)	Aritomi-san's M (2)	Nothing (1)	Okada-san (2)	ANU, Takano-san's D (3)
	12	9	8	6	11

My concerning points

- MI is simple, easy to operate and sensitive, so it makes sense that it has been used in many exp. so far
- Most sensitive exp.: MINT?

  2×10^{-10} rad/√Hz at 0.2-30 Hz at room temp.

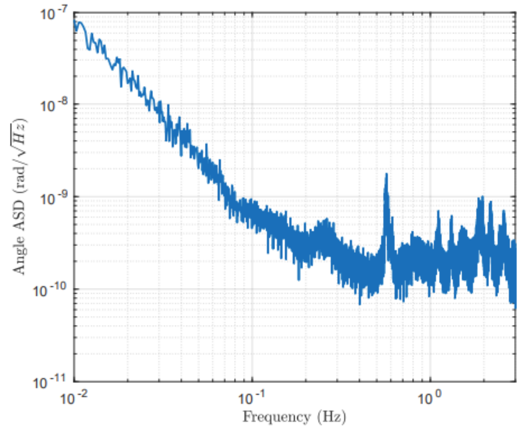


FIG. 7: Amplitude spectral density of the inertial angular noise.

Ross+ (2021)

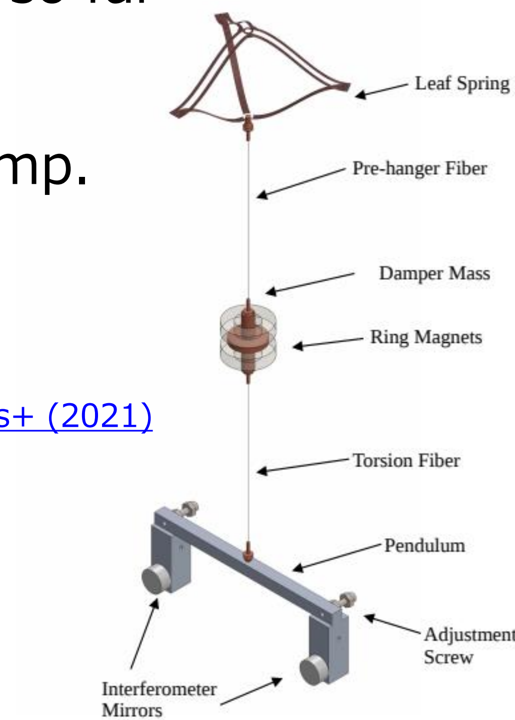


FIG. 2: CAD rendering of the MINT pendulum and eddy current damper assembly.

- Is MI not impactful enough for Ph.D.?

  Difficult if not the highest sensitivity in the world?
- Is it first in the world to build a torsion pendulum with cryo. temp. and two cavities to readout angles?

Contents

Two kinds of seismic noise

- We should take into account of two kinds of seismic noise to design TOBA's suspension

- Rotational seismic noise
  - Assuming 10^{-9} rad/$\sqrt{\text{Hz}}$ at 0.1 Hz
  - Need to be isolated to 10^{-15} rad/$\sqrt{\text{Hz}}$
    (= reduce 6 orders of magnitude)

- Translational seismic noise
  - 5×10^{-6} m/$\sqrt{\text{Hz}}$ at 0.1 Hz
  - Need to reduce the coupling (transfer function) to $<10^{-10}$ rad/m

Rotational seismic noise

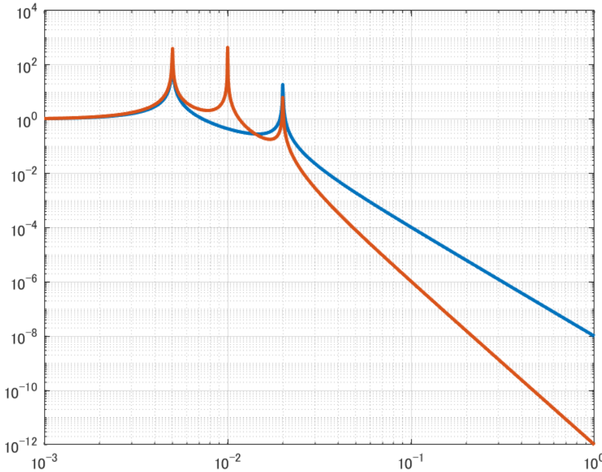
- Requirement: reduce 6 orders of magnitude

- Multi-stage torsion pendulum can isolate torsional seismic noise
- 3-stage pendulum: 6 orders of magnitude (enough)
- 2-stage pendulum: 4 orders of magnitude
  - Isolate with CMRR for two more orders of magnitude
  - CMRR = 1/100

    → Match resonant freq. of TM and OB with the accuracy of 0.5%

    → Moment of inertia 1% (10 g scale)

$$\mathrm{CMRR} \sim \frac{2\Delta f}{f_{\mathrm{yaw}}} \qquad f_{\mathrm{yaw}} = \frac{1}{2\pi}\sqrt{\frac{\pi E d^4}{64 l (1+\nu) I}}$$



Translational seismic noise (1)

- Requirement: coupling < 10^{-10} rad/m
- Coupling path 1: tilt of the two mirrors

$$\delta\theta_{\mathrm{seis}} = \frac{\theta_{\mathrm{mir}}}{L}\delta x_{\mathrm{seis}}\mathrm{CMRR} = 2\times10^{-13}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$$

Angle of two mirrors: $\theta_{\mathrm{mir}} = 10^{-6}\,\mathrm{rad}$

Arm length: $L = 30\,\mathrm{cm}$

Translational seismic noise:

$$\delta x_{\mathrm{seis}} = 5\times10^{-6}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$$

Common mode rejection ratio
between TM and OB: $\mathrm{CMRR} = 1/100$

↓

Requirement: $\mathrm{CMRR} < 10^{-4}$

↓

Match the distance from the clamping point to the center of gravity for TM and OB with an accuracy of 1 mm

Translational seismic noise (2)

- Requirement: coupling < 10^{-10} rad/m
- Coupling path 2: shift of center of gravity and suspension point

Requirement: $\Delta\phi_P < 2.5 \times 10^{-9}$

(Shimoda-san's M thesis)

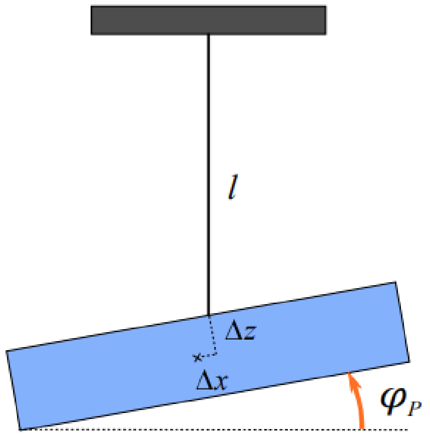
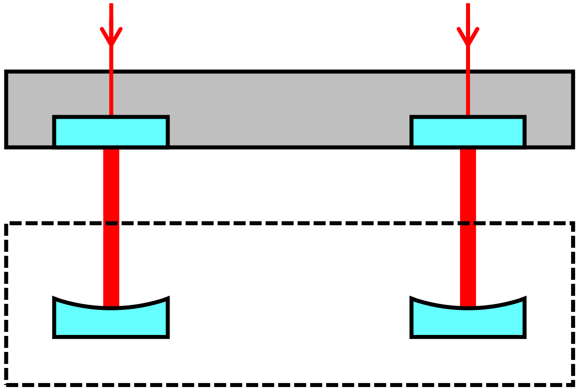


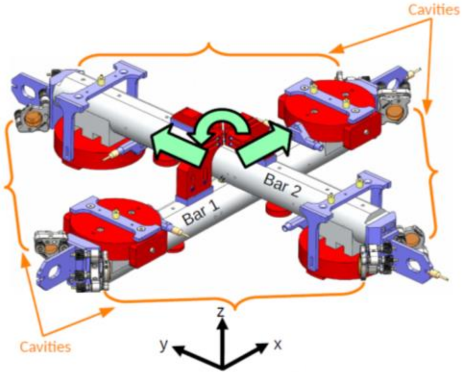
図 4.6: 重心と懸架点のズレ

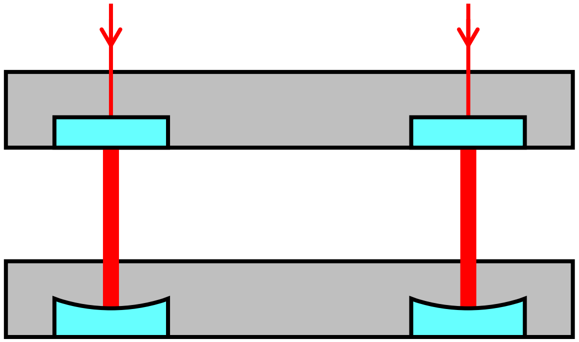
- How to reduce
  - Counter-weight
  - Actuate
  - But feasibility is not clear…

Two cavities: optical bench



Reference:
optical bench or another test mass?



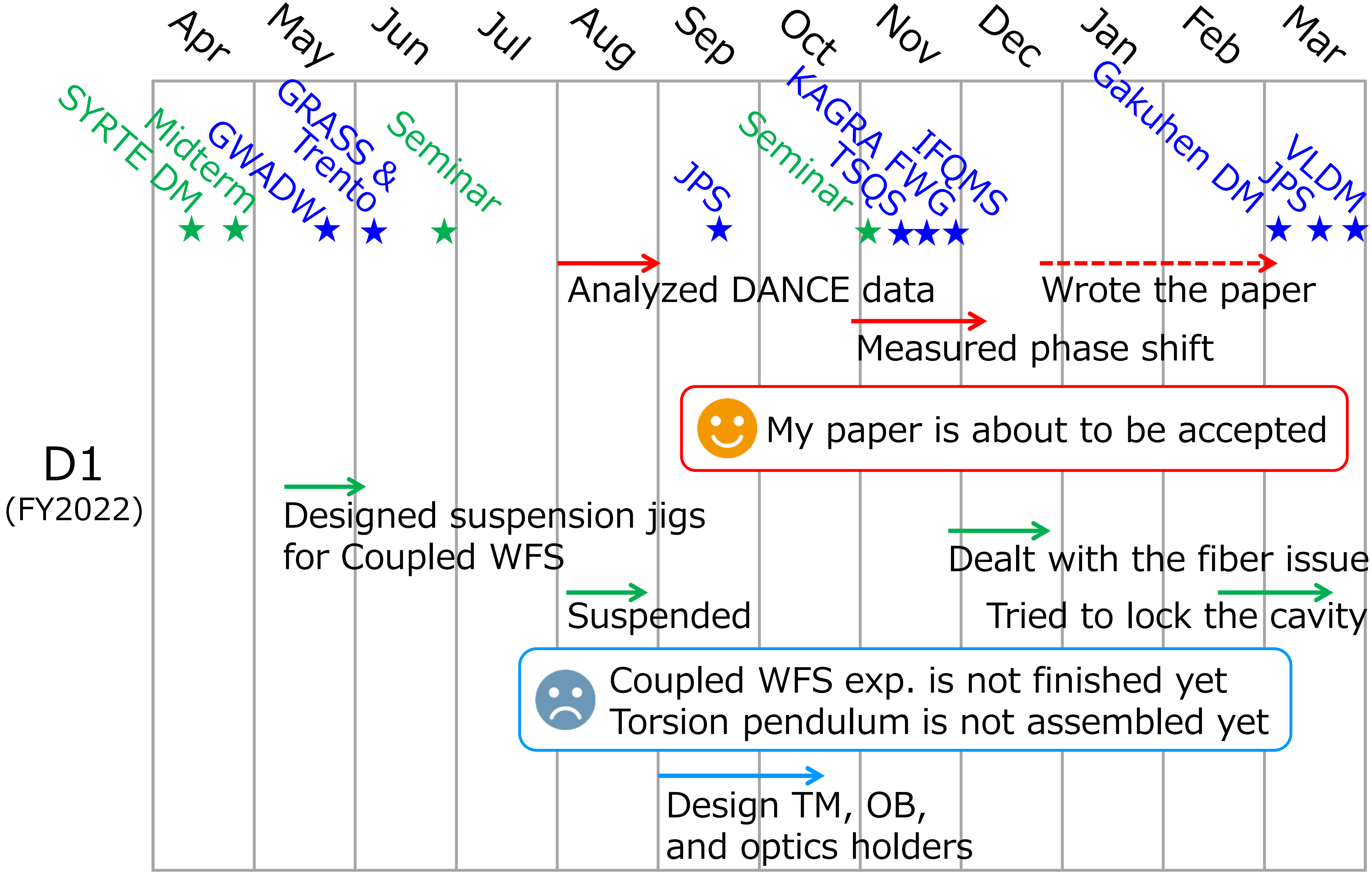


	Optical bench	Another test mass
Experience in Ando Lab	Many (All TOBA)	Few (Ono-kun's M, Komori-san's D?)
Common mode rejection	Difficult because test masses are different	Easier because the same two suspension

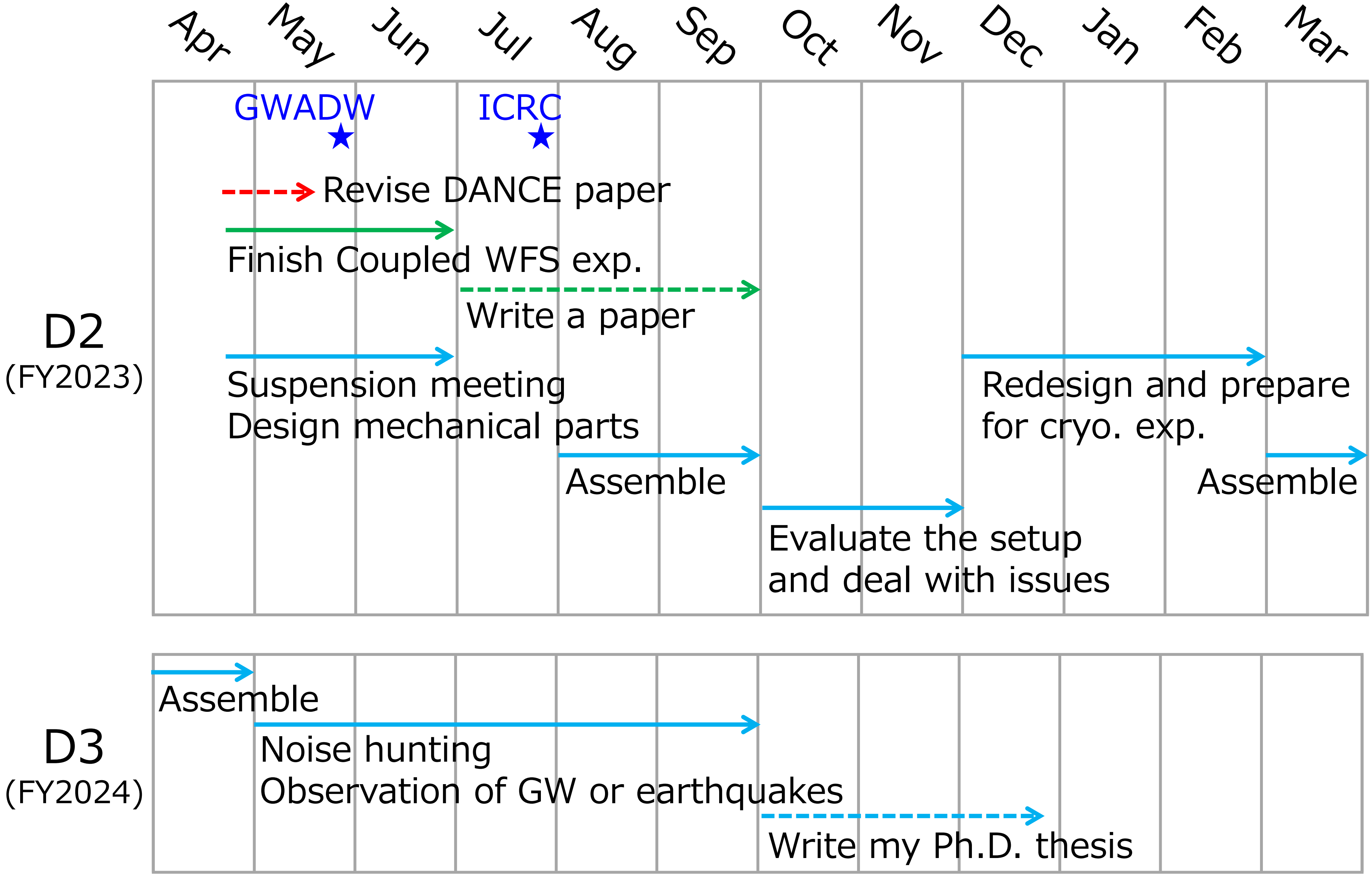
Contents

- Comparison of the readout optical systems
- Requirement of the suspension system
- Timeline for the next two years

Reflections on FY2022



Plans of the next two years



Summary

- Comparison of the readout optical systems
  - MI is quite simple, but two cavities are suitable for my Ph.D.?
- Requirement of the suspension system
- Timeline for the next two years
  - I should start the engine to finish the experiment in a year and a half

また1年間よろしくお願いします