

# **Stellar-mass black holes in young massive and open stellar clusters and their role in gravitational-wave generation I -IV**

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# Why star clusters?



Cancelled due to COVID-19



I saw beautiful すばる with the naked eye in Kiso  
Star clusters are my favorites

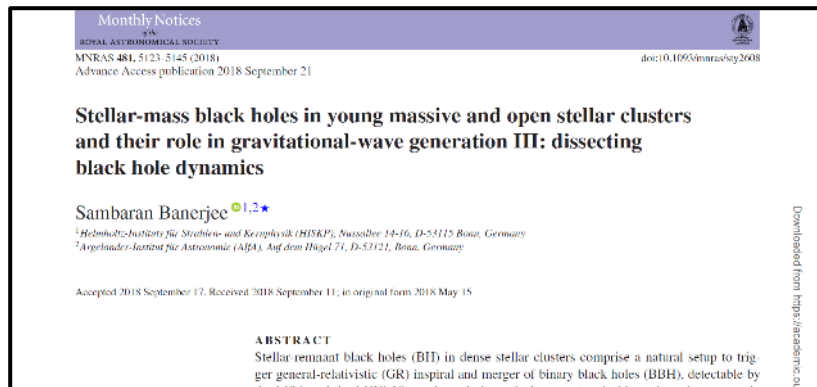
→ I looked for papers about “star clusters × GWs”

 not galaxy clusters

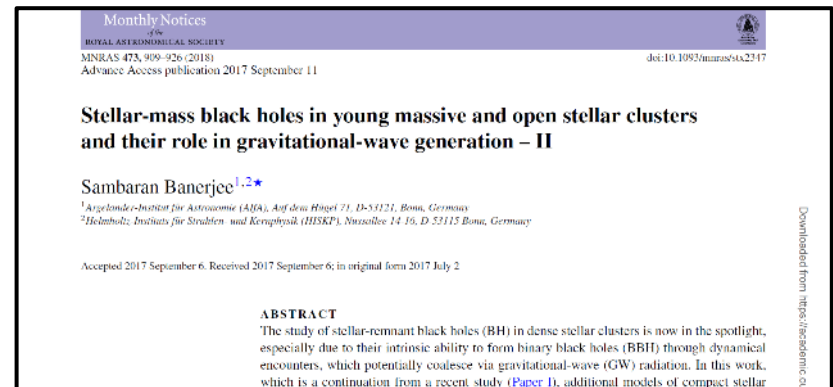
# Today's papers

## Stellar-mass black holes in young massive and open stellar clusters and their role in gravitational-wave generation by [Sambaran Banerjee](#)

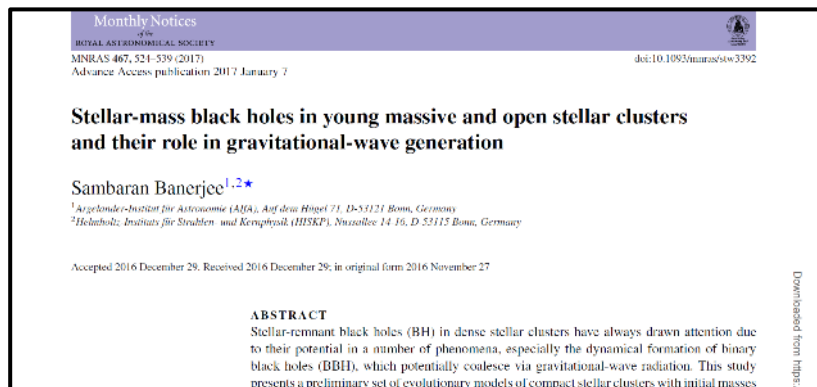
I : [MNRAS 467, 524–539 \(2017\)](#)



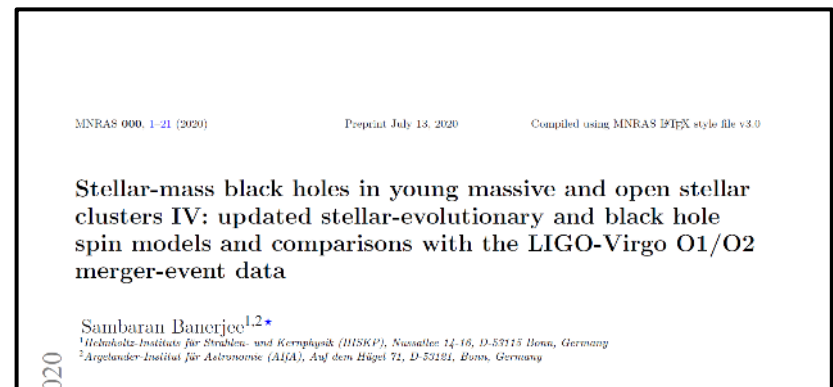
II : [MNRAS 473, 909–926 \(2018\)](#)



III : [MNRAS 481, 5123–5145 \(2018\)](#)



IV : [arXiv:2004.07382v3](#)



# Summary

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- In general, star clusters are classified into open clusters and globular clusters by their age
- Many models of young massive and open clusters are prepared
- The dynamical formation of BBHs in clusters are simulated by N-body evolution program
- The results are similar to LIGO-Virgo data

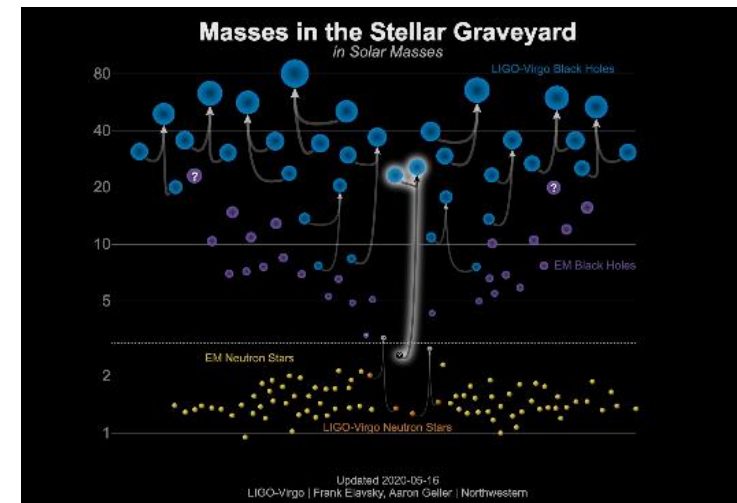
# Contents

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- Introduction
  - Classification of star clusters
- Methods
  - N-body evolution program: NBODY7
  - Natal kicks & merger kicks
  - Natal spins & final spins
  - Primordial-binary fraction
  - Initial conditions of model clusters
- Results
  - Basic quantities
  - Comparisons with LIGO-Virgo data
  - Interesting phenomena

# Introduction

- The study of dynamical interactions of BHs in star clusters has been started since 1993 ([Nature 364, 421–423](#))
- BH mass detected by LIGO is typically  $10 - 100 M_{\odot}$
- The scenario of formation of stellar-mass BBHs in star clusters is simple and easy



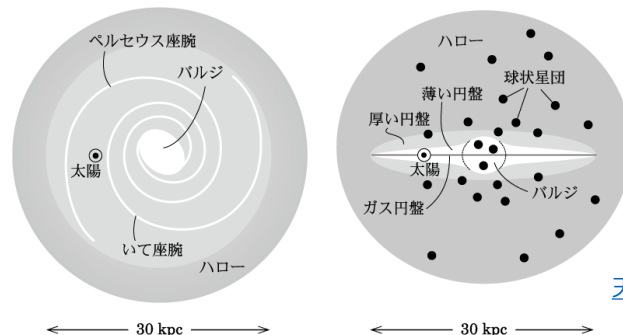
# Classification of star clusters

	Age	Total mass	Number of stars	Metallicity	Shape & Location	Example
Young massive cluster	$\lesssim 100 \text{ Myr}$	$10^4 M_{\odot} - 10^5 M_{\odot}$	—	$\gtrsim 1 Z_{\odot}$	—	—
Open cluster	$\lesssim 300 \text{ Myr}$	$\lesssim 10^4 M_{\odot}$	$10^2 - 10^3$	$\sim 1 Z_{\odot}$	Irregular, Disk	Pleiades, M45, すばる <small>©Antonio Fernandez-Sanchez</small>
Globular cluster	$\gtrsim 10 \text{ Gyr}$	$10^5 M_{\odot} - 10^6 M_{\odot}$	$10^5 - 10^6$	$\lesssim 1 Z_{\odot}$	Globe, Halo	Omega Centauri, NGC 5139 <small>©Martin Pugh</small>

Definition of the papers

$$\text{Metallicity : } Z = \sum_{i>\text{He}} \frac{m_i}{M_{\text{total}}}$$

$(Z_{\odot} = 0.02)$

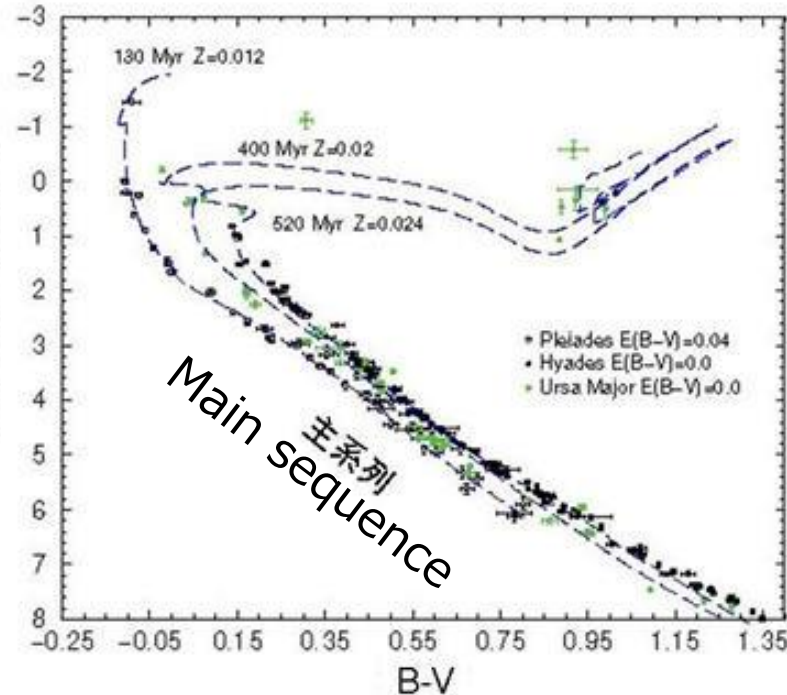


天文学辞典

# Star cluster's age

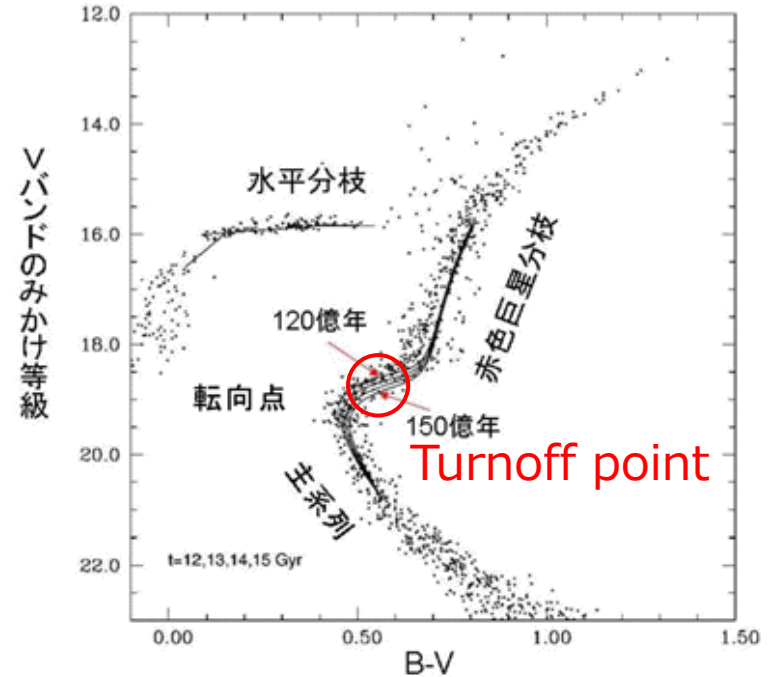
- Make Hertzsprung-Russell Diagram
- Estimate cluster's age by location of turnoff point

Open cluster (young)



blue ← color → red  
(or temperature, spectrum type)

Globular cluster (old)



[理科年表オフィシャルサイト](#)

- Born in the same place and time  
→ Separate in long time
- The way to be born and evolve is not understood well

# NBODY7

- NBODY7:  
N-body evolution program,  
descendant of NBODY6

TABLE 1  
MAIN *N*-BODY CODES

Keyword	Period	Name
Primitive beginnings .....	1961–1969	NBODY1
Two-body regularization .....	1969–1974	NBODY3
Cosmological experiments .....	1974–1983	NBODY2
Star cluster simulations .....	1979–1992	NBODY5
Hermite integration .....	1993–1999	NBODY6
The HARP challenge .....	1994–1999	NBODY4

[S. J. Aarseth \(1999\)](#)

- 4th-order Hermite integrator
- Neighbor-based scheme
- Post-Newtonian approximation

# 4th-order Hermite integrator

- ① Predict positions and velocities of all particles at time  $t_{n+1} = t_n + \Delta t$

$$\begin{aligned} \mathbf{x}_{p,j} &= \mathbf{x}_{n,j} + \mathbf{v}_{n,j}\Delta t + \mathbf{a}_{n,j}\frac{\Delta t^2}{2} + \dot{\mathbf{a}}_{n,j}\frac{\Delta t^3}{6} + O(\Delta t^4) \\ \mathbf{v}_{p,j} &= \mathbf{v}_{n,j} + \mathbf{a}_{n,j}\Delta t + \dot{\mathbf{a}}_{n,j}\frac{\Delta t^2}{2} \end{aligned}$$

predictor      particle  $j$       time  $t_n$

- ② Calculate the acceleration and its time derivative for particle  $i$  at time  $t_{n+1}$  using predictors

$$\begin{aligned} \mathbf{a}_{n+1,i} &= \sum_j Gm_j \frac{\mathbf{r}_{ij}}{(r_{ij}^2 + \varepsilon^2)^{3/2}} \\ \dot{\mathbf{a}}_{n+1,i} &= \sum_j Gm_j \left[ \frac{\mathbf{v}_{ij}}{(r_{ij}^2 + \varepsilon^2)^{3/2}} + \frac{3(\mathbf{v}_{ij} \cdot \mathbf{r}_{ij})\mathbf{r}_{ij}}{(r_{ij}^2 + \varepsilon^2)^{5/2}} \right] \end{aligned}$$

where  $\mathbf{r}_{ij} = \mathbf{x}_{p,j} - \mathbf{x}_{p,i}$ ,  $\mathbf{v}_{ij} = \mathbf{v}_{p,j} - \mathbf{v}_{p,i}$   
 $\varepsilon$  : softening parameter

# 4th-order Hermite integrator

- ③ Construct 3rd-order Hermite interpolation polynomial


$$\mathbf{a}_i(t) = \mathbf{a}_{n,i} + \dot{\mathbf{a}}_{n,i}(t - t_n) + \mathbf{a}_{n,i}^{(2)} \frac{(t - t_n)^2}{2} + \mathbf{a}_{n,i}^{(3)} \frac{(t - t_n)^3}{6}$$

where  $\mathbf{a}_{n,i}^{(2)} = \frac{-6(\mathbf{a}_{n,i} - \mathbf{a}_{n+1,i}) - \Delta t(4\dot{\mathbf{a}}_{n,i} + 2\dot{\mathbf{a}}_{n+1,i})}{\Delta t^2}$

$$\mathbf{a}_{n,i}^{(3)} = \frac{12(\mathbf{a}_{n,i} - \mathbf{a}_{n+1,i}) + 6\Delta t(\dot{\mathbf{a}}_{n,i} + \dot{\mathbf{a}}_{n+1,i})}{\Delta t^3}$$

- ④ Integrate  $\mathbf{a}_i(t)$  from  $t_n$  to  $t_{n+1}$  to obtain correctors and set  $\mathbf{x}_{n+1,i} = \mathbf{x}_{c,i}$  and  $\mathbf{v}_{n+1,i} = \mathbf{v}_{c,i}$

$$\begin{aligned} \mathbf{x}_{c,i} &= \mathbf{x}_{p,i} + \mathbf{a}_{n,i}^{(2)} \frac{\Delta t^4}{24} + \mathbf{a}_{n,i}^{(3)} \frac{\Delta t^5}{120} \\ \mathbf{v}_{c,i} &= \mathbf{v}_{p,i} + \mathbf{a}_{n,i}^{(2)} \frac{\Delta t^3}{6} + \mathbf{a}_{n,i}^{(3)} \frac{\Delta t^4}{24} \end{aligned}$$

corrector 

- ⑤ Update  $t_n$  and go back to step ①

# Neighbor-based scheme

- Neighbor-based scheme is used in order to ease computing time
- Each CPU stores a copy of its local checkpoint in the memory of its neighbor CPU
- Whenever a CPU fails, the lost local checkpoint data can be recovered from its neighbor CPU

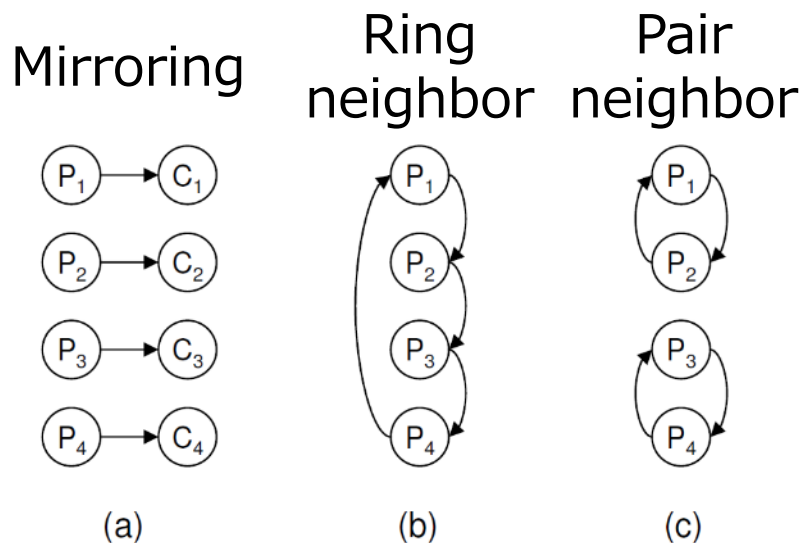


Figure 1: Neighbor-Based Schemes

[Z. Chen+ \(2005\)](#)

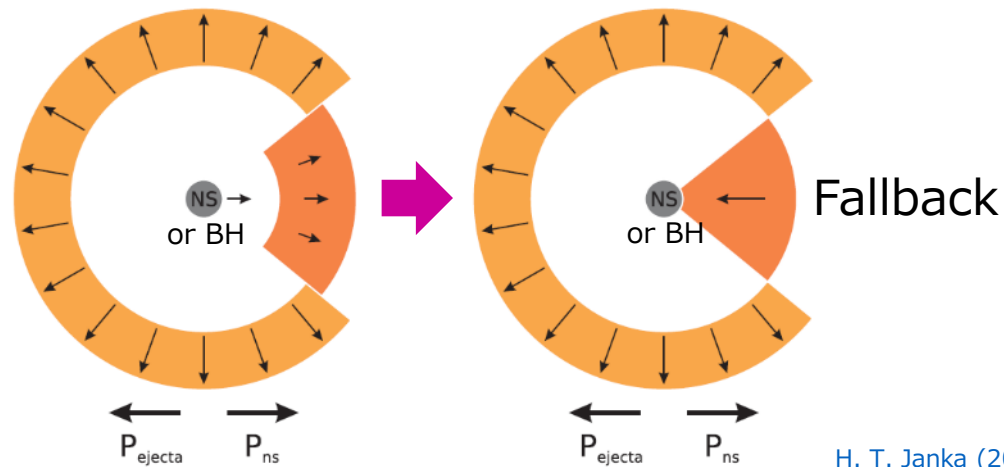
# Post-Newtonian approximation

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- Post-Newtonian approximation:
  - an approximate solution of Einstein's equation in the case of weak field
- Starting at Newton's law of gravity, higher order terms can be added to increase accuracy
  - Expansion parameter:  $\varepsilon = \left(\frac{v}{c}\right)^2 \ll 1$
  - PN-1, PN-2: GR periastron precession
  - PN-2.5: orbital shrinking due to GW radiation
  - PN-3, PN-3.5: spin-orbit coupling
- PN terms up to the order 2.5 are applied
- BH spins are taken to be zero for the economy in computing time

# Remnant masses

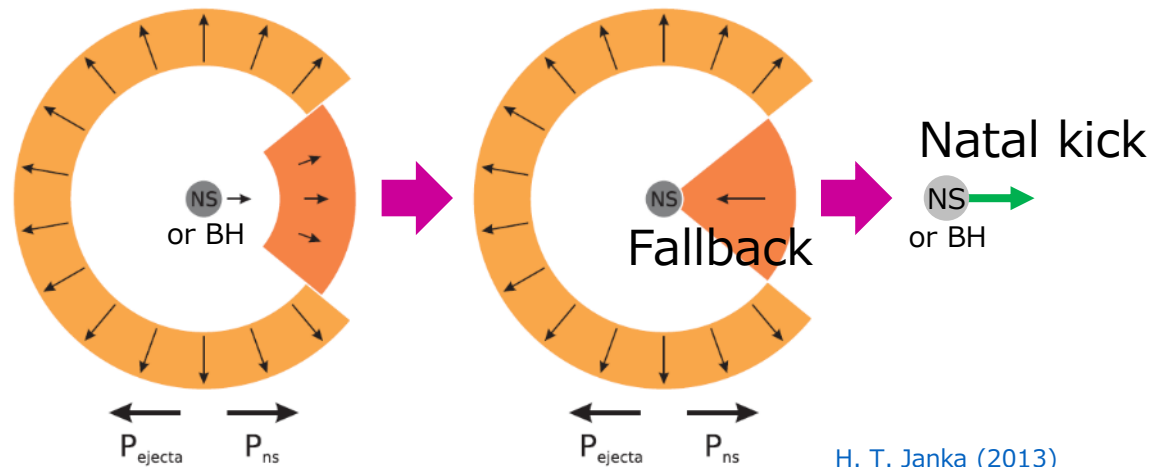
- The remnant (NS or BH) masses are determined by
  - Wind mass-loss until core collapse
  - CO and FeNi core mass
  - The amount of material fallback
- “Fallback”:  
during supernovae, some of the stellar material does not receive enough energy to escape the potential of NS and it falls back on to the core



[H. T. Janka \(2013\)](#)

# Natal kicks

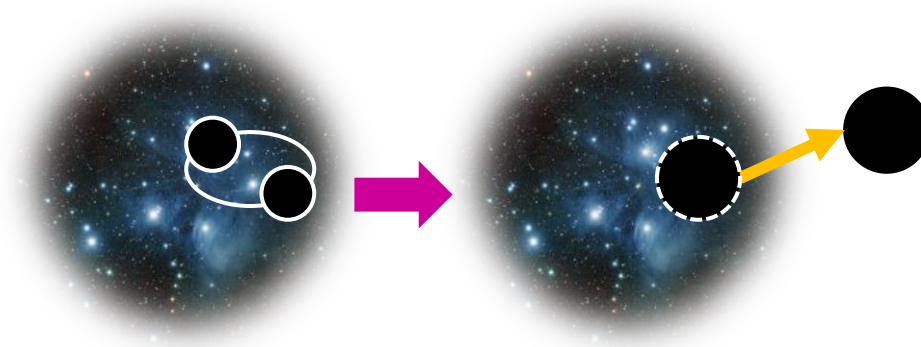
- Asymmetric mass ejection will lead to material fallback and natal kicks
- If NSs and BHs receive a high natal kick velocity, they will escape from the host clusters



# Merger kicks ( I - III )

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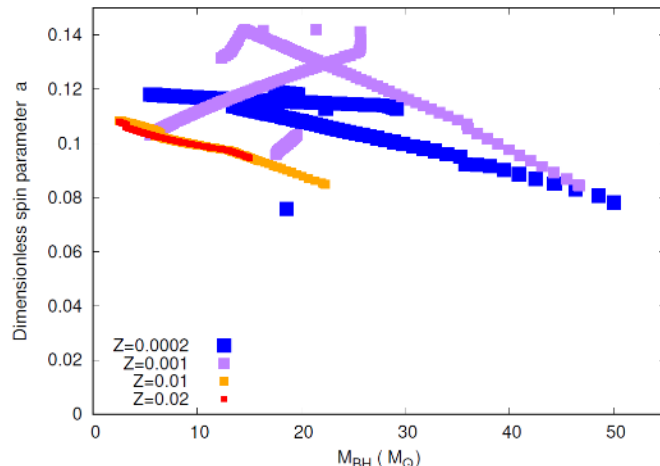
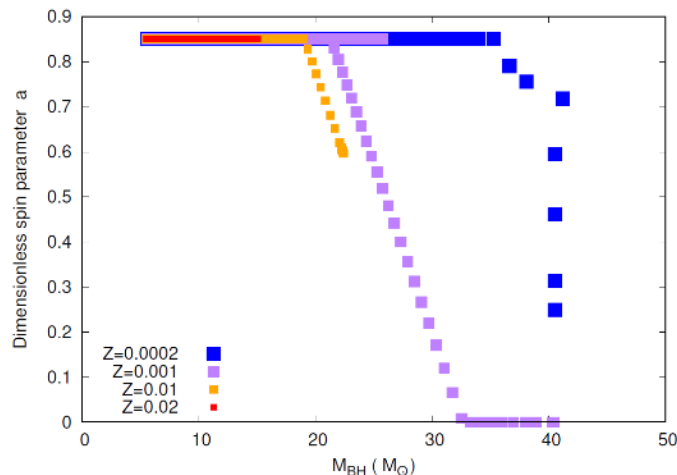
- PN-2.5  $\rightarrow$  BH spins are not taken
- In reality, when BHs have spins, BBHs will receive a large GW merger kick (100 – 1000 km/s) during inspiral phase
- Merged BH will escape from the cluster
- A kick velocity is applied on to the merged BH immediately after a coalescence in order to eject the merged BH out of the cluster



# Natal spins (IV)

- BH spins are applied in IV
- Geneva BH-spin model
  - Not include magnetic field
  - Angular momentum transport from core to envelope is purely convective
  - BHs have a high spin ( $a = 0.85$ )
- MESA BH-spin model
  - Include magnetic field
  - Angular momentum transport from core to envelope is much more efficient
  - BHs have a small spin ( $a = 0.1$ )

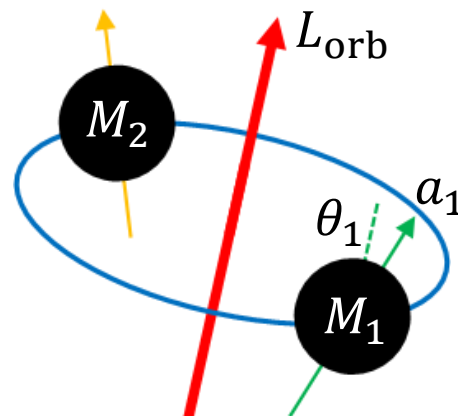
$$\text{Dimensionless spin parameter } a = \frac{cS_{\text{BH}}}{GM_{\text{BH}}^2}$$



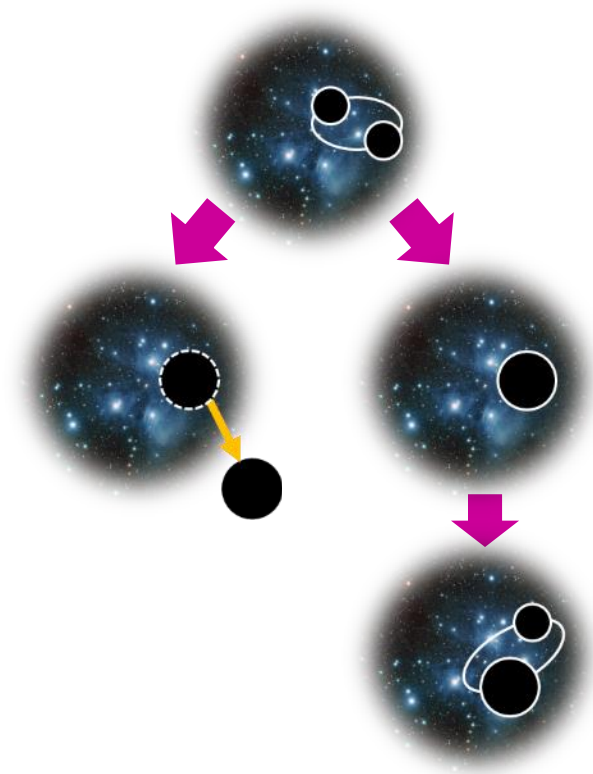
# Final spins & merger kicks (IV)

- For all mergers, effective spin parameter is evaluated

$$\chi_{\text{eff}} = \frac{M_1 a_1 \cos \theta_1 + M_2 a_2 \cos \theta_2}{M_1 + M_2}$$



- Orbital angular momentum  $L_{\text{orb}}$  and natal spins  $a_1, a_2$  determine merger kick velocity  $v_k$
- Merger kick velocity  $v_k$  determines whether the merged BH should stay in the clusters or escape
- If the BH stays, it is allowed for second-generation BBH mergers



# Primordial-binary fraction ( II -IV)

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- Binary stars are initially included
- This makes the models more realistic
- However, the system will become more complicated  
→ A reduction in computation is needed
- All stars are ZAMS (Zero-Age Main Sequence) stars
- Initial masses of stars are distributed over  $0.08 - 150 M_{\odot}$
- For  $M_{\text{ZAMS}} \geq 16 M_{\odot}$  (O-type stars)  
primordial-binary fraction  $f_{\text{bin},\text{O}} \approx 100 \%$
- For  $M_{\text{ZAMS}} < 16 M_{\odot}$   
primordial-binary fraction  $f_{\text{bin}} \approx 2 - 50 \%$
- The population of O-type stars is much smaller than the total stellar population ( $N_{\text{O}} \ll N_{\text{total}}$ )
- Note that  $f_{\text{bin}}$  represents the overall primordial-binary fraction ( $f_{\text{bin}} \approx f_{\text{bin},\text{total}}$ )

# Initial conditions of model clusters

Paper	Number of models	Total mass [ $M_{\odot}$ ]	Half-mass radius [pc]	Metallicity [ $Z_{\odot}$ ]	Primordial -binary fraction
I	12	$7 \times 10^3 - 5 \times 10^4$	1, 2	0.05 – 1.0	0
II	23 (include I)	$7 \times 10^3 - 1 \times 10^5$	1, 2	0.05 – 1.0	0, 0.02, 0.05, 0.1
III	22	$7.5 \times 10^3 - 5 \times 10^4$	1, 1.5, 2	0.05 – 1.0	0.05, 0.1, 0.3, 0.5
IV	65 (spins)	$1 \times 10^4 - 1 \times 10^5$	1, 1.5, 2, 3	0.0001 – 0.02	0, 0.05, 0.1

- Initial conditions are typical for young massive and open clusters
- All stars in the clusters are initially unsegregated
- All models are evolved for 10 – 13.7 Gyr or until they dissolve completely
- Half-mass radius: the radius that contains half the total mass (c.f. Half-light radius)

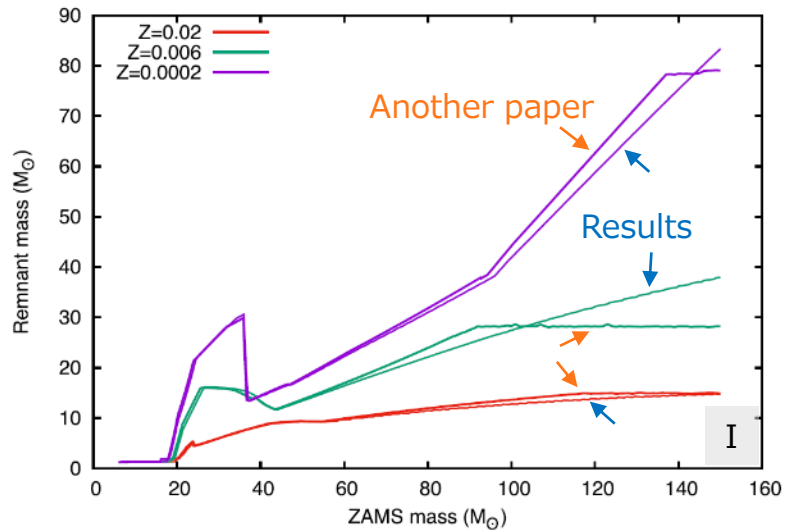
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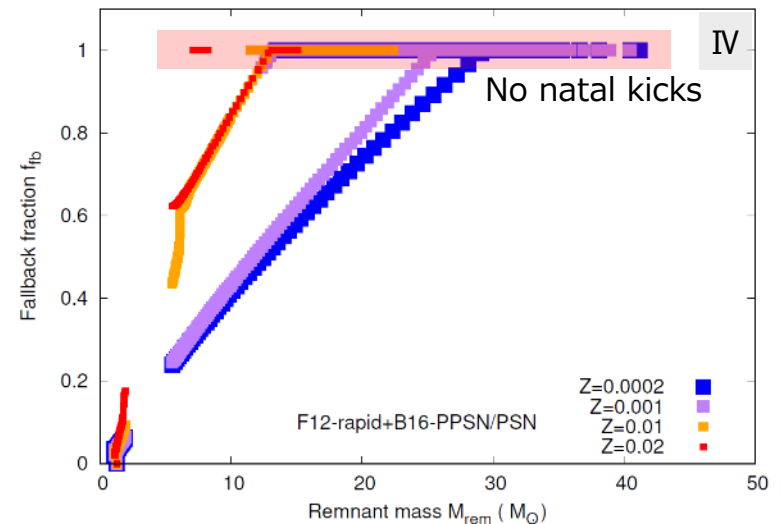
- I : not important
- II : large total mass, small primordial-binary fraction
- III : large primordial-binary fraction
- IV : many models, spins, low metallicity

# Basic quantities

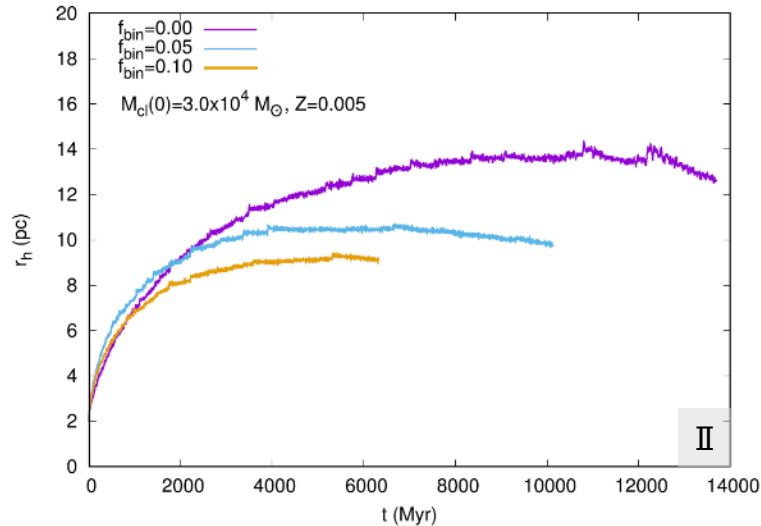
Remnant mass – ZAMS mass



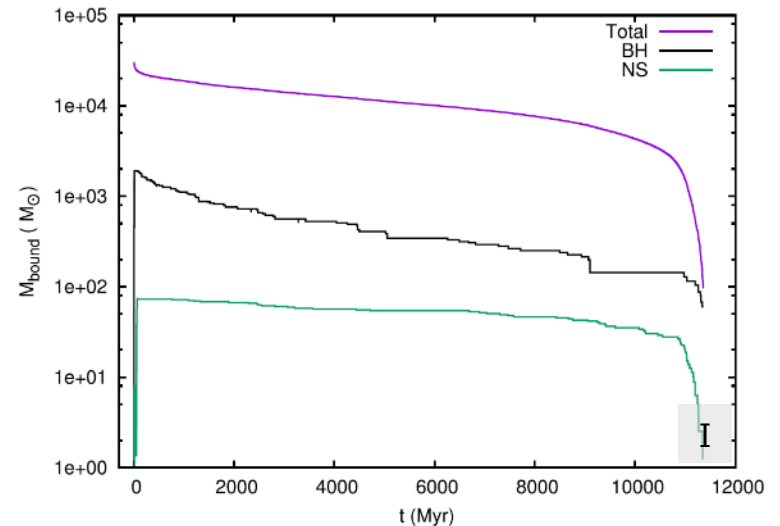
Fallback fraction - remnant mass



Evolution of half-mass radius

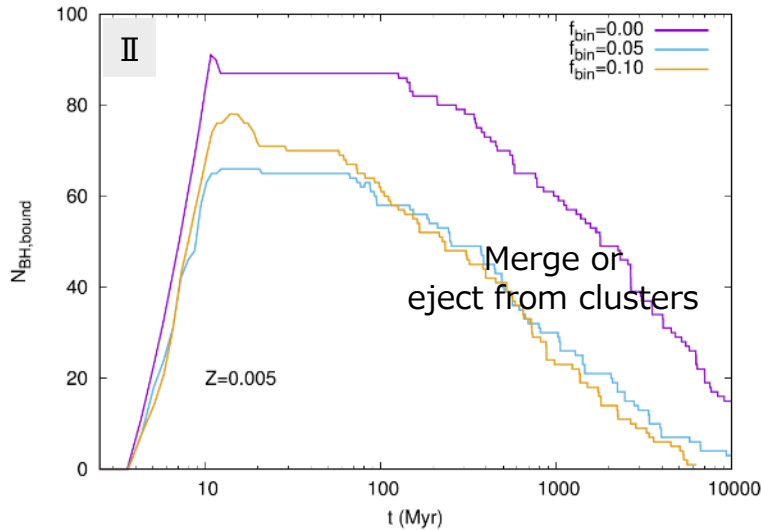


Evolution of cluster mass

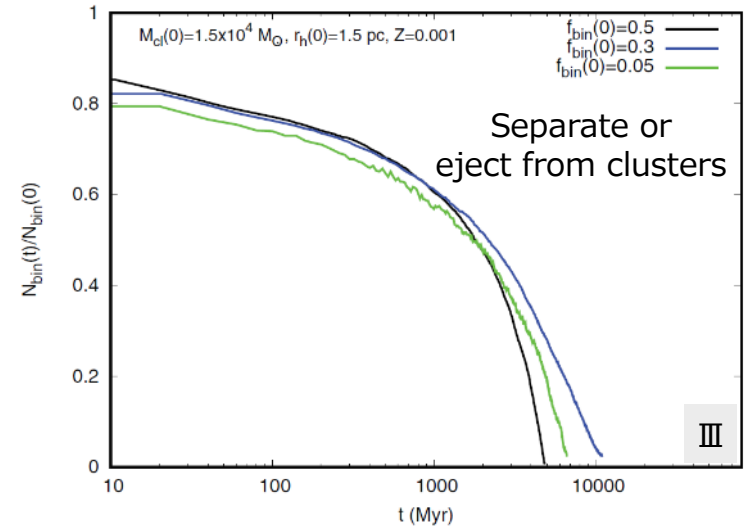


# Basic quantities

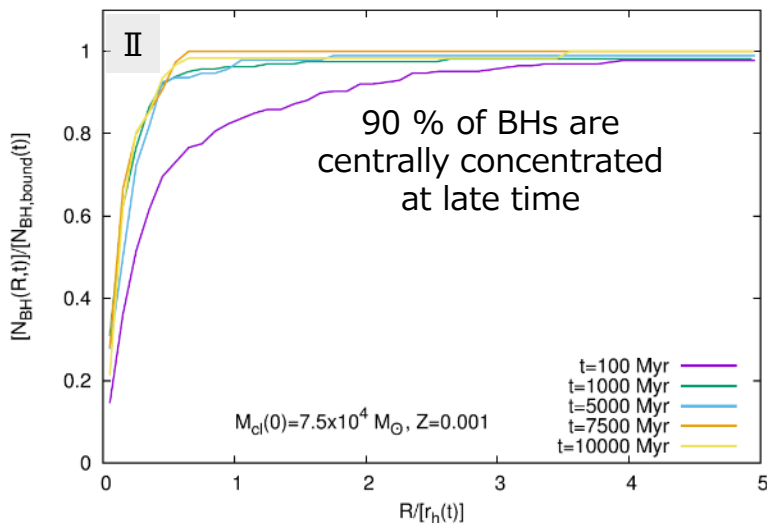
Evolution of BH number in cluster



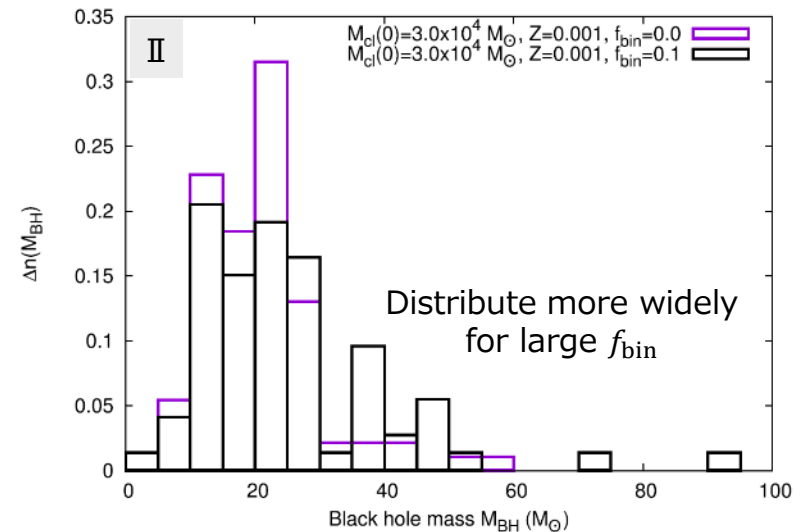
Evolution of binary number in cluster



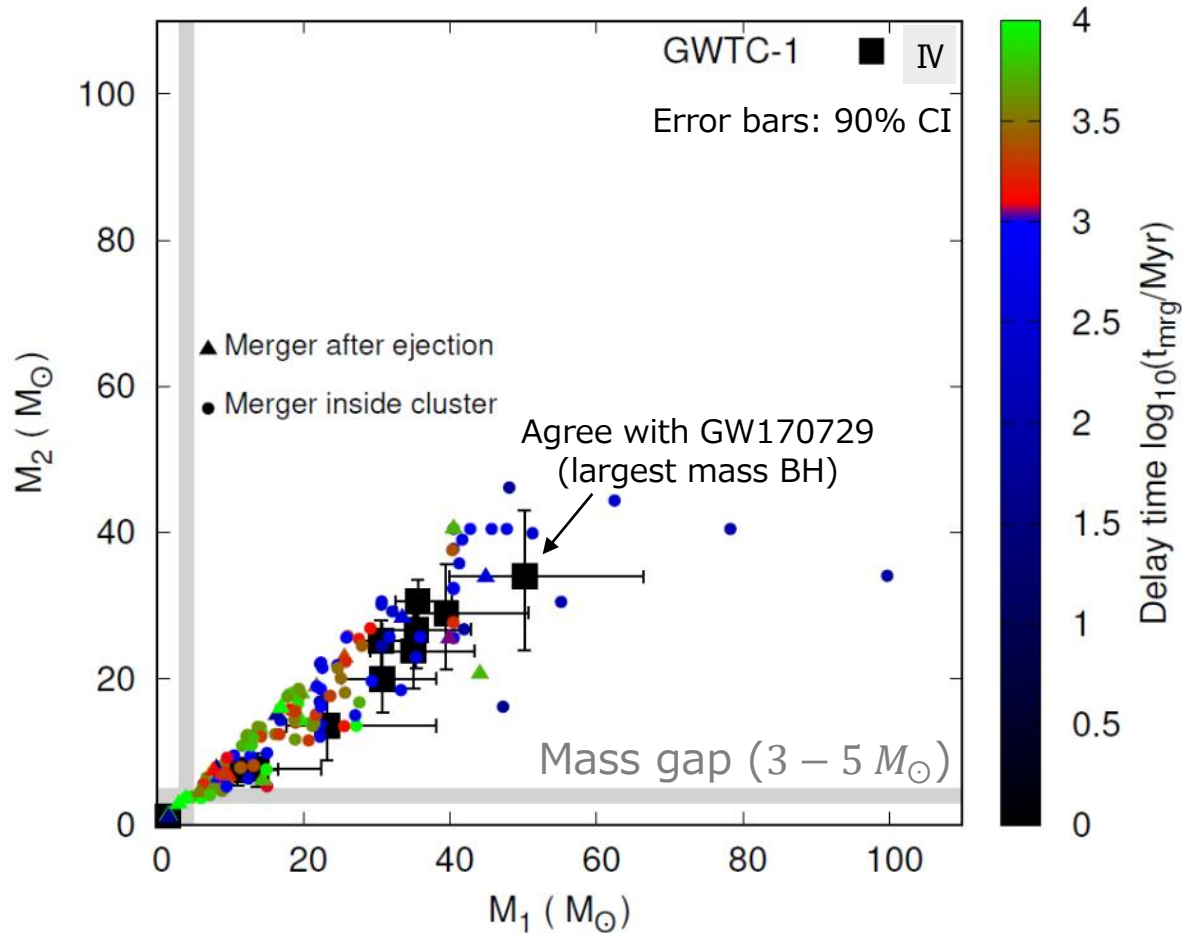
Cumulative radial distribution of BH in cluster



BH mass distribution



# Primary mass - secondary mass



$t_{\text{mrg}}$ : time interval between the beginning of cluster evolutions and the occurrence of mergers

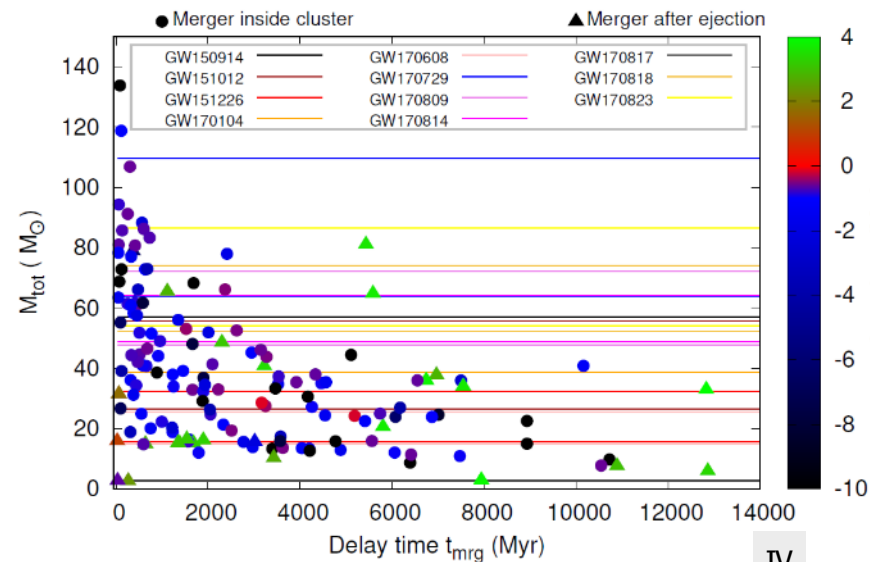
Event

GW150914  
GW151012  
GW151226  
GW170104  
GW170608  
GW170729  
GW170809  
GW170814  
GW170817  
GW170818  
GW170823

[B. P. Abbott+ \(2019\)](#)

GWTC (GW Transient Catalog) -1:  
11 events with LIGO and Virgo during O1/O2

# Total mass & mass ratio



IV

Color horizontal lines: 90% CI

Coalescence time:

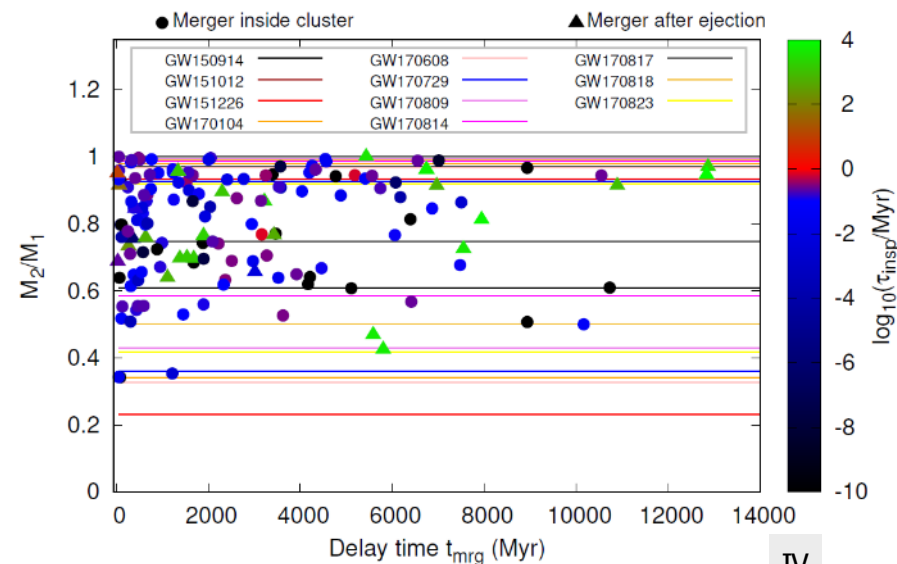
$$\tau_{\text{insp}} \approx \frac{5}{64} \frac{c^5 a_{\text{ej}}^4 (1 - e_{\text{ej}}^2)^{7/2}}{G^3 M_1 M_2 (M_1 + M_2)} \left( 1 + \frac{73}{24} e_{\text{ej}}^2 + \frac{37}{96} e_{\text{ej}}^4 \right)^{-1}$$

●  $\tau_{\text{insp}} \ll 1$  Myr  
for in-cluster mergers

●  $1 \text{ Gyr} < \tau_{\text{insp}} < 14 \text{ Gyr}$   
for ejected mergers

● Agree with O1/O2 data

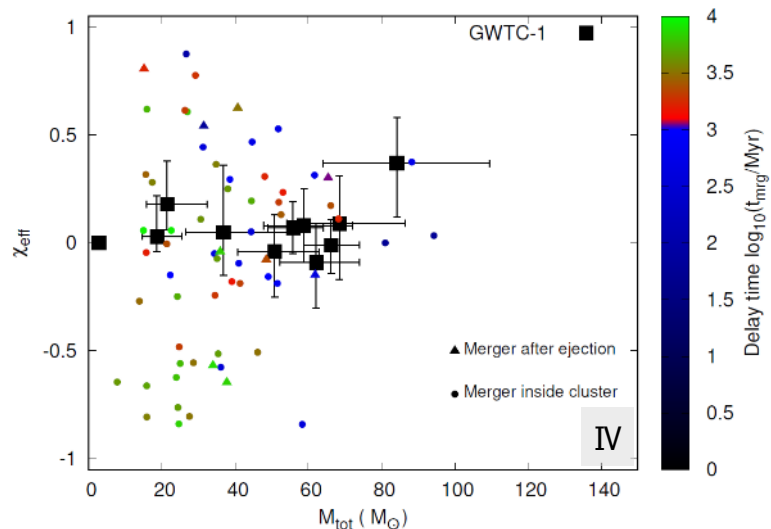
● Not agree with  
GW190814 ( $M_2/M_1 = 0.112^{+0.008}_{-0.009}$ )



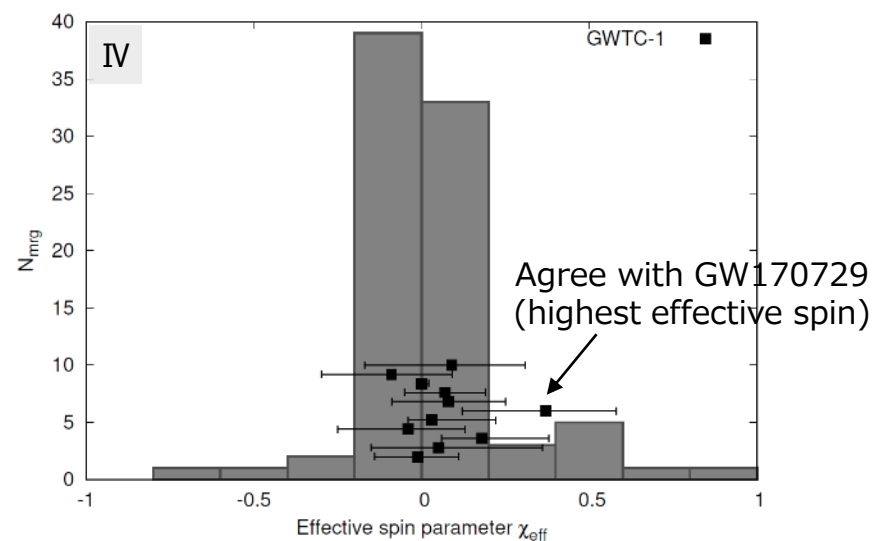
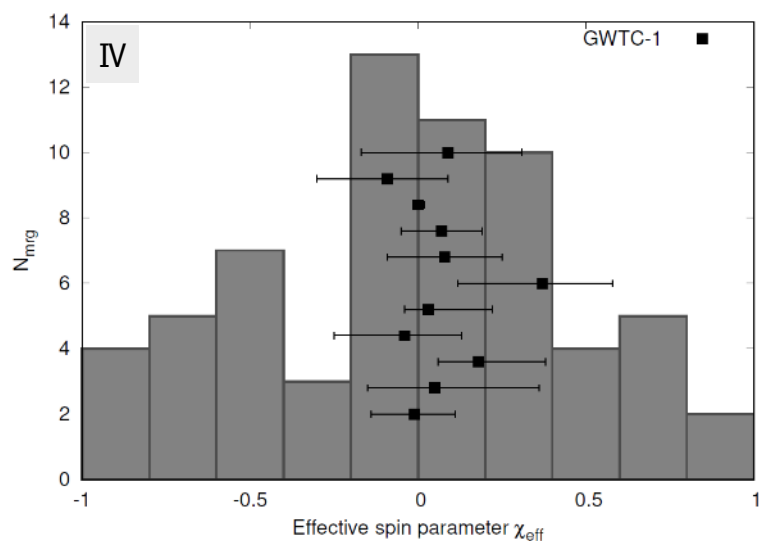
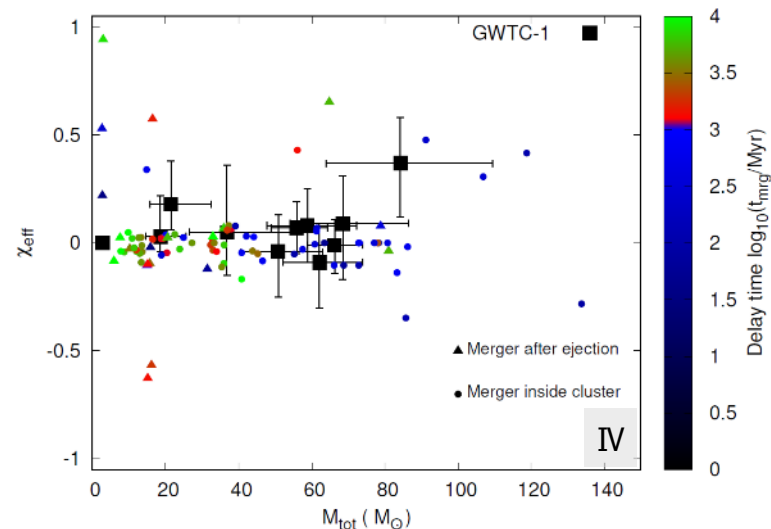
IV

# Effective spin parameter

Geneva (high spin)

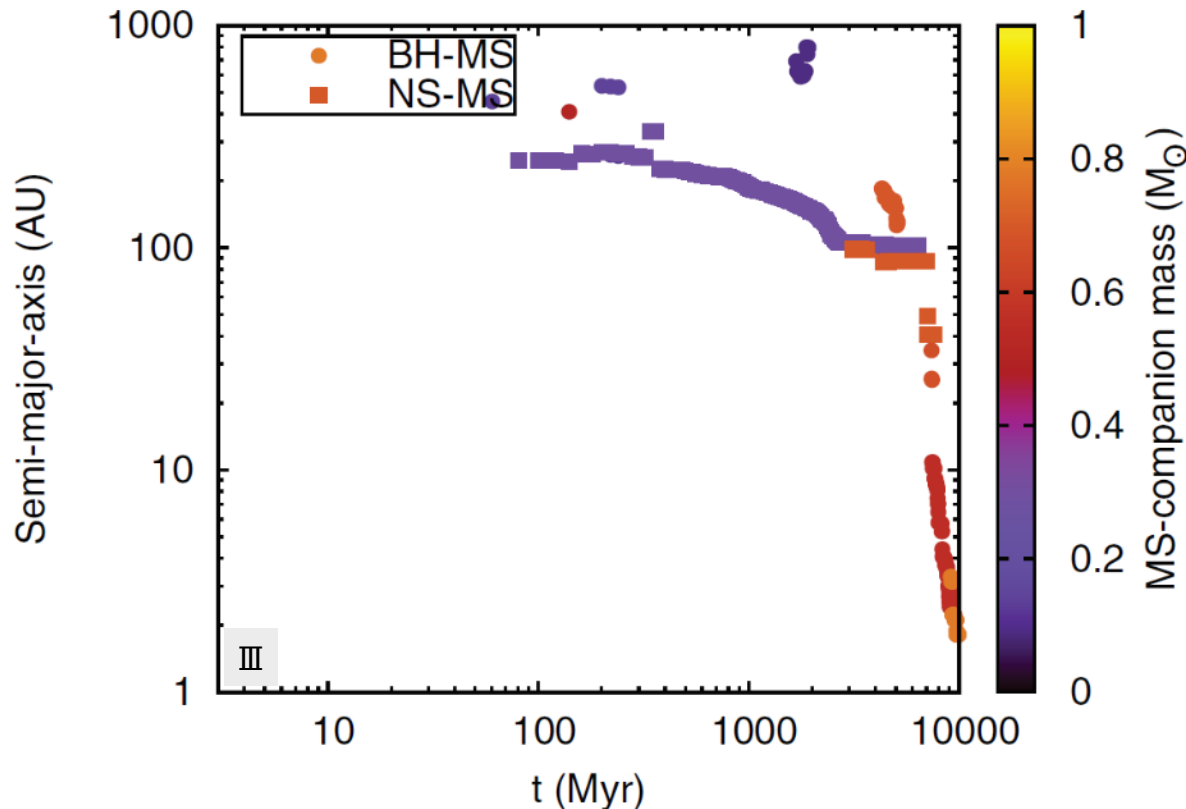


MESA (low spin)



# non-BH binaries

- The majority of binaries are BBHs, but non-BH binaries are found
- For example, WD-WD, BH-WD and BH-MS (main sequence)
- Their mergers will leave electromagnetic signatures

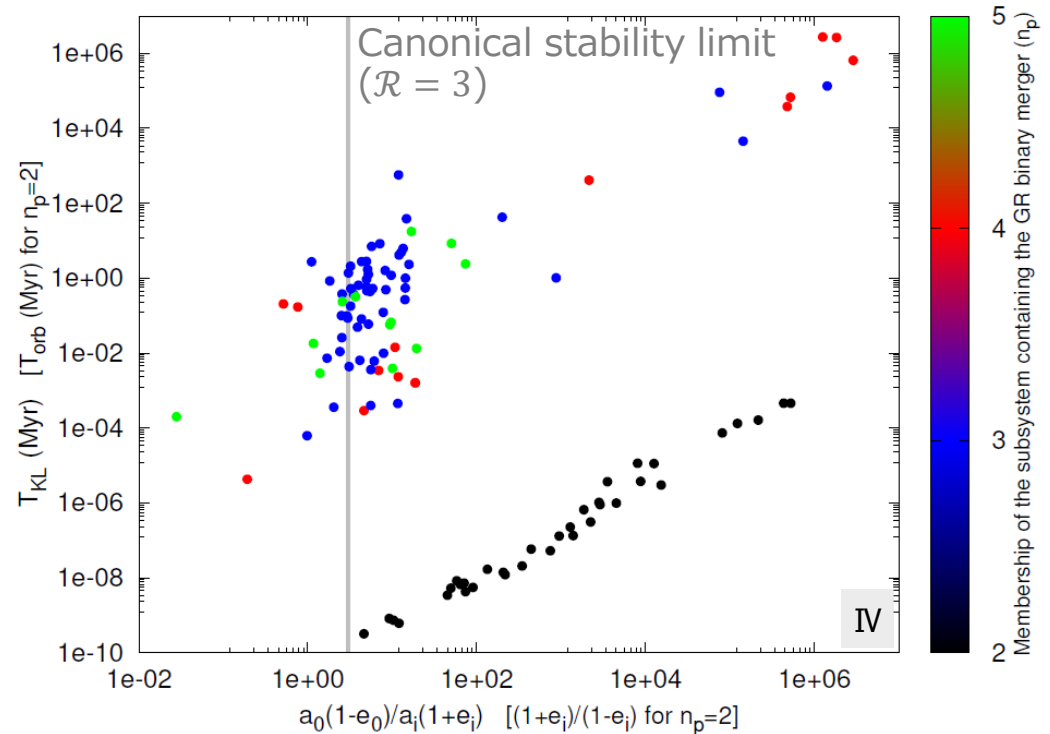


# Triples or higher-order subsystems

- Some binary mergers occur in a triple or higher-order subsystems
- These subsystems are strongly perturbed and chaotic

Kozai-Lidov time period of the triple  
(innermost triple)

$$T_{\text{KL}} = \frac{2P_o^2}{3\pi P_i} (1 - e_o^2)^{3/2} \frac{M_1 + M_2 + m_o}{m_o}$$



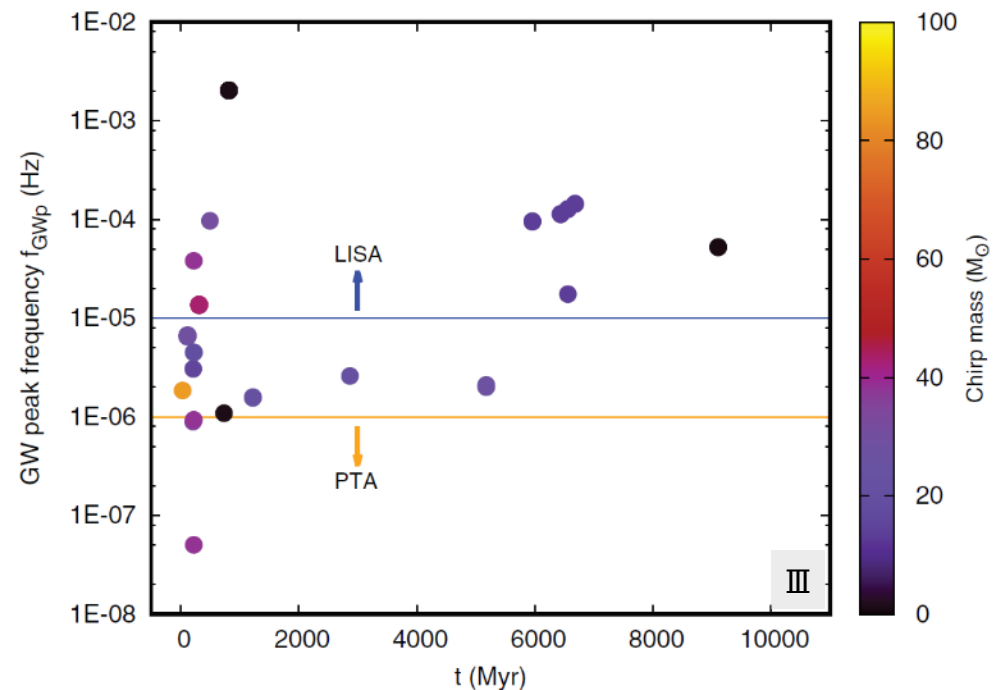
Ratio of the outer periastron to the inner apoastron of the triple (innermost triple)  $\mathcal{R} = \frac{a_o(1 - e_o)}{a_i(1 + e_i)}$

# Slingshot events

- “Slingshot” events:  
close fly-by interactions between a binary and a single object
- Slingshot events are highly eccentric ( $e > 0.9$ )
- The “proper” detection will be difficult
- They might contribute to GW background noise for LISA and PTA

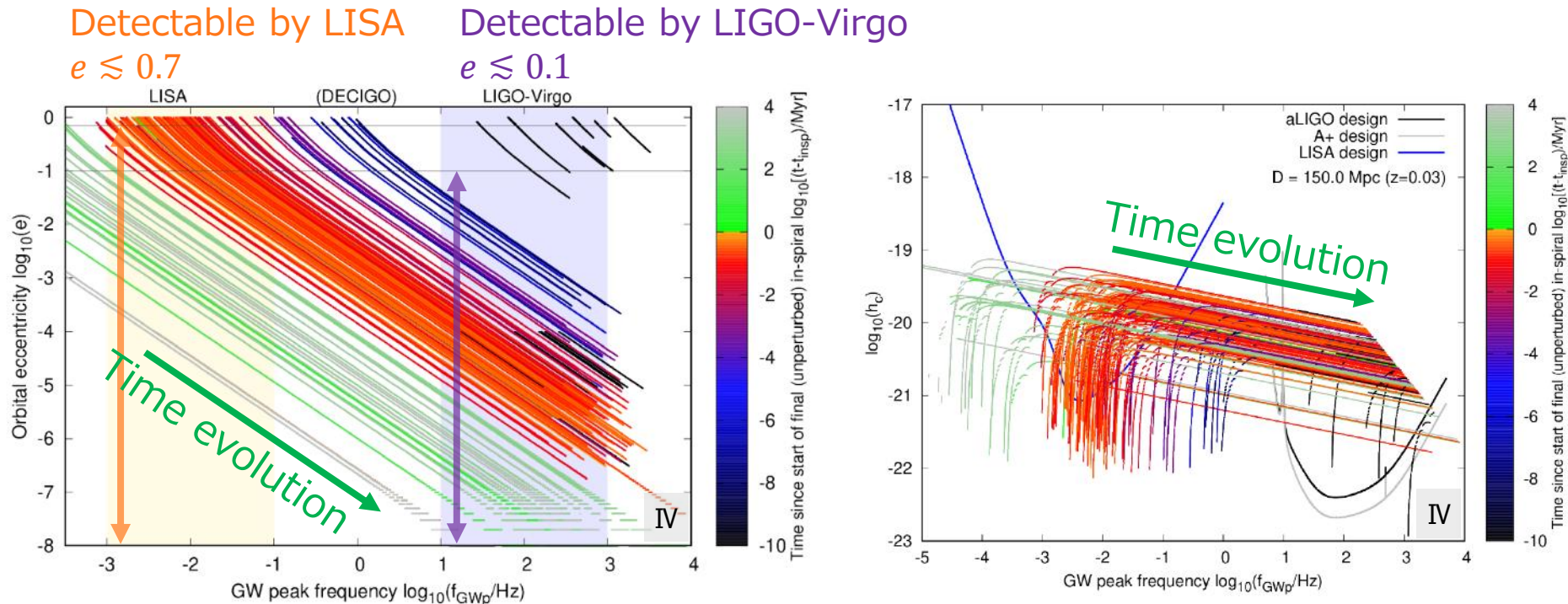


[Wikipedia: Slingshot](#)



# Evolution of orbital eccentricity

- Almost all BBHs have very high eccentricities
- The reason is slingshot events or in a triple



Peak-power GW frequency:

$$f_{\text{GWp}} = \frac{\sqrt{G(M_1 + M_2)}}{\pi} \frac{(1 + e_t)^{1.1954}}{|a_t(1 - e_t^2)|^{1.5}}$$

# Summary

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- In general, star clusters are classified into open clusters and globular clusters by their age
- Many models of young massive and open clusters are prepared
- The dynamical formation of BBHs in clusters are simulated by N-body evolution program
- The results are similar to LIGO-Virgo data