Estimation of Noises

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Abstract

- I will explain some theories to derive thermal noise
  - Fluctuation-Dissipation Theorem
  - Viscous damping model
  - Structure damping model

- Deriving…
  - Brownian motion in a dilute gas
  - Suspension thermal noise
  - Mirror thermal noise

- Apply these noises calculations to my DPFP cavity setup
  - Ploted shot noise and radiation pressure noise too

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Mind map to understand thermal noises

3 kinds of thermal noises will be derived from The Fluctuation-Dissipation Theorem

The Fluctuation-Dissipation Theorem
揺動散逸定理

Viscous damping model

Loss angle : $\phi = \frac{\omega}{\omega_0}\frac{1}{Q}$

Brownian motion in a dilute gas

Structure damping model

$\phi = \frac{1}{Q}$

$\omega \gg \omega_0$

$\omega \ll \omega_0$

Suspension thermal noise

Mirror thermal noise

The Fluctuation-Dissipation Theorem　揺動散逸定理

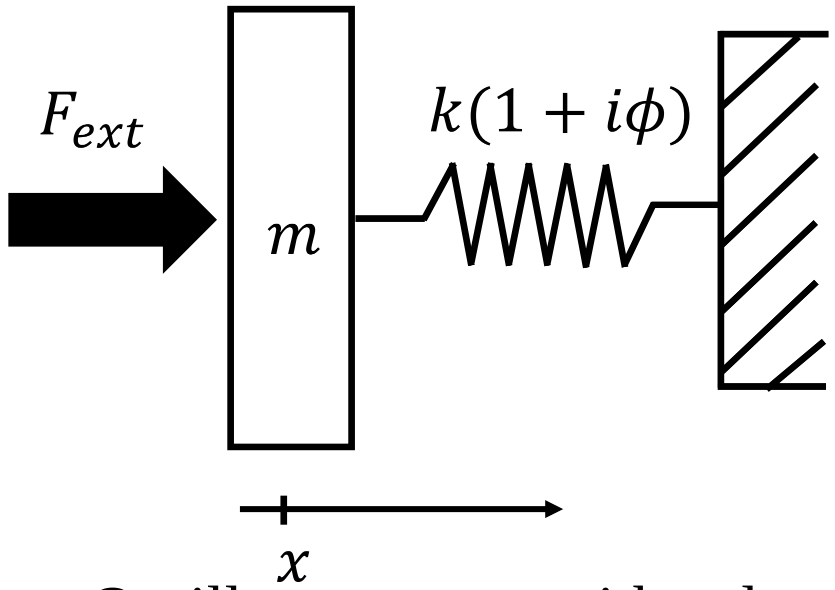
Equation of motion of the oscillator system having a dissipation

$$m\ddot{x} = F_{ext} - k(1 + i\phi)x$$

$$F_{ext} = m\ddot{x} + k(1 + i\phi)x$$

$$\tilde{F}_{ext} = \left(i\omega m + \frac{k}{i\omega} + \frac{k\phi}{\omega}\right)v$$

$\equiv Z(\omega)$: Mechanical impedance



Oscillator system with a damper

Mechanical admittance $\mathrm{Y}(\omega) = Z^{-1}(\omega)$

The Fluctuation-Dissipation Theorem

Using Z and Y, power spectrum can be written

Force power spectrum $\mathrm{F}^2(f) = 4k_B T\ \mathrm{Re}[Z]\ \ [N^2/\mathrm{Hz}]$

Displacement power spectrum $x^2(f) = \frac{k_B T}{\pi^2 f^2}\ \mathrm{Re}[Y]\ \ [\mathrm{m}^2/\mathrm{Hz}]$

Resonant frequecy

How much dissipation

If we decide on ϕ and k,
We can derive spectrum

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$\phi = \frac{1}{Q}$

$\omega \gg \omega_0$

Suspension thermal noise

$\omega \ll \omega_0$

Mirror thermal noise

Viscous damping model for thermal noise

In viscous damping, the loss angle : $\phi = \frac{\omega}{\omega_0}\frac{1}{Q}$

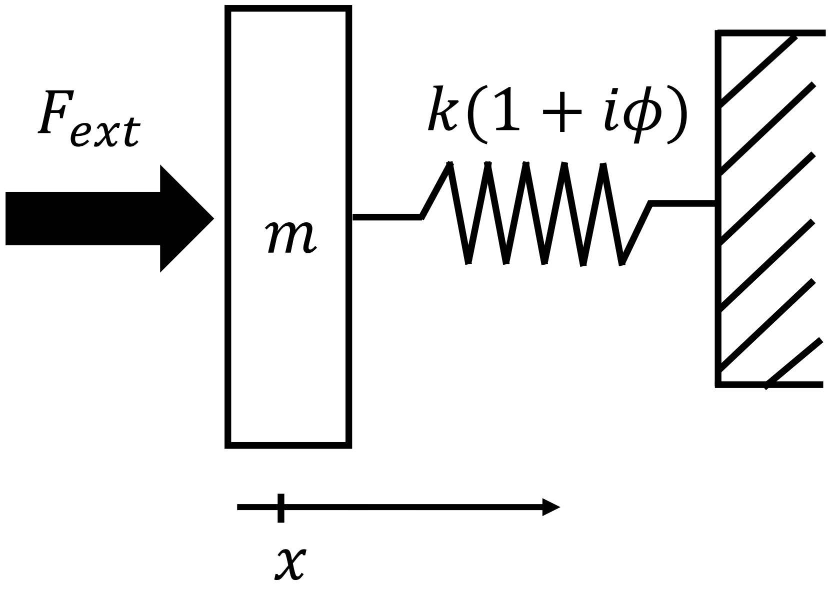
It is equal to consider an oscillator system with a spring constant of k and a damping constant of $b = \frac{k}{\omega_0 Q}$

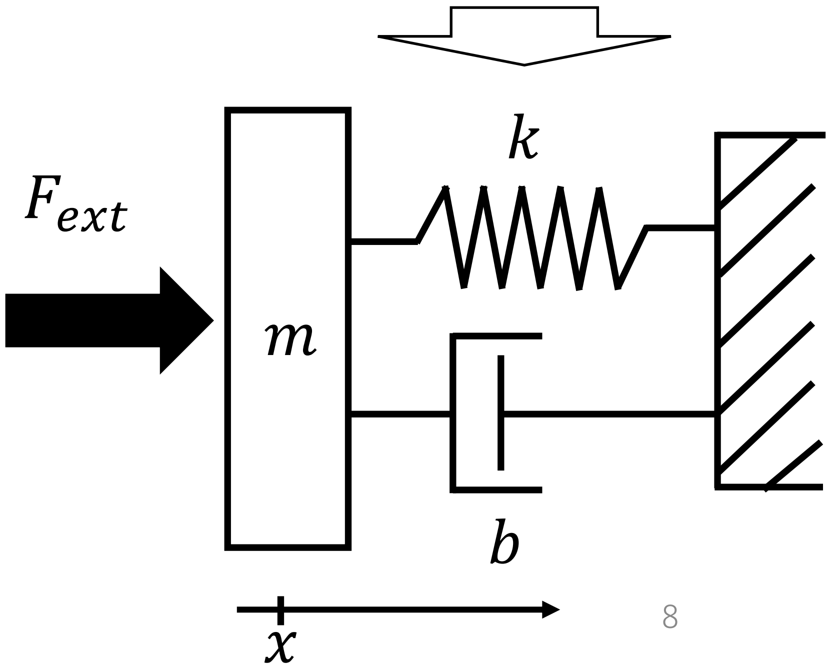
$$m\ddot{x} = F_{ext} - b\dot{x} - kx$$

$$F_{ext} = m\ddot{x} + b\dot{x} + kx$$

$$\tilde{F}_{ext} = \left(i\omega m + b + \frac{k}{i\omega}\right) v$$

$\equiv Z(\omega)$: Mechanical impedance





Kinetic theory of gases 気体分子運動論

Considering kinetic theory of gases for how much test mass fluctuates due to molecules in the gas

Equation of State

$$p = nk_B T$$

Maxwell-Boltzmann distribution describes a distribution of velocity of molecules

$$f(v) = \left(\frac{\mu}{2\pi k_B T}\right)^{\frac{3}{2}} \exp\left(-\frac{\mu(v_x^2 + v_y^2 + v_z^2)}{2k_B T}\right)$$

Average speed of particles

$$\bar{v} = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \; v f(v)$$

$$= \sqrt{\frac{8k_B T}{\pi\mu}}$$

p : Pressure [Pa]

n : The number of density of molecules in the gas [/m^3]

k_B : Boltzmann constant [m^2 kg s^{-2}K^{-1}]

T : Temperature [K]

μ : The mass of an individual molecule [kg]

A : Cross sectional are [m^2]

Brownian motion in a dilute gas

The rate of collisions with molecules arriving form one side

単位時間当たりの衝突個数

$$N = \int_0^\infty n v_x A\, f(v)\, dv_x dv_y dv_z$$

$$= \frac{1}{4} n A \bar{v}$$

Force of friction between the mass swinging at speed of v_p and particles

$$F_{\text{friction}} = -\mu N v_p = -\underline{\frac{1}{4}\mu n \bar{v} A} v_p$$

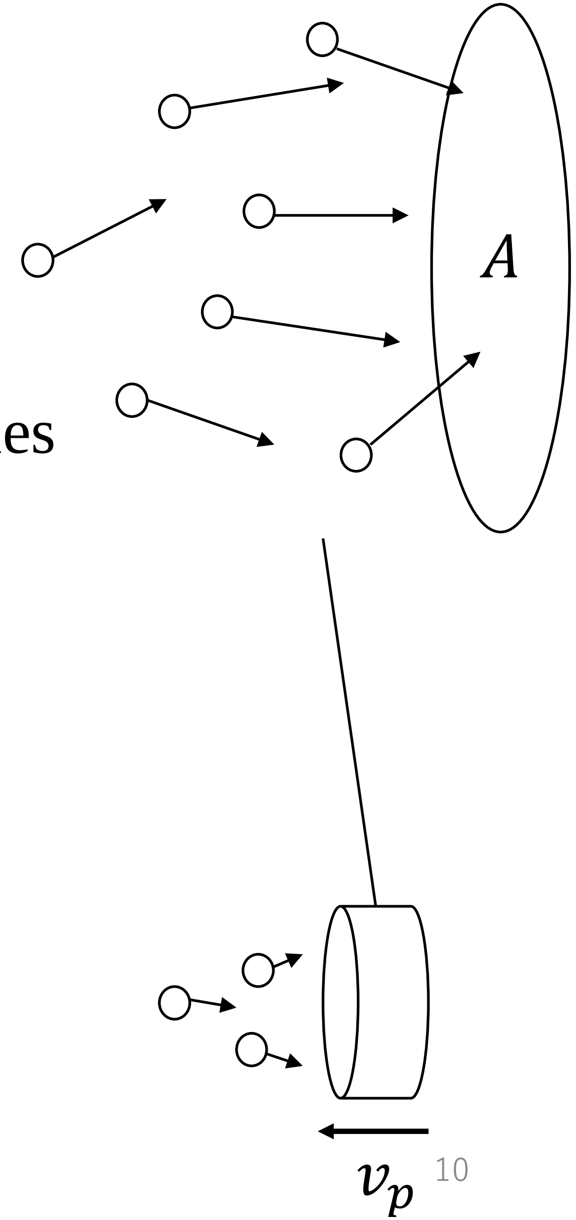
$$\equiv b_{gas}$$

Spring constant of the harmonic oscillator of suspension

$$k = \frac{mg}{l}$$

The Fluctuation-Dissipation Theorem $x^2(f) = \frac{k_B T}{\pi^2 f^2} \mathrm{Re}[Y]$

Displacement power spectrum $x^2_{viscous}(f) = \frac{4 k_B T b_{gas}}{\omega^2 b_{gas}^2 + (k - m\omega^2)^2}$



Mind map to understand thermal noises

3 kinds of thermal noises will be derived from The Fluctuation-Dissipation Theorem

The Fluctuation-Dissipation Theorem
摇動散逸定理

Viscous damping model

Loss angle : $\phi = \frac{\omega}{\omega_0}\frac{1}{Q}$

Brownian motion in a dilute gas

Structure damping model

$\phi = \frac{1}{Q}$

$\omega \gg \omega_0$

Suspension thermal noise

$\omega \ll \omega_0$

Mirror thermal noise

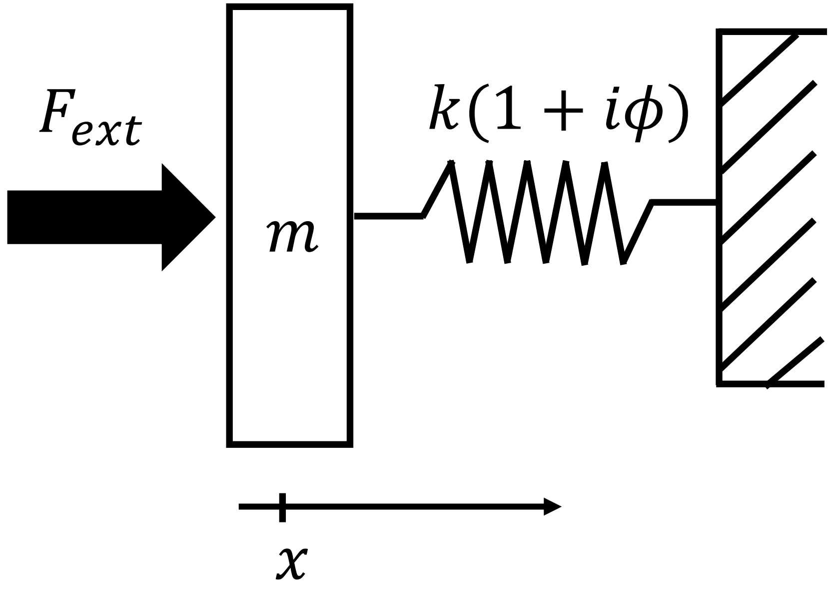
Structure damping model for thermal noise

In structure damping, the loss angle : $\phi = \dfrac{1}{Q}$

$$m\ddot{x} = F_{ext} - k(1 + i\phi)x$$

$$F_{ext} = m\ddot{x} + k(1 + i\phi)x$$

$$\tilde{F}_{ext} = \underbrace{\left(i\omega m + \frac{k}{i\omega} + \frac{k\phi}{\omega}\right)}_{= Z(\omega)} v \qquad \left(b = \frac{k\phi}{\omega}\right)$$



The Fluctuation-Dissipation Theorem $\quad x^2(f) = \dfrac{k_B T}{\pi^2 f^2}\,\mathrm{Re}[Y]$

Displacement power spectrum $x^2_{structure}(f) = \dfrac{4 k_B T k\phi}{\omega(k^2\phi^2 + (k - m\omega^2)^2}$

Suspension thermal noise and Mirror thermal noise

Considering the situation at low and high frequencies separately

Displacement power spectrum $x^2_{structure}(f) = \frac{4k_B T k\phi}{\omega(k^2\phi^2 + (k - m\omega^2)^2}$

$$\simeq \frac{4k_B T\phi}{m\omega\omega_0^2} \propto \frac{1}{\omega} \qquad (\omega \ll \omega_0)$$

$$\simeq \frac{4k_B T\omega_0^2\phi}{m\omega^5} \propto \frac{1}{\omega^5} \qquad (\omega \gg \omega_0)$$

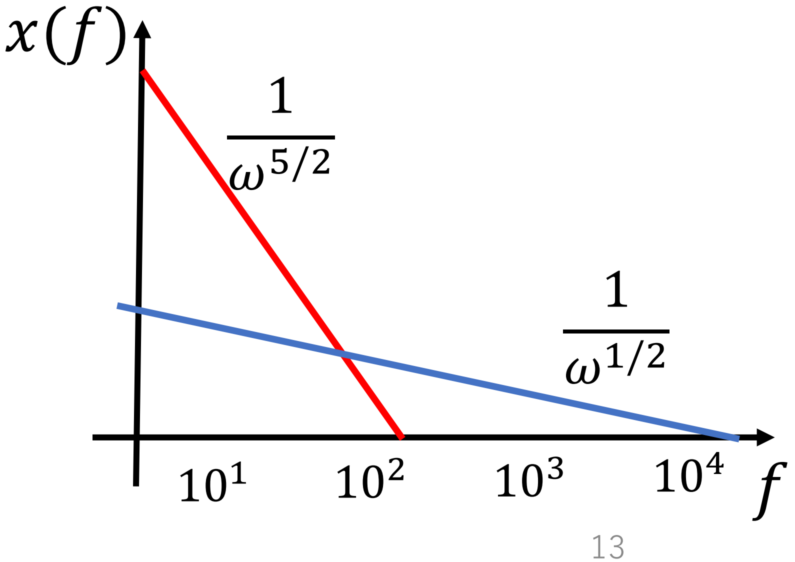
For suspension thermal noise, $\omega_0 = \mathcal{O}(1)$ [Hz]

For mirror thermal noise, $\omega_0 = \mathcal{O}(10^4 \sim 10^5)$ [Hz]

In our main interesting frequency region $1 \sim 10^4$ [Hz]

Suspension thermal noise is proportional to $\frac{1}{\omega^5}$

Mirror thermal noise is proportional to $\frac{1}{\omega}$



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- Application to my setup
- Summary

How much order is the level of thermal noises in my setup?

Roughly assuming some parameters…

$p = 1[\mathrm{Pa}]$ $T = 300[\mathrm{K}]$

Resonant frequencies

$f_{0sus} = 1[\mathrm{Hz}]$

$f_{0mirror} = 100[\mathrm{kHz}]$

$\phi_{sus} = 10^{-4}$ Tungsten wire with a diameter of 0.2 mm

$\phi_{mirror} = 10^{-8}$

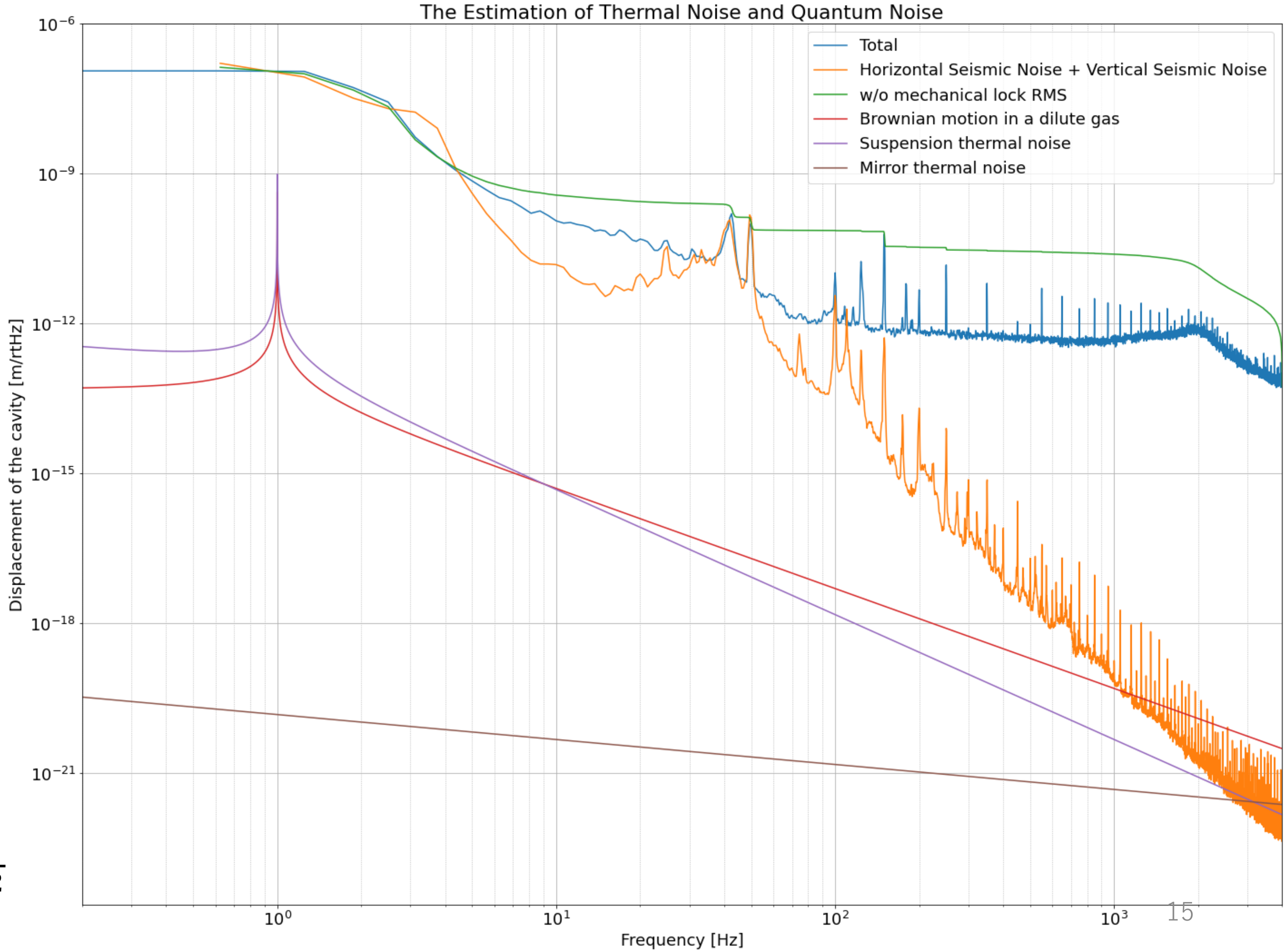
Fused silica substrate

For Brownian motion in a dilute gas

$$x^2_{viscous}(f) = \frac{4k_BTb_{gas}}{\omega^2 b^2_{gas} + (k - m\omega^2)^2}$$

For Suspension and Mirror thermal noise

$$x^2_{structure}(f) = \frac{4k_BTk\phi}{\omega(k^2\phi^2 + (k - m\omega^2)^2}$$



How much order is the level of quantum noises in my setup?

Shot noise

$$x_{shot}(f) = L\sqrt{\frac{\hbar\lambda}{8\pi c P}\left(\frac{1}{\tau^2} + \omega^2\right)}$$

$$\tau = \frac{2L\mathcal{F}}{\pi c}$$

Radiation pressure noise

$$x_{rp}(f) = \frac{1}{m\omega^2}\frac{2\mathcal{F}}{\pi}\sqrt{\frac{32\pi\hbar P}{c\lambda}}\frac{1}{\sqrt{1+(\omega\tau)^2}}$$

Used parameters

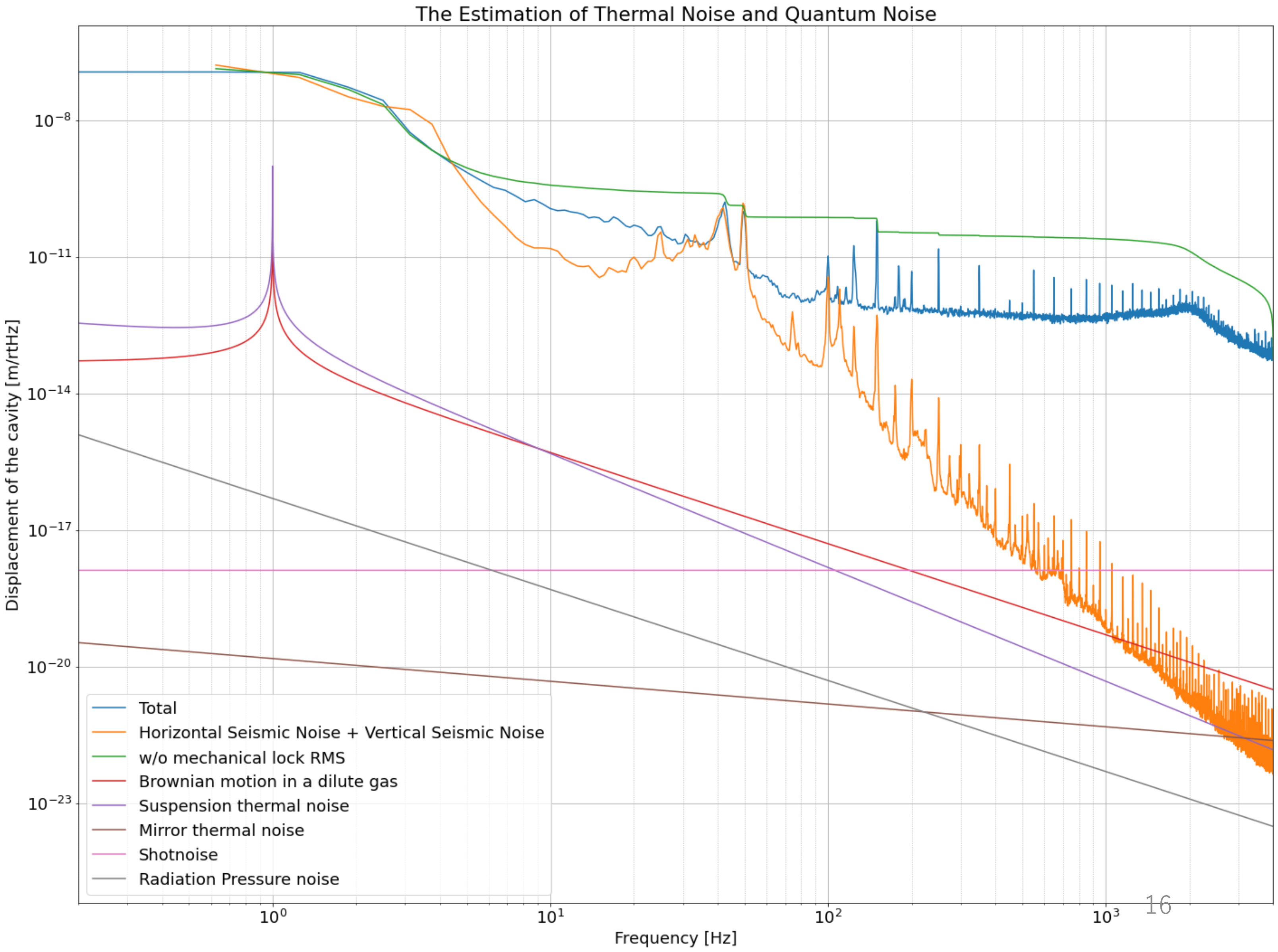
$\mathcal{F} = 314$

$L = 0.56$ [m]

$P = 10$ [mW]

$\lambda = 1064$ [nm]

$m = 0.3$ [kg]



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Summary

- Introduced The Fluctuation-Dissipation Theorem

- Derived power spectra of 2 models
    - Viscous damping model
    - Structure damping model

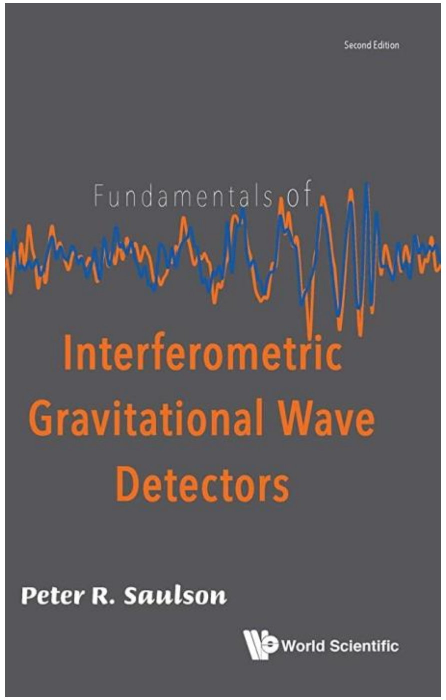
- Calculated and estimated roughly thermal noise level of my DPFP cavity setup

References

- Ando-san’s Master degree’s thesis

- Saulson, Fundamentals of Interferometric Gravitational Wave Detectors, World Scientific

- Saulson輪講のレジュメ



Thank you for listening