観測天文学特別講義I @ 理1-1043
Observational Astronomy Advanced Course I

November 25, 2025

レーザー干渉計型重力波検出器
Laser interferometric gravitational wave detectors

5. Quantum Noise

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https://yutamich.gitlab.io/lectures.html#lectures

Assignment for Nov 25

- Explain what the standard quantum limit (SQL) is, and why it is possible to surpass the SQL.
- You may also answer from the Google Form below
  https://forms.gle/6AwJ48XcpWQXqMon9

Don’t forget to put your name and student#

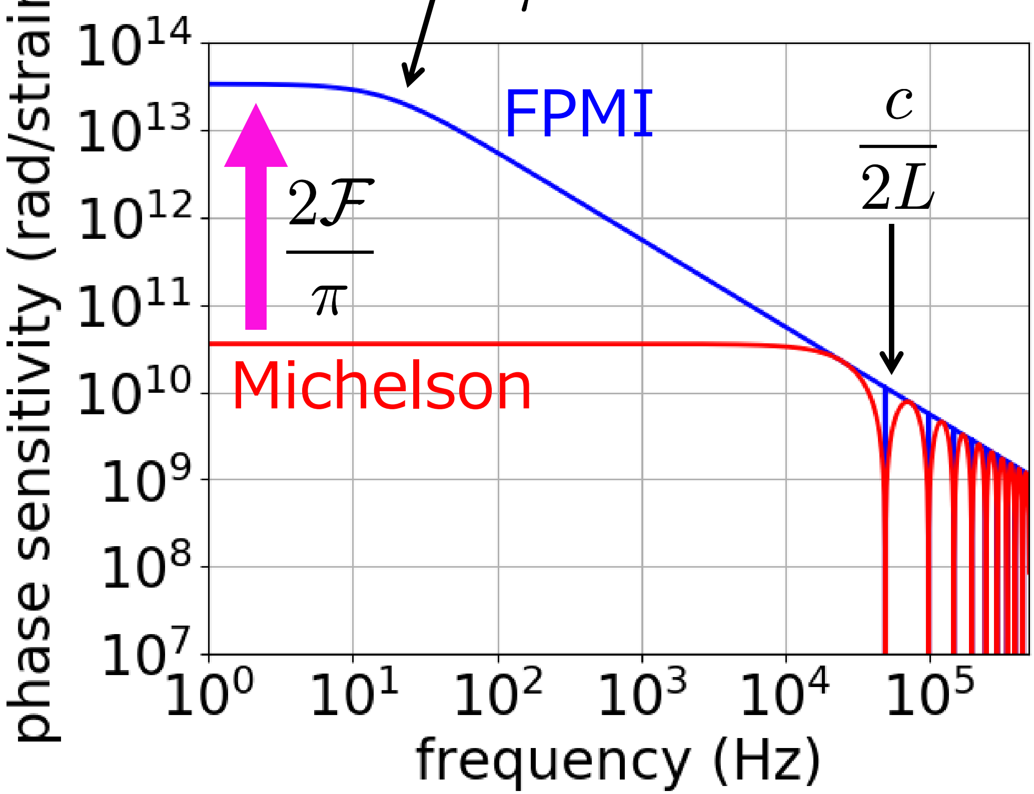
You may answer in any language

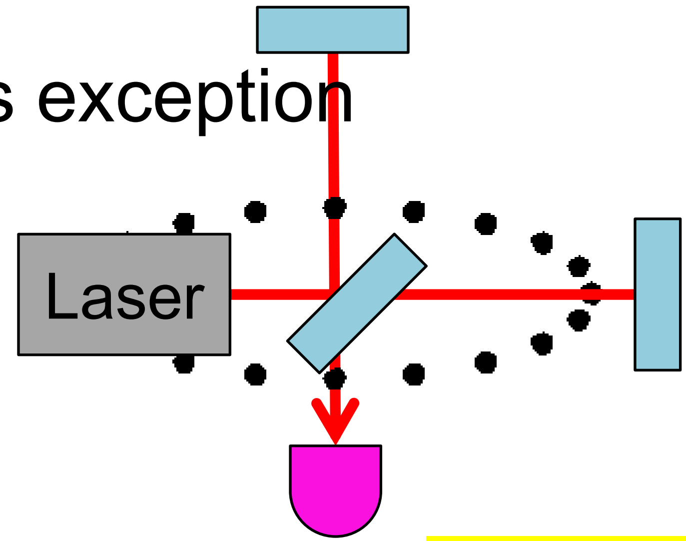
FORM

Bigger the Better?

- Basically, yes
- High frequency response is exception

$$f_{\mathrm{c}} = \frac{1}{2\pi}\Bigg(\underbrace{\frac{2\mathcal{F}}{\pi}\frac{L}{c}}_{\tau}\Bigg)^{-1} = \frac{c}{4L\mathcal{F}}$$





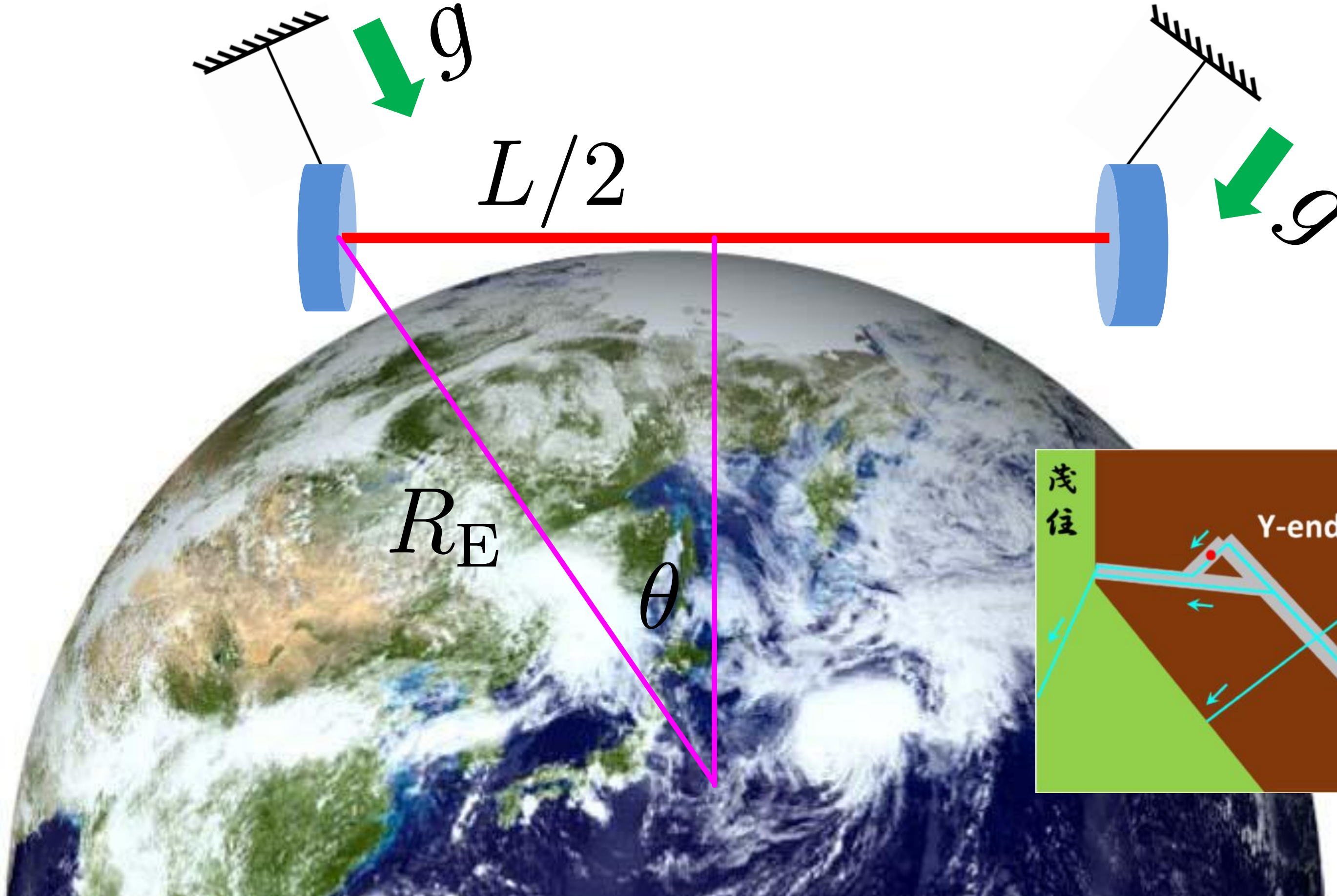
Recall Lecture 2

For a given frequency, there is a limit where longer arm length and higher finesse won’t help increasing the sensitivity

Curvature of the Earth

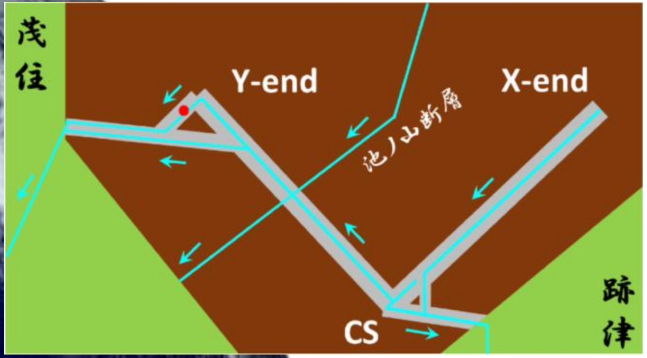
- Noise from vertical to horizontal coupling

$$h_{\mathrm{vert}} = \frac{\delta L_{\mathrm{vert}}}{L} = \frac{\sin\theta\,\delta x_{\mathrm{vert}}}{L} = \frac{1}{2R_{\mathrm{E}}}$$



Independent on the arm length

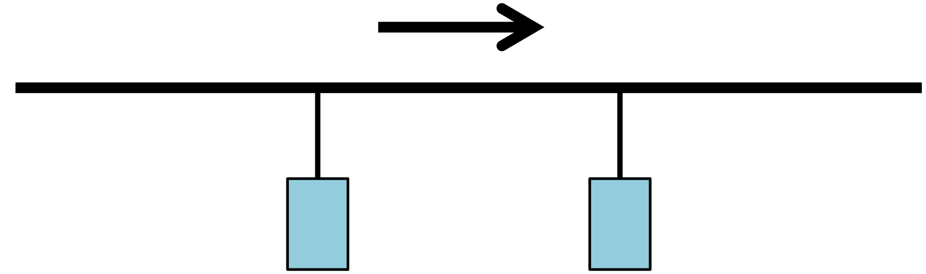
Recall Lecture 4



Seismic Noise Common Mode

- Seismic motion is common for $\omega < L/v$
- But in terms of strain, longer the better

Ground motion phase velocity: v



Moves largely in common if $\omega < L/v$

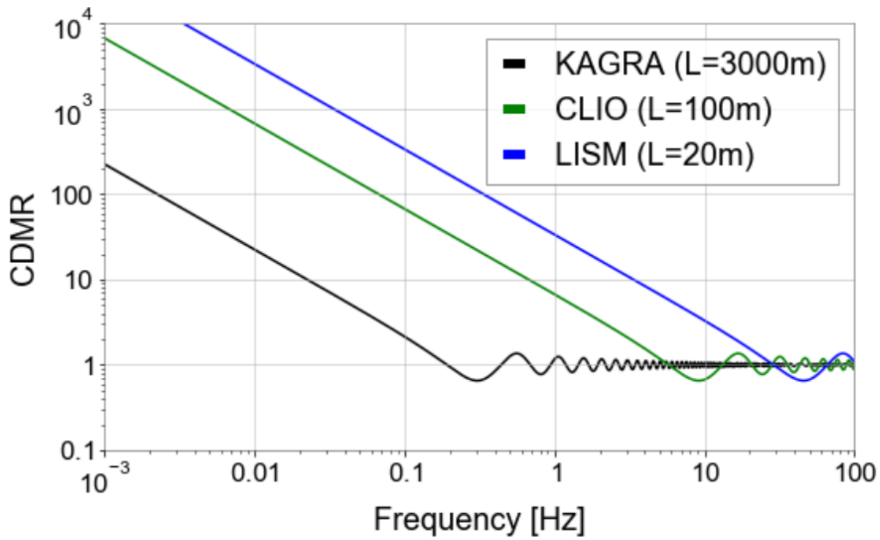


Figure 2.6: CDMR given in Eq.(2.25), of the underground GW detectors assuming the uniform plane waves model with phase velocity of 3 km/s. Black is the CDMR of KAGRA with the 3000 m baseline, green is the CDMR of CLIO with the 100 m baseline, and blue is the CDMR of LISM with the 20 m baseline.

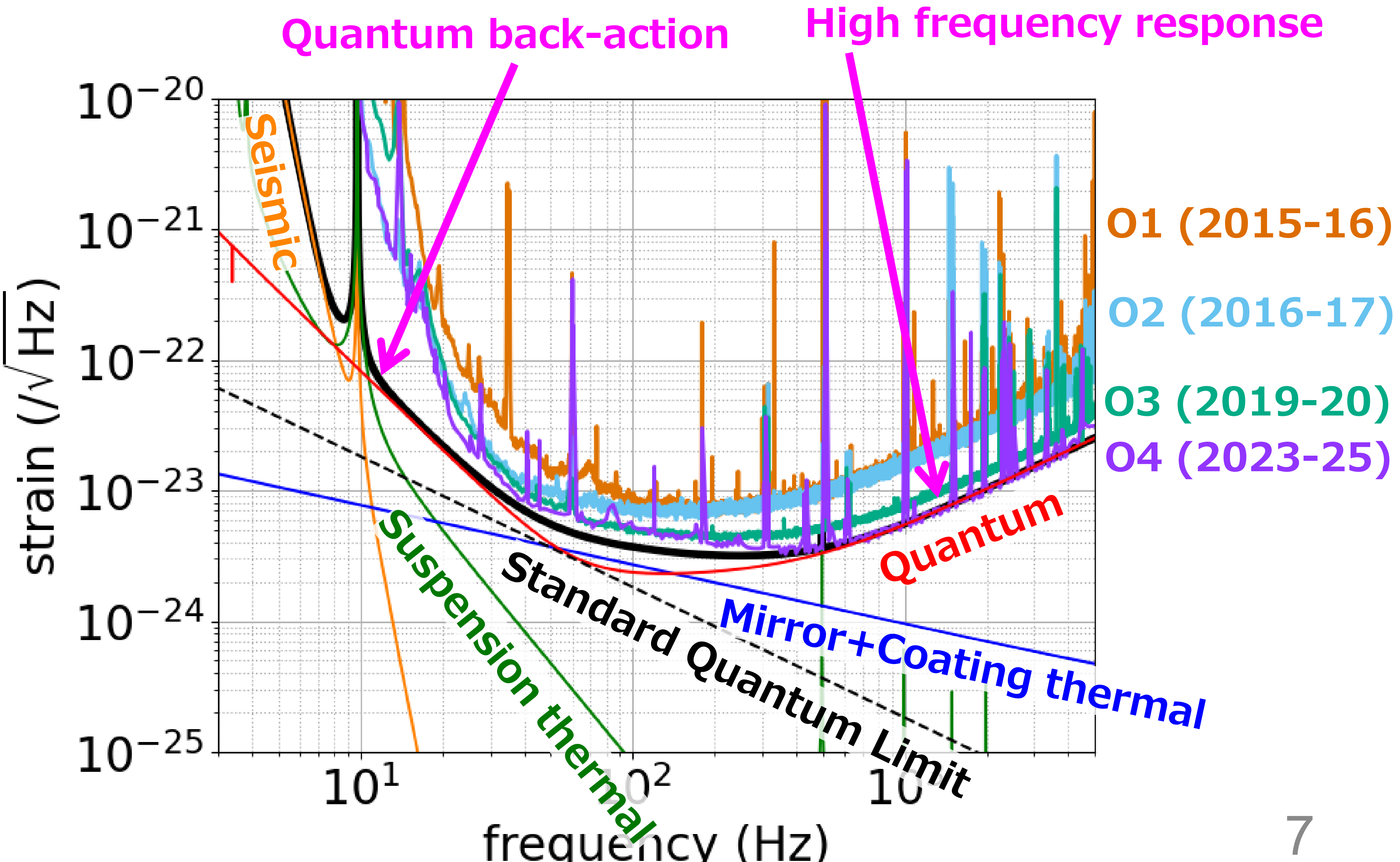
Koseki Miyo, Doctoral Thesis (UTokyo 2020) [JGW-P1911125]

Plan of the Lecture Today

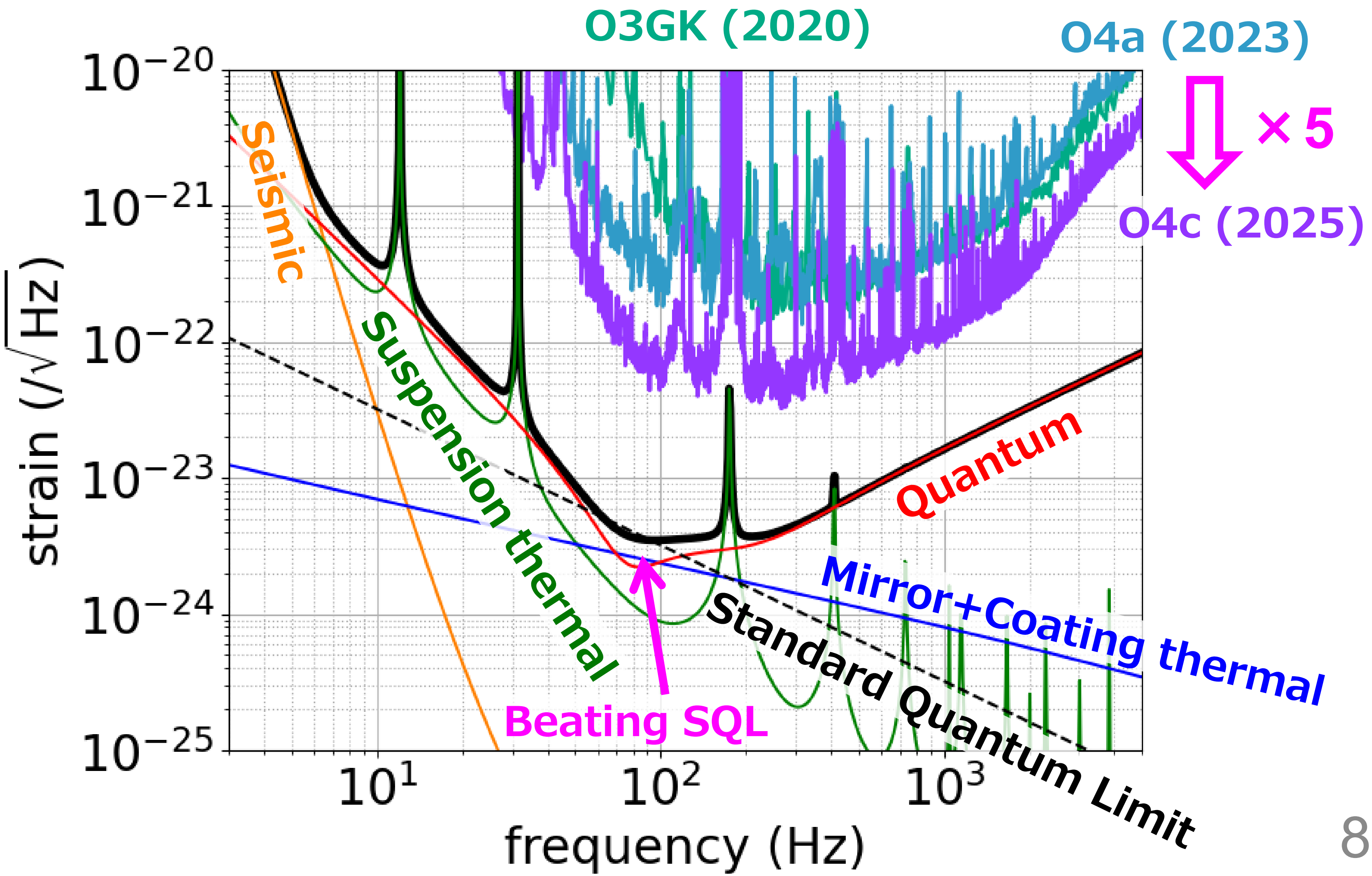
- Quantum noise of laser interferometer
  - Michelson
  - Fabry-Perot Michelson
  - Power recycling
  - Resonant sideband extraction
- Squeezing techniques

- **Goal**: Understand how current generation of interferometers work in terms of quantum noise

Advanced LIGO Design

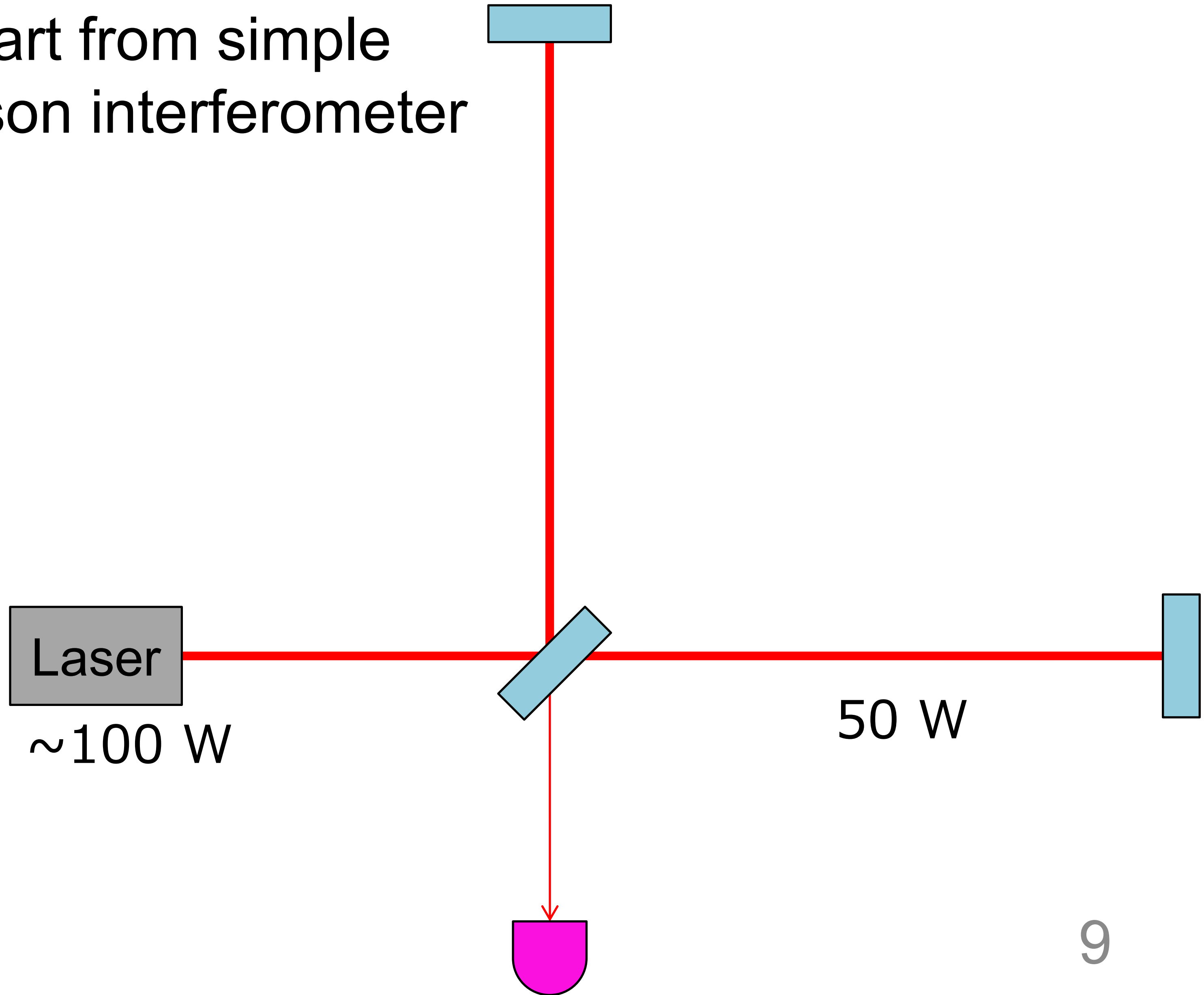


KAGRA Design



Michelson Interferometer

- Let's start from simple Michelson interferometer

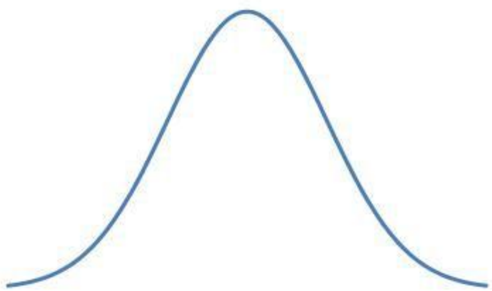


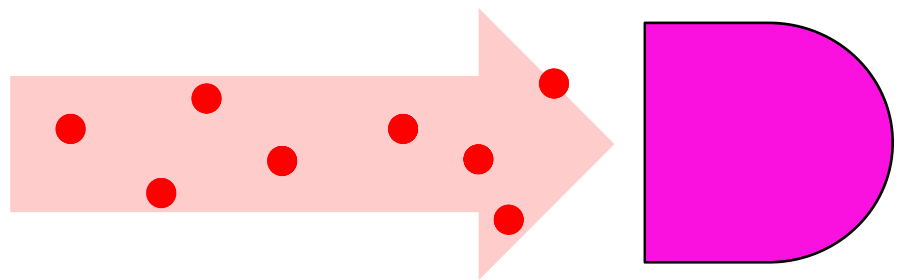
Recall Lecture 2

Photon Shot Noise

- Number of photons to photodiodes fluctuates

$$N \pm \sqrt{N}$$





Photodiode

Photon energy

$$p_1 = \frac{hc}{\lambda}$$

Number of photons

$$N = \frac{P_{\mathrm{PD}}}{p_1}$$

- Quantum fluctuation of power

$$\delta P_{\mathrm{shot}} = \sqrt{\frac{2hcP_{\mathrm{PD}}}{\eta\lambda}}$$

Shot noise Spectrum (one-sided ASD)

Quantum efficiency

Shot Noise Limit of Michelson

- Power change $$\frac{\partial P_{\mathrm{PD}}}{\partial L_-} = \frac{2\pi P_0}{\lambda} \sin \frac{4\pi L_-}{\lambda}$$
- Shot noise

Recall Lecture 2

$$\delta P_{\mathrm{shot}} = \sqrt{\frac{2hcP_{\mathrm{PD}}}{\eta\lambda}} = \sqrt{\frac{hcP_0}{\eta\lambda}\left(1 - \cos\frac{4\pi L_-}{\lambda}\right)}$$

- Shot noise limited sensitivity

$$\delta L_{\mathrm{shot}} = \delta P_{\mathrm{shot}} \left(\frac{\partial P_{\mathrm{PD}}}{\partial L_-}\right)^{-1} \rightarrow \frac{1}{2\pi}\sqrt{\frac{hc\lambda}{2\eta P_0}}$$

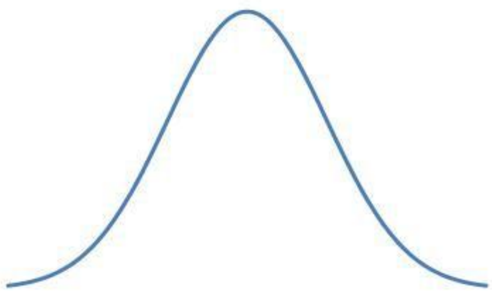
$$\frac{\sqrt{1-\cos\phi}}{\sin\phi} = \frac{\sqrt{2\sin^2\frac{\phi}{2}}}{2\sin\frac{\phi}{2}\cos\frac{\phi}{2}} = \frac{1}{\sqrt{2}\cos\frac{\phi}{2}}$$

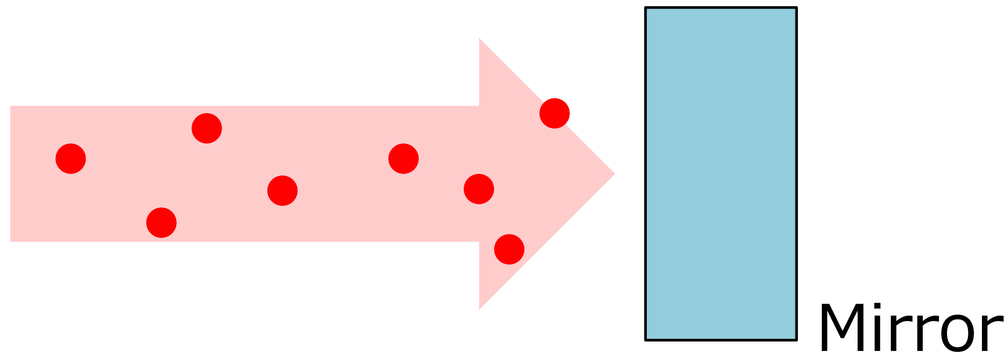
Better shot noise with higher input power
Best at dark fringe (where $P_{\mathrm{PD}}=0$)

Radiation Pressure Noise

- Number of photons to mirror fluctuates

$$N \pm \sqrt{N}$$





- Power fluctuation

Assumed Michelson with input power P_0

Recall [Lecture 3]

$$\delta P_{\rm rad} = \sqrt{\frac{2hcP_0}{\lambda}}$$

- Mirror displacement

Force on mirror

Force to displacement (susceptibility)

free-falling mirror

$$\delta L_{\rm rad} = \frac{2\delta P_{\rm rad}}{c} |\chi(\omega)| \simeq \sqrt{\frac{8hP_0}{c\lambda}} \frac{1}{m\omega^2}$$

Standard Quantum Limit

- Shot noise is lower with higher power

$$h_{\mathrm{shot}} = \frac{1}{2\pi L}\sqrt{\frac{hc\lambda}{2P_0}}$$

Trade-off

- Radiation pressure noise is lower with lower power

$$h_{\mathrm{rad}} = \frac{1}{m\omega^2 L}\sqrt{\frac{8hP_0}{c\lambda}}$$

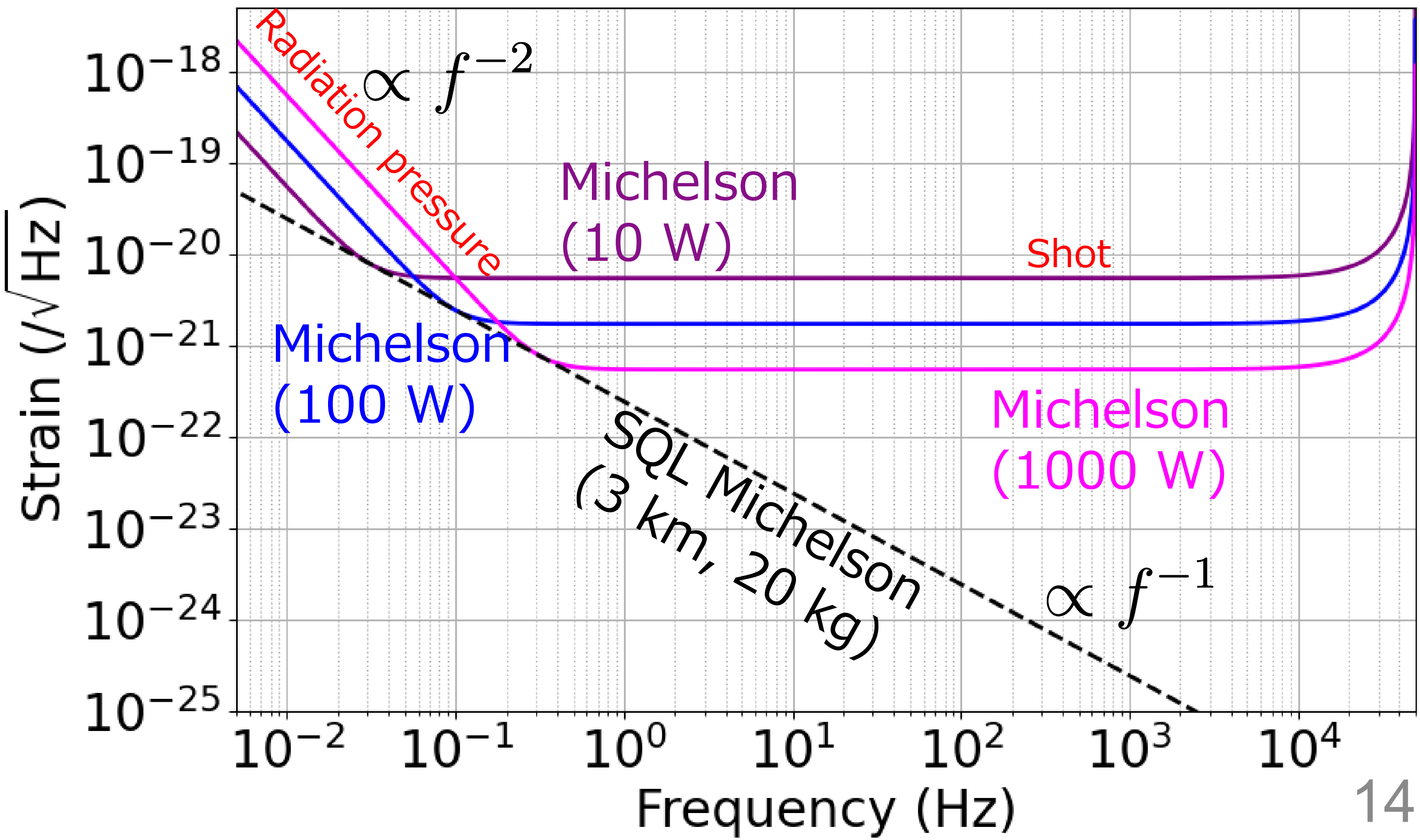
√2 for two arms
m is mirror mass
(m/2 is reduced mass for MI)
Assuming BS with infinite mass

- Standard Quantum Limit (SQL) for Michelson

$$h_{\mathrm{SQL}} = \sqrt{2h_{\mathrm{shot}}h_{\mathrm{rad}}} = \sqrt{\frac{4\hbar}{m\omega^2 L^2}}$$

Input Power and Sensitivity

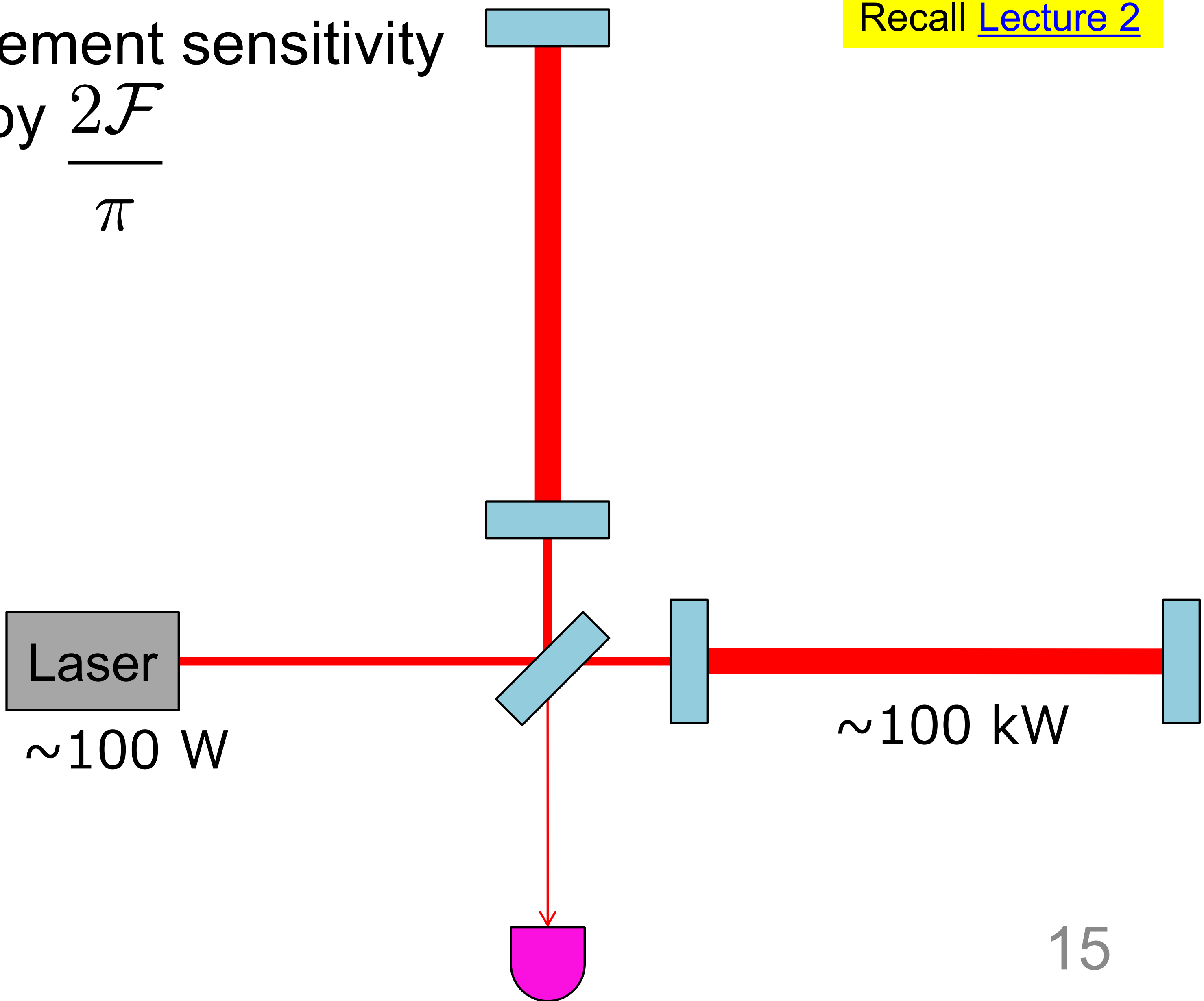
- SQL cannot be beaten just by changing power



Fabry-Pérot-Michelson Interferometer

Recall [Lecture 2]

- Displacement sensitivity higher by $\dfrac{2\mathcal{F}}{\pi}$
- FPMI

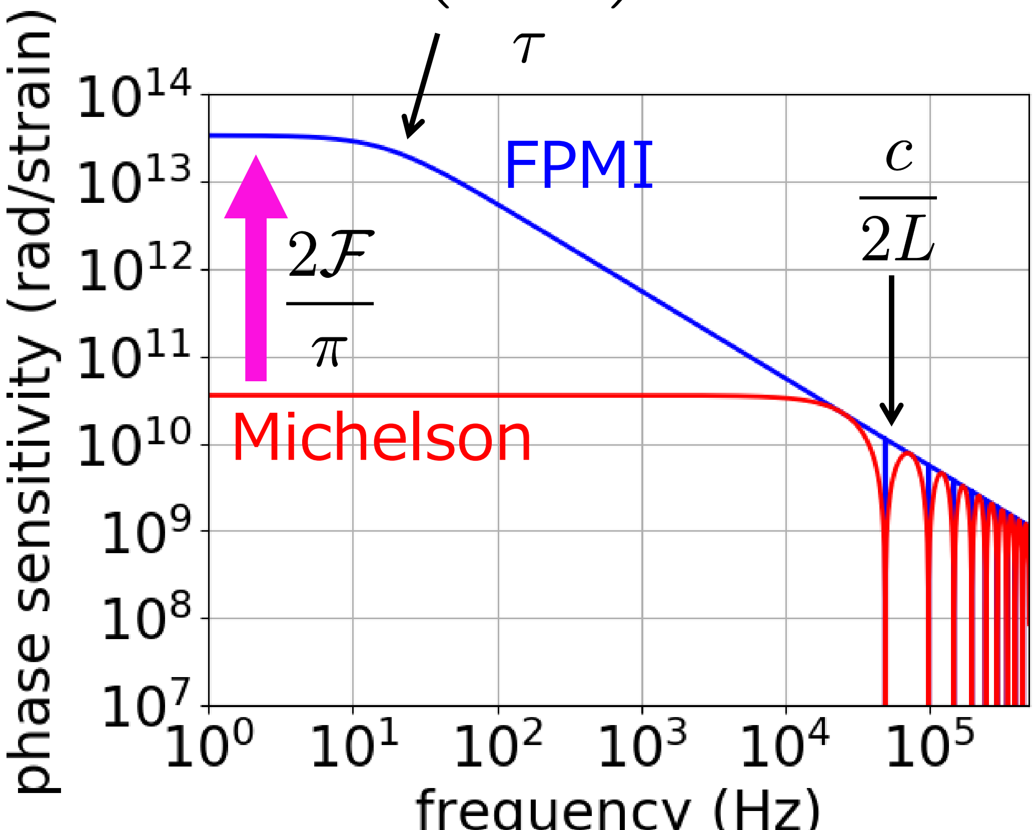


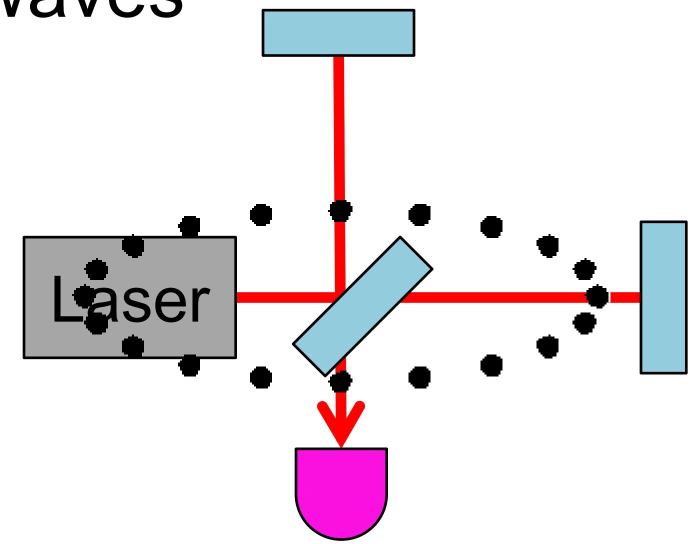
High-Frequency Response

Recall Lecture 2

- The effect of gravitational waves cancel at high frequencies

$$f_c = \frac{1}{2\pi}\underbrace{\left(\frac{2\mathcal{F}}{\pi}\frac{L}{c}\right)}_{\tau}^{-1} = \frac{c}{4L\mathcal{F}}$$





For a given frequency, there is a limit where longer arm length and higher finesse won't help increasing the sensitivity

FPMI Quantum Noise

- Shot noise

$$h_{\rm shot} = \frac{1}{2\pi L}\sqrt{\frac{hc\lambda}{2P_0}}\frac{\pi}{2\mathcal{F}}\sqrt{1+(\omega\tau)^2} \qquad \tau \equiv \frac{2L\mathcal{F}}{\pi c}$$

- Radiation pressure noise

$$h_{\rm rad} = \frac{2}{m\omega^2 L}\sqrt{\frac{8hP_0}{c\lambda}}\frac{2\mathcal{F}}{\pi}\frac{1}{\sqrt{1+(\omega\tau)^2}}$$

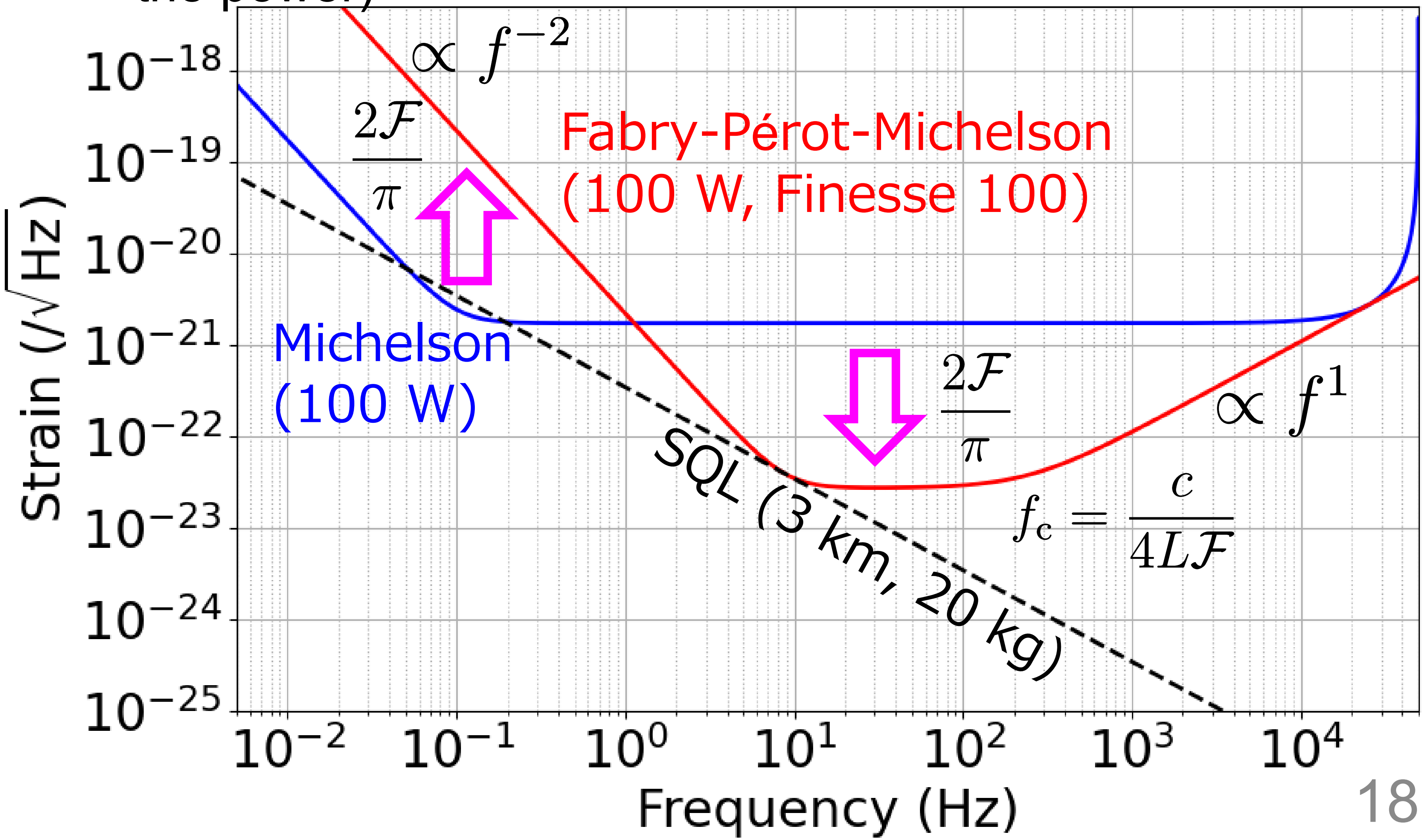
2 for two mirrors of a cavity
m is mirror mass (m/4 is reduced mass for FPMI)

- Standard Quantum Limit (SQL) for FPMI

$$h_{\rm SQL} = \sqrt{2h_{\rm shot}h_{\rm rad}} = \sqrt{\frac{8\hbar}{m\omega^2 L^2}}$$

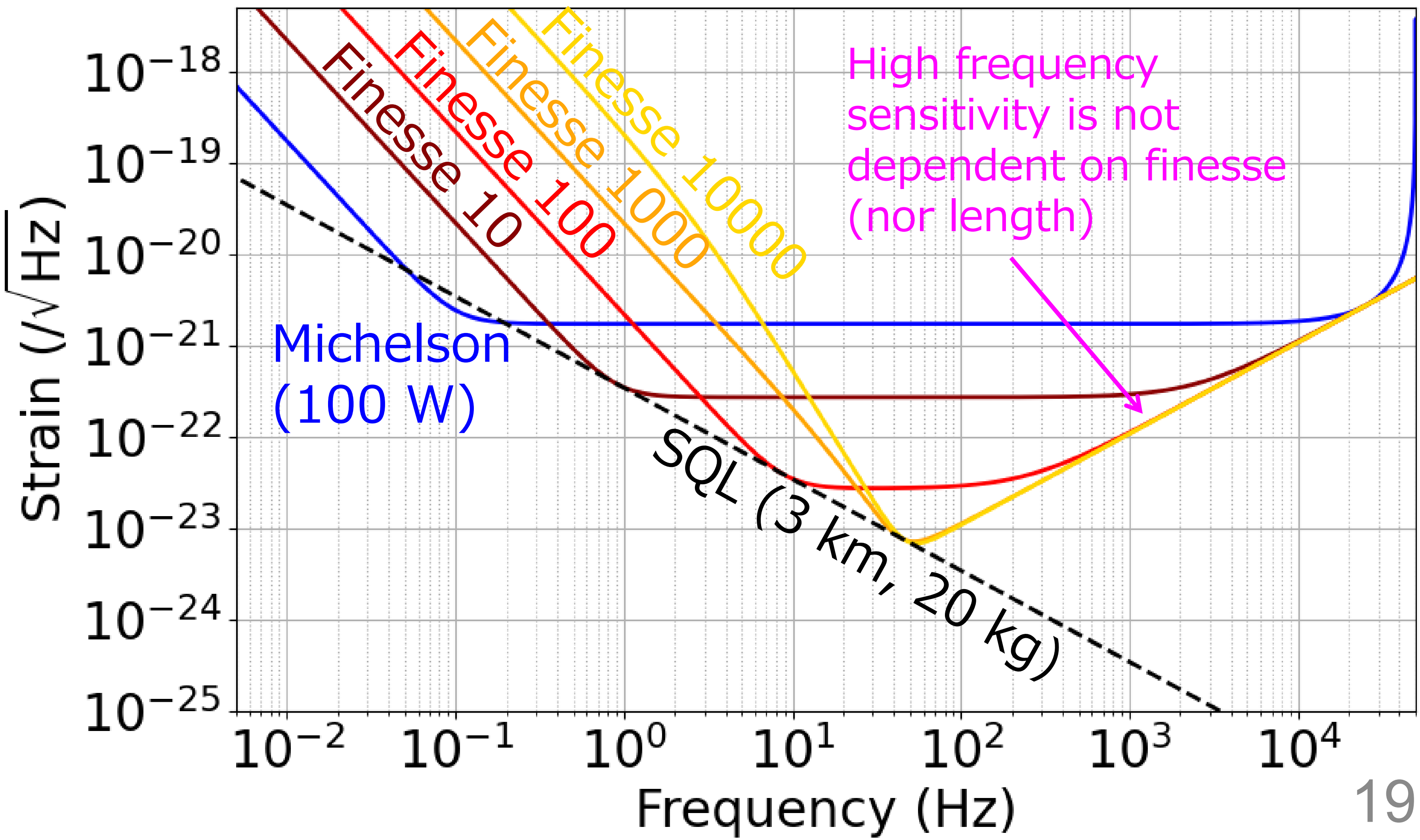
Use of Fabry-Pérot Cavities

- Still, SQL cannot be beaten (similar effect to increasing the power)



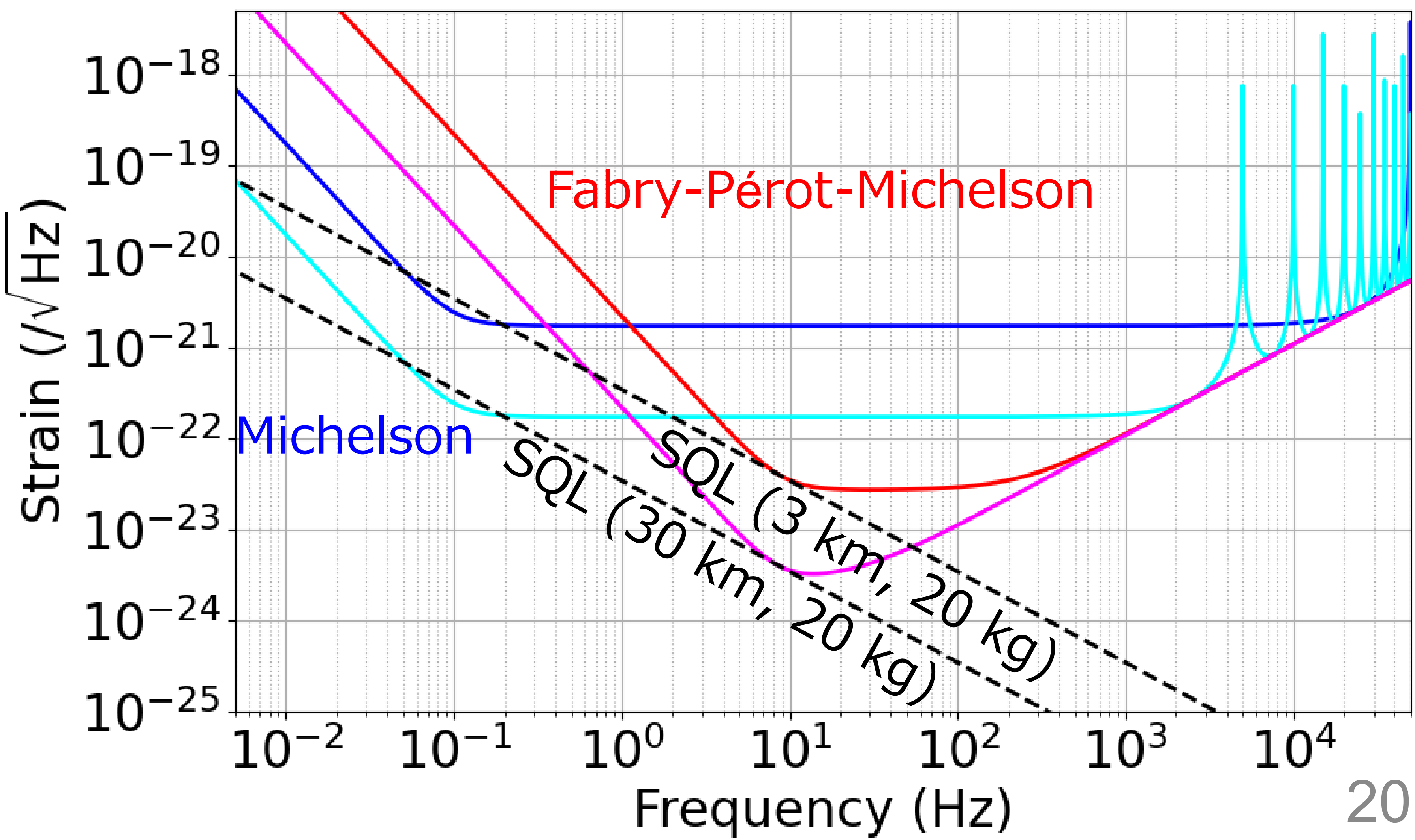
Finesse Dependence

- Higher the finesse, narrower the bandwidth



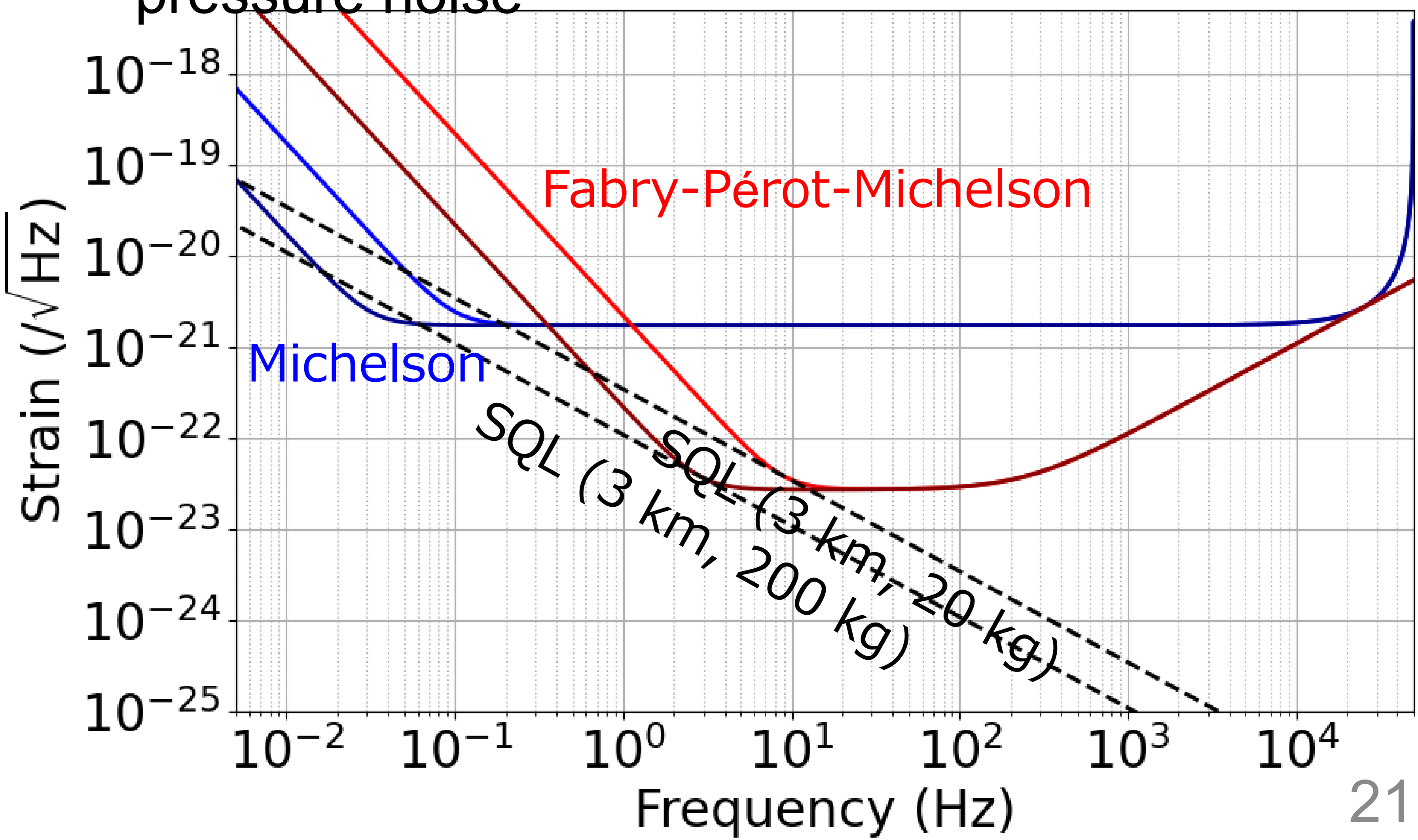
Arm Length Dependence

- Longer arm is better (but not at high frequencies)



Mirror Mass Dependence

- Heavier mass is better for reducing radiation pressure noise

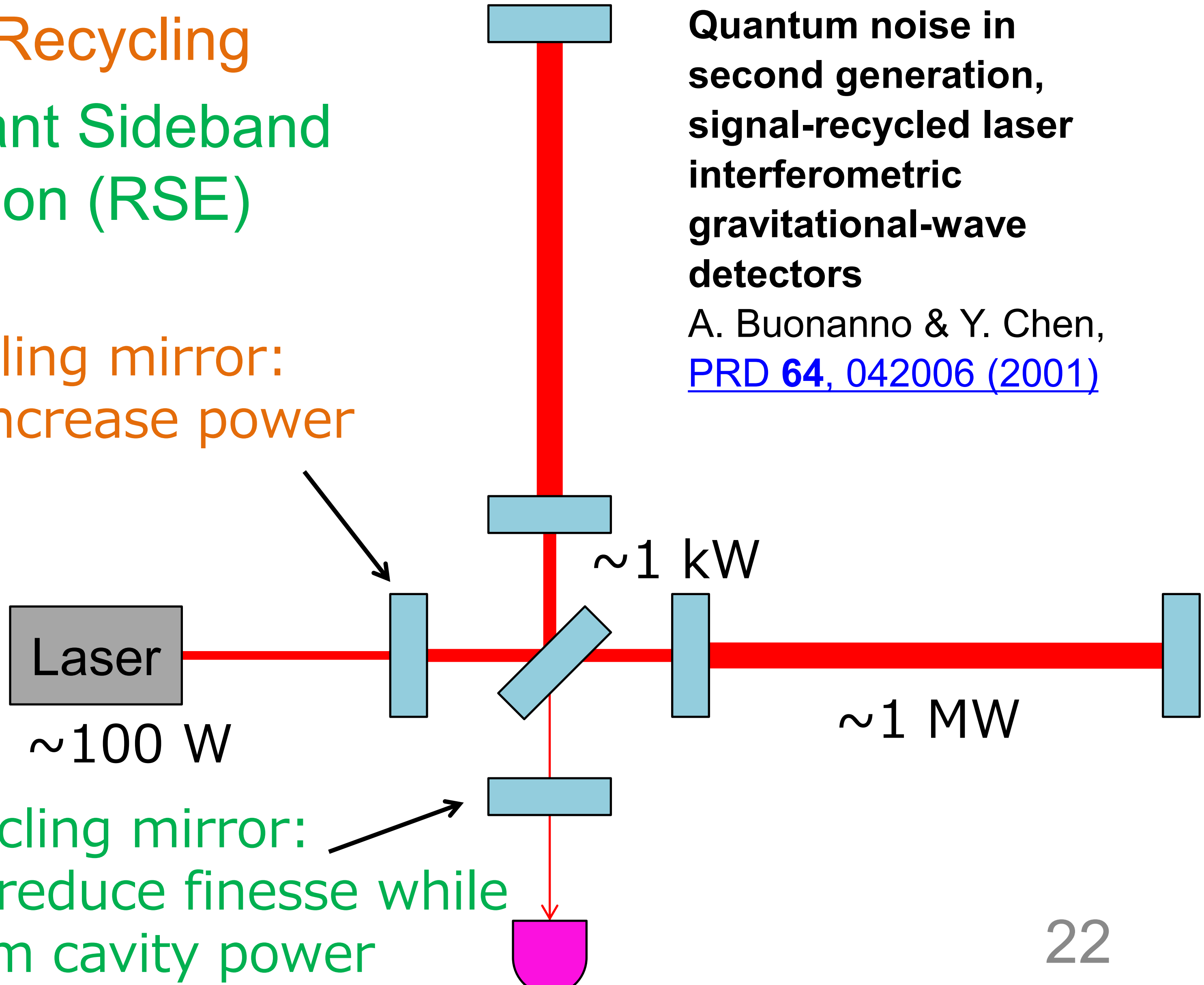


Resonant Sideband Extraction

- Power Recycling
- Resonant Sideband Extraction (RSE)

Quantum noise in second generation, signal-recycled laser interferometric gravitational-wave detectors
A. Buonanno & Y. Chen, [PRD **64**, 042006 (2001)]

Power recycling mirror:
Effectively increase power

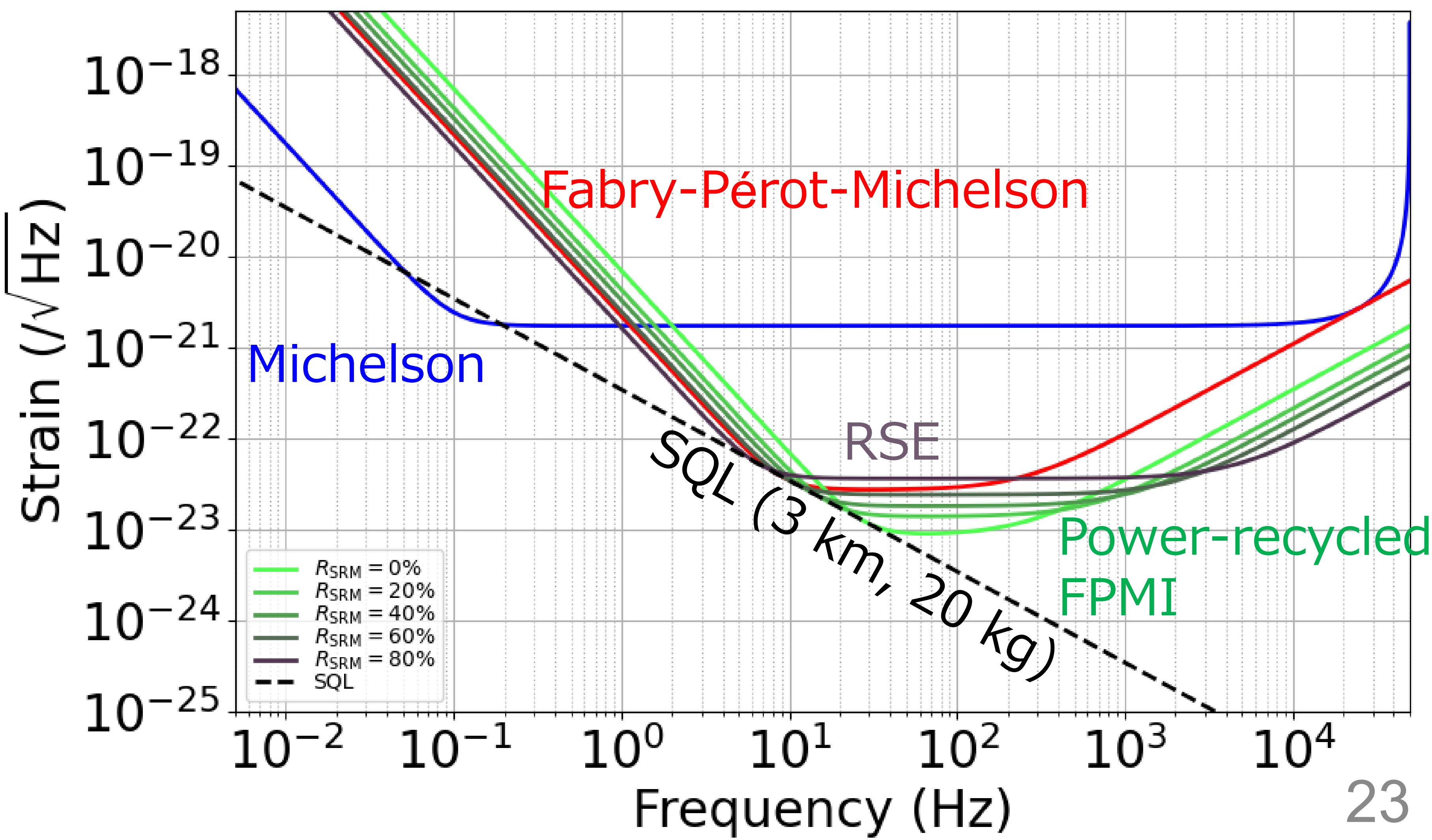


※ Actually, this is signal extraction mirror, but we call this recycling

Signal recycling mirror:
Effectively reduce finesse while keeping arm cavity power

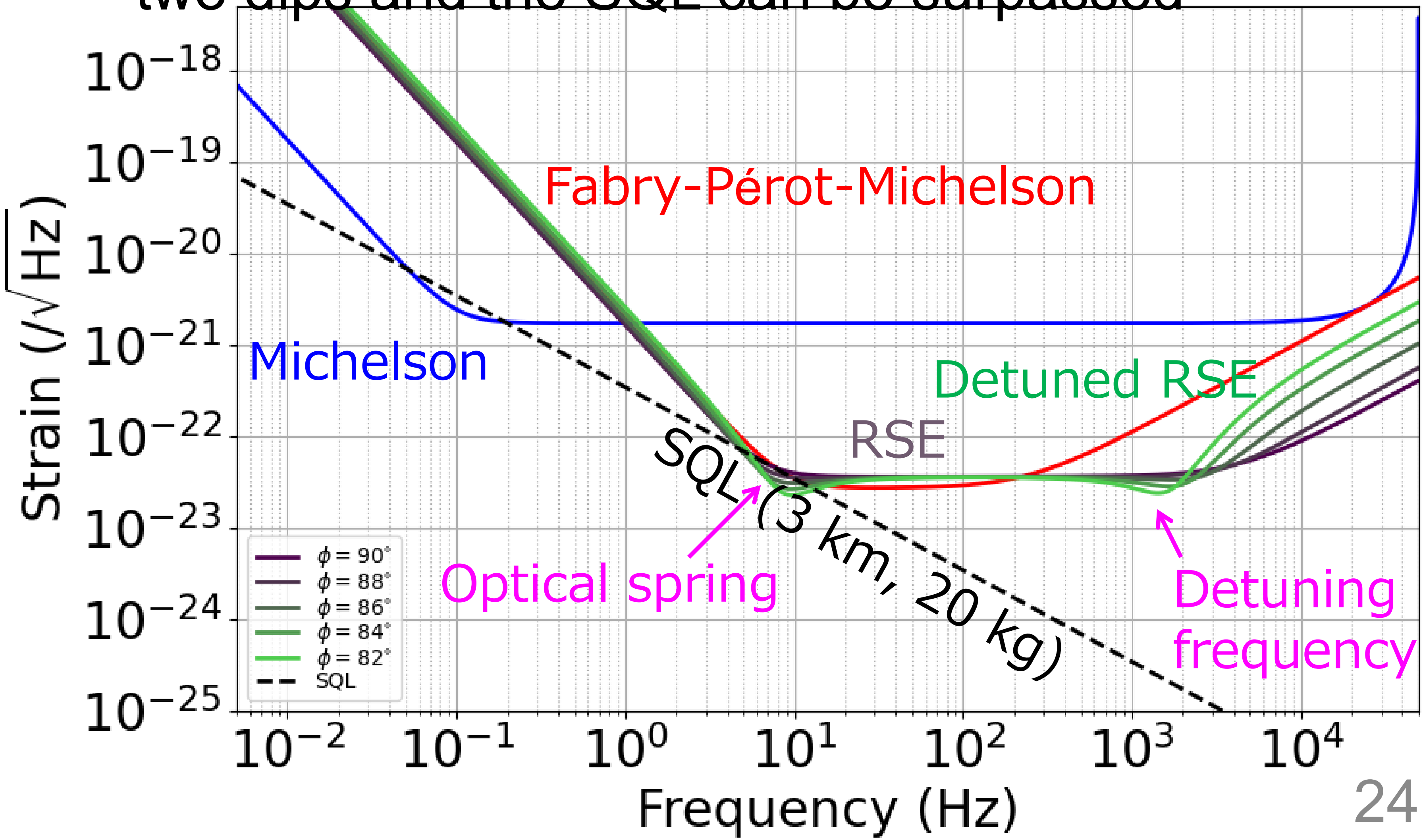
Resonant Sideband Extraction

- RSE increases the detector bandwidth



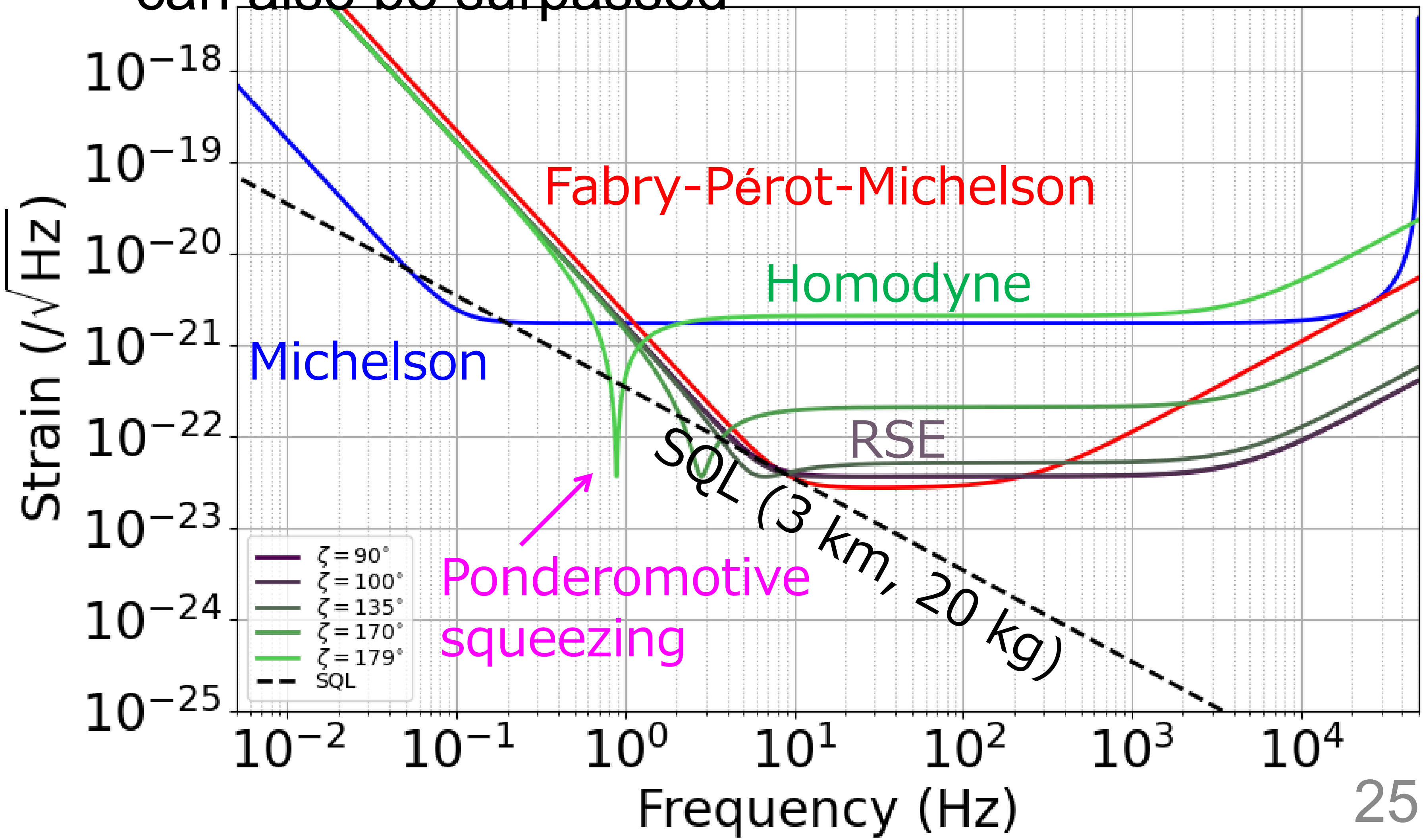
Detuned RSE

- Detuning of signal recycling cavity (SRC) creates two dips and the SQL can be surpassed



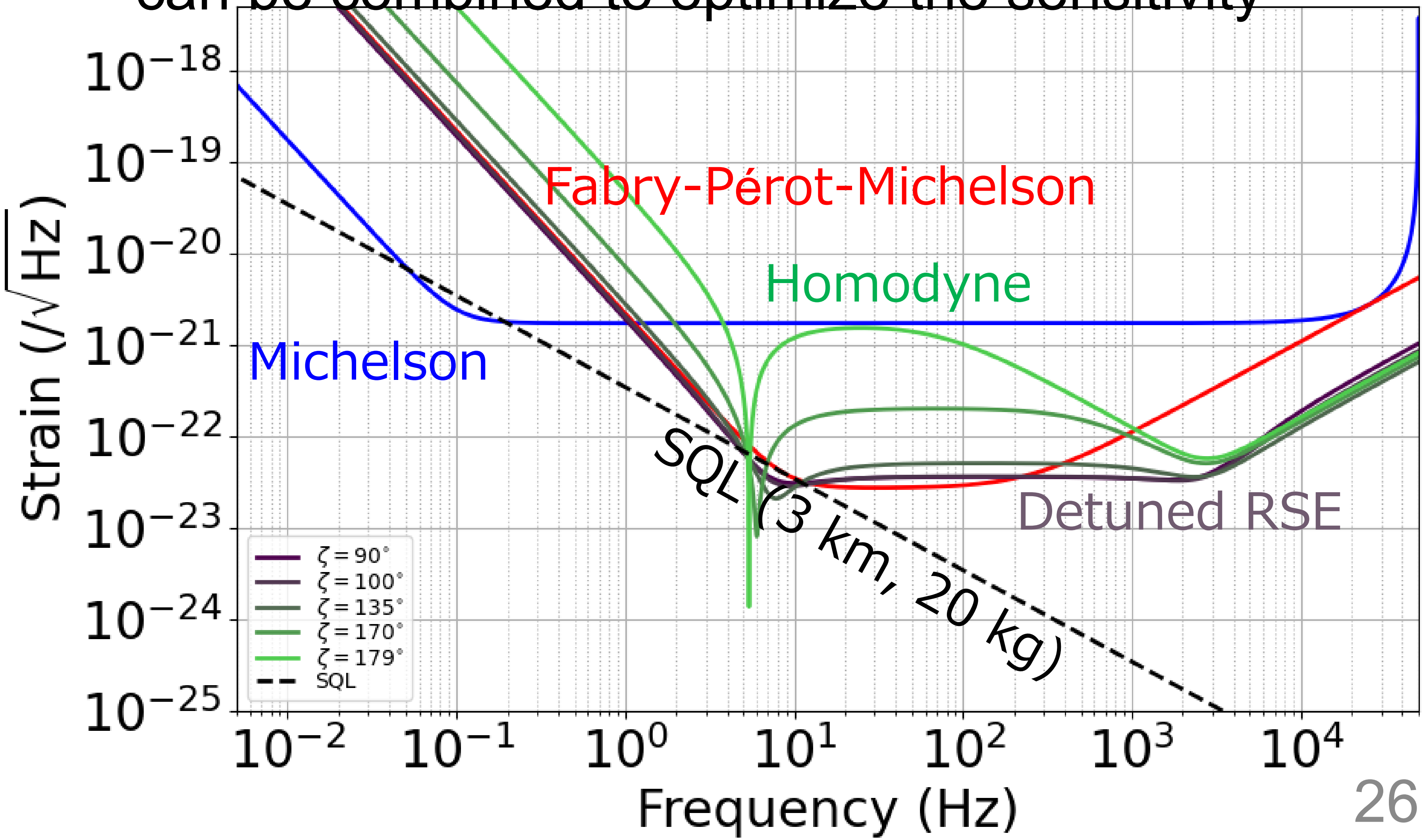
Changing Homodyne Angle

- By changing the readout homodyne angle, SQL can also be surpassed



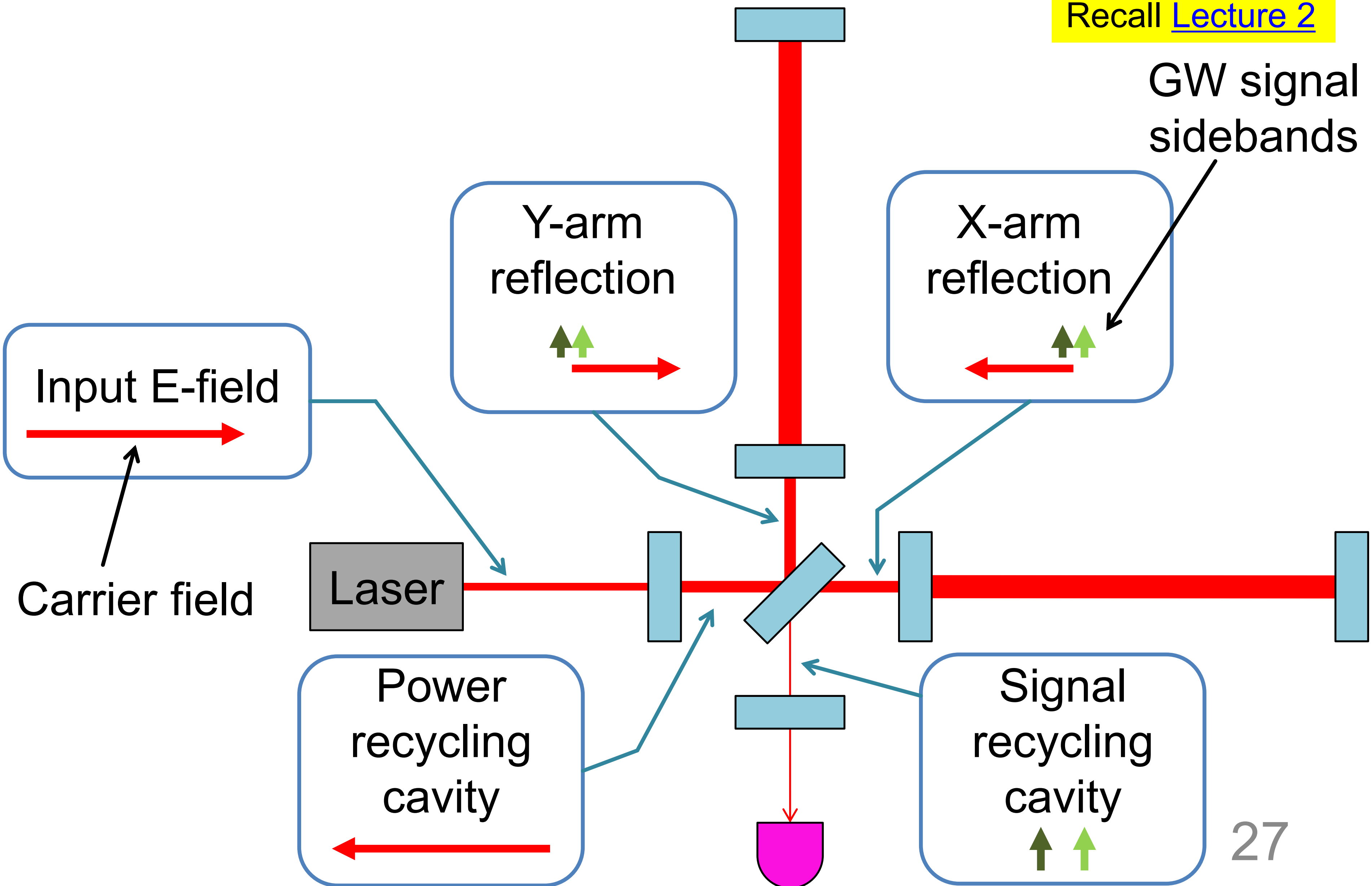
Detuning + Homodyne

- Detuning of SRC and changing homodyne angle can be combined to optimize the sensitivity



Magic of Michelson Interferometer

Recall Lecture 2

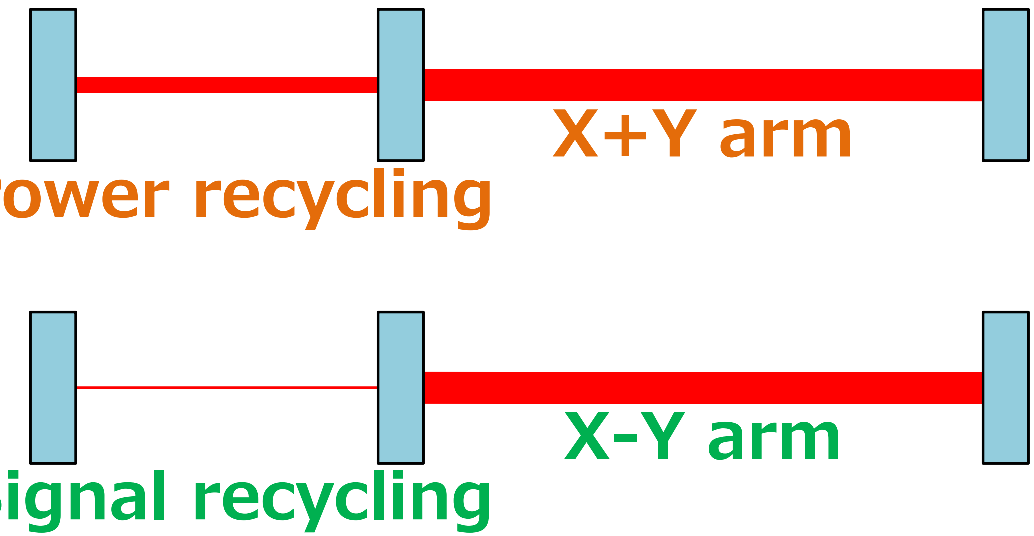


Magic of Michelson Interferometer

- Michelson interferometer effectively splits X and Y arms into common mode and differential mode

Power recycling cavity

Carrier field response determined by power recycling side



Signal recycling cavity

Signal field response determined by signal recycling side

Recap of Cavity Response

Recall Lecture 2

- Cavity reflection

$$E_{\text{refl}} = \underbrace{\left(-r_1 + \frac{t_1^2 r_2 e^{i\frac{4\pi L}{\lambda}}}{1 - r_1 r_2 e^{i\frac{4\pi L}{\lambda}}} \right)}_{\text{Cavity reflectivity}} E_{\text{in}}$$

On resonance(~+1) Off resonance(~-1)

$$r_a = \frac{r_2(r_1^2 + t_1^2) - r_1}{1 - r_1 r_2} \qquad r_a' = -\frac{r_2(r_1^2 + t_1^2) + r_1}{1 + r_1 r_2}$$

- Cavity gain

$$E_{\text{cav}} = \underbrace{\frac{t_1}{1 - r_1 r_2 e^{i\frac{4\pi L}{\lambda}}}}_{\text{Cavity gain}} E_{\text{in}}$$

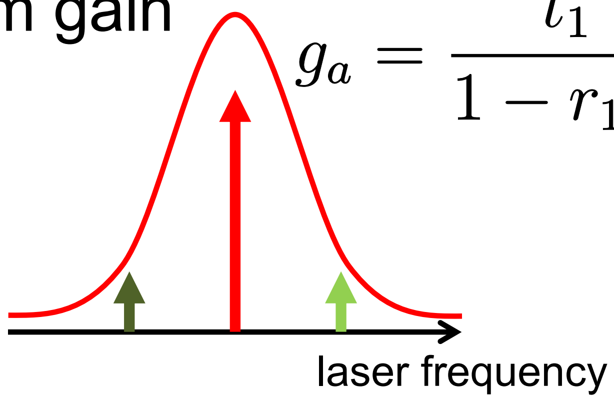
On resonance(~big) Off resonance(~small)

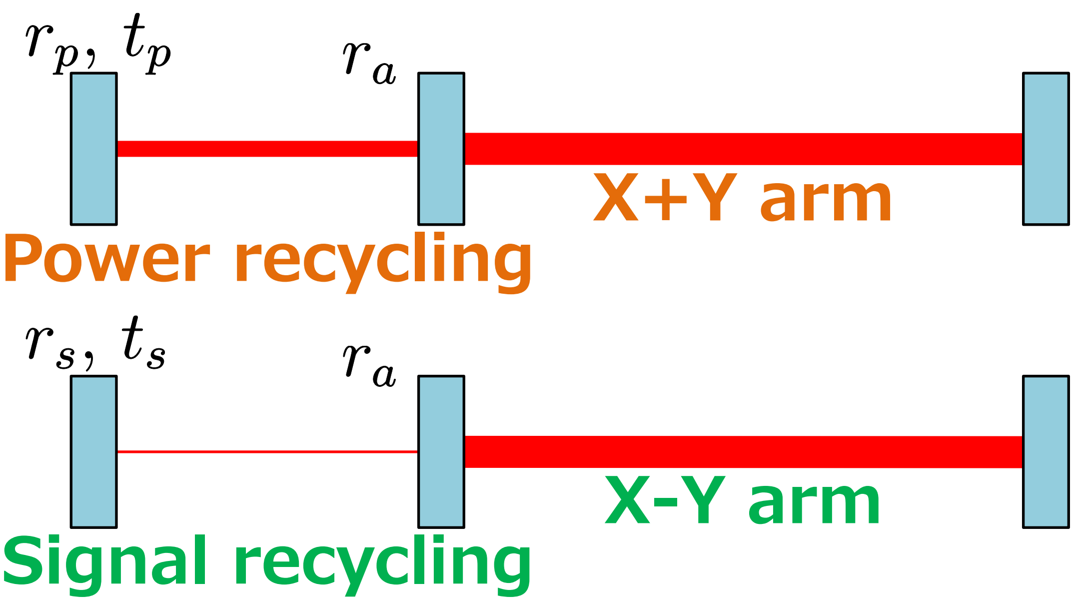
$$g_a = \frac{t_1}{1 - r_1 r_2} \qquad g_a' = \frac{t_1}{1 + r_1 r_2}$$

Coupled Cavity Response

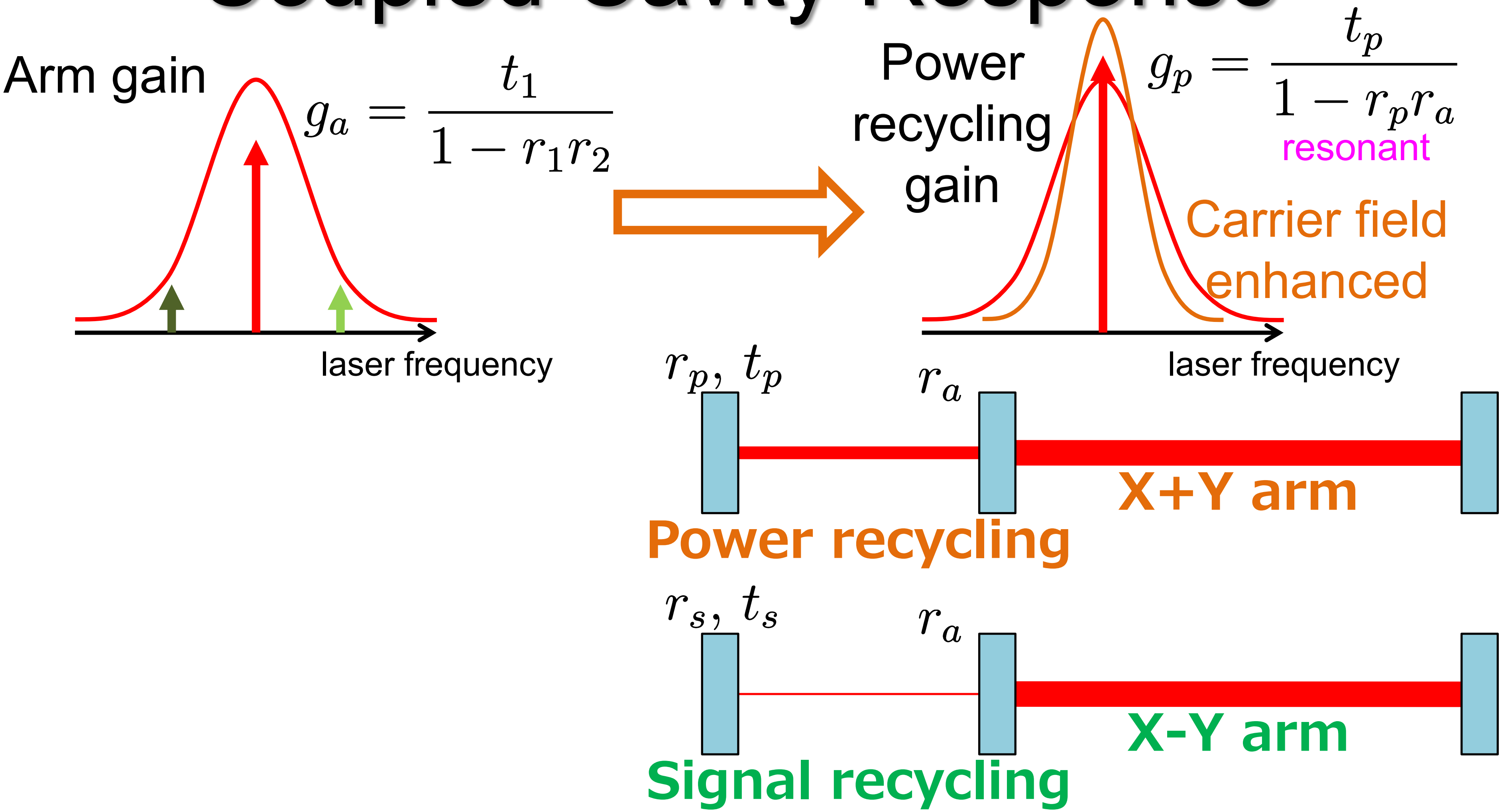
Arm gain

$$g_a = \frac{t_1}{1 - r_1 r_2}$$



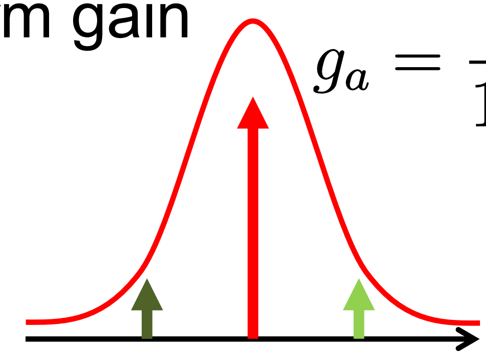


Coupled Cavity Response



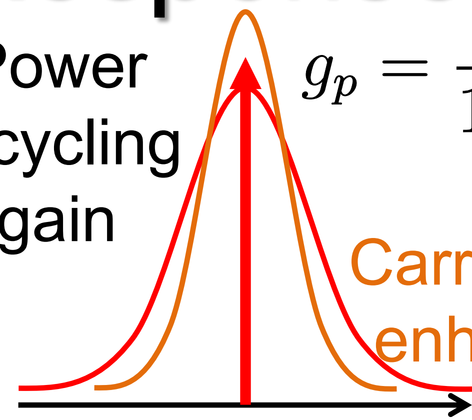
Coupled Cavity Response

Arm gain



$$g_a = \frac{t_1}{1 - r_1 r_2}$$

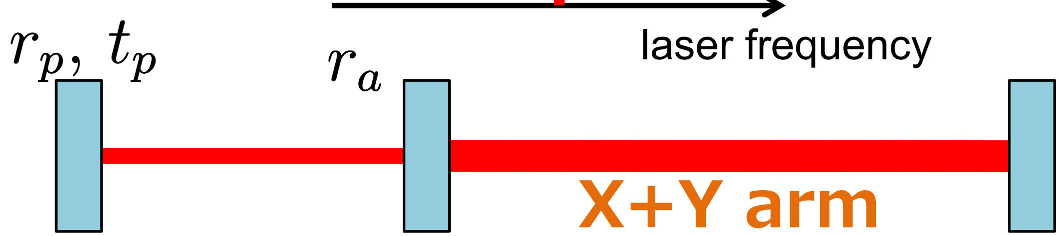
Power recycling gain



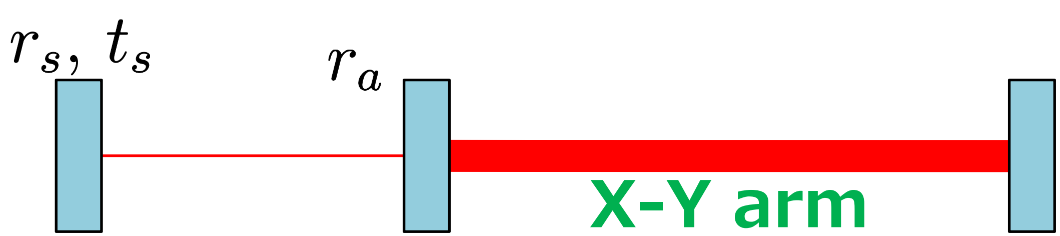
$$g_p = \frac{t_p}{1 - r_p r_a}$$

resonant

Carrier field enhanced

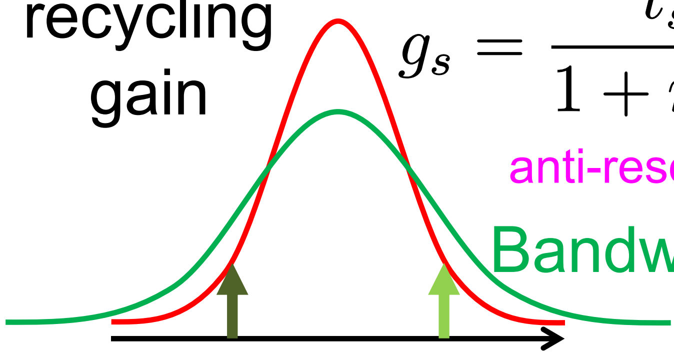


Power recycling



Signal recycling

Signal recycling gain



$$g_s = \frac{t_s}{1 + r_s r_a}$$

anti-resonant

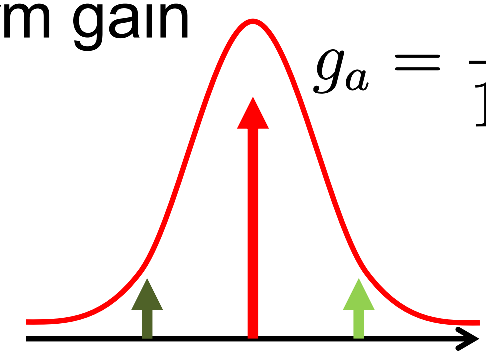
Bandwidth widened (Resonant Sideband Extraction)

Low frequency signal gain is reduced, but high frequency signal gain is increased

Detuned Signal Recycling Cavity

Arm gain

$$g_a = \frac{t_1}{1 - r_1 r_2}$$

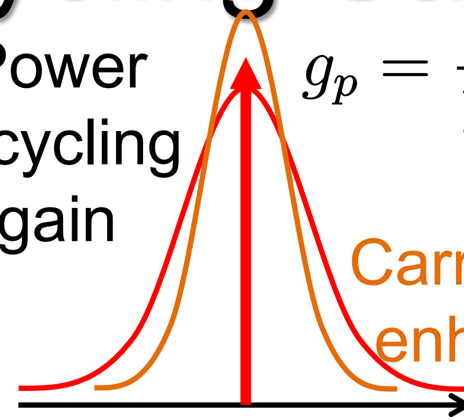


Power recycling gain

$$g_p = \frac{t_p}{1 - r_p r_a}$$

resonant

Carrier field enhanced

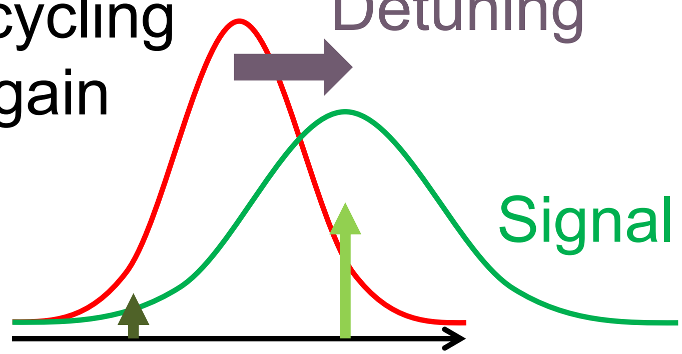


Power recycling

Signal recycling

Signal recycling gain

Detuning



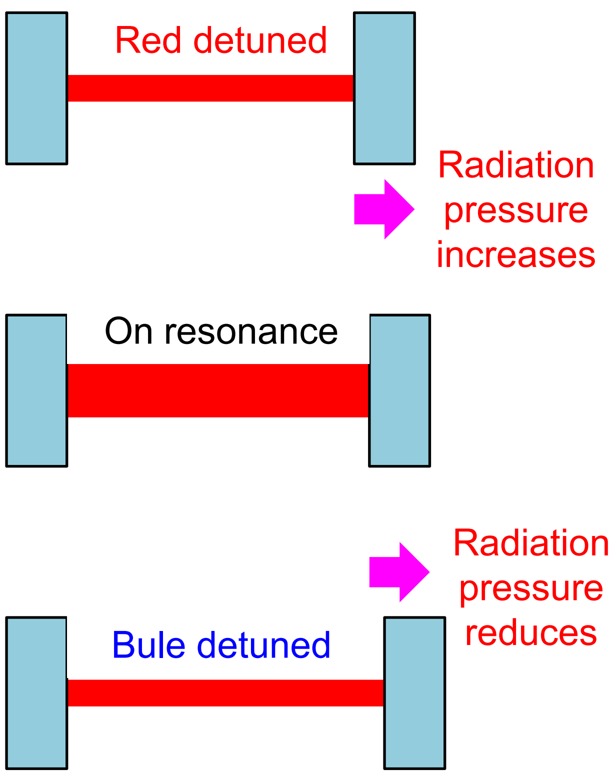
Signal at detuning frequency enhances

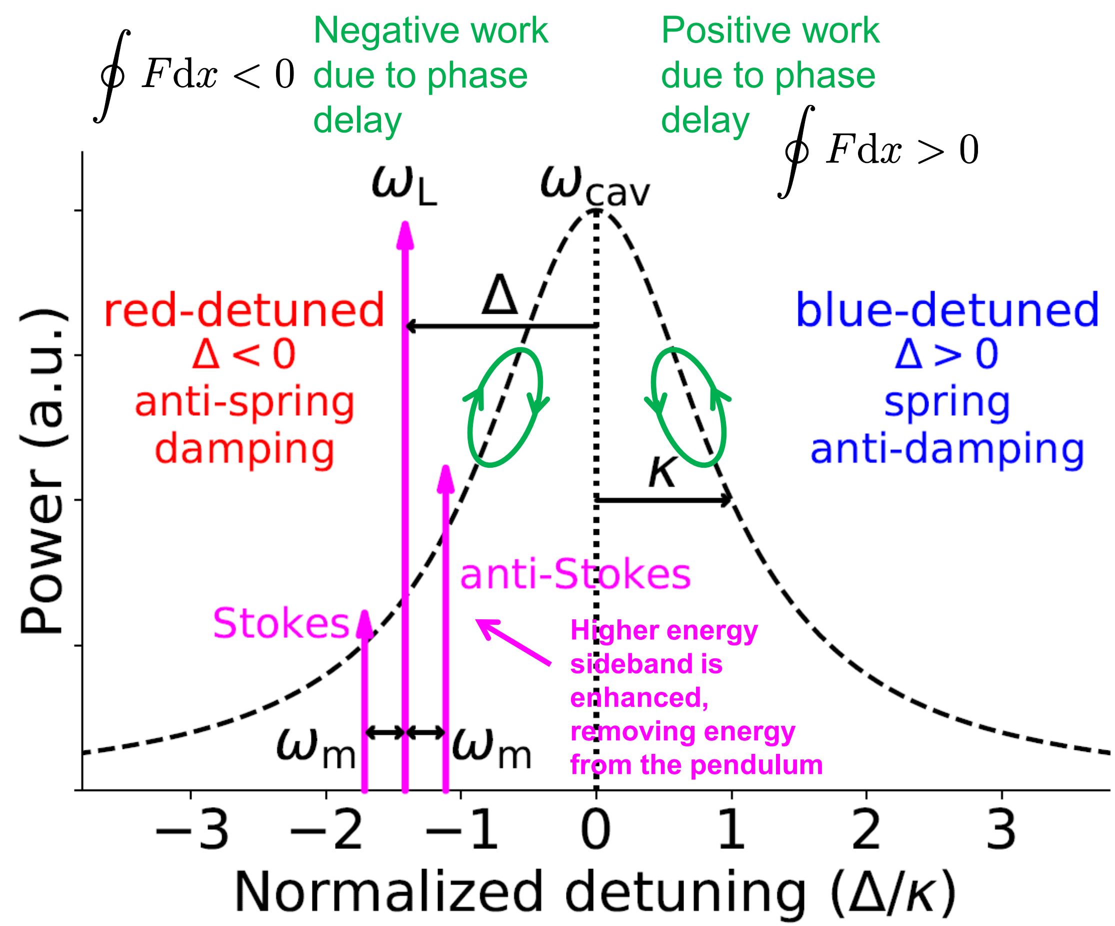
Detuning also creates signal sideband asymmetry, which creates "optical spring" dip, beating the SQL

Optical Spring and Damping

- Radiation pressure force inside optical cavity works as a spring and damping

Recall Lecture 3



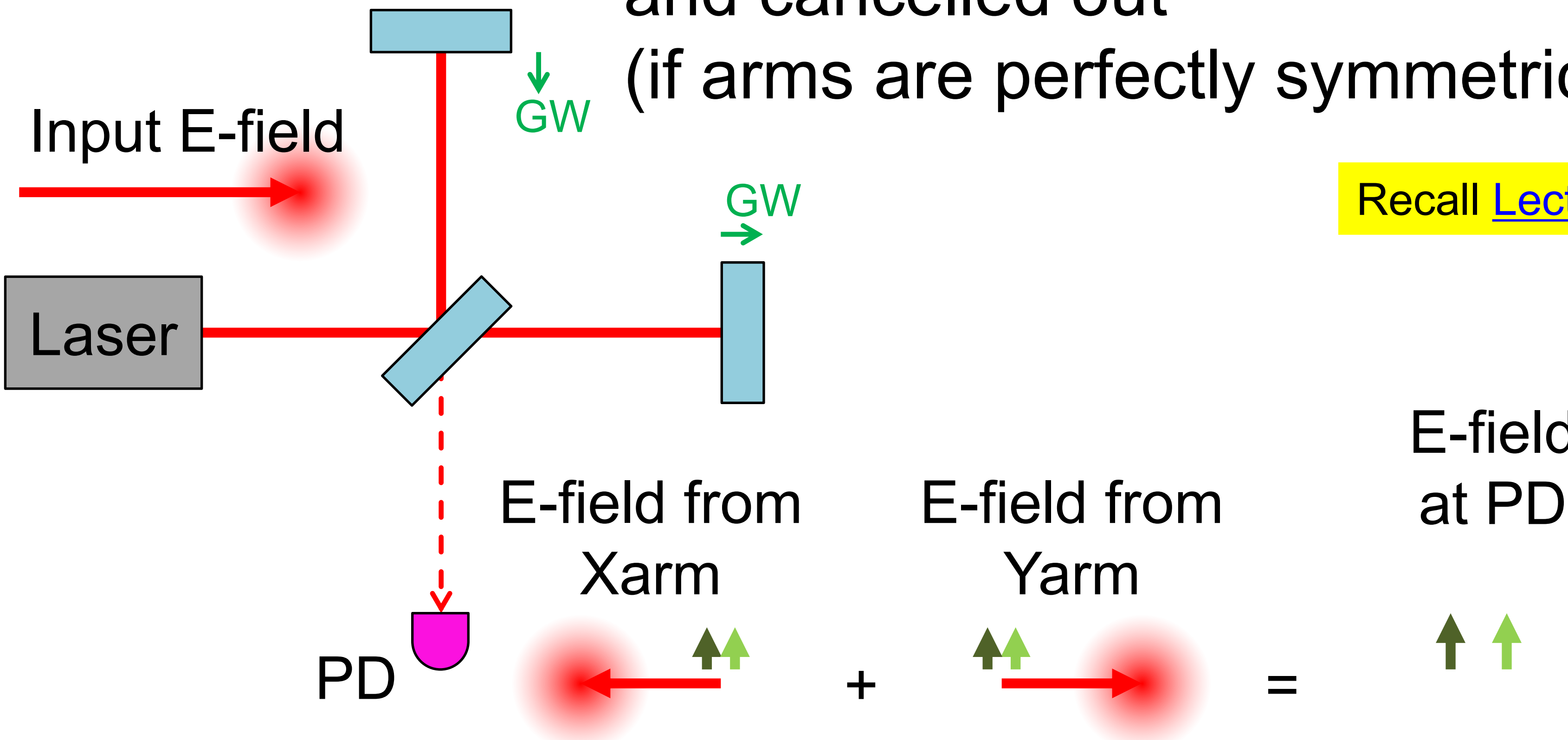


Y. Michimura, K. Komori, EPJD 74, 126 (2020)
M. Aspelmeyer, T. J. Kippenberg, F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014)

Now Let's Consider Noise

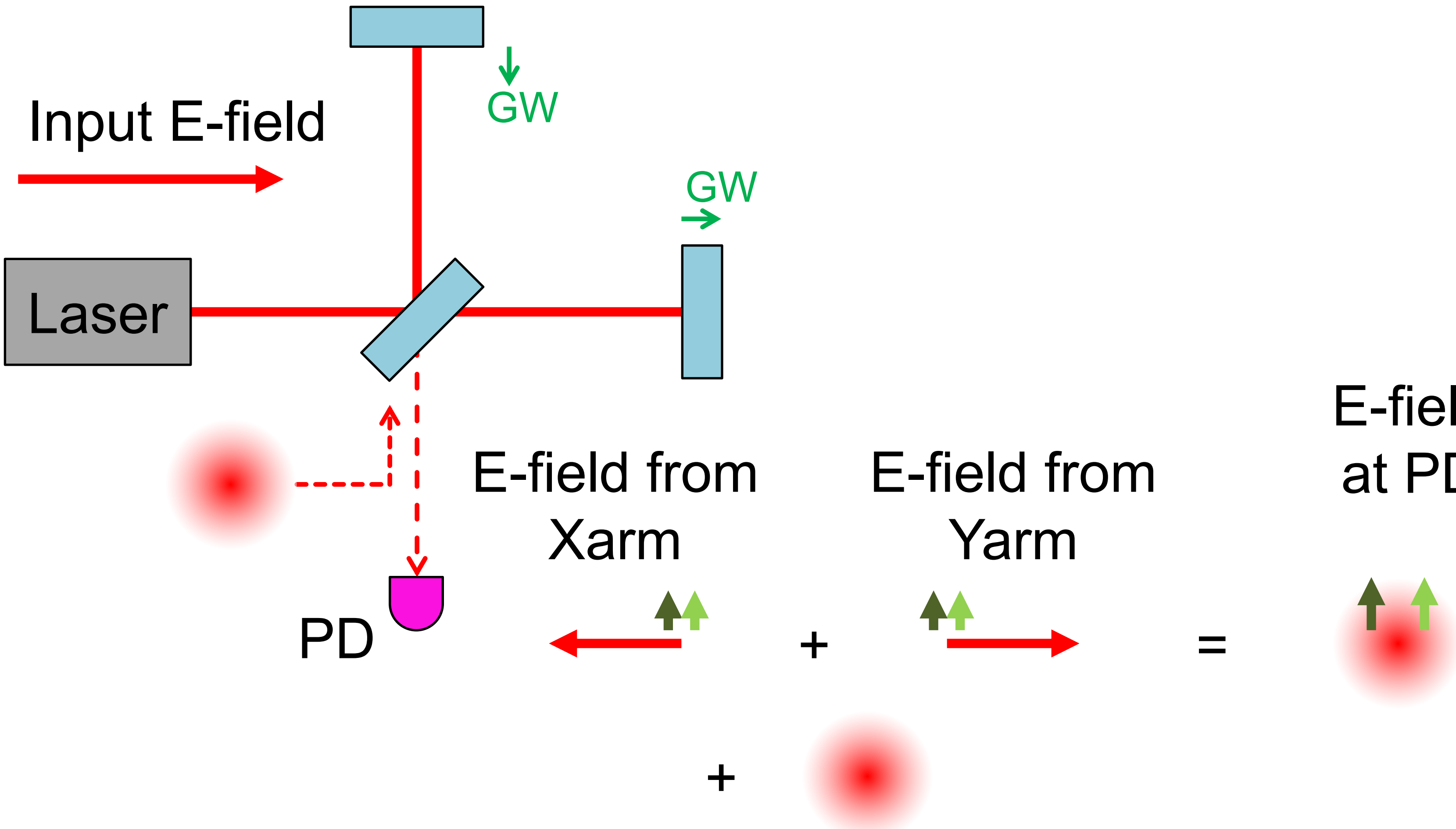
- Actually, noise from laser are common and cancelled out (if arms are perfectly symmetric)

Recall Lecture 2



Noise Comes from Dark Port

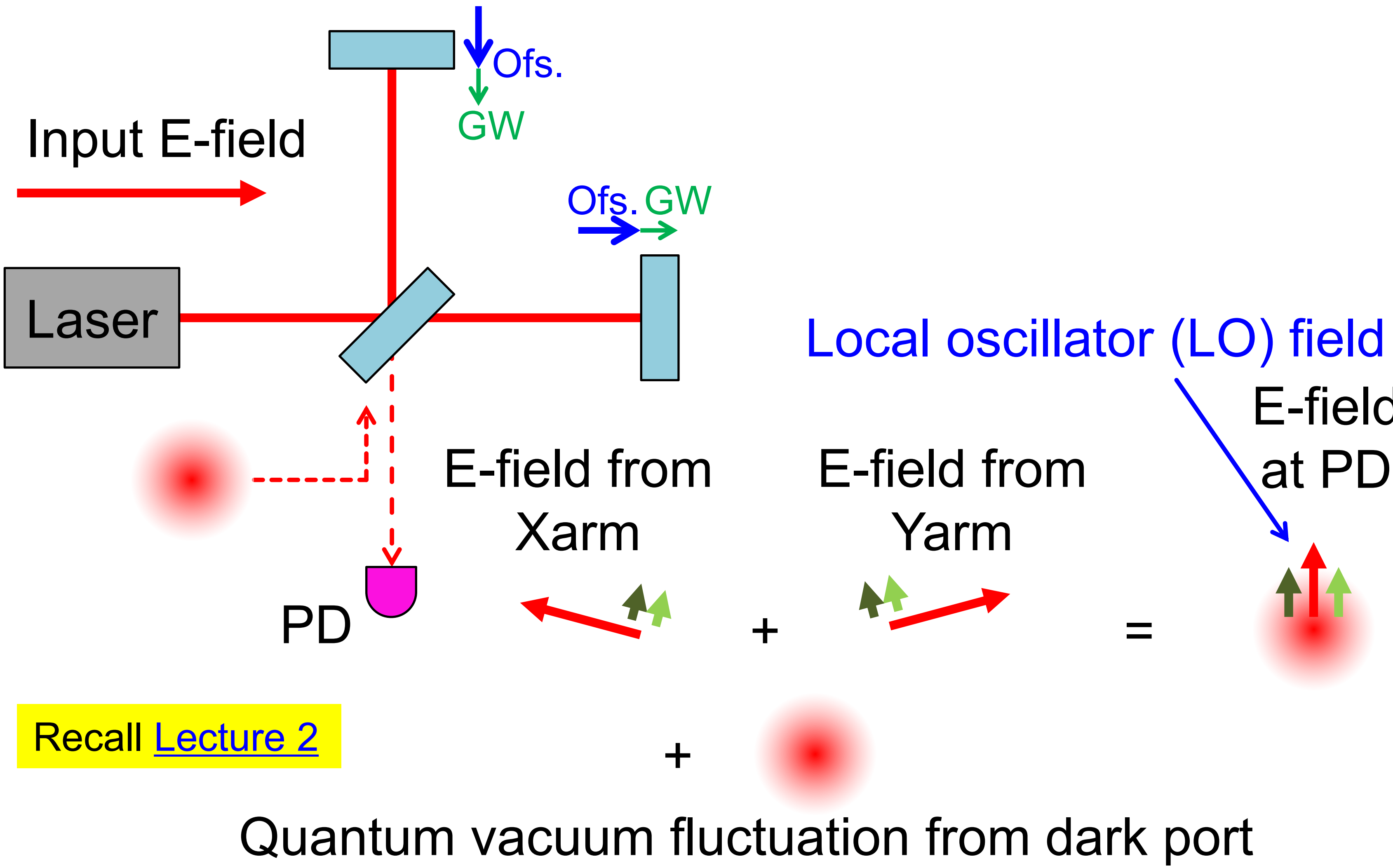
- Quantum vacuum fluctuation from dark port matters



Quantum vacuum fluctuation from dark port

Arm Offset for DC Readout

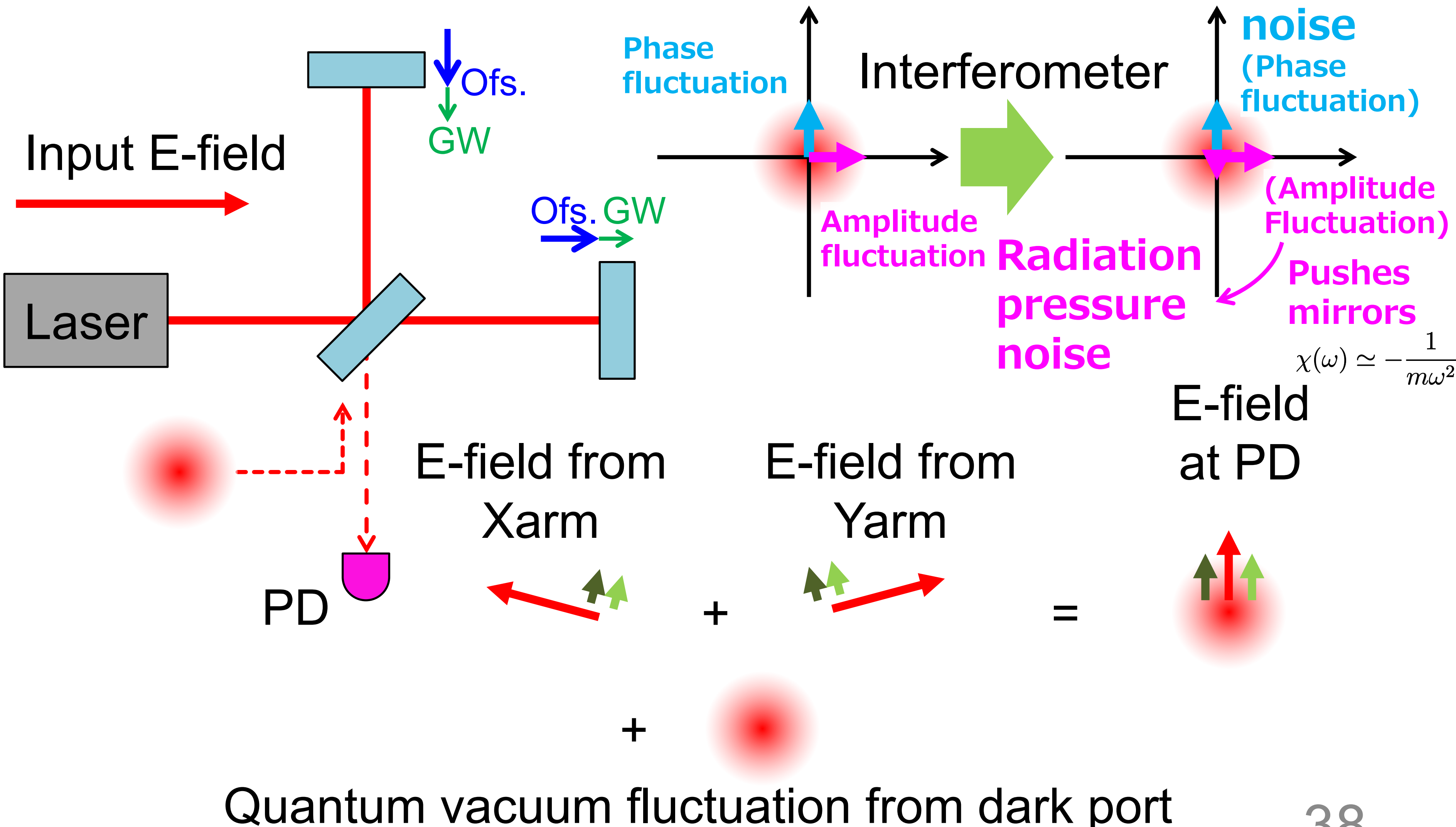
- Arm offset makes LO for homodyne readout



Recall Lecture 2

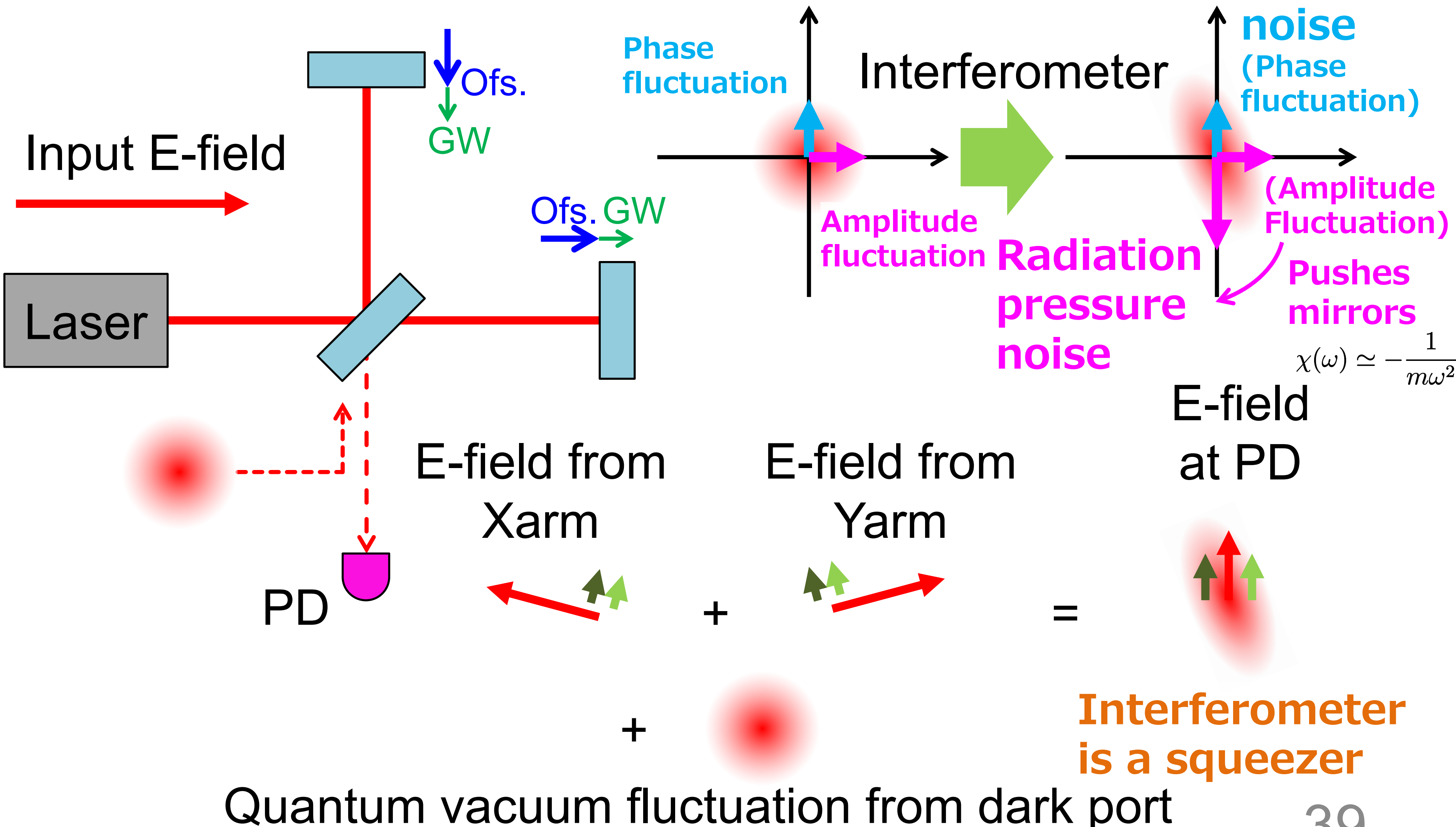
Shot Noise at High Frequencies

- Shot noise dominates at high frequencies

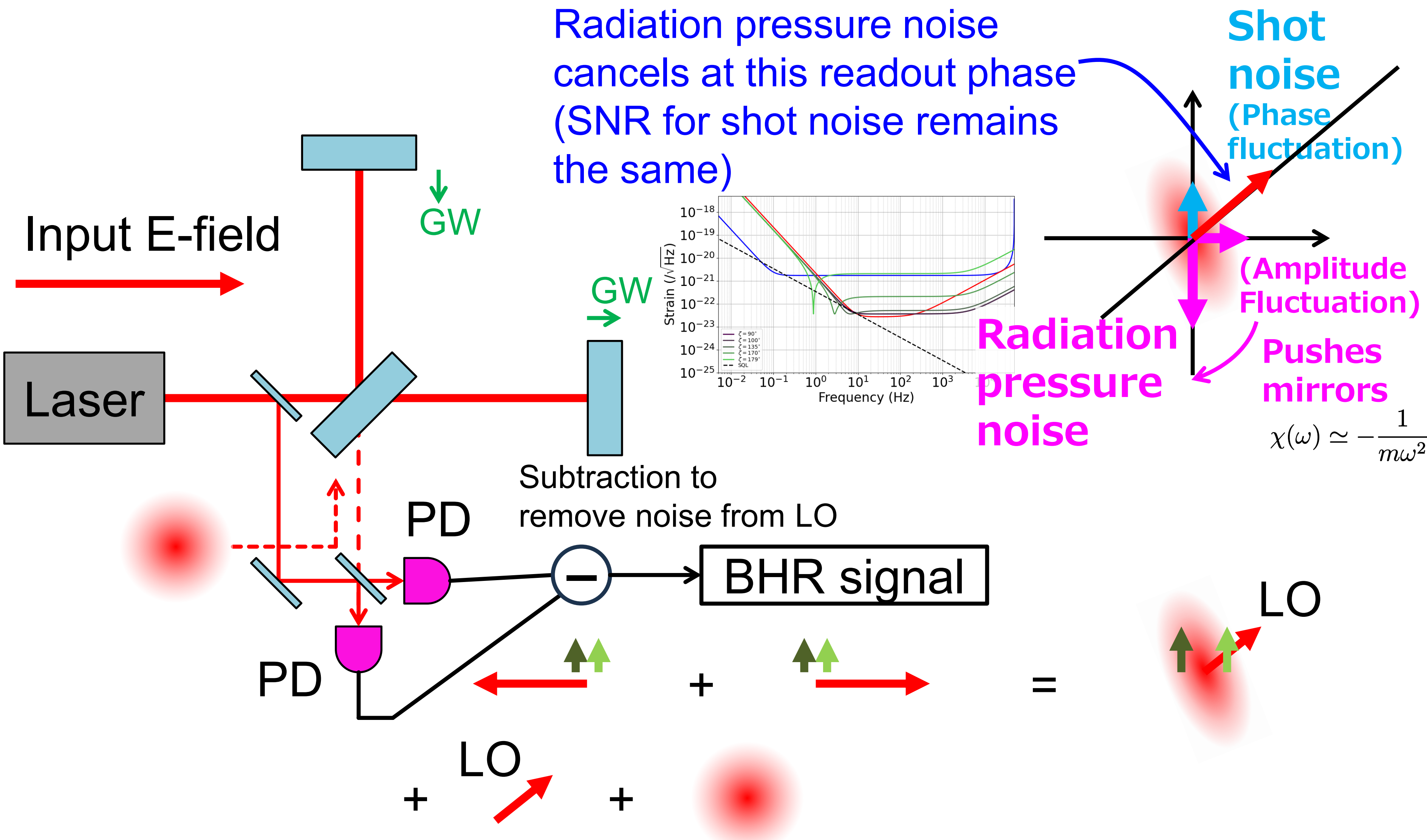


Ponderomotive Squeezing

- Radiation pressure makes squeezing



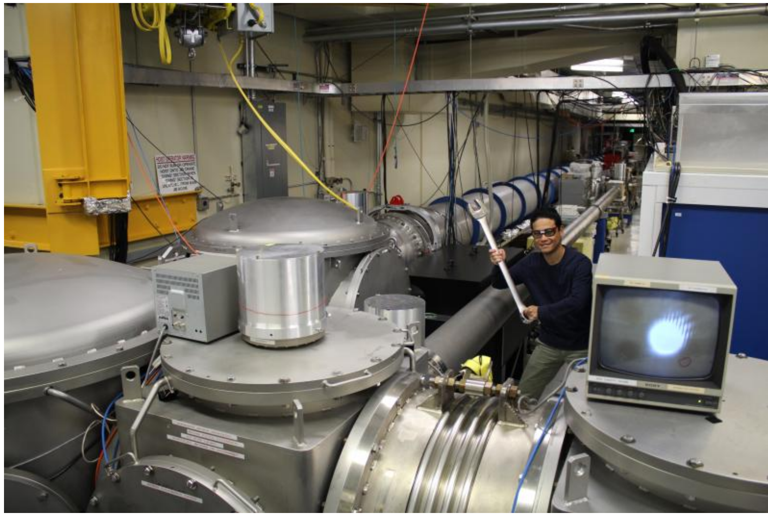
Balanced Homodyne Readout



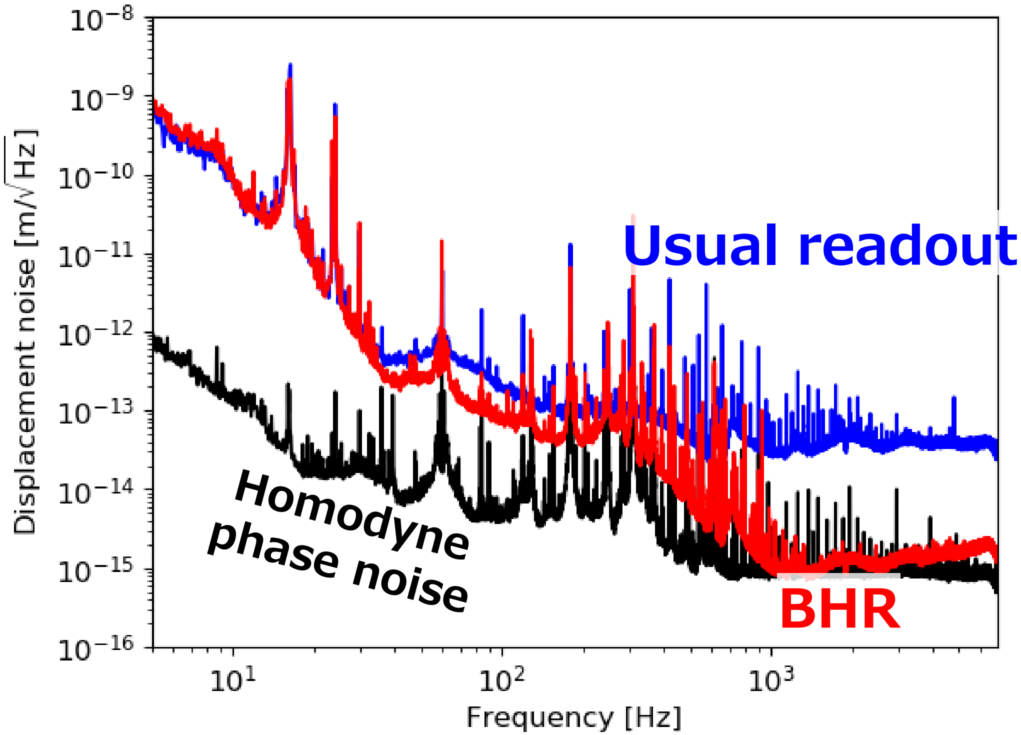
Quantum vacuum fluctuation from dark port

BHD at Caltech 40m Prototype

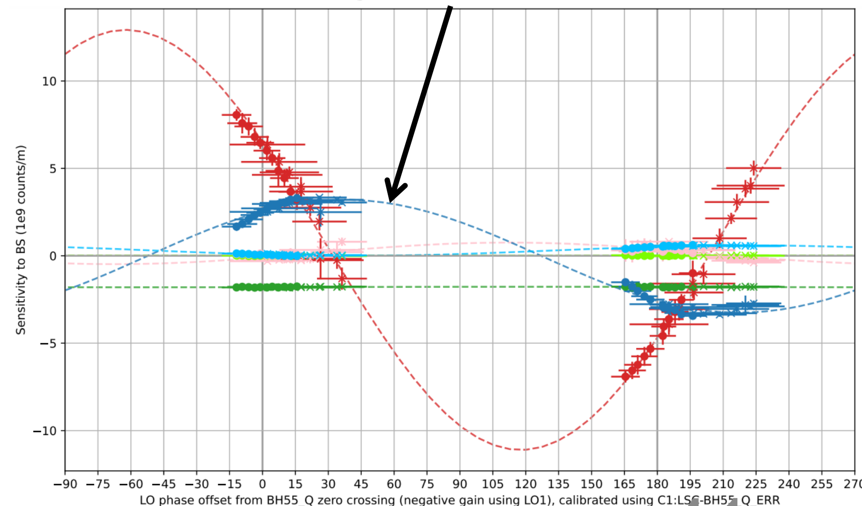
- LIGO plans to implement BHD for O5
- Prototype experiment happening at Caltech 40m



Caltech

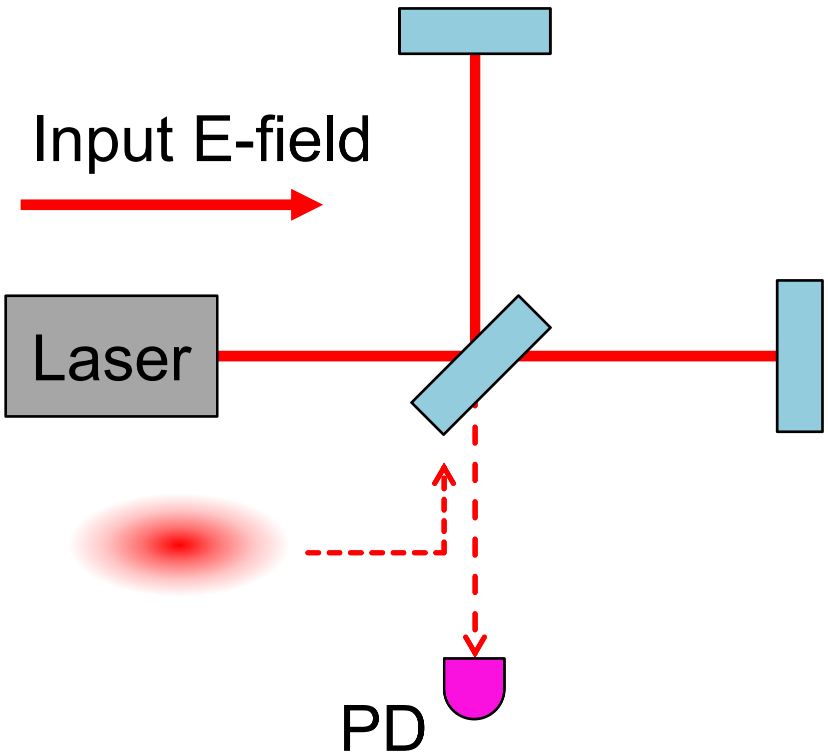


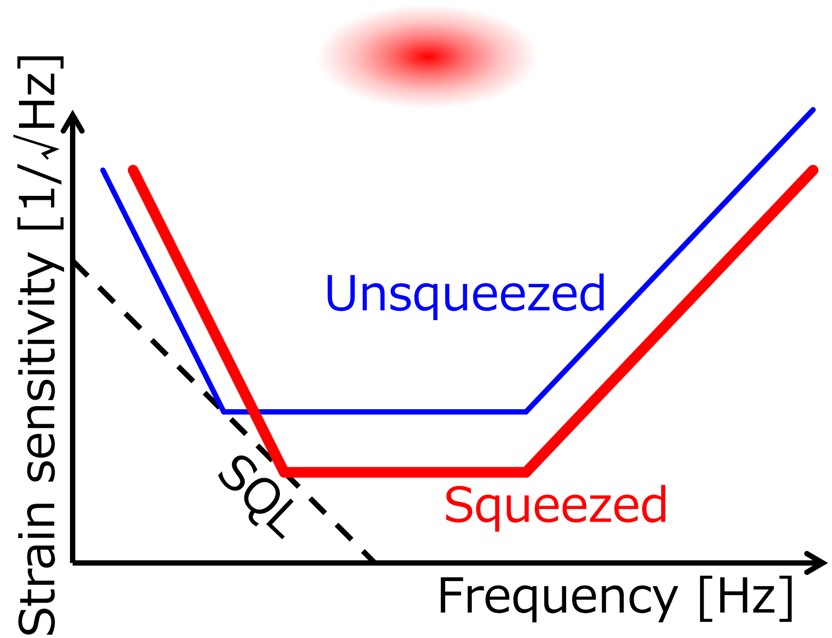
Homodyne angle changes the response



Squeezed Vacuum Injection

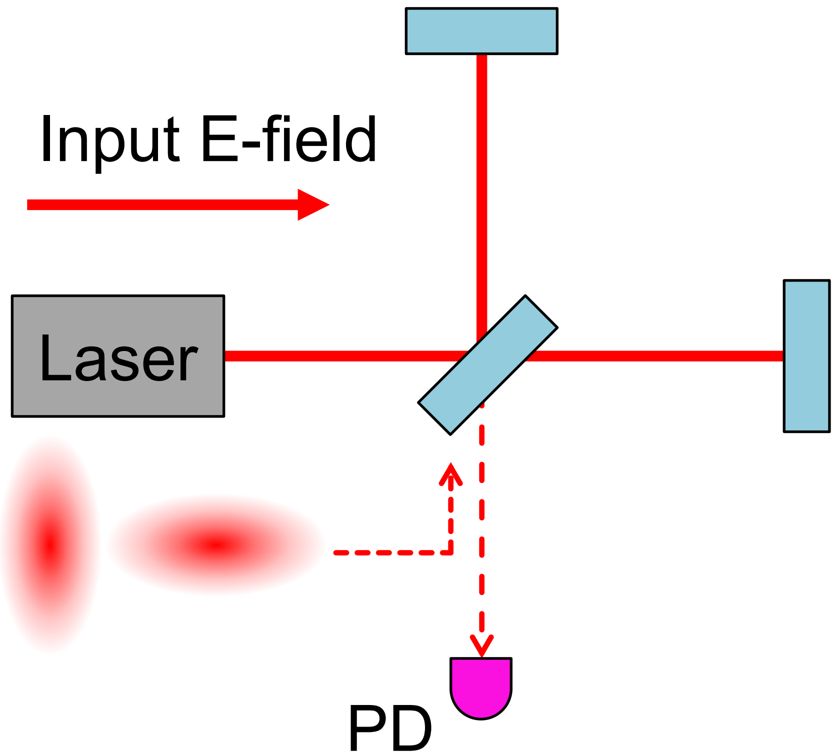
- Shot noise or radiation pressure noise can be reduced by squeezed vacuum injection
- SQL cannot be beaten

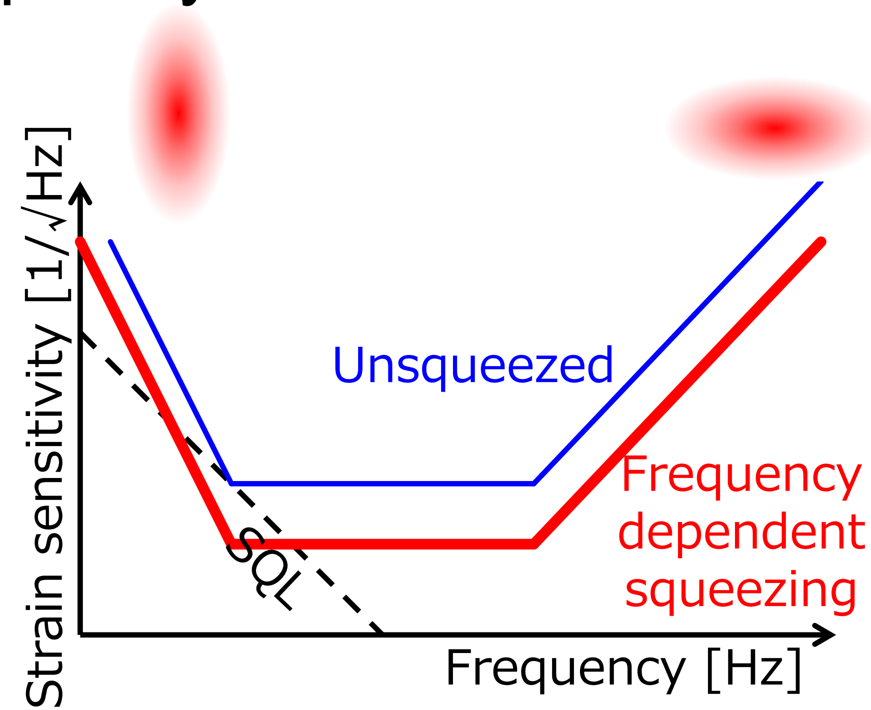




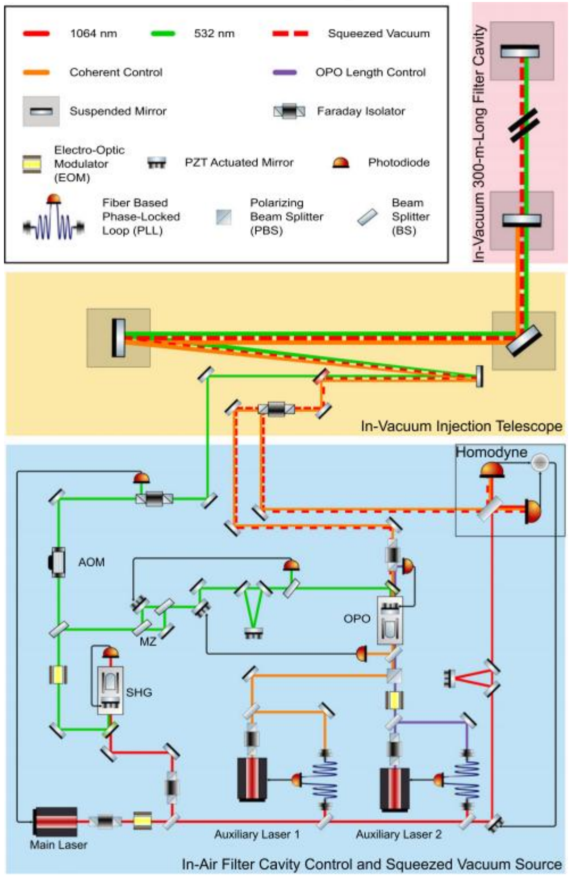
Frequency Dependent Squeezing

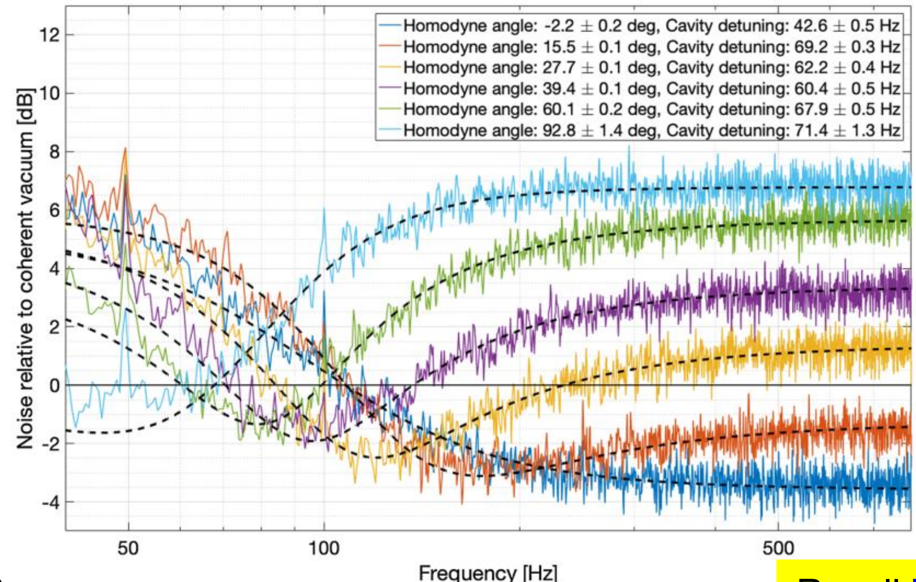
- Both can be reduced by changing the squeezing angle as a function of frequency
- Beats SQL





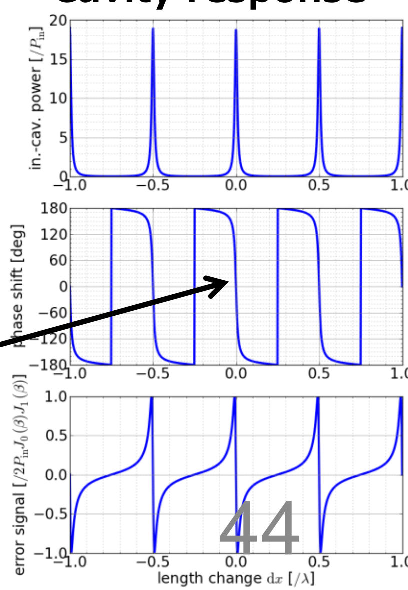
Freq. Dep. Sqz. Demonstration





Recall [Lecture 2](#)

Cavity response



Use detuned filter cavity to rotate the squeezing angle

Detuning creates different phase rotation for upper and lower sidebands

Demonstrated at TAMA300 and MIT

Y. Zhao+, [PRL **124**, 171101 (2020)](#)

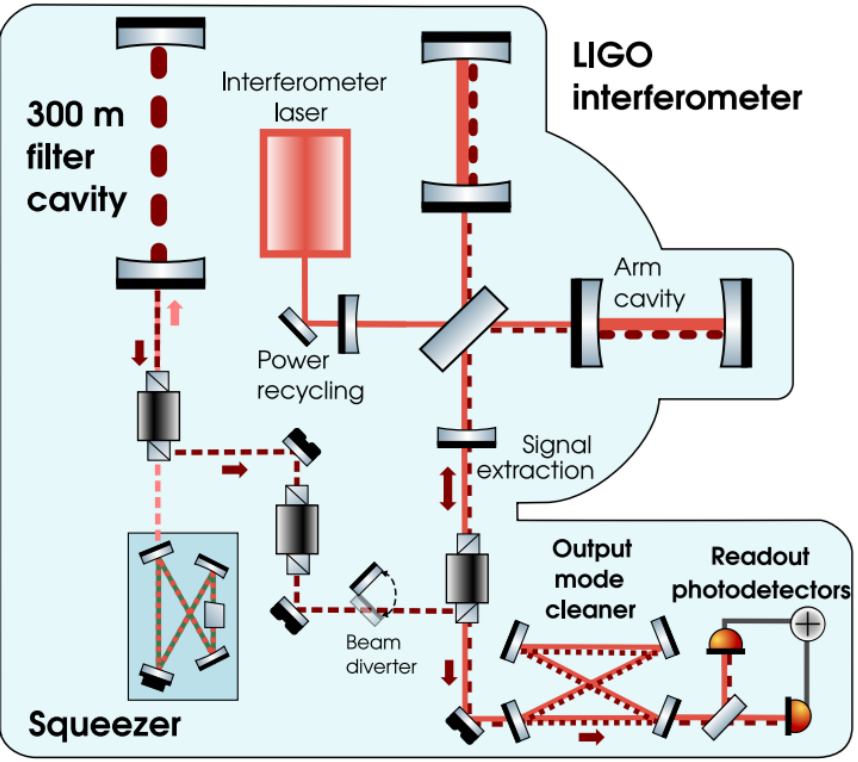
L. McCuller+, [PRL **124**, 171102 (2020)](#)

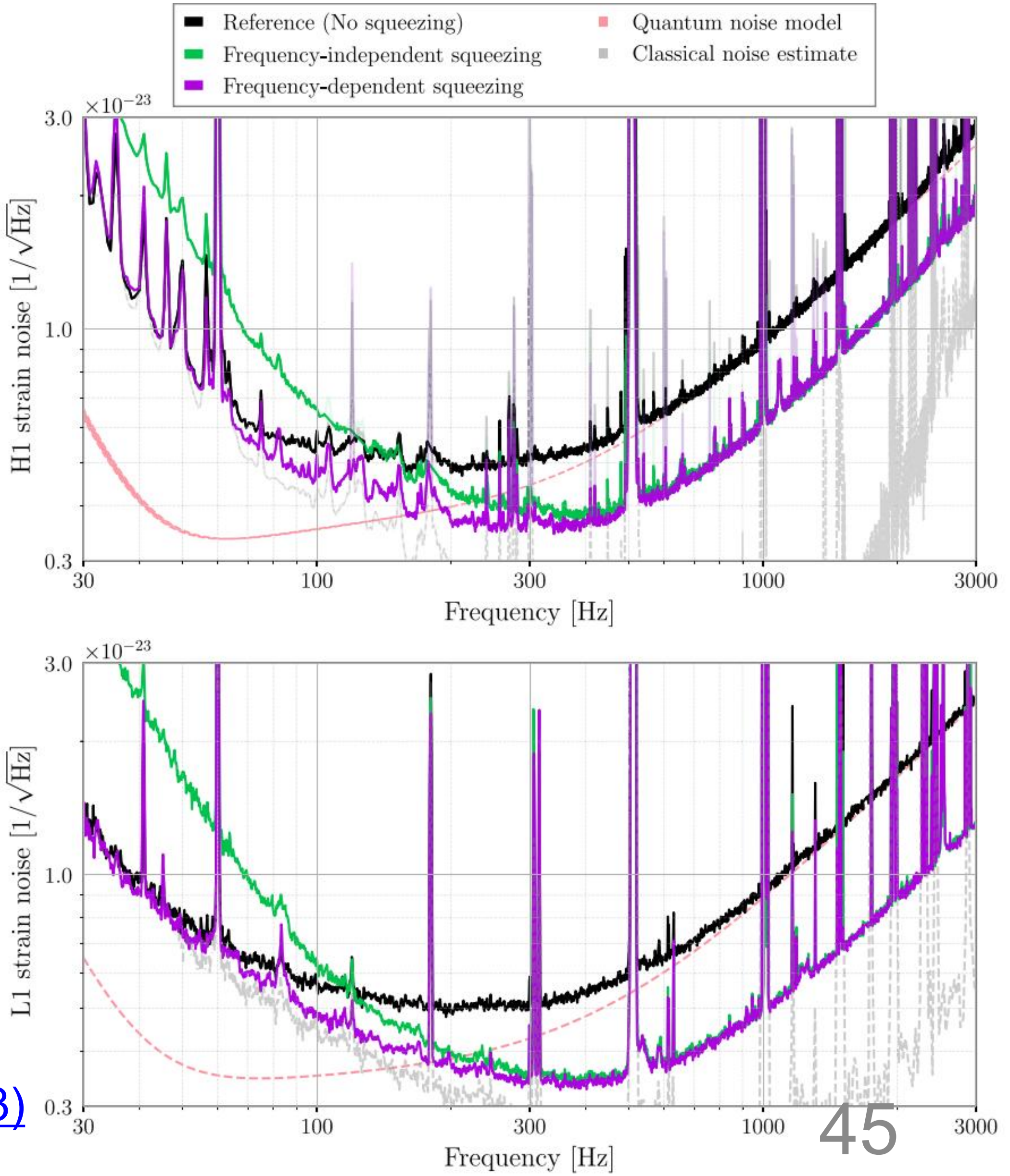
Installed for LIGO and Virgo

D. Ganapathy+, [PRX 13, 041021 (2023)](#)

Virgo, [PRL 131, 041403 (2023)](#)

Installed for Advanced LIGO



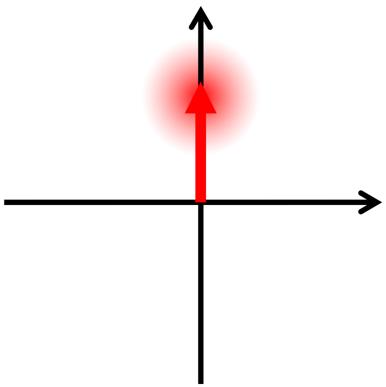


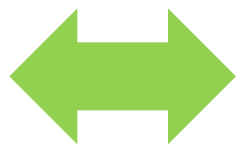
Squeezing angle rotates

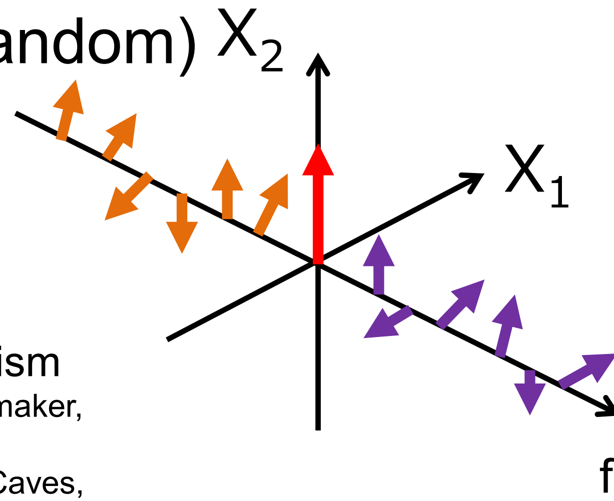
D. Ganapathy+, [PRX 13, 041021 (2023)]

Squeezed Light

- Coherent light has random noises (phases of upper and lower sidebands are random)

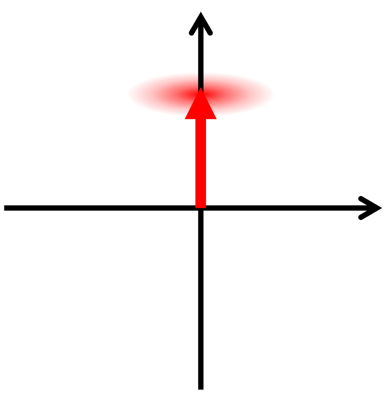


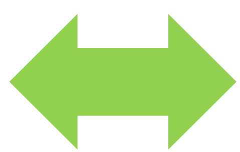




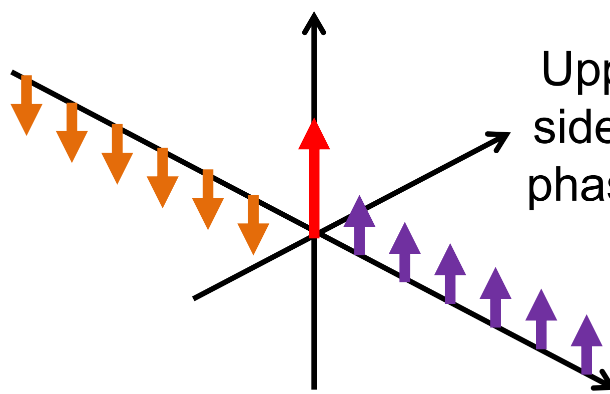
Two-photon formalism
C.M. Caves & B.L. Schumaker, [PRA 31, 3068 (1985)]
B.L. Schumaker & C.M. Caves, [PRA 31, 3093 (1985)]

- Squeezed light has correlated upper and lower sidebands





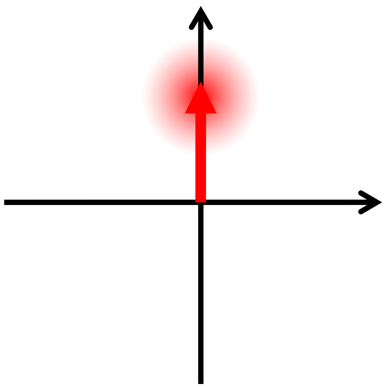
Amplitude squeezing

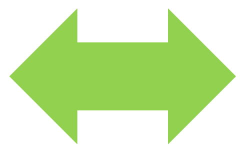


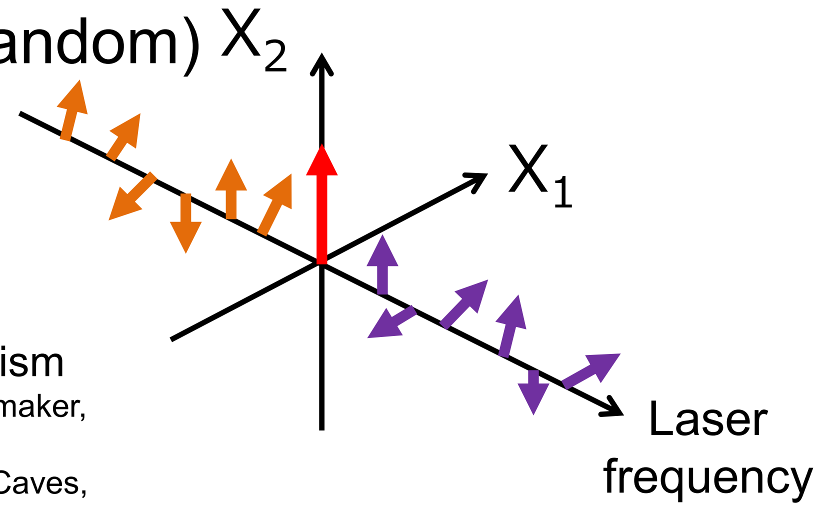
※ Phases between sidebands at different frequencies are not important

Squeezed Light

- Coherent light has random noises (phases of upper and lower sidebands are random)







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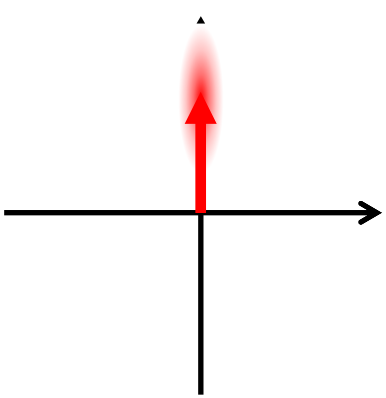
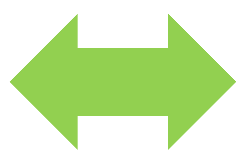
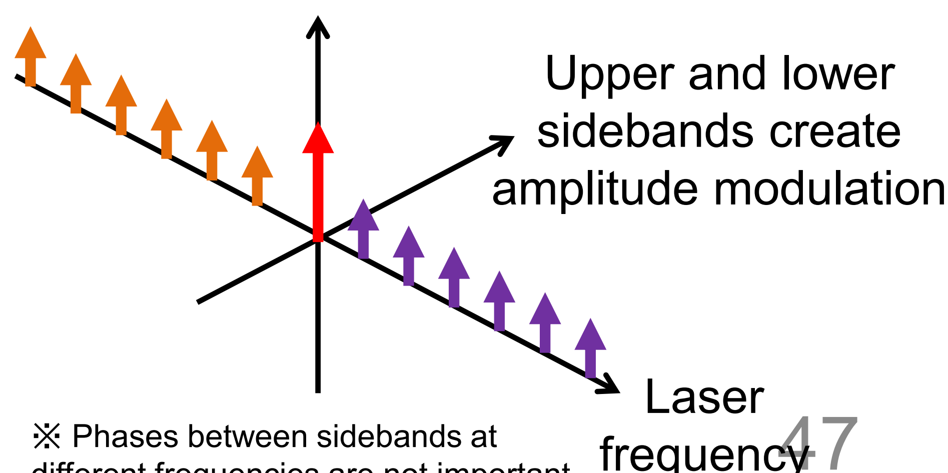




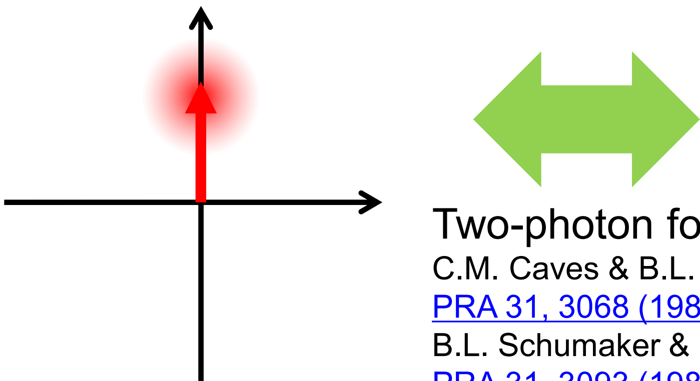
Phase squeezing

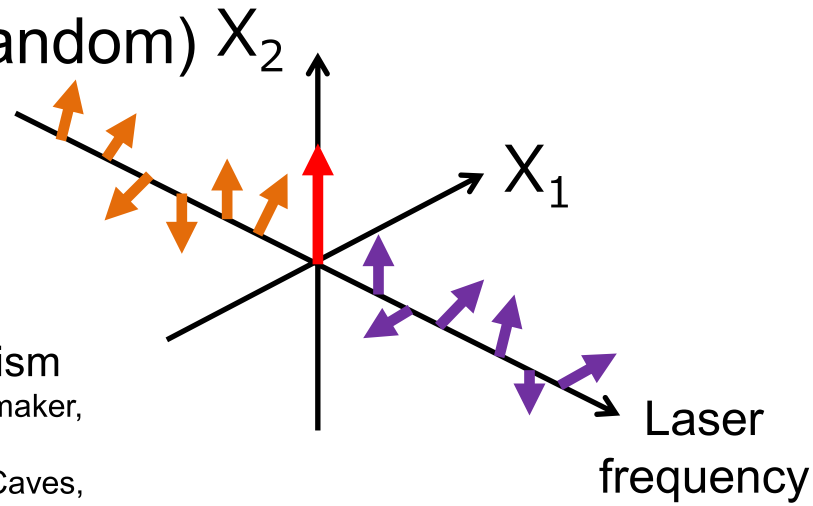


※ Phases between sidebands at different frequencies are not important

Frequency Dependent Squeezing

- Coherent light has random noises (phases of upper and lower sidebands are random)





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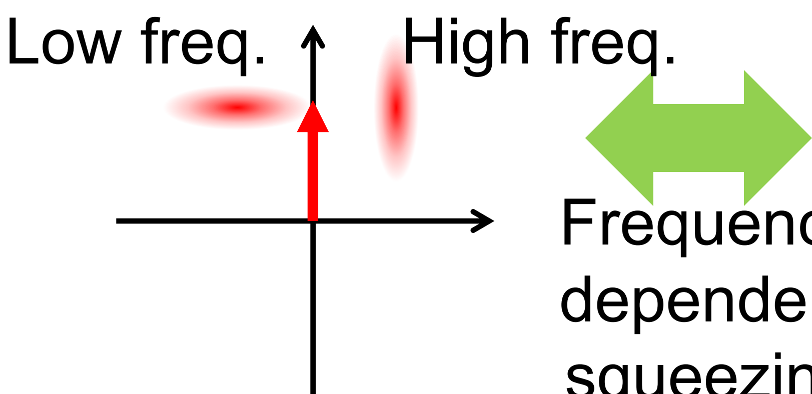
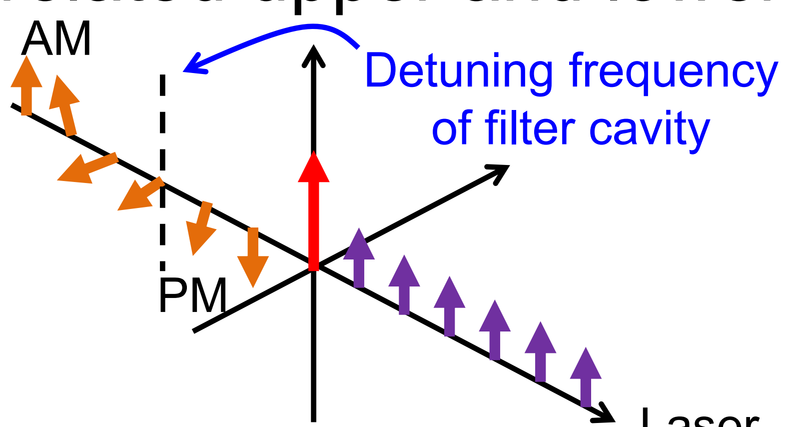


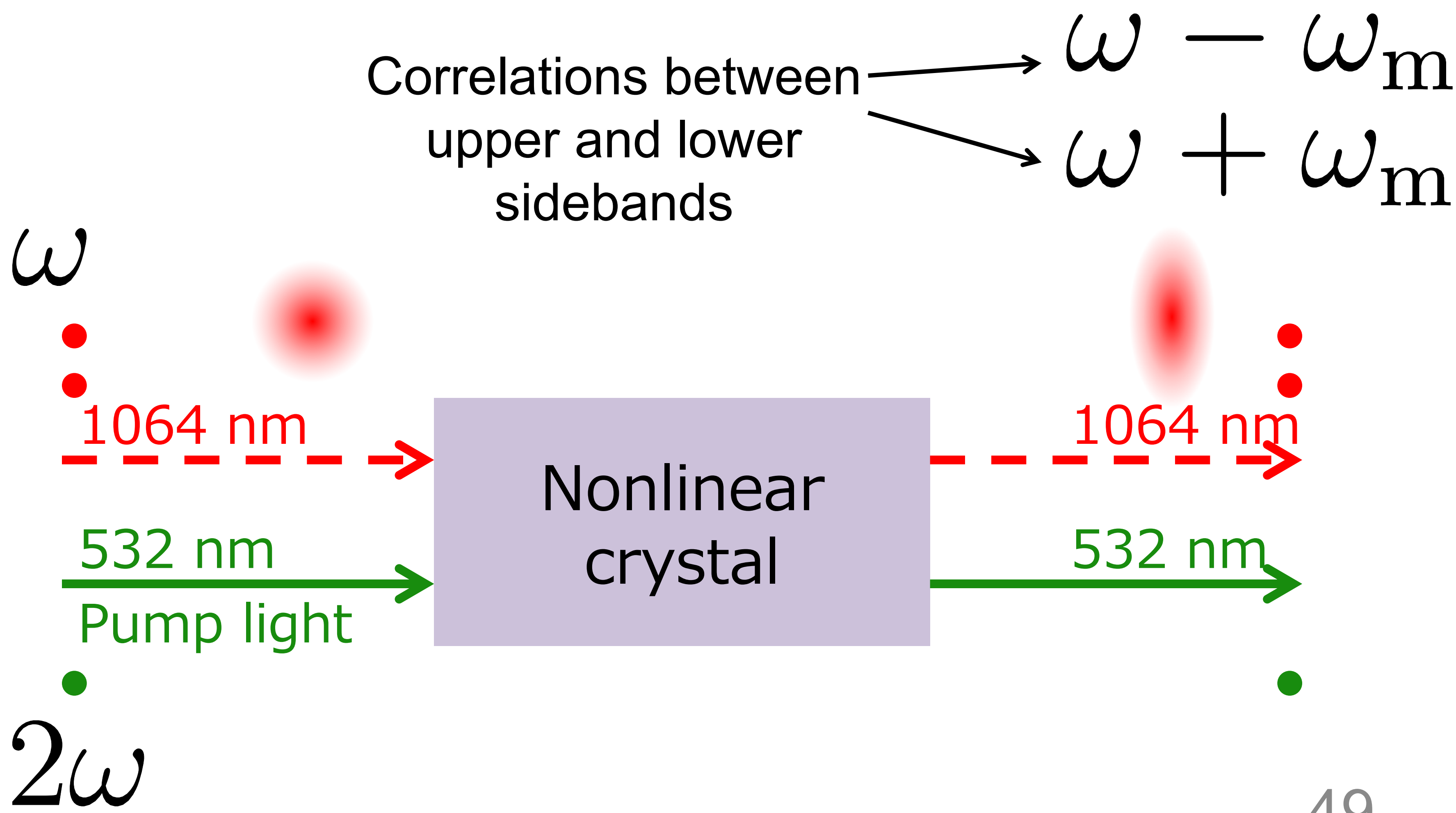
Frequency dependent squeezing



※ Phases between sidebands at different frequencies are not important

Generating Squeezed Vacuum

- Optical Parametric Amplifiers are used



Summary

- Many tricks can be done to reduce quantum noise
- Many tricks can be understood by mastering
  - Michelson response
  - Cavity response
  - Sideband picture
- For more concrete understanding…
  - A. Buonanno & Y. Chen, PRD **64**, 042006 (2001)
  - Y. Chen, J. Phys. B: At. Mol. Opt. Phys. **46**, 104001 (2013)
  - M. Aspelmeyer, T. J. Kippenberg, F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014)
  - Y. Michimura, K. Komori, EPJD **74**, 126 (2020)

Assignment for Nov 25

- Explain what the standard quantum limit (SQL) is, and why it is possible to surpass the SQL.
- You may also answer from the Google Form below https://forms.gle/6AwJ48XcpWQXqMon9

Don’t forget to put your name and student#

You may answer in any language

FORM