観測天文学特別講義I @ 理1-1043

Observational Astronomy Advanced Course I

November 17, 2025

レーザー干渉計型重力波検出器
Laser interferometric gravitational wave detectors
4. Seismic Noise and Thermal Noise

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https://yutamich.gitlab.io/lectures.html#lectures

Assignment for Nov 17

- Give an example where bigger is not necessarily better for a gravitational wave detector, and explain the reason.
- You may also answer from the Google Form below
  https://forms.gle/6AwJ48XcpWQXqMon9

Don't forget to put your name and student#

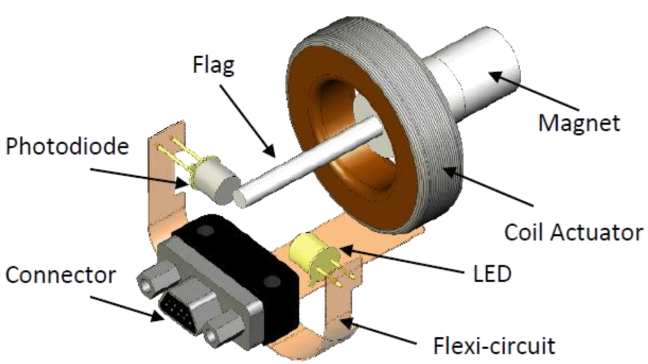
You may answer in any language

FORM

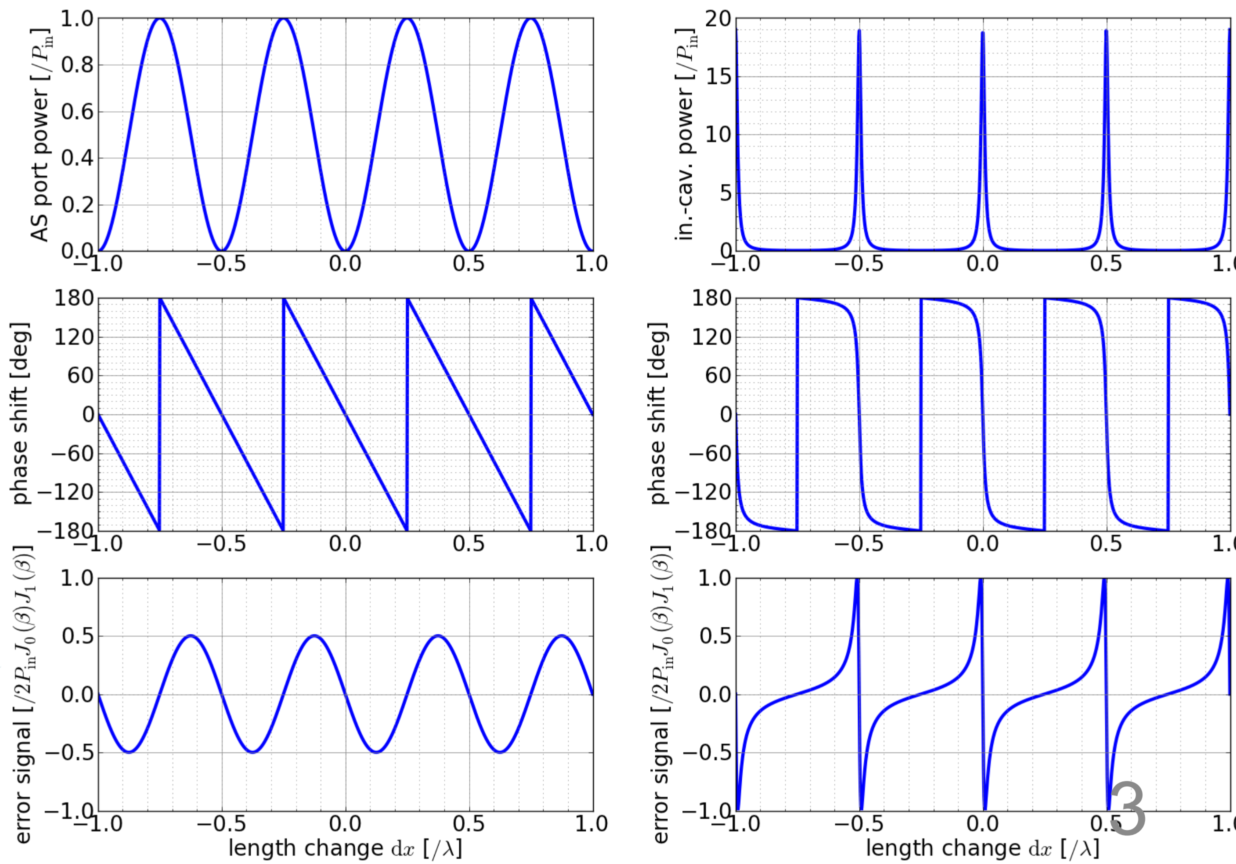
Nonlinearity in GW detectors

- Assignment for the previous lecture was about nonlinearity
- Actually, everything is nonlinear to some extent
  - Detuning
  - Actuation
  - Scattered light noise

  etc…



CQG **29**, 115005 (2012)



I will try to finish this lecture by 12:00

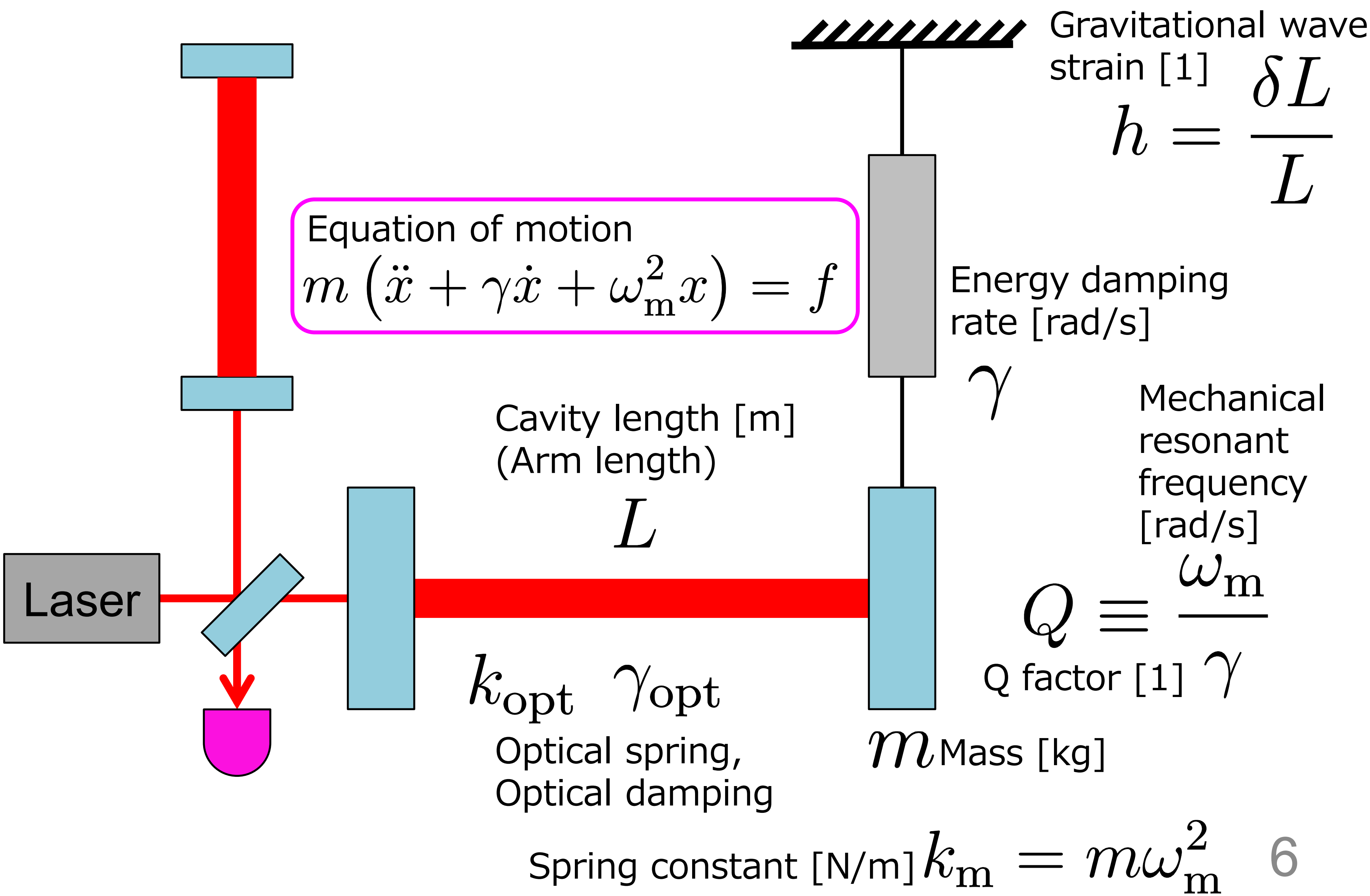
Plan of the Lecture Today

- Seismic noise
- Newtonian noise
- Thermal noise
  - Suspension thermal noise
  - Mirror substrate thermal noise
  - Mirror coating thermal noise

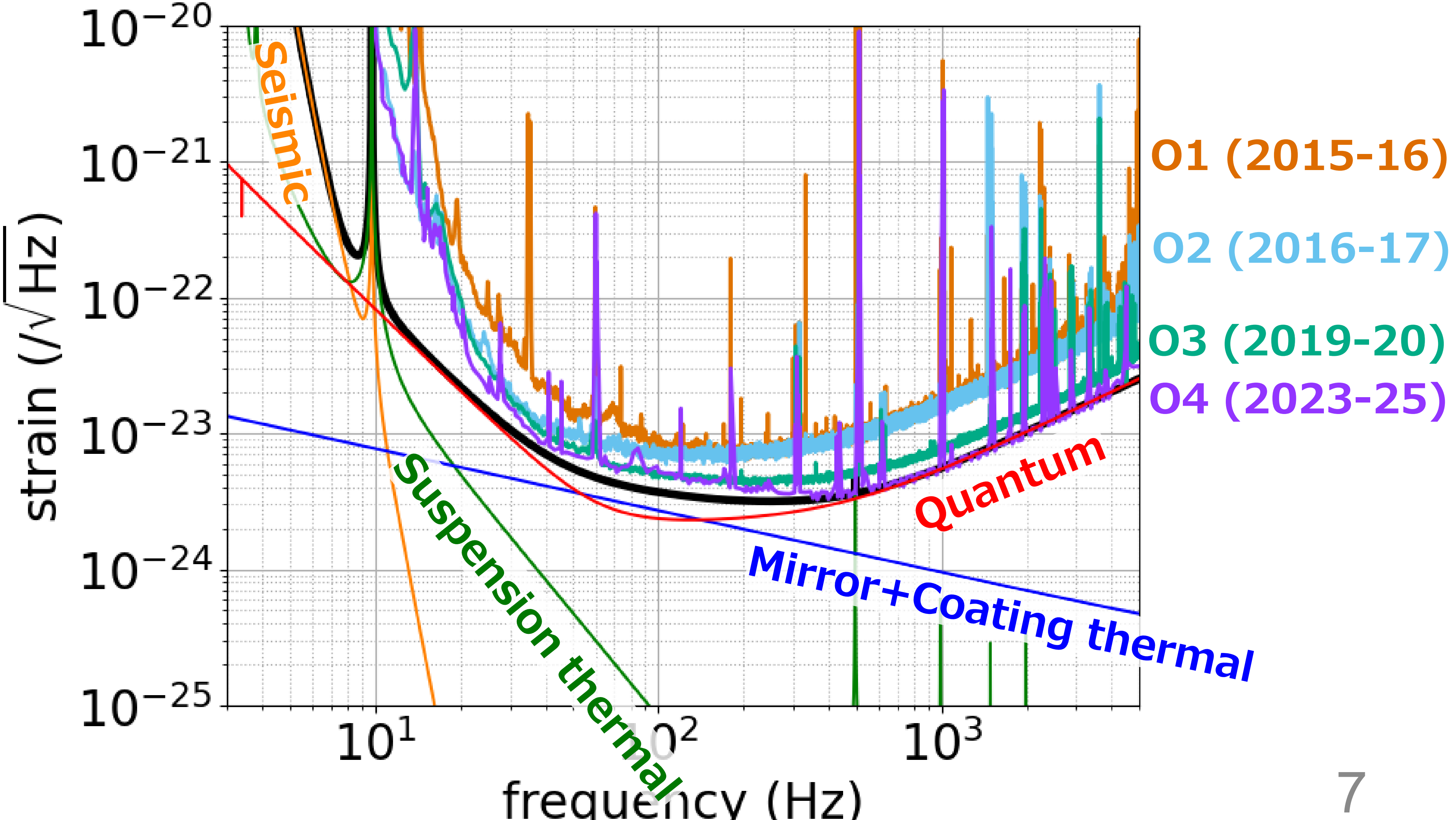
Note: I have updated the definition of Fourier transformation by 2π in Lecture 3 for better explanation of thermal noise today

- **Goal**: Understand the mechanism and mitigation schemes for seismic noise, thermal noise and other displacement noises

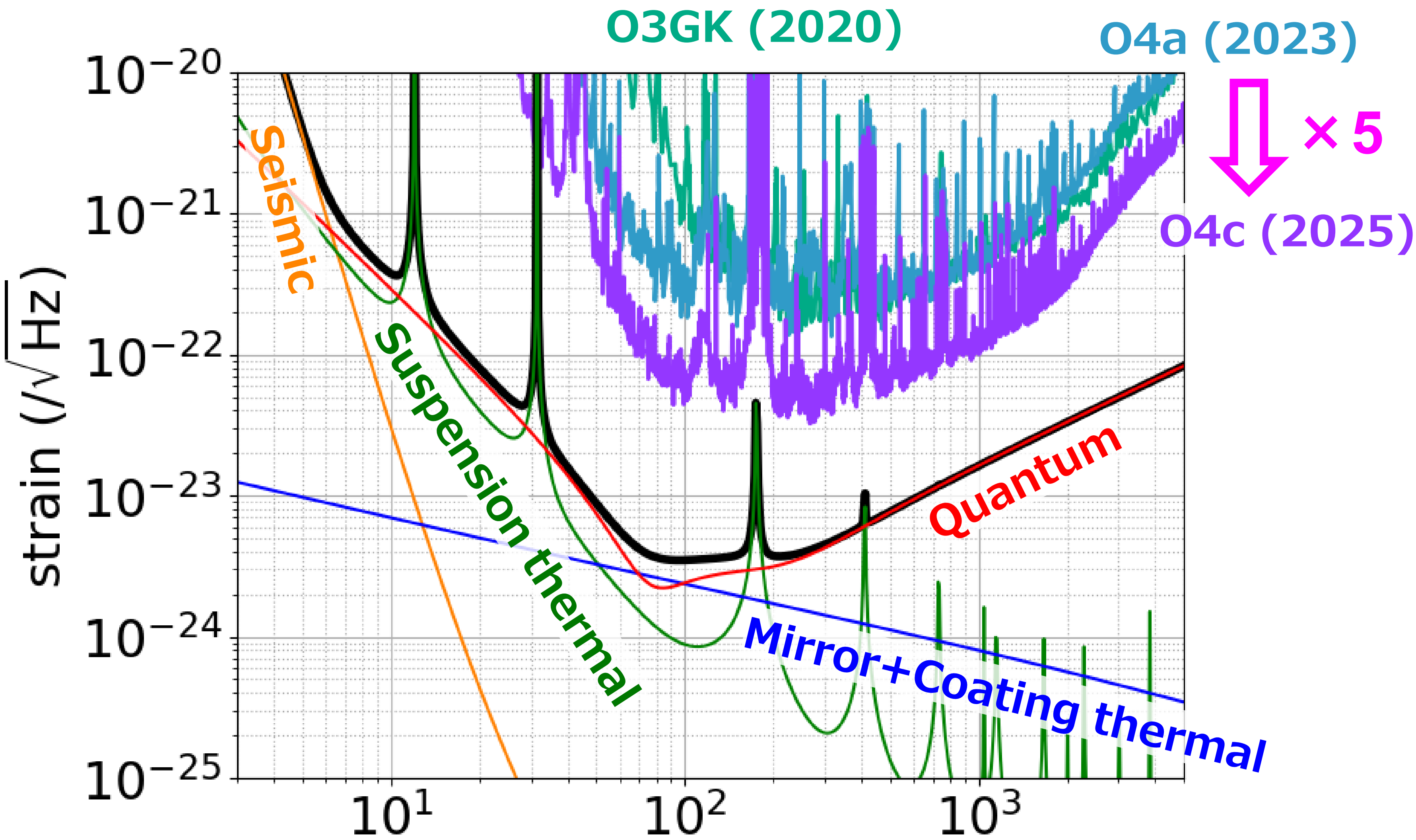
List of Parameters



Advanced LIGO Design

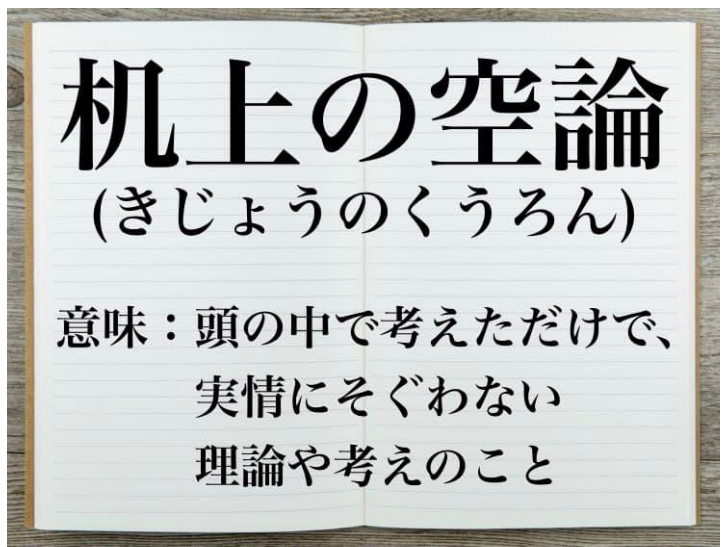


KAGRA Design



Key Points

- Actual noise spectrum is much more complicated than the design, and so many kinds of noise contributes to the spectrum
- In the design, quantum noise limits the sensitivity in most of the frequency band, but this is only true when classical displacement noises are reduced
- This is why we do “commissioning”

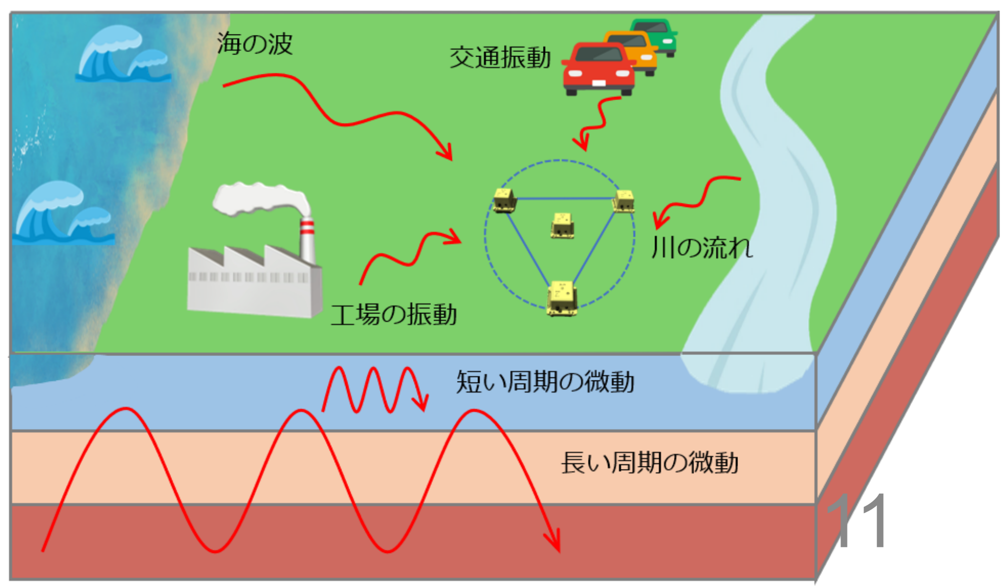


Kijo no Kuron

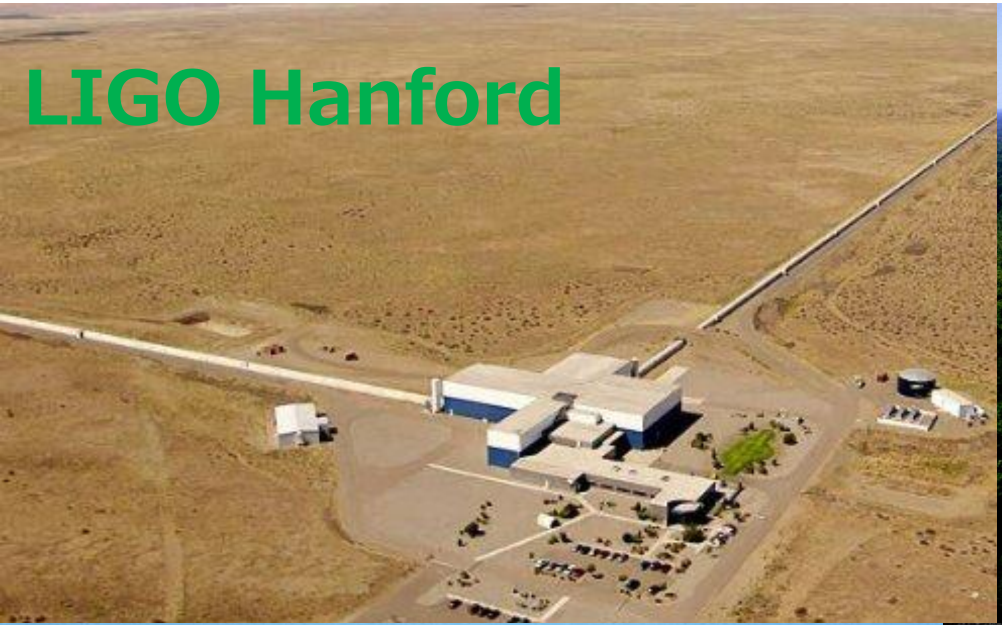
Seismic Noise

Seismic Vibration

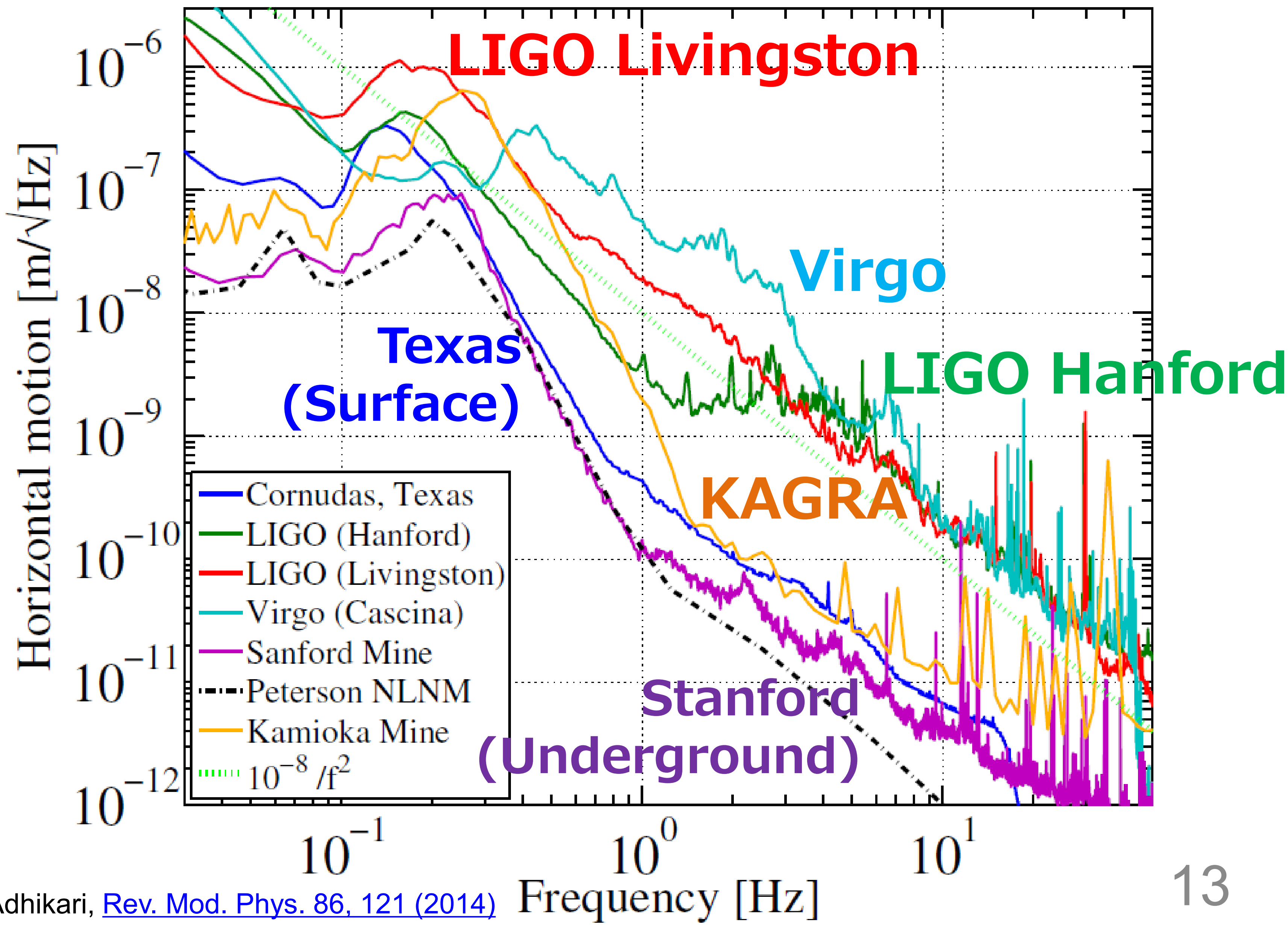
- Ground always vibrates even if there are no earthquakes (常時微動, micro-tremor)
- Vibration level differs by
  - day and night
  - stormy day and calm day
  - urban place and countryside
  - surface and underground
- Site selection is very important for GW detectors



GW Detector Sites



Seismic Vibration Between Sites



R. X. Adhikari, Rev. Mod. Phys. 86, 121 (2014)

Vibration Isolation

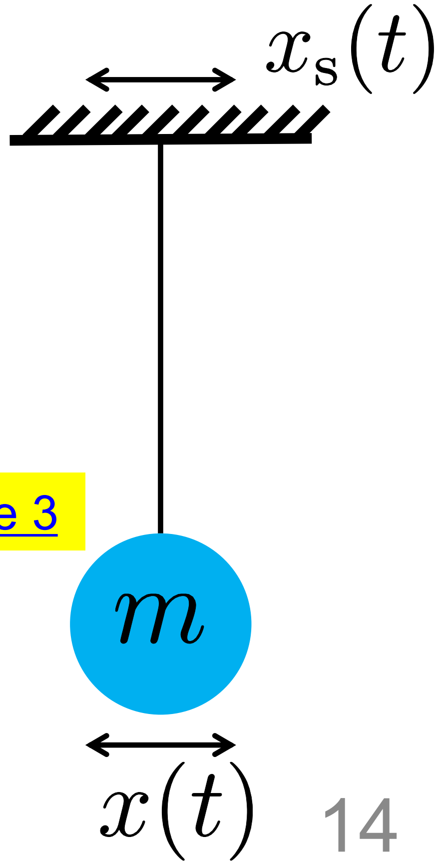
- Typical seismic noise (according to Saulson)

$$x_{\rm seis}(\omega) = \begin{cases} 10^{-9}\,{\rm m}/\sqrt{\rm Hz} & \text{1-10 Hz} \\ 10^{-9}\,(10\,{\rm Hz}/f)^2\ {\rm m}/\sqrt{\rm Hz} & \text{>10 Hz} \end{cases}$$

- Needs vibration attenuation by ~9 orders of magnitude
- Use pendulums to attenuate the ground vibration
  - Low frequency & Multi-stage

Recall Lecture 3

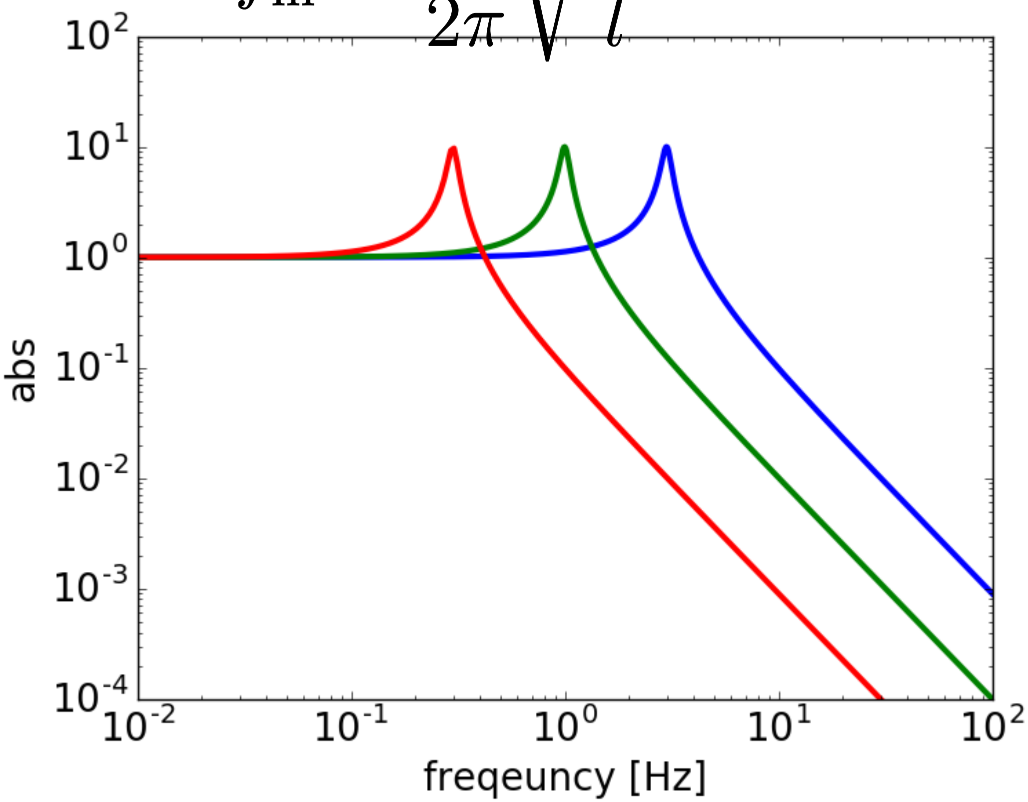
$$H_s(\omega) = \frac{\omega_{\rm m}^2}{\omega_{\rm m}^2 - \omega^2 + i\frac{\omega\omega_{\rm m}}{Q}}$$

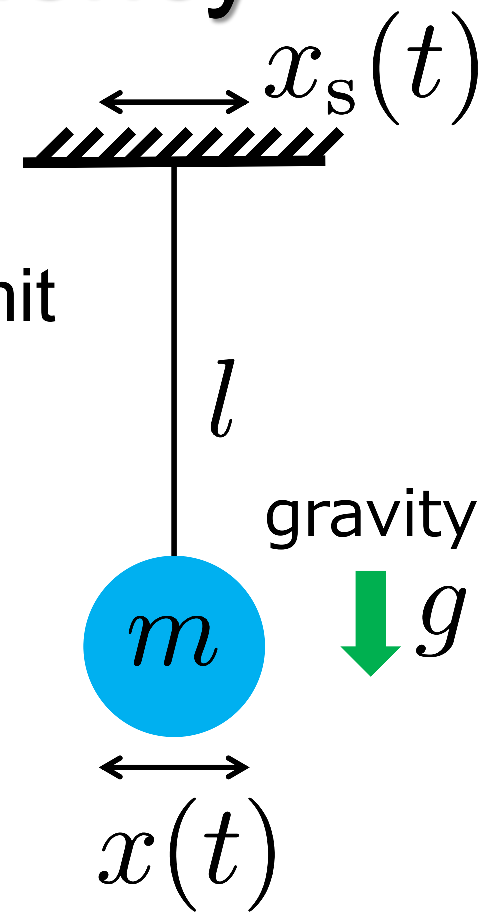


Lower Resonant Frequency

- Better vibration isolation with a lower resonant frequency
- But on Earth, $l \sim 1\,\mathrm{m}$ is a rough limit

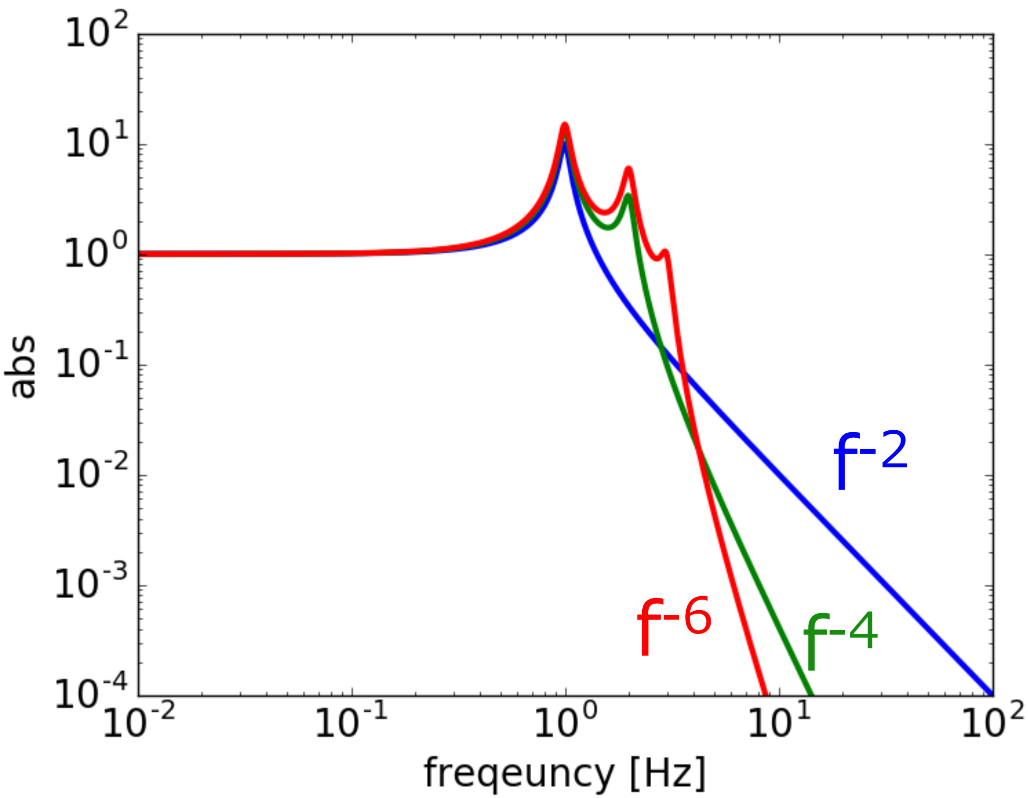
$$f_\mathrm{m} = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \simeq 0.5\ \mathrm{Hz}$$

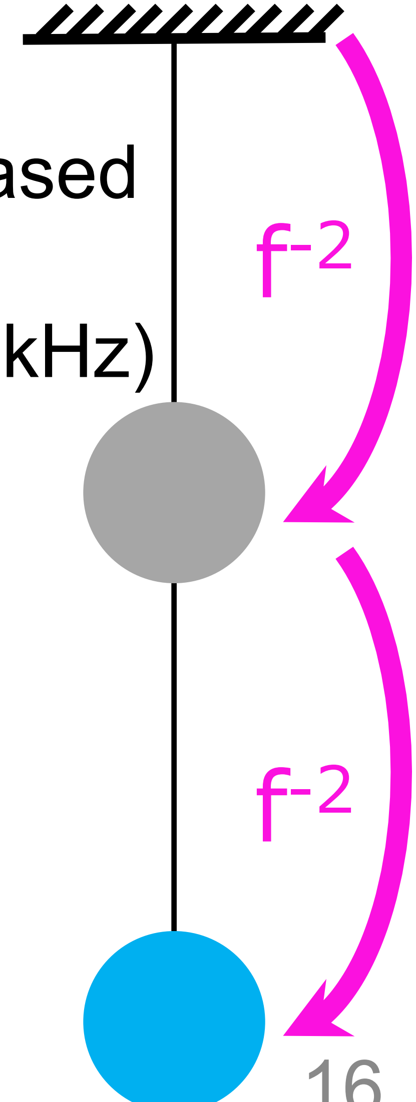




Multi-Stage Vibration Isolation

- Multi-stage pendulum for better isolation at higher frequencies
- Good vibration isolation for ground-based gravitational-wave detector band (10 Hz ~ 1 kHz)

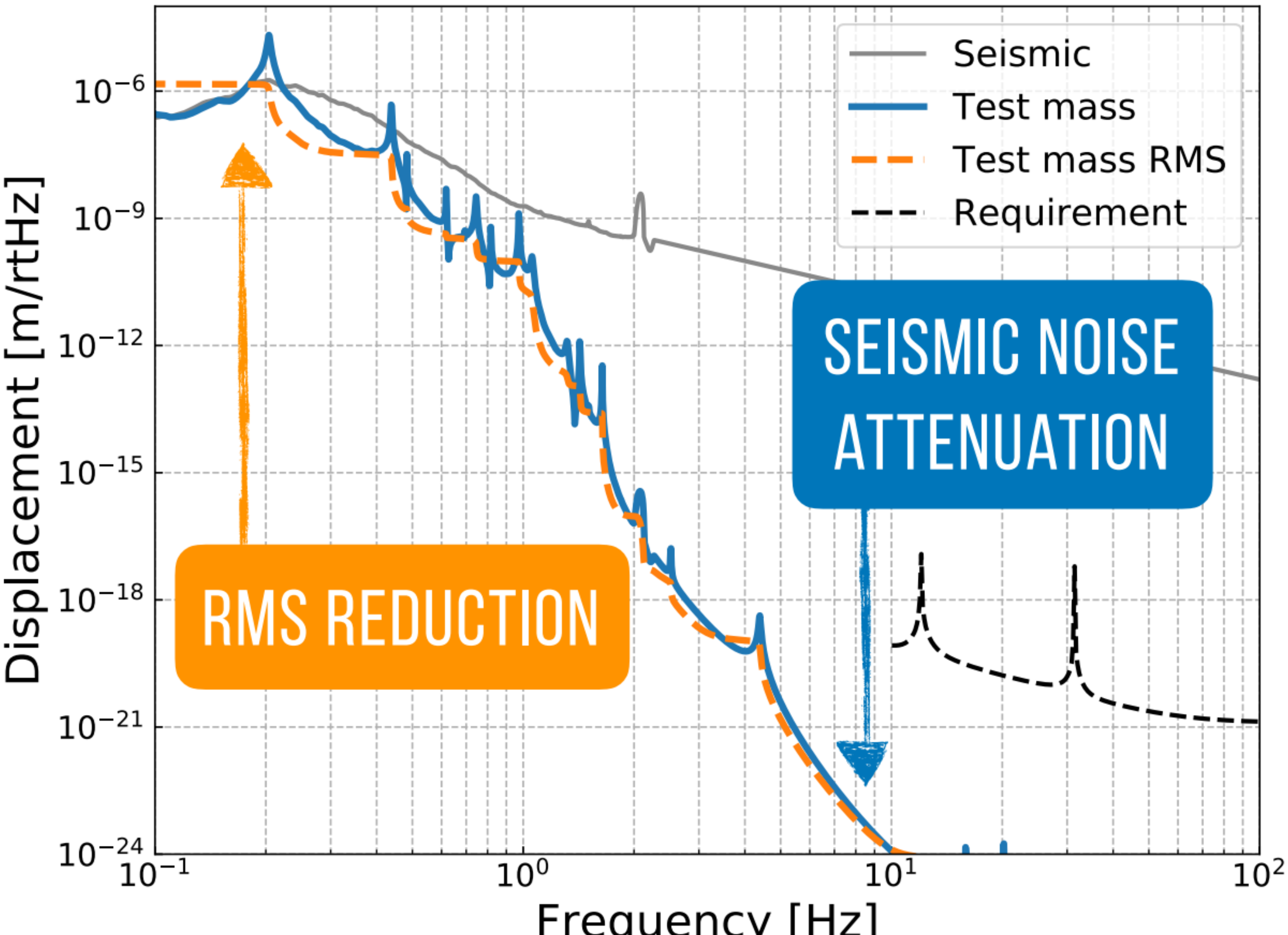


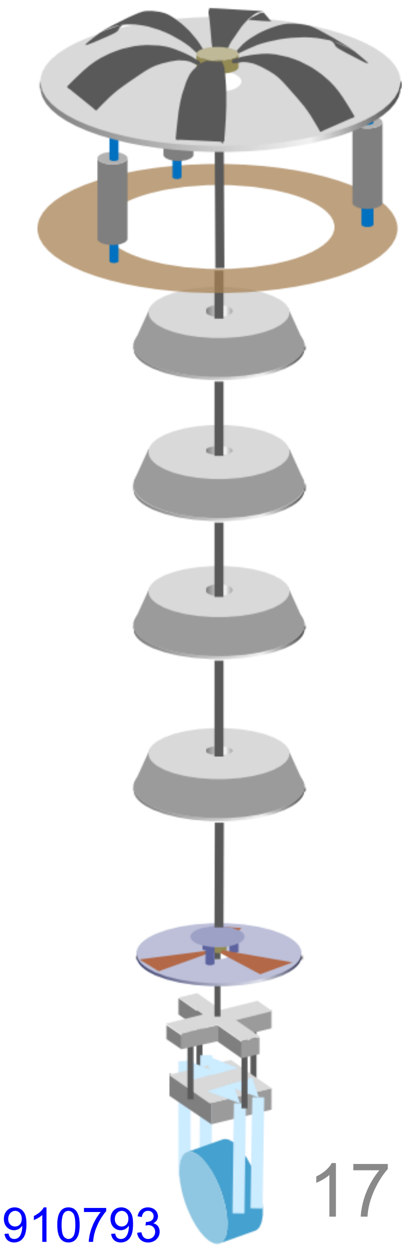


KAGRA Vibration Isolation

- 13.5 m tall, 9-stage pendulum

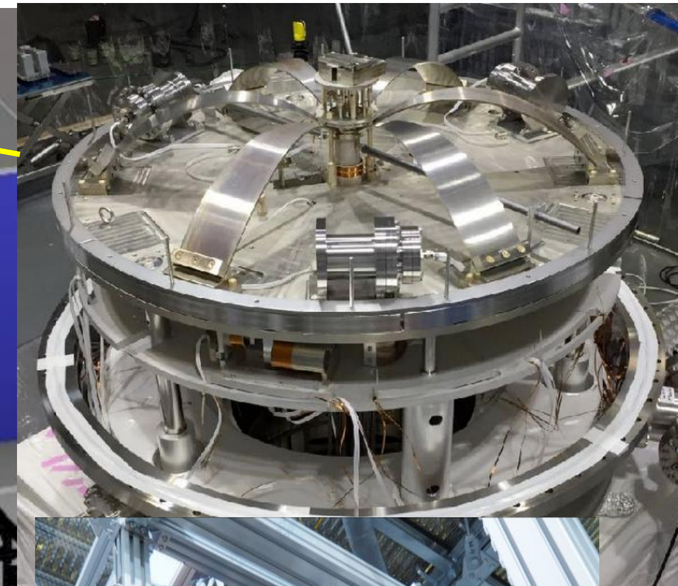
including inverted pendulum on top

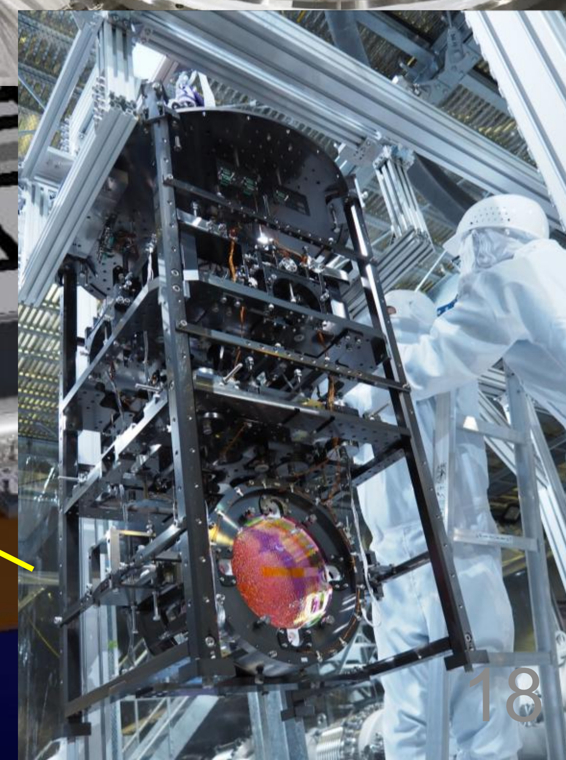




K. Okutomi, JGW-G1910793

KAGRA Vibration Isolation





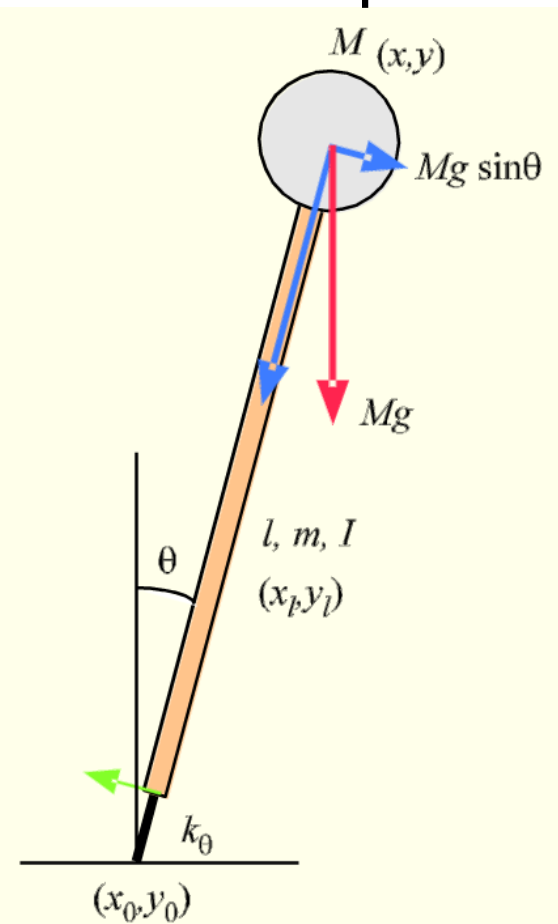
Low Frequency Vibration Isolation

- Anti-springs are used for making resonant frequency low

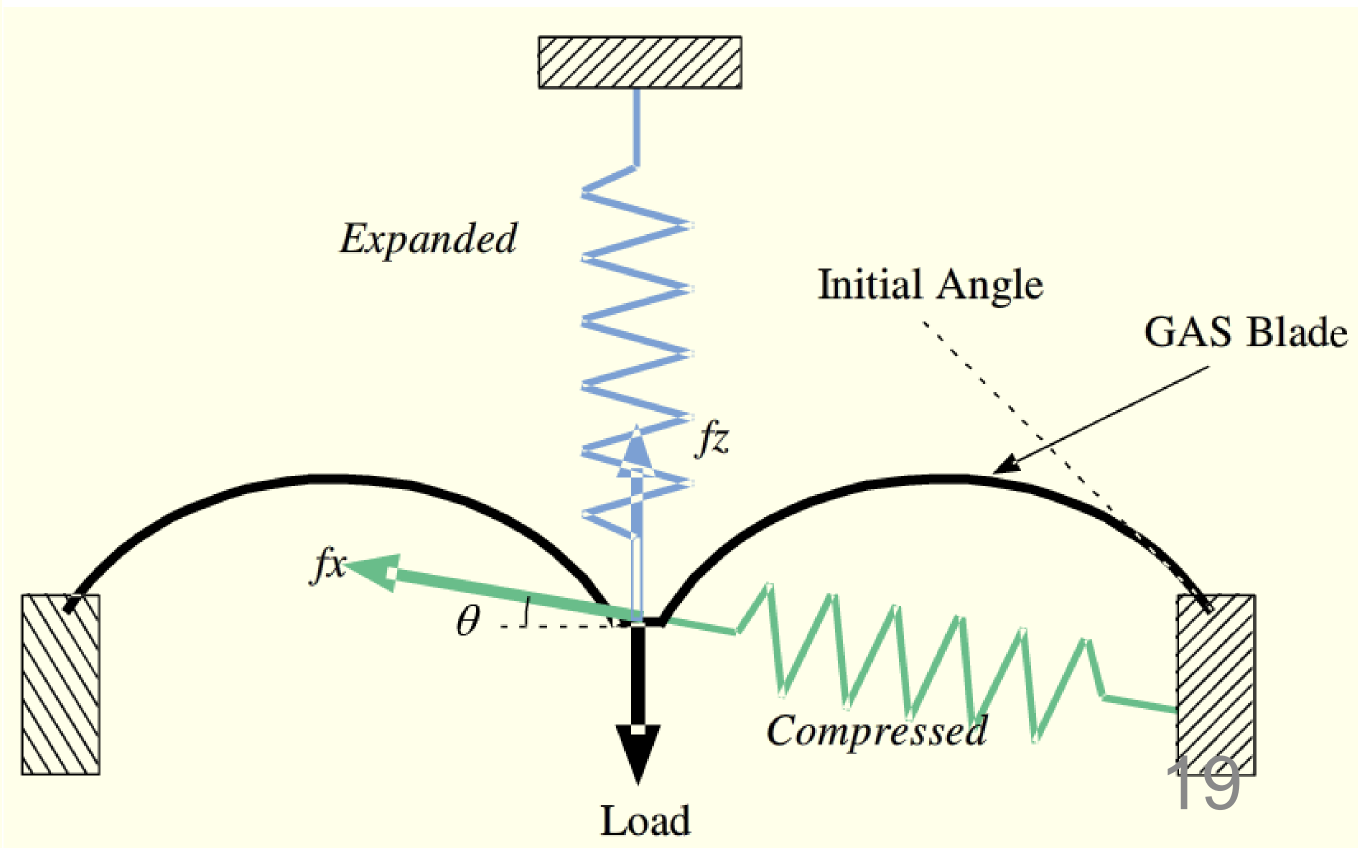
$$f_{\mathrm{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

~0.5 Hz for pendulums

Inverted pendulum

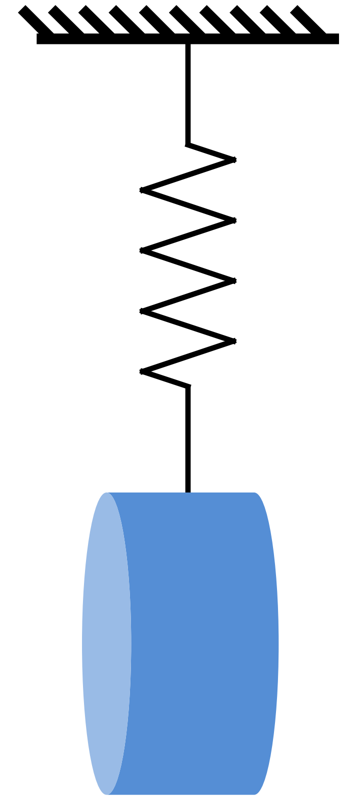


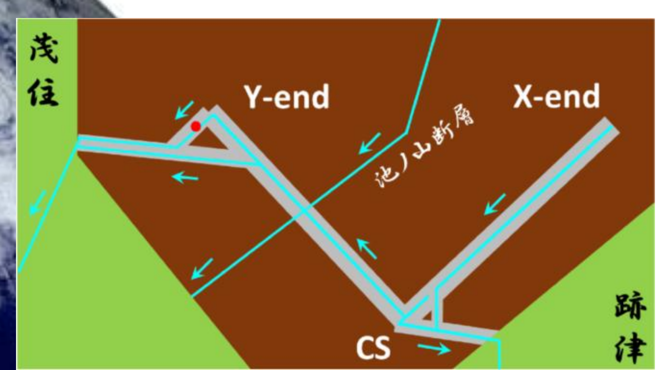
Geometric-Anti-Spring filter



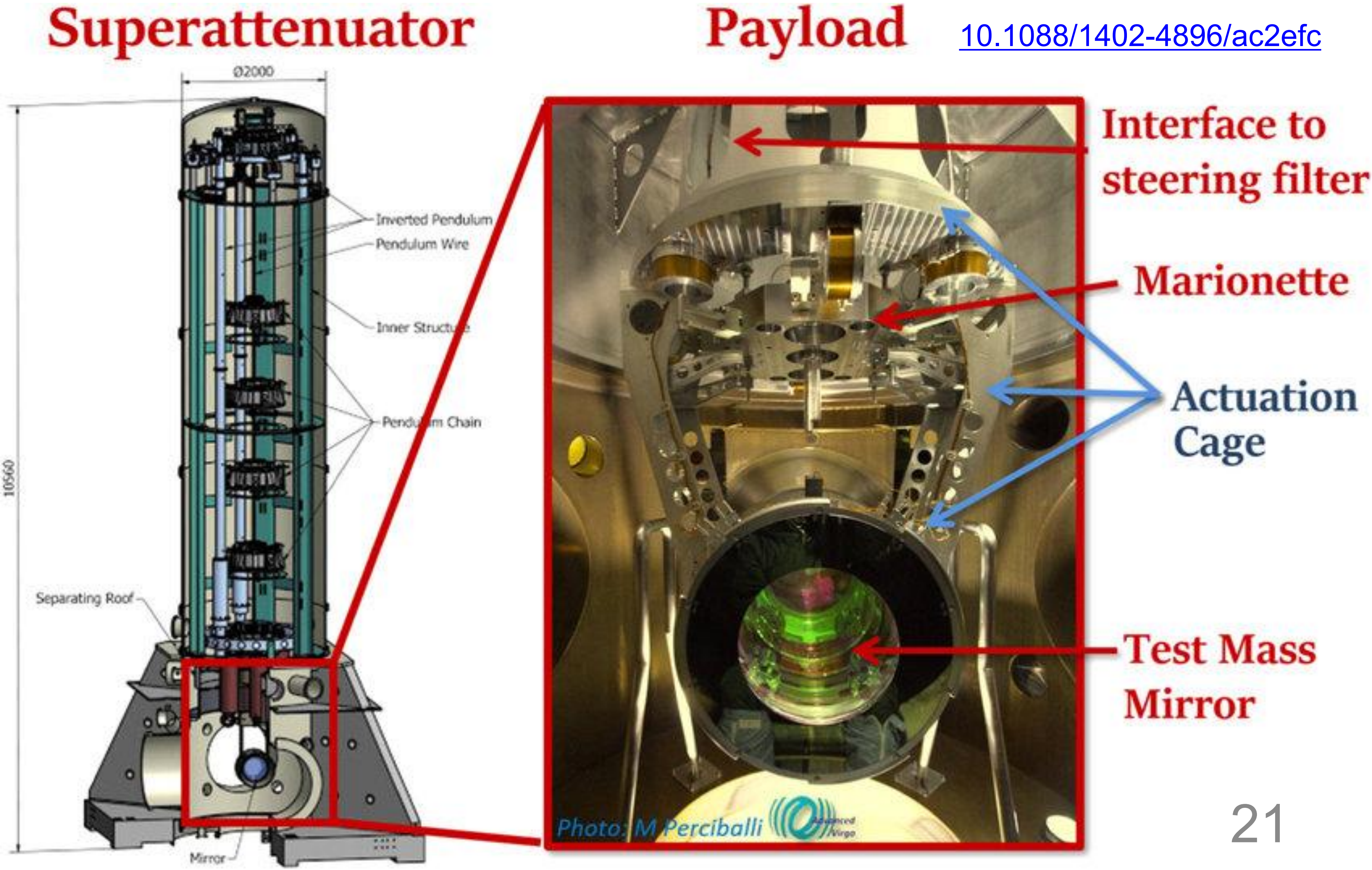
Vertical Vibration Isolation

- Vertical vibration isolation is also necessary since the beam axis is not perfectly horizontal (also ground tilt in KAGRA case to drain water)

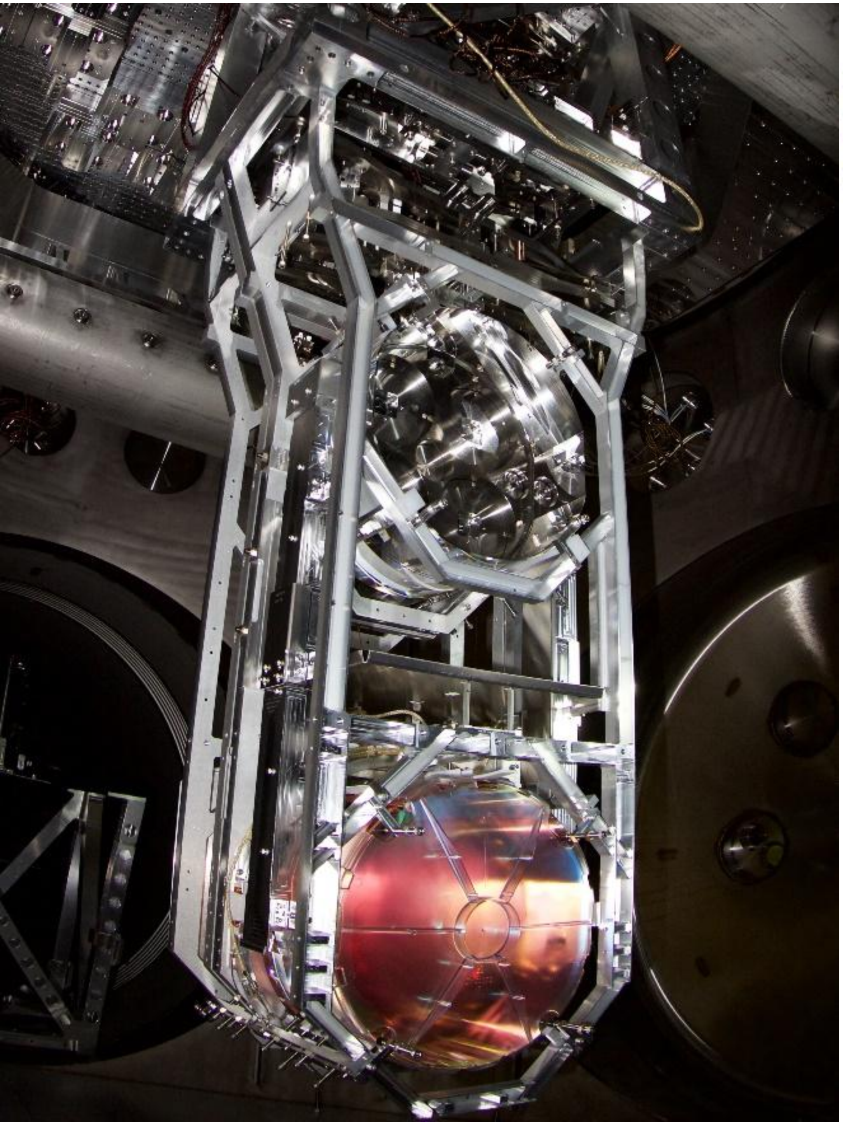


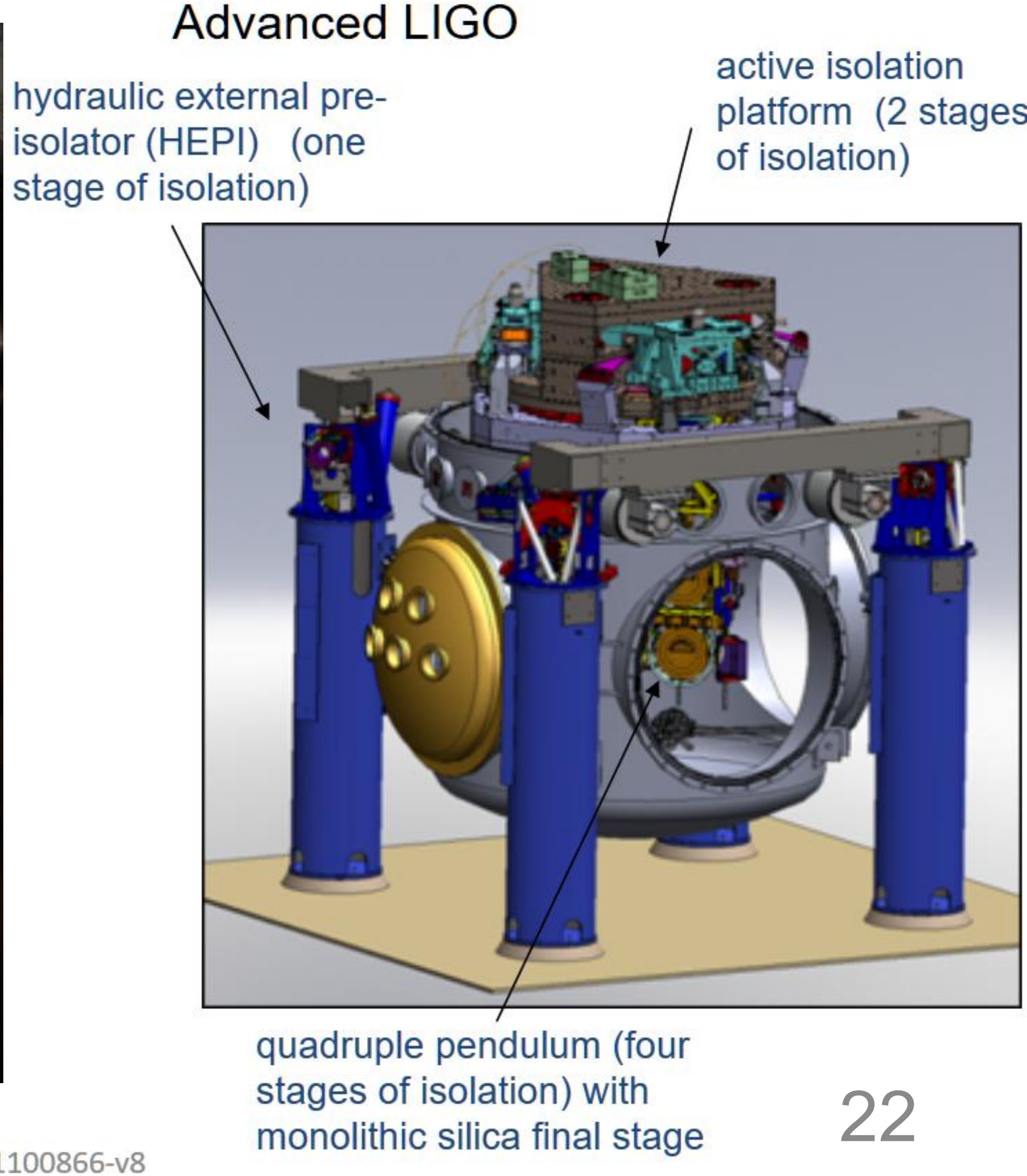


Virgo Superattenuator

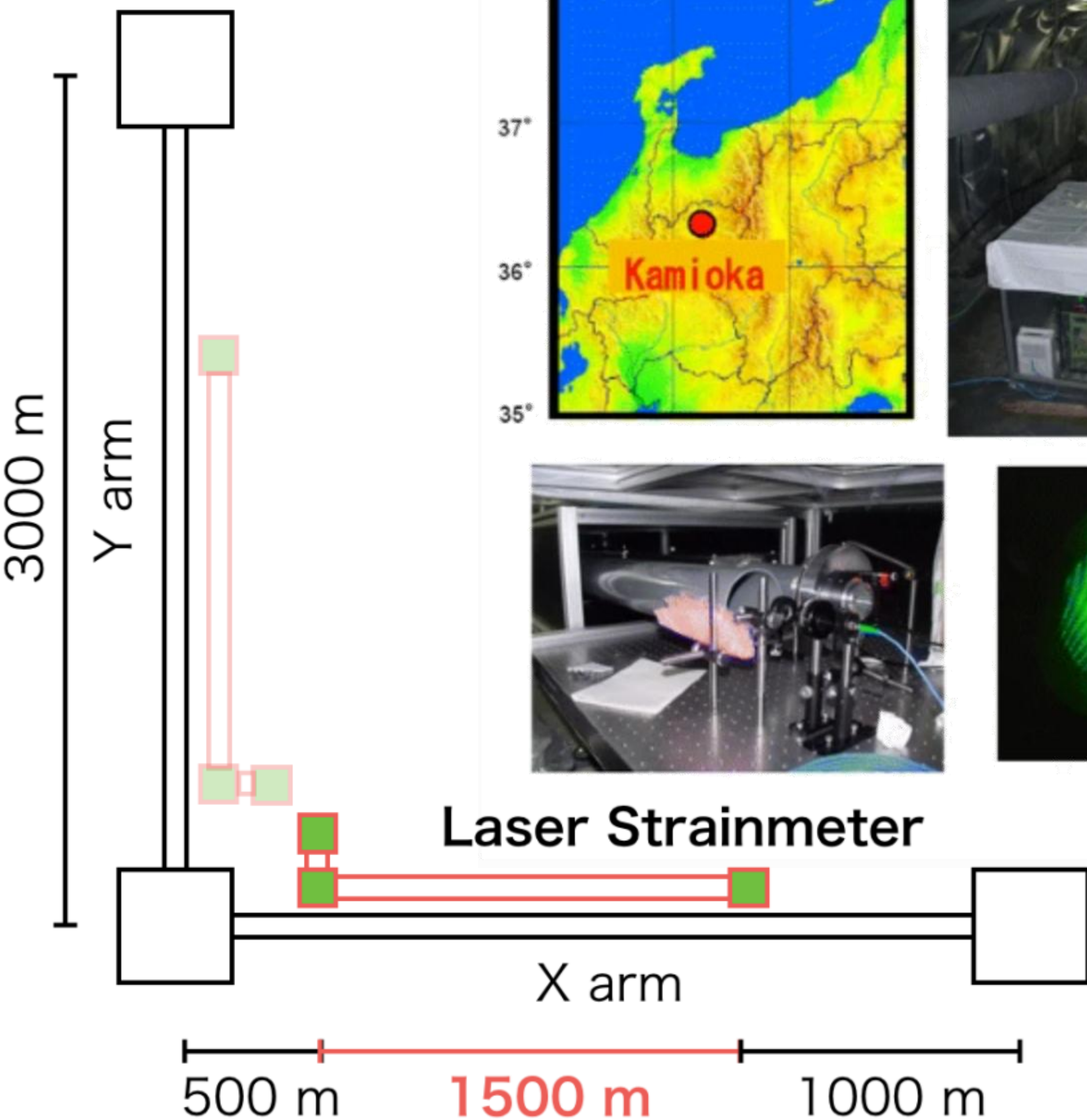


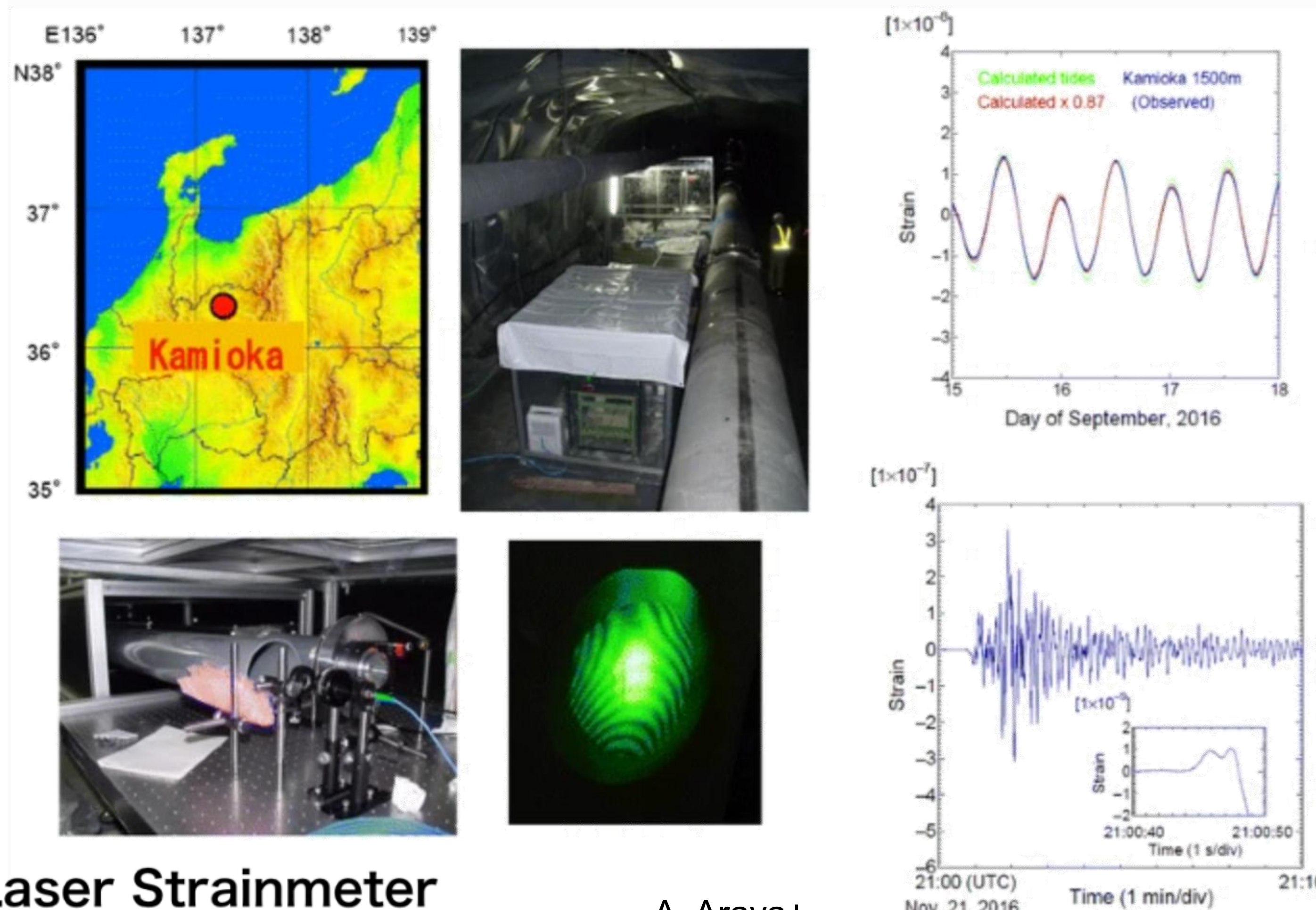
LIGO Quads & Active Isolation





KAGRA Geophysics Interferometer



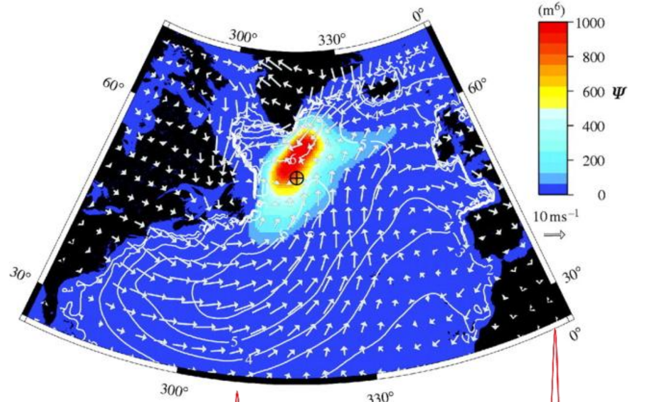


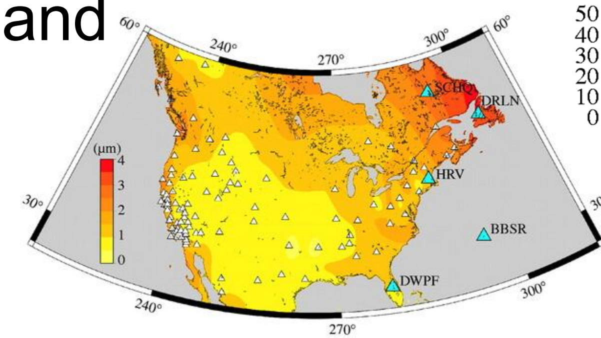
A. Araya+,
[Earth, Planets and Space **69**, 77 (2017)]

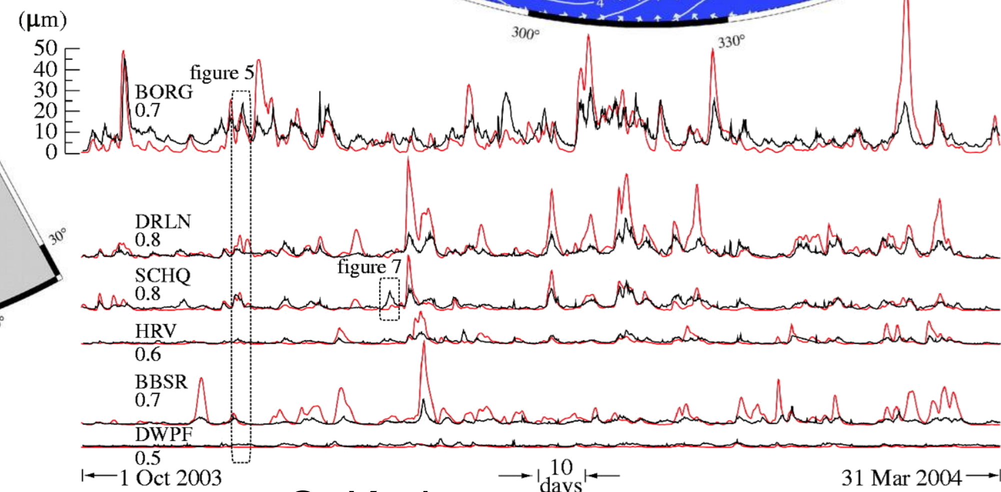
K. Miyo (KAGRA Collaboration)
[JPCS **1468**, 012221 (2020)]

Seismic Forecast

- Microseism at Livingston is affected by weather in Greenland

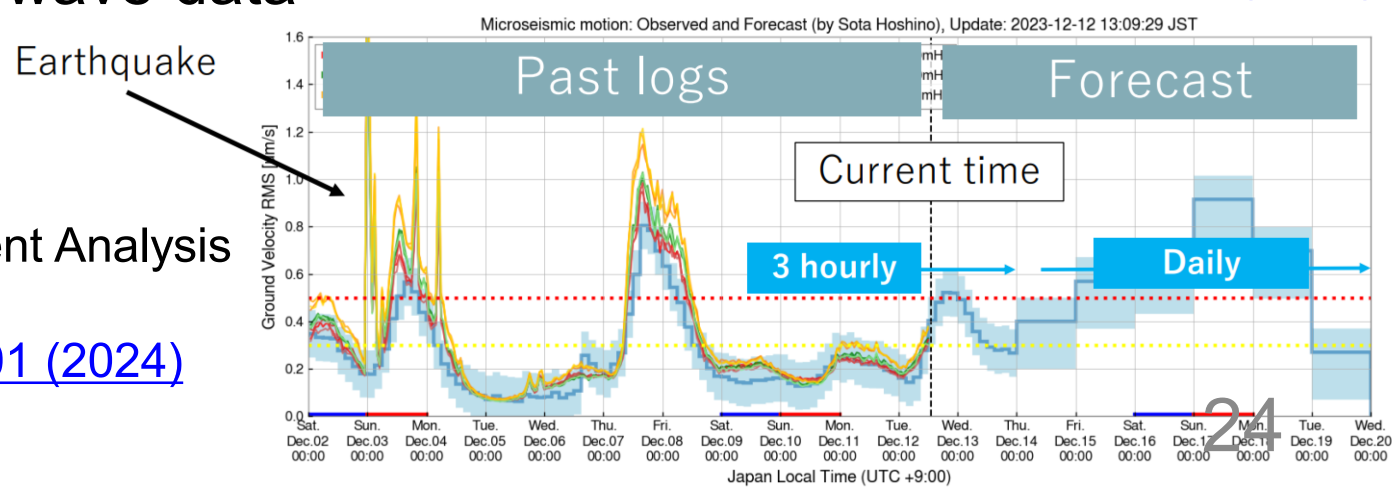






S. Kedar+,
[Proc. R. Soc. A. **464**, 777 (2008)]()

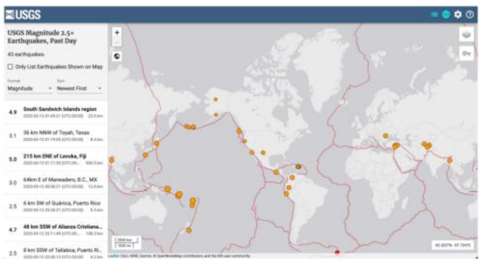
- Microseism level at KAGRA can be predicted from ocean wave data forecast



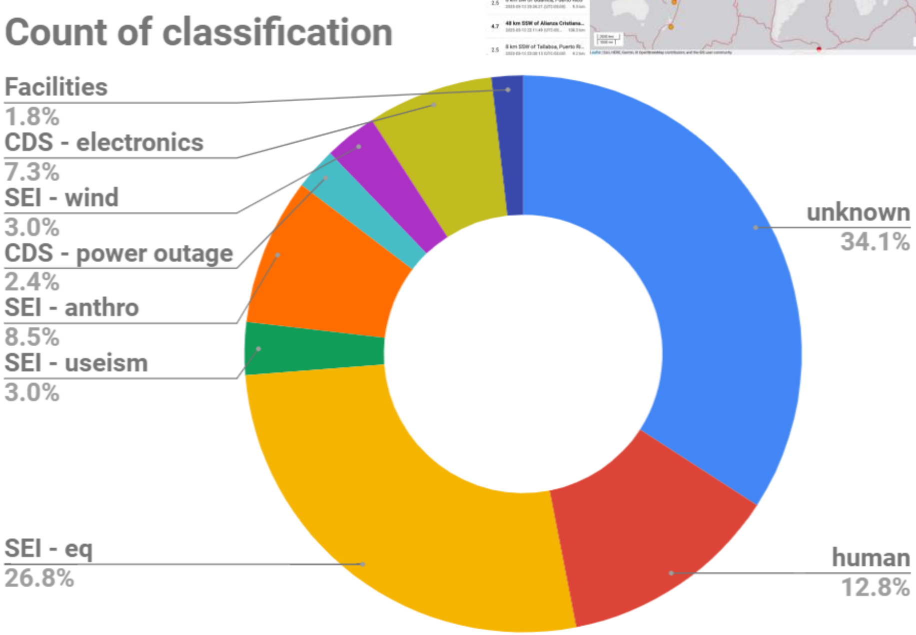
Principal Component Analysis
S. Hoshino+,
[PTEP **2024**, 103F01 (2024)]()
[JGW-G2315480]()

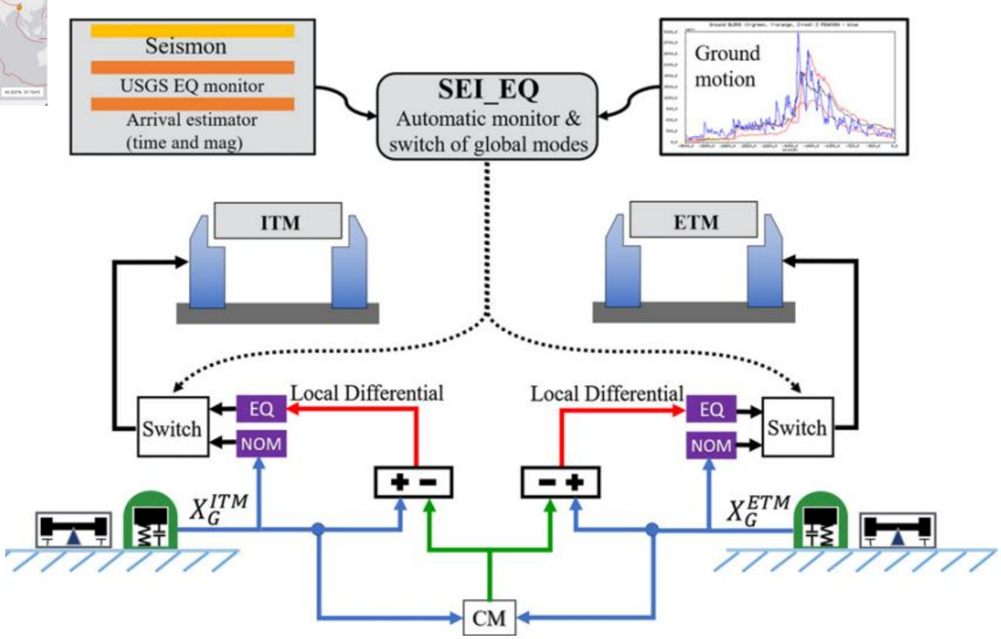
Protection from Seismic

- LIGO: SEISMON + Picket Fense
  - Switch suspension control modes based on seismic data around the detector (low noise mode or robust mode)



E. Bonilla+, LIGO-G2300553
E Schwartz+, CQG **37**, 235007 (2020)



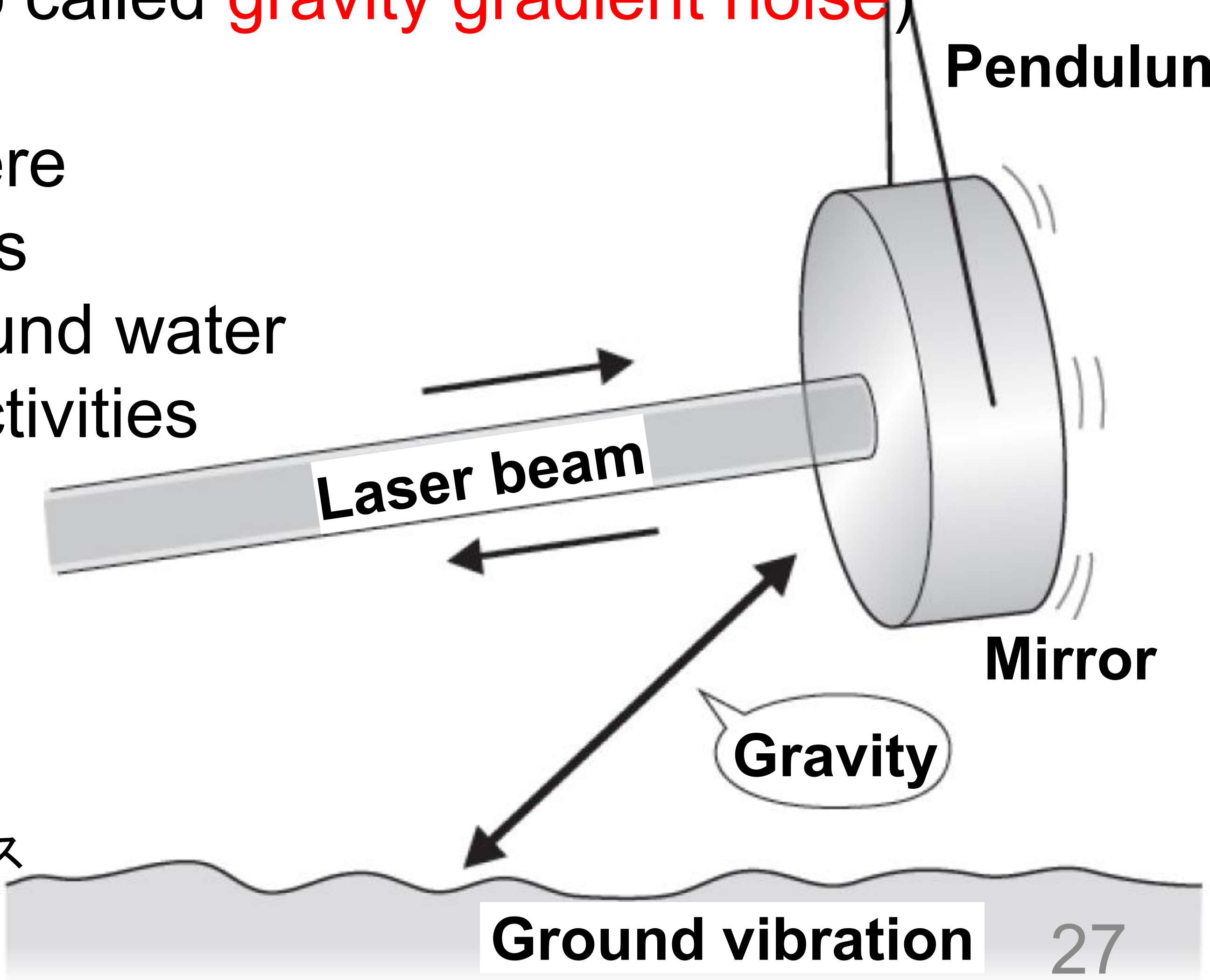


- Useful to improve duty factor

Newtonian Noise

Newtonian Noise

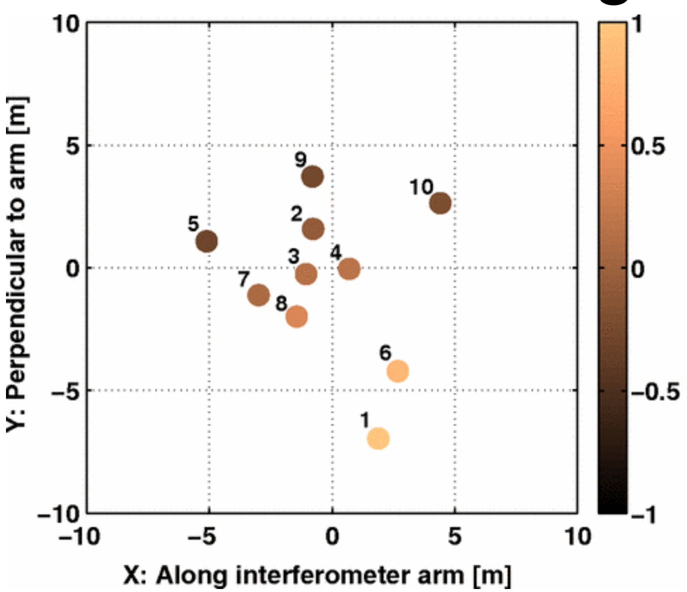
- Newtonian gravitational force fluctuations on mirrors (also called gravity gradient noise)
  - ground
  - atmosphere
  - sea waves
  - underground water
  - human activities

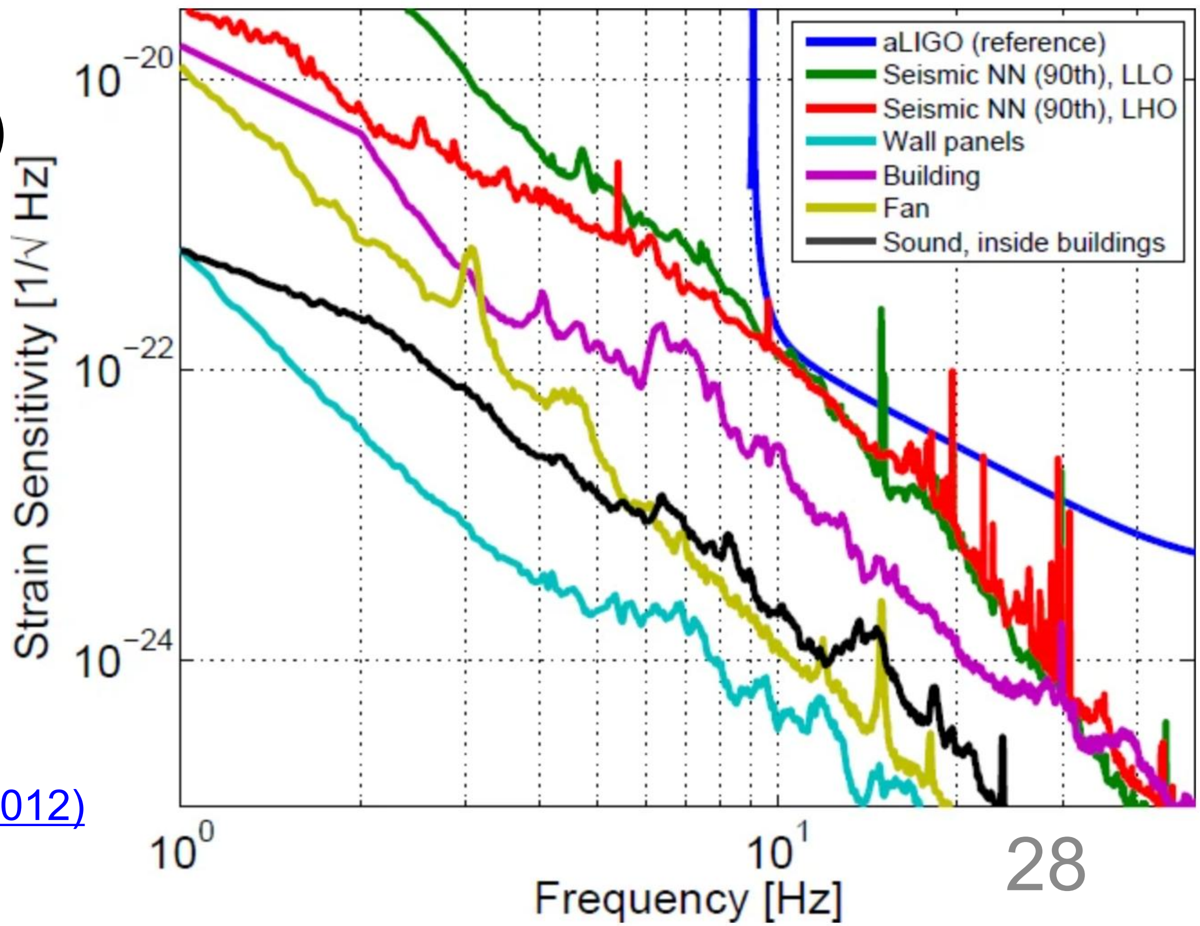


Modified from
講談社ブルーバックス
『重力波とはなにか』
安東正樹著

Estimation and Cancellation

- Estimations say they could contribute to sensitivity, especially for next generation detectors
- Could be subtracted with, e.g, seismometer array (but acoustic could be tough)

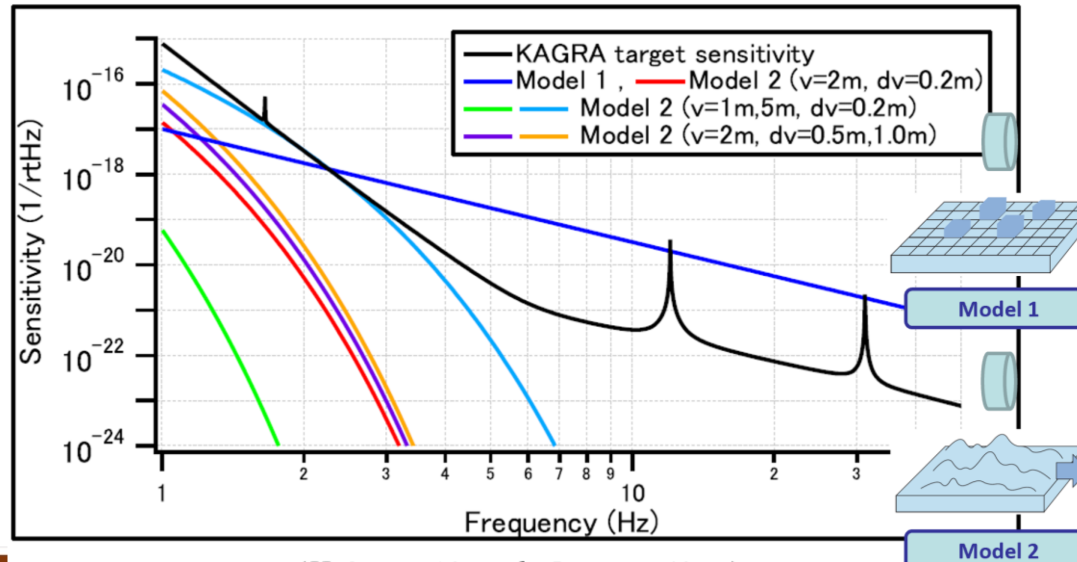




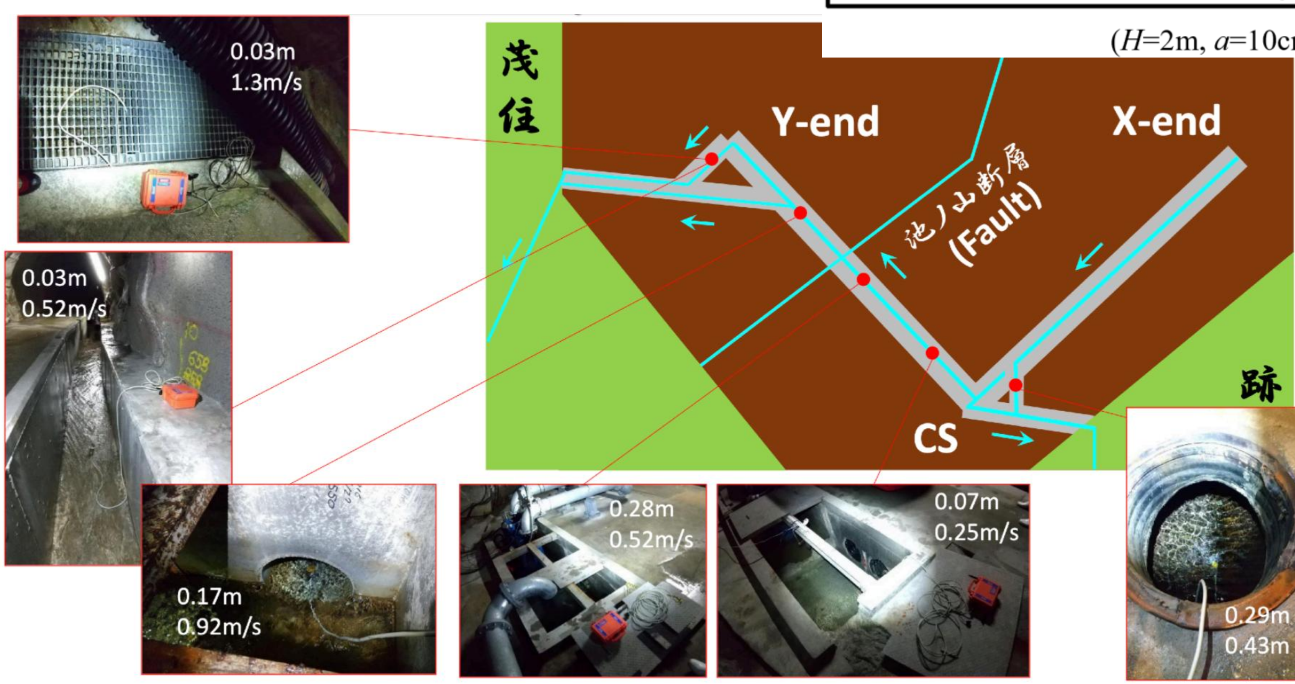
J. C. Driggers+, [PRD **86**, 102001 (2012)]
J. Harms, [LLR **18**, 3 (2015)]

Water Newtonian Noise

- Flow of underground water could be bad



(H=2m, a=10cm, b=5mm, w=40cm)



K. Somiya, JGW-G1910792

T. Yokozawa, JGW-G2415673

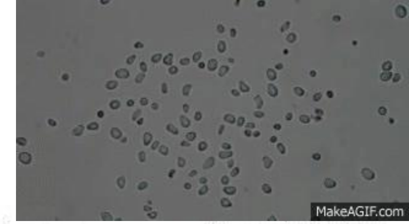
Underground GW Detector

- Underground site gives lower seismic noise and lower Newtonian noise
- KAGRA is the first km-scale underground detector
- Future GW project Einstein Telescope also considers using underground site

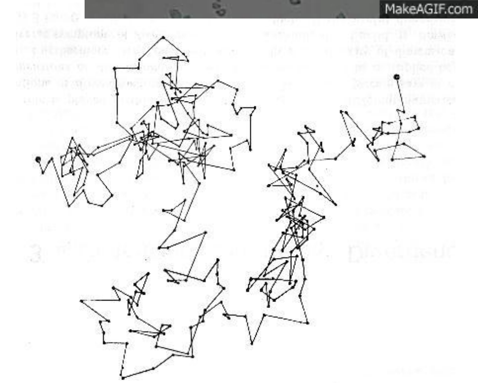
KAGRA site (March 2014)

Suspension Thermal Noise

Brownian Motion

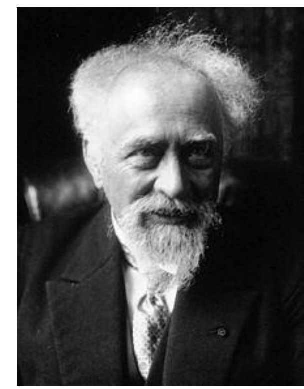


- 1827 Robert Brown
  Discovered random motion
  of pollen grains in water



- 1905 Albert Einstein
  Molecular-Kinetic Theory
  → Quantitative calculations on
  the movement of small particles in
  a liquid (Ph. D. Thesis)

- 1908 Jean Perrin
  Experimental verification using colloid liquid



- Lead to proof for existence of atoms, statistical physics

Johnson-Nyquist Noise

- 1926 John B. Johnson
  Pbserved voltage fluctuation
  across a resister
- 1928 Harry Nyquist

$$\overline{V_{\mathrm{n}}^2} = 4k_{\mathrm{B}}TR\Delta f$$

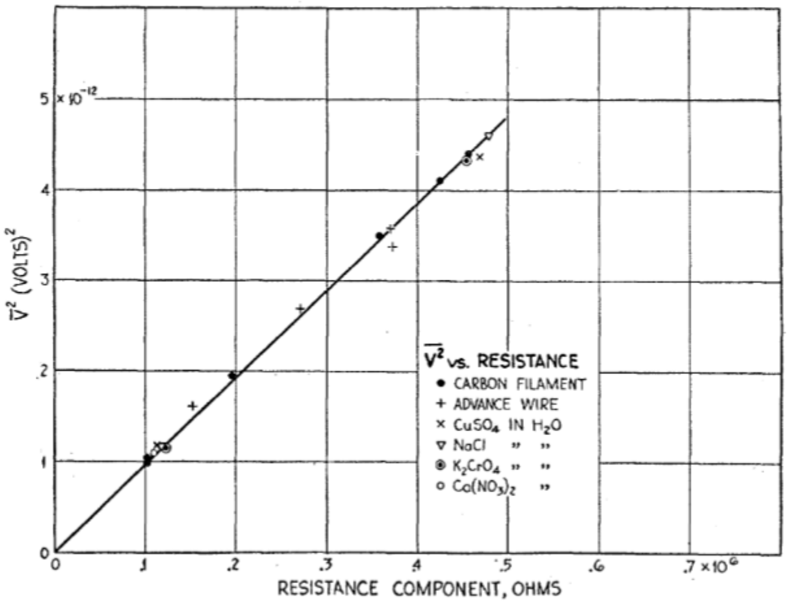
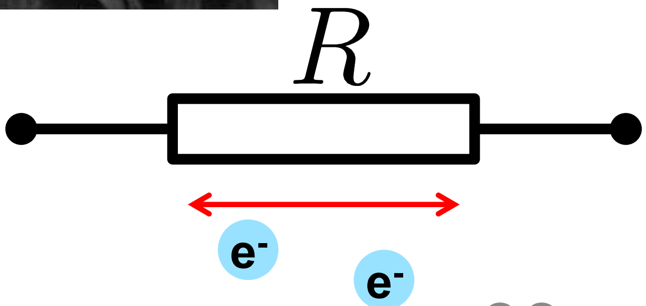


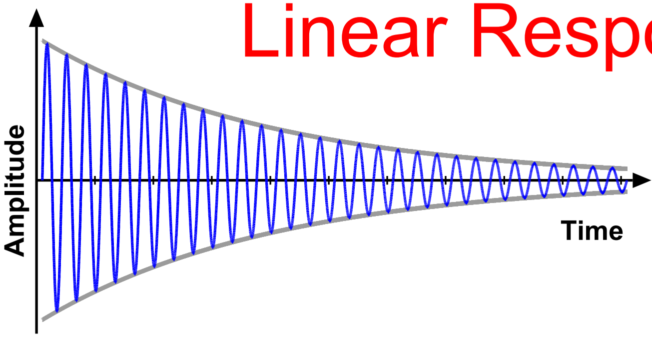
Fig. 4. Voltage-squared vs. resistance component for various kinds of conductors.

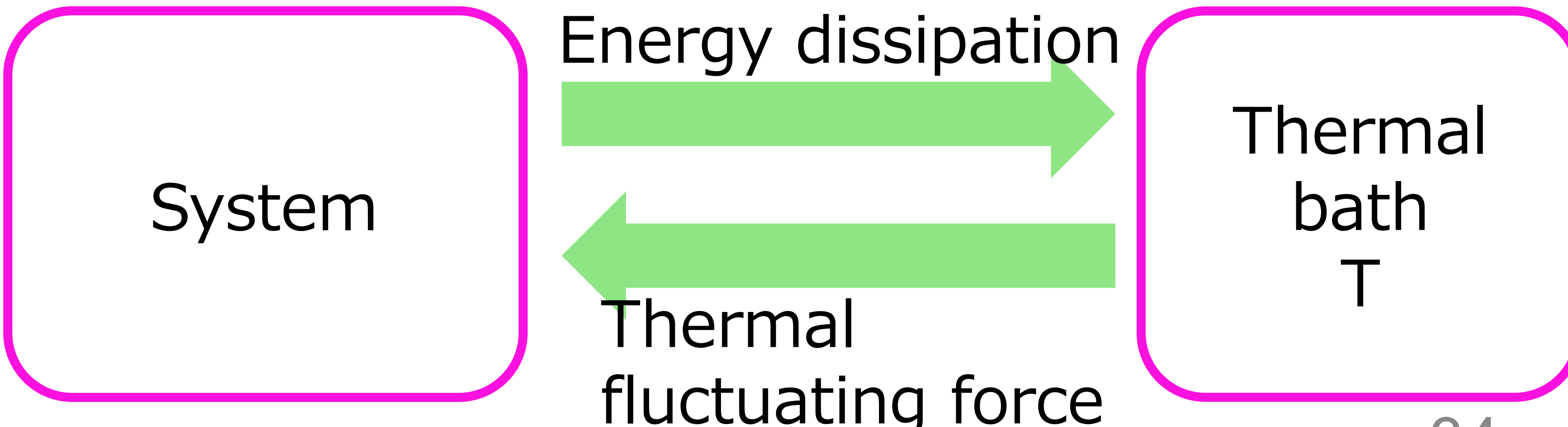


J. B. Johnson, Phys. Rev. 32, 97 (1928)

Generalization of Brownian Motion

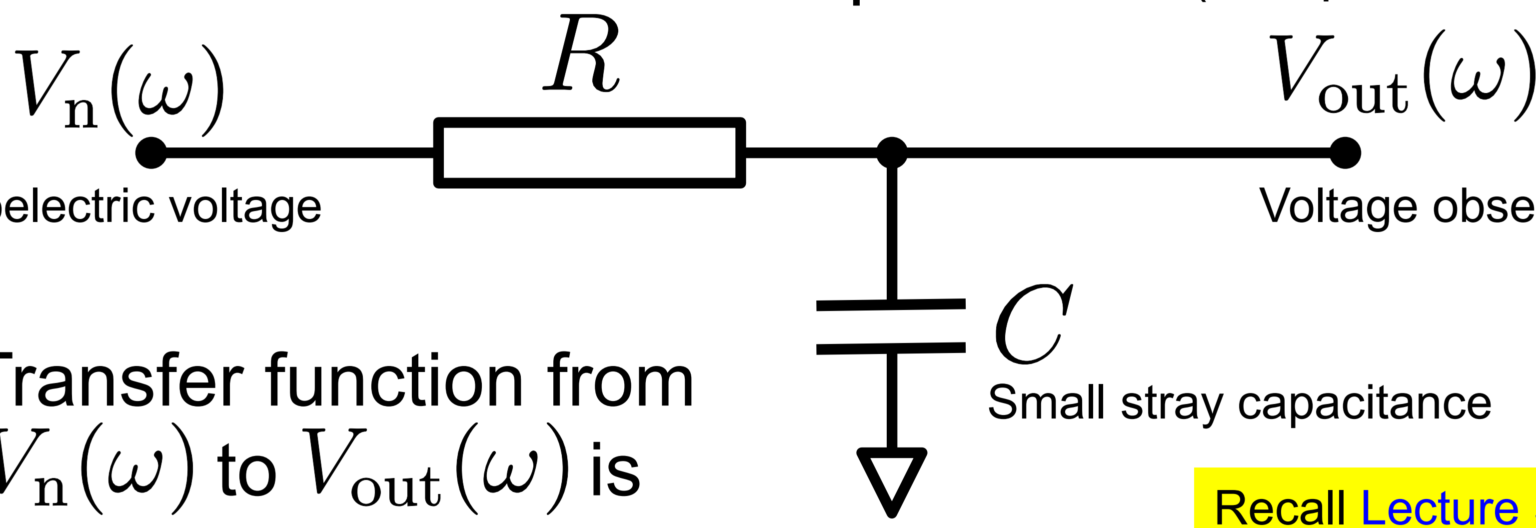
- 1951 Herbert B. Callen and Theodore A. Welton
  Fluctuation-Dissipation Theorem
- 1957- Expanded by Ryogo Kubo
  Linear Response Theory





Johnson-Nyquist Noise Derivation

- Resistance with a small capacitance (Low pass filter)



Recall Lecture 3

- Transfer function from $V_{\mathrm{n}}(\omega)$ to $V_{\mathrm{out}}(\omega)$ is

$$H(\omega) = \frac{1}{1 + i\omega RC}$$

- Assume that thermoelectric voltage is a white noise

$$S_{V_{\mathrm{n}}}(\omega) = D$$

Two-sided power spectral density of V_{n}

Johnson-Nyquist Noise Derivation

- Observed voltage fluctuations will be

$$\overline{V_{\mathrm{out}}^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\mathrm{V_{out}}}(\omega)\mathrm{d}\omega \quad \text{Perseval's theorem}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 S_{\mathrm{V_n}}(\omega)\mathrm{d}\omega \quad \text{Linear response}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{D}{1+(\omega CR)^2}\mathrm{d}\omega$$

$$= \frac{D}{2CR}$$

- From equipartition of energy $\frac{1}{2}C\overline{V_{\mathrm{out}}^2(t)} = \frac{1}{2}k_{\mathrm{B}}T$
- Therefore

$$S_{V_{\mathrm{n}}}(\omega) = D = 2k_{\mathrm{B}}TR$$

Johnson-Nyquist Noise Derivation

- Usually, we use two-sided power spectral density

$$S_{V_\mathrm{n}}(\omega) = 2k_\mathrm{B}TR$$ Two-sided power spectral density

$$\boxed{G_{V_\mathrm{n}}(f) = 4k_\mathrm{B}TR}$$ One-sided power spectral density

$$\begin{aligned}\overline{V_\mathrm{n}^2(t)} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{V_\mathrm{n}}(\omega)\mathrm{d}\omega \\ &= \int_0^{\infty} G_{V_\mathrm{n}}(f)\mathrm{d}f\end{aligned}$$

- For limited bandwidth Δf

$$\overline{V_\mathrm{n}^2} = 4k_\mathrm{B}TR\Delta f$$ Johnson-Nyquist noise

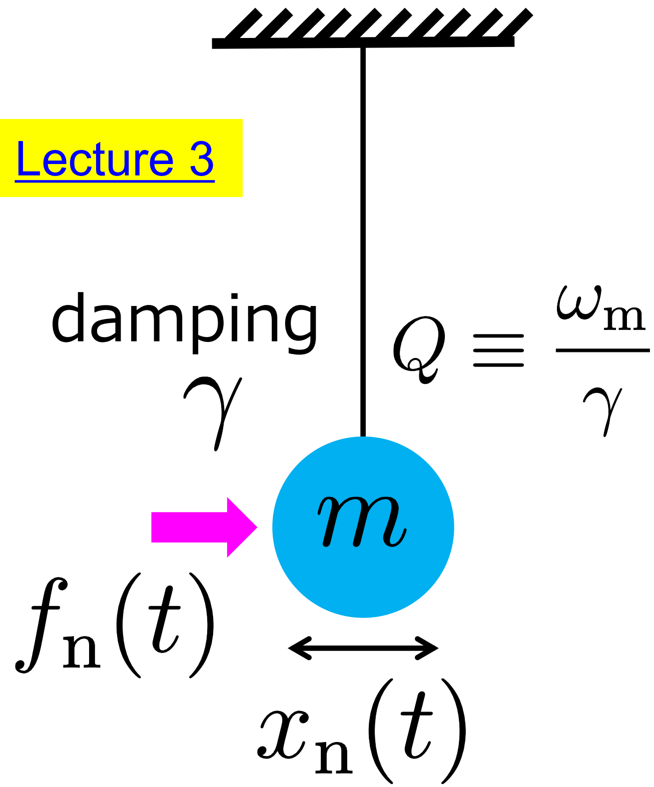
Brownian Noise of Oscillators

- Transfer function from force to displacement is

Recall Lecture 3

$$\chi(\omega) = \frac{1}{m}\frac{1}{\omega_{\mathrm{m}}^2 - \omega^2 + i\frac{\omega\omega_{\mathrm{m}}}{Q}}$$

Susceptibility
(感受率)



- Assume thermal fluctuation force is a white noise

$$S_{f_{\mathrm{n}}}(\omega) = D$$

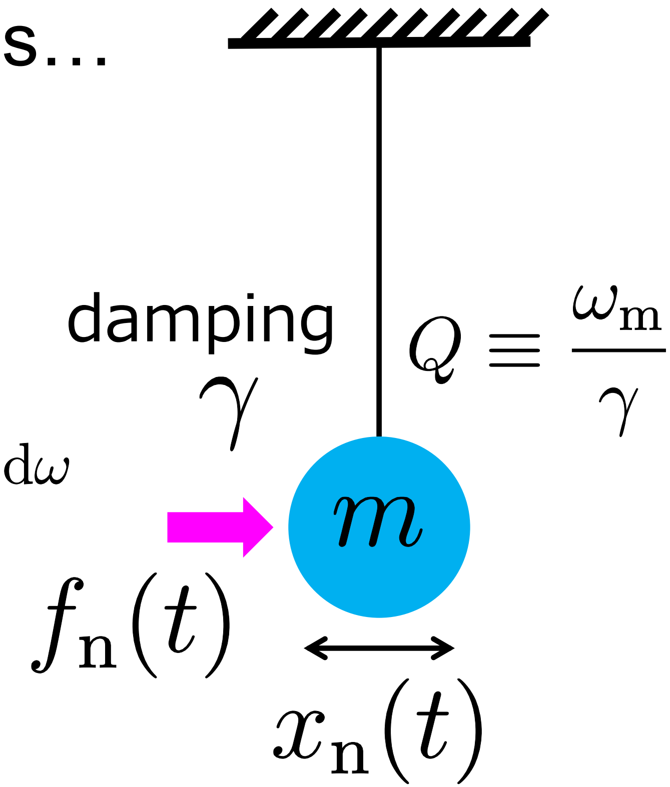
- From equipartition of energy

$$\frac{1}{2}m\omega_{\mathrm{m}}^2\overline{x_{\mathrm{n}}^2(t)} = \frac{1}{2}k_{\mathrm{B}}T$$

Brownian Noise of Oscillators

- By repeating similar calculations…

$$
\begin{aligned}
\overline{x_{\mathrm{n}}^2(t)} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{x_{\mathrm{n}}}(\omega)\mathrm{d}\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} |\chi(\omega)|^2 S_{f_{\mathrm{n}}}(\omega)\mathrm{d}\omega \\
&= \frac{1}{2\pi m^2}\int_{-\infty}^{\infty} \frac{D}{(\omega^2-\omega_{\mathrm{m}}^2)^2+(\omega^2\omega_{\mathrm{m}}^2/Q)^2}\mathrm{d}\omega \\
&= \frac{QD}{2m^2\omega_{\mathrm{m}}^3}
\end{aligned}
$$



- Therefore

$$S_{f_{\mathrm{n}}}(\omega) = D = 2k_{\mathrm{B}}Tm\gamma$$

$$\boxed{G_{f_{\mathrm{n}}}(f) = 4k_{\mathrm{B}}Tm\gamma}$$

Fluctuation Dissipation Theorem

- Linear system

$$\underset{\text{Generalized force}}{\xrightarrow{F(\omega)}} \boxed{\text{System}} \underset{\text{Generalized coordinate}}{\xrightarrow{x(\omega)}}$$

- Susceptibility

$$\chi(\omega) = \frac{x(\omega)}{F(\omega)}$$

Impedance

$$Z(\omega) = \frac{F(\omega)}{v(\omega)} = \frac{F(\omega)}{i\omega x(\omega)}$$

Inverse of impedance is called admittance

- Thermal fluctuating force will be

$$G_{F_\mathrm{n}}(f) = 4k_\mathrm{B}T\underline{\mathrm{Re}(Z(\omega))}$$

Real part of impedance (called resistance)

$$= -4k_\mathrm{B}T\frac{1}{\omega\mathrm{Im}(\chi(\omega))}$$

Derivation from FDT

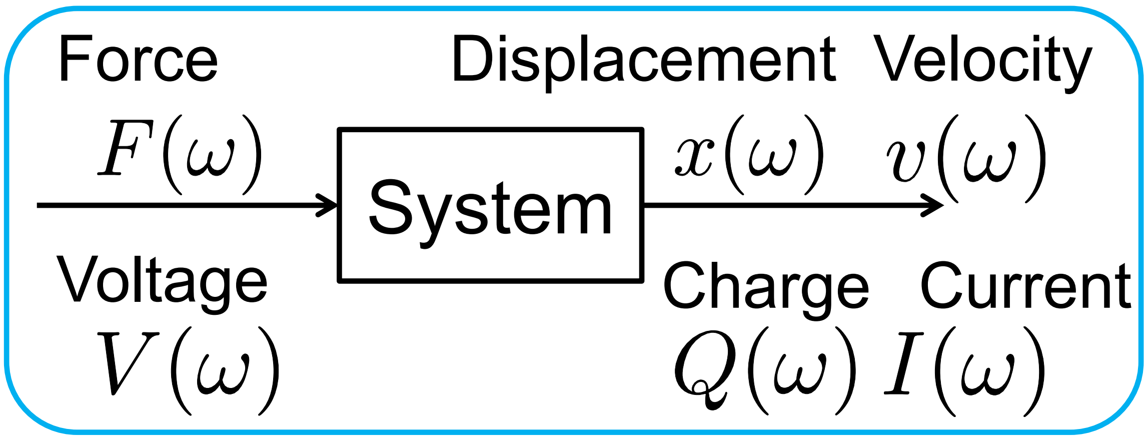
- **Harmonic Oscillator**

$$G_{F_\mathrm{n}}(f) = -4k_\mathrm{B}T\frac{1}{\omega\mathrm{Im}(\chi(\omega))}$$

$$\chi(\omega) = \frac{1}{m}\frac{1}{\omega_\mathrm{m}^2 - \omega^2 + i\frac{\omega\omega_\mathrm{m}}{Q}}$$

$$= 4k_\mathrm{B}T\frac{1}{\omega/(m\omega\omega_\mathrm{m}/Q)}$$

$$= 4k_\mathrm{B}Tm\gamma$$



- **Johnson-Nyquist Noise**

$$G_{V_\mathrm{n}}(f) = 4k_\mathrm{B}T\mathrm{Re}(Z(\omega))$$

$$= 4k_\mathrm{B}TR$$

$$Z(\omega) = \frac{V_\mathrm{n}}{I} \qquad H(\omega) = \frac{1}{1+i\omega RC}$$

$$= \frac{V_\mathrm{n}}{\dot{V}_\mathrm{out}C}$$

$$= \frac{1}{i\omega C}H(\omega)$$

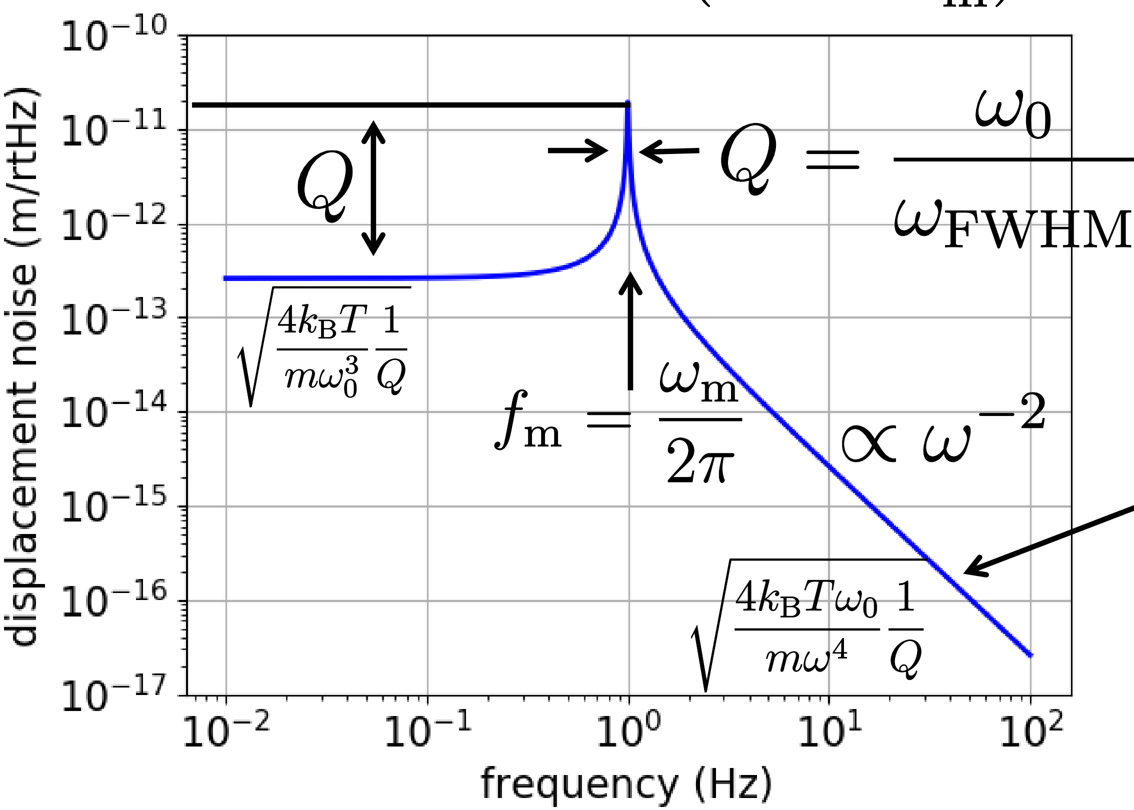
$$= R + \frac{1}{i\omega C}$$

Thermal Noise in Displacement

- Brownian thermal noise will be

$$G_{x_\mathrm{n}}(f) = |\chi(\omega)|^2 G_{F_\mathrm{n}}(f)$$

$$= \frac{4k_\mathrm{B}Tm\gamma}{(\omega^2-\omega_\mathrm{m}^2)^2+(\omega^2\omega_\mathrm{m}^2/Q)^2}$$



$$\overline{x_\mathrm{n}^2(t)} = \frac{k_\mathrm{B}T}{m\omega_\mathrm{m}^2}$$

This do not depend on Q

When Q is high, most of the power is in the resonant peak, and thermal noise at higher frequencies is reduced

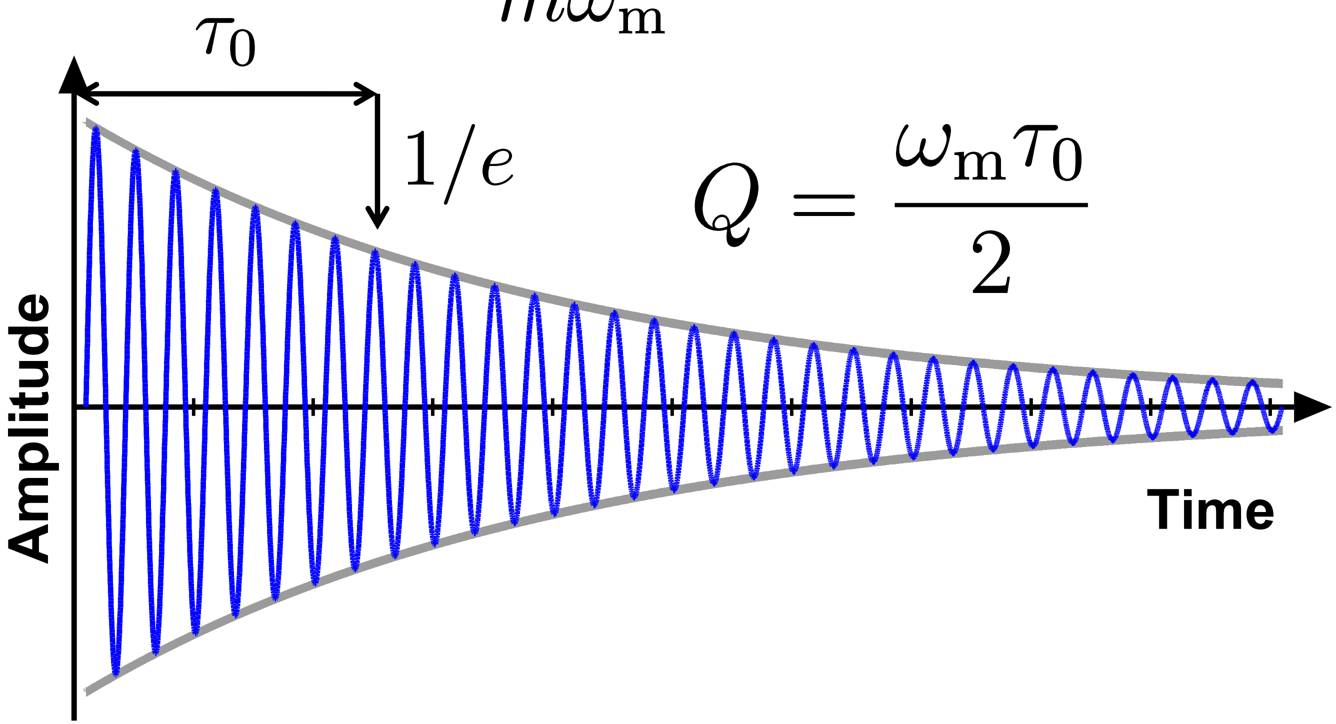
Q Measurement

- Resonant frequency and Q-value can be measured from the impulse response

$$x(t) = \frac{1}{m\omega_\mathrm{m}} e^{-\frac{\omega_\mathrm{m} t}{2Q}} \sin \omega_\mathrm{m} t$$

Fourier transform of

$$\chi(\omega) = \frac{1}{m} \frac{1}{\omega_\mathrm{m}^2 - \omega^2 + i\frac{\omega\omega_\mathrm{m}}{Q}}$$



$$Q = \frac{\omega_\mathrm{m} \tau_0}{2}$$

tuning fork

Viscous and Structure Damping

- Viscous damping
  - Damping by a force proportional to velocity
  - e.g. gas damping, eddy-current damping

$$m\left(\ddot{x} + \gamma\dot{x} + \omega_{\mathrm{m}}^2 x\right) = f$$

$$\gamma = \frac{\omega_{\mathrm{m}}}{Q}$$

Frequency independent

- Structure damping
  - Energy loss due to internal loss
  - e.g. internal friction of materials
  - ideal elastic material

$$F_{\mathrm{spring}} = -kx \quad \text{(Hooke's Law)}$$

  - anelastic material

loss angle

$$F_{\mathrm{spring}} = -k(1 + i\phi)x$$

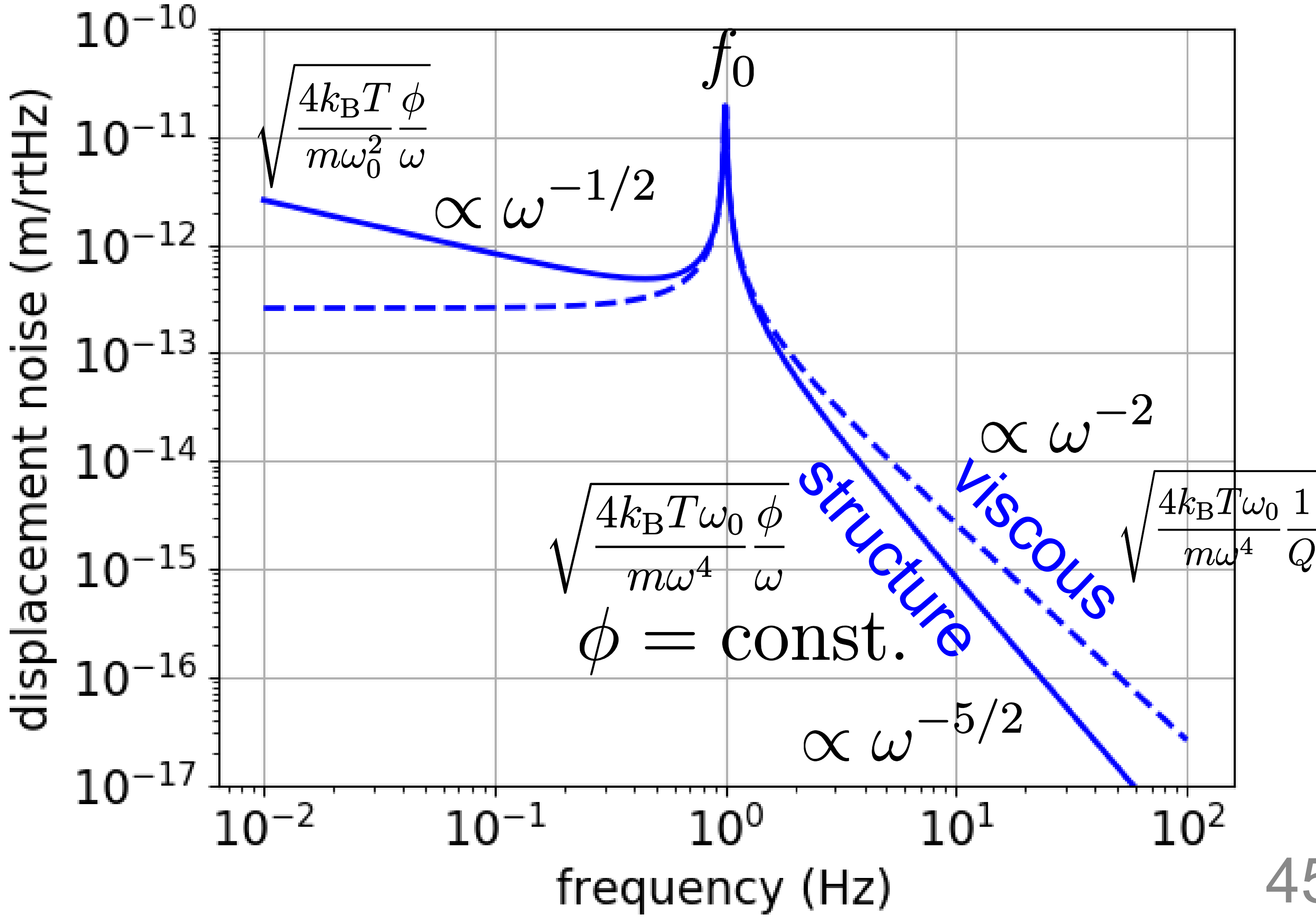
$$\phi = \frac{1}{Q}$$

Frequency dependent

$$\gamma(\omega) = \frac{\omega_{\mathrm{m}}^2}{\omega Q}$$

Viscous and Structure Damping

- Structure has smaller noise above resonance



Pendulum Thermal Noise

- Restoring force also from elasticity

$$F_{\mathrm{spring}} = -(mg/l + k_{\mathrm{el}}(1 + i\phi))x$$
$$= -(mg/l + k_{\mathrm{el}})(1 + i\phi_{\mathrm{p}})x$$

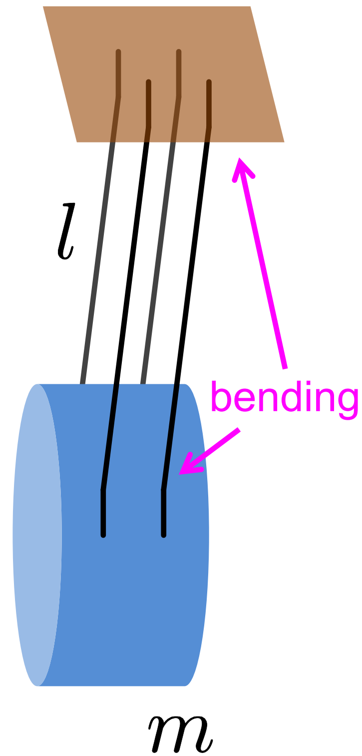
- Gravitational dilution

$$\phi_{\mathrm{p}} \equiv \underbrace{\frac{k_{\mathrm{el}}}{mg/l + k_{\mathrm{el}}}}_{\text{dilution factor}} \phi \simeq \frac{\phi}{2l}\sqrt{\frac{EI}{mg}}$$

loss angle of pendulum 10^{-5}~10^{-7}

dilution factor 10^{-3}~10^{-2}

intrinsic loss angle of wire 10^{-4}~10^{-3}



$$k_{\mathrm{el}} = \frac{\sqrt{TEI}}{2l^2} = \frac{\sqrt{mgEI}}{2l^2}$$

T: wire tension
E: wire Young's modulus
I: wire moment of inertia

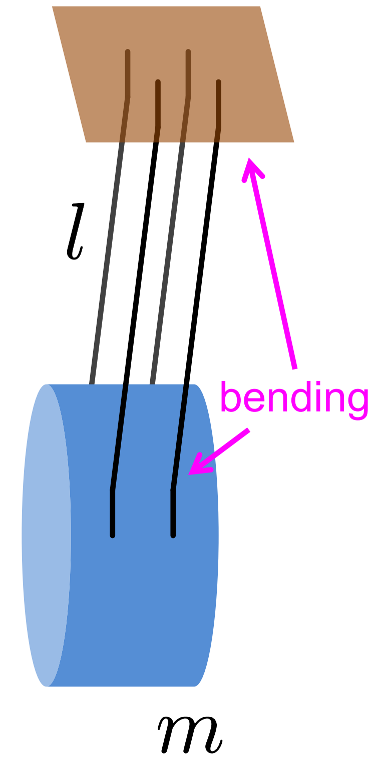
Pendulum Thermal Noise

- Suspension thermal noise ($\omega \gg \omega_\mathrm{m}$)

$$x_\mathrm{susp}^2(\omega) = \frac{4k_\mathrm{B}T\omega_\mathrm{m}}{m\omega^4}\frac{\phi_\mathrm{p}}{\omega}$$

From now on we will say that this is one-sided power spectral density

$$= \frac{4k_\mathrm{B}T}{m\omega^5}\sqrt{\frac{g}{l}}\frac{\phi}{2l\omega}\sqrt{\frac{EI}{mg}}$$



- To lower the thermal noise
  - lower temperature $\quad x_\mathrm{susp}(\omega) \propto \sqrt{T\phi}$
  - lower loss angle
  - longer suspension $\quad x_\mathrm{susp}(\omega) \propto l^{-3/2}$
  - heavier mirror

$x_\mathrm{susp}(\omega) \propto m^{-1/4}$ assuming $I \propto A^2$

A: wire cross section $\quad m \propto A$

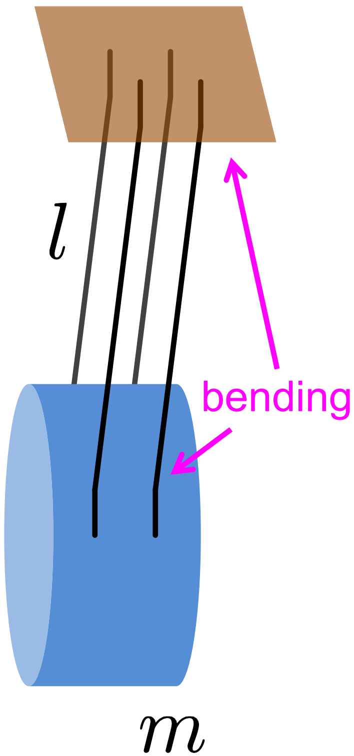
(we will need thicker wire to suspend heavier mirror)

Pendulum Thermal Noise

- Suspension thermal noise ($\omega \gg \omega_\mathrm{m}$)

$$x^2_\mathrm{susp}(\omega) = \frac{4k_\mathrm{B}T\omega_\mathrm{m}}{m\omega^4}\frac{\phi_\mathrm{p}}{\omega}$$

- Calculate suspension thermal noise for m=10 kg, f_0= 1Hz, T = 300 K, φ_p = 10^{-6} at 100 Hz. $k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J/K}$

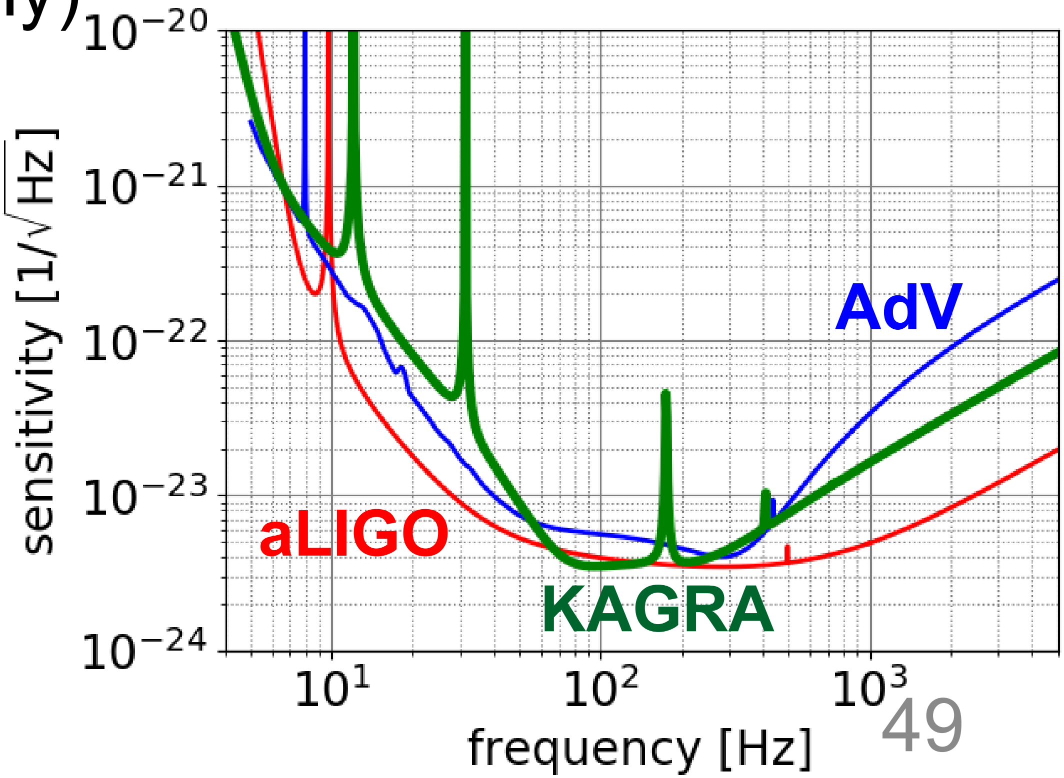


$$x_\mathrm{susp}(\omega) = 1.0 \times 10^{-20} \left(\frac{100\,\mathrm{Hz}}{f}\right)^{5/2}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

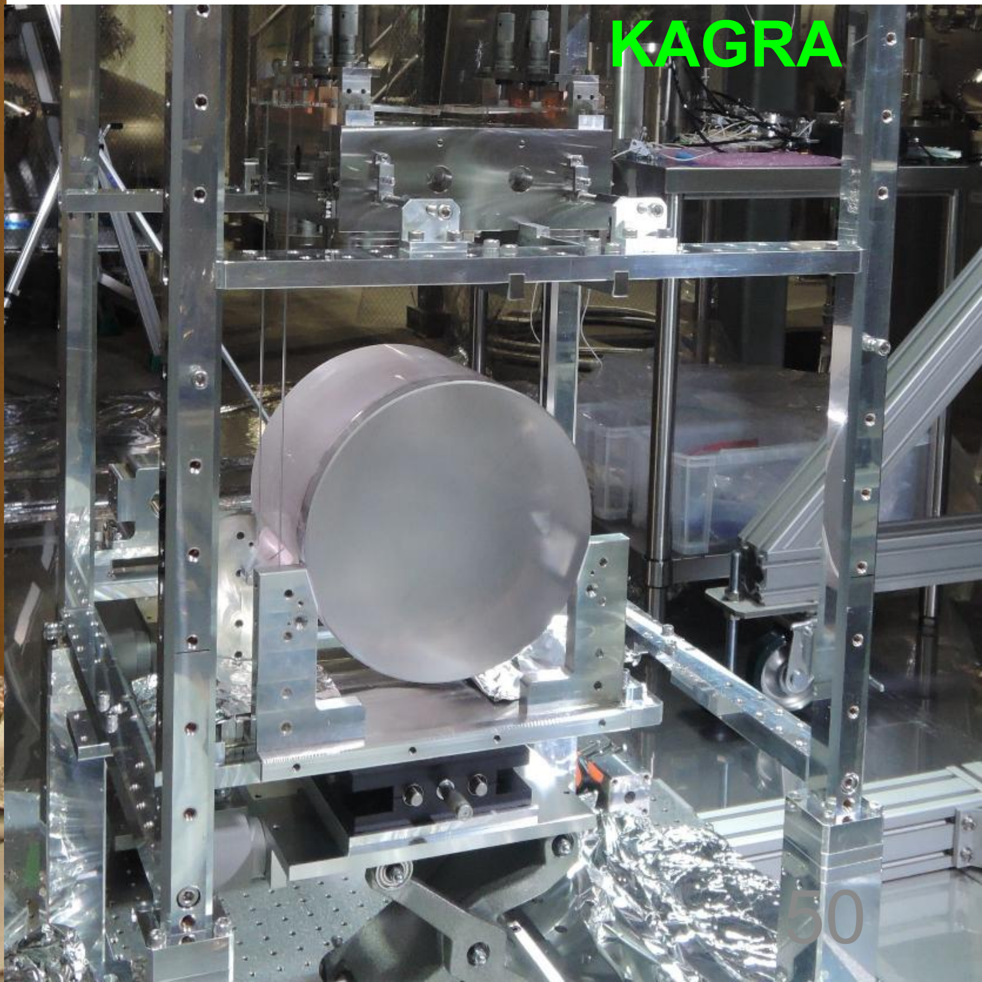
Sufficient for GW detection!

Suspension Thermal Noise in LVK

- Advanced LIGO (US)
  40 kg, 60 cm,
  fused silica,
  295 K, φw=~1e-7
- Advanced Virgo (Italy)
  42 kg, 70 cm,
  fused silica,
  295 K, φw=~1e-7
- KAGRA (Japan)
  23 kg, 35 cm,
  sapphire,
  22 K, φw=2e-7

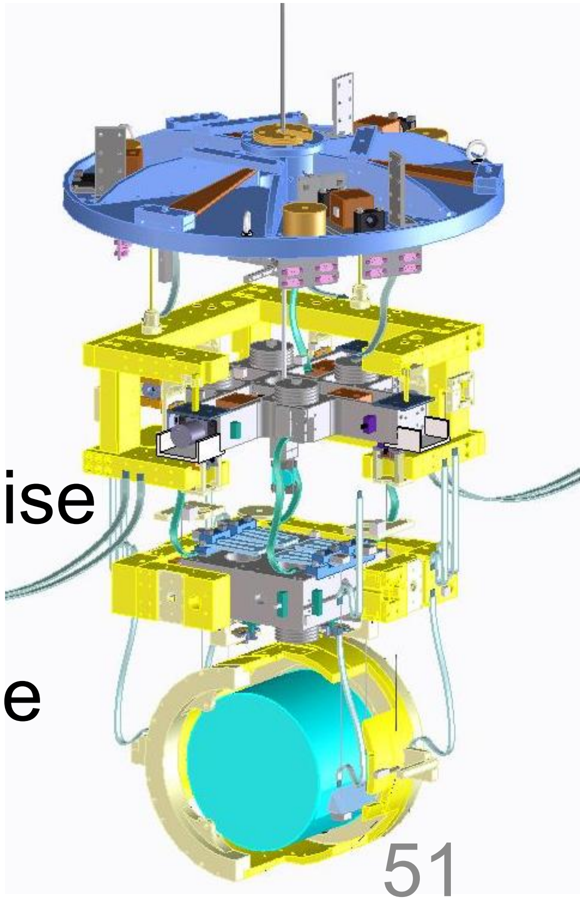


Suspensions



Challenges (Opportunities) in Cryogenics

- KAGRA is the first km-scale GW detector to use cryogenics to reduce thermal noise
- Fused silica has large mechanical loss at cryogenic temperatures
  → sapphire
- Limitation in mirror size: 23 kg
  (small compared with aLIGO/AdV)
  → smaller beam size
  worse quantum and thermal noise
- Thicker wire for heat extraction
  → larger suspension thermal noise
- Vibration of cryocoolers



Material Search is Important

- Thermal noise $\propto \sqrt{T\phi}$

Temperature

Mechanical loss angle (temperature dependent)

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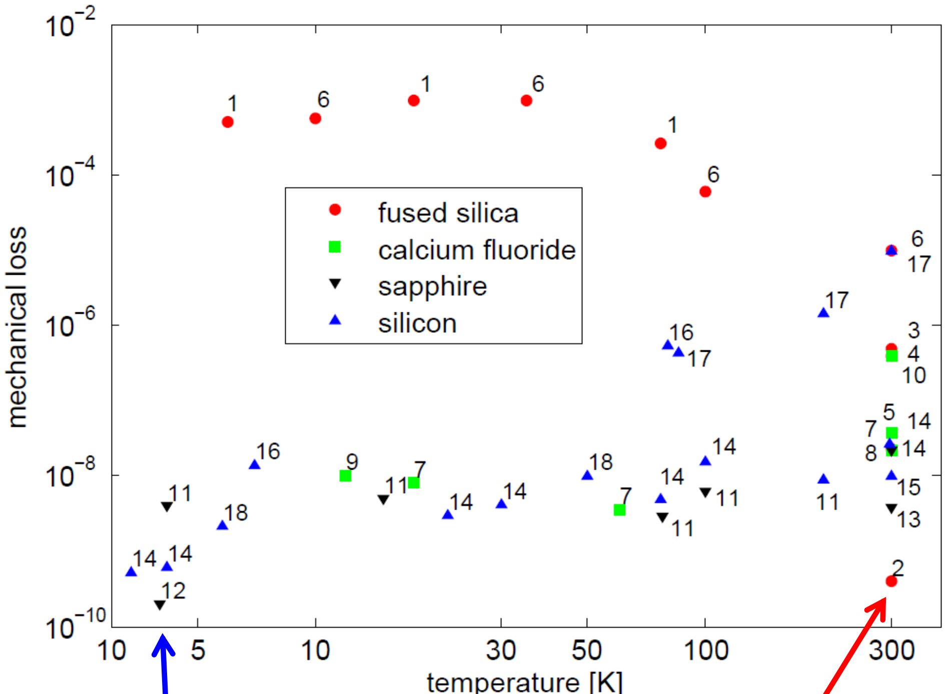
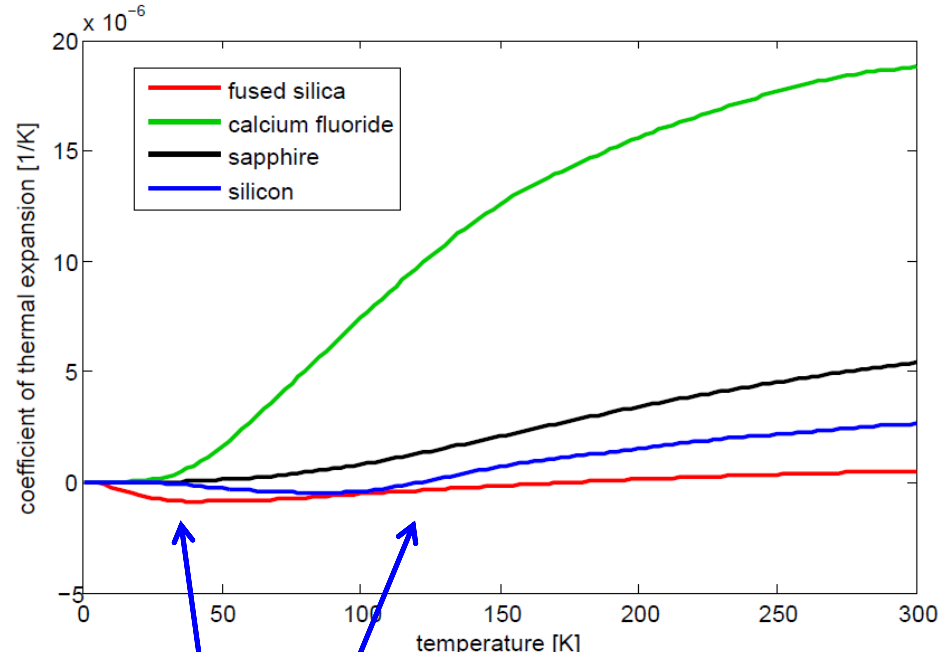




Sapphire / silicon is good at cryogenic temp.

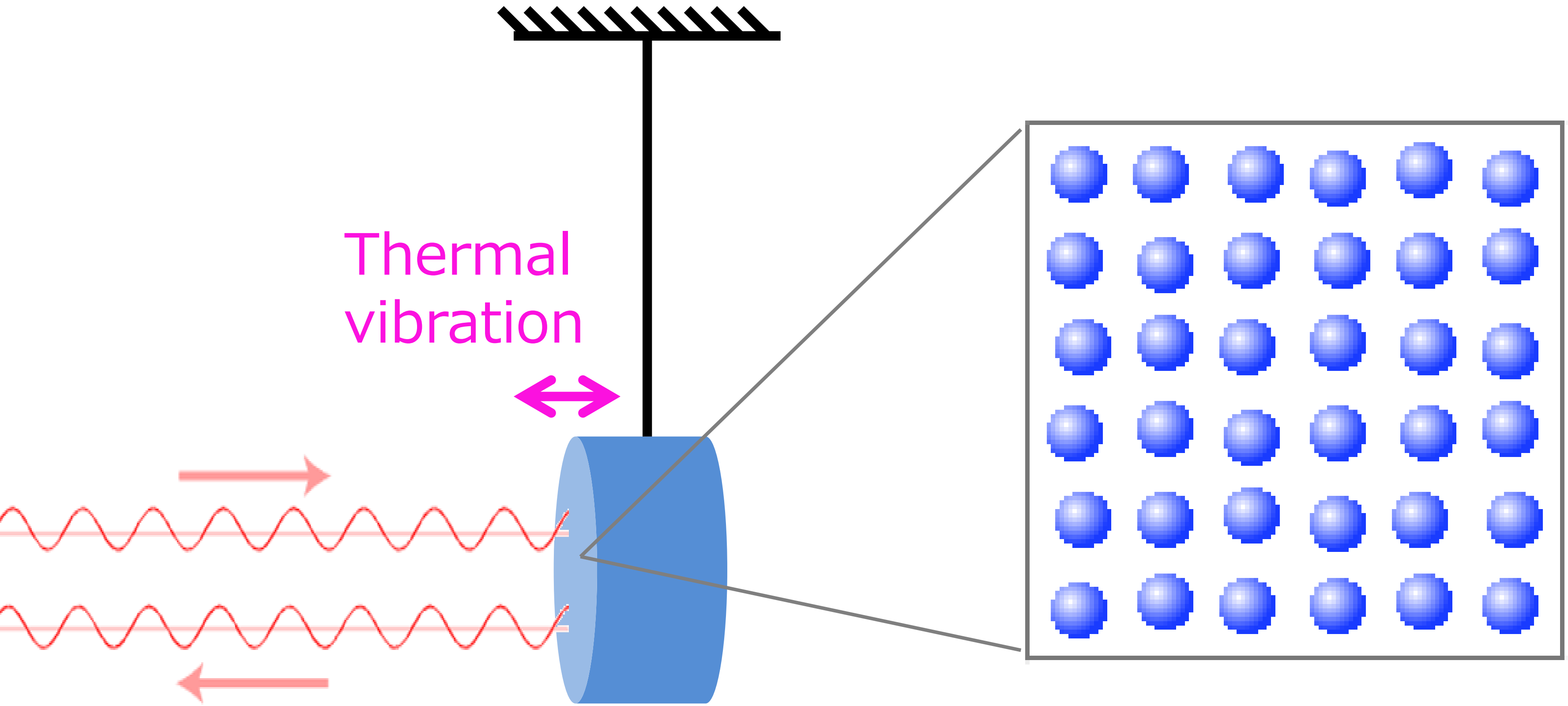
Fused silica is good at room temp.

Silicon has zero thermal expansion at 18 K and 123 K

Mirror and Coating Thermal Noises

Mirror and Coating Thermal Noise

- Thermal vibration of mirror surface due to
  - Mirror substrate
  - Mirror high-reflective coating

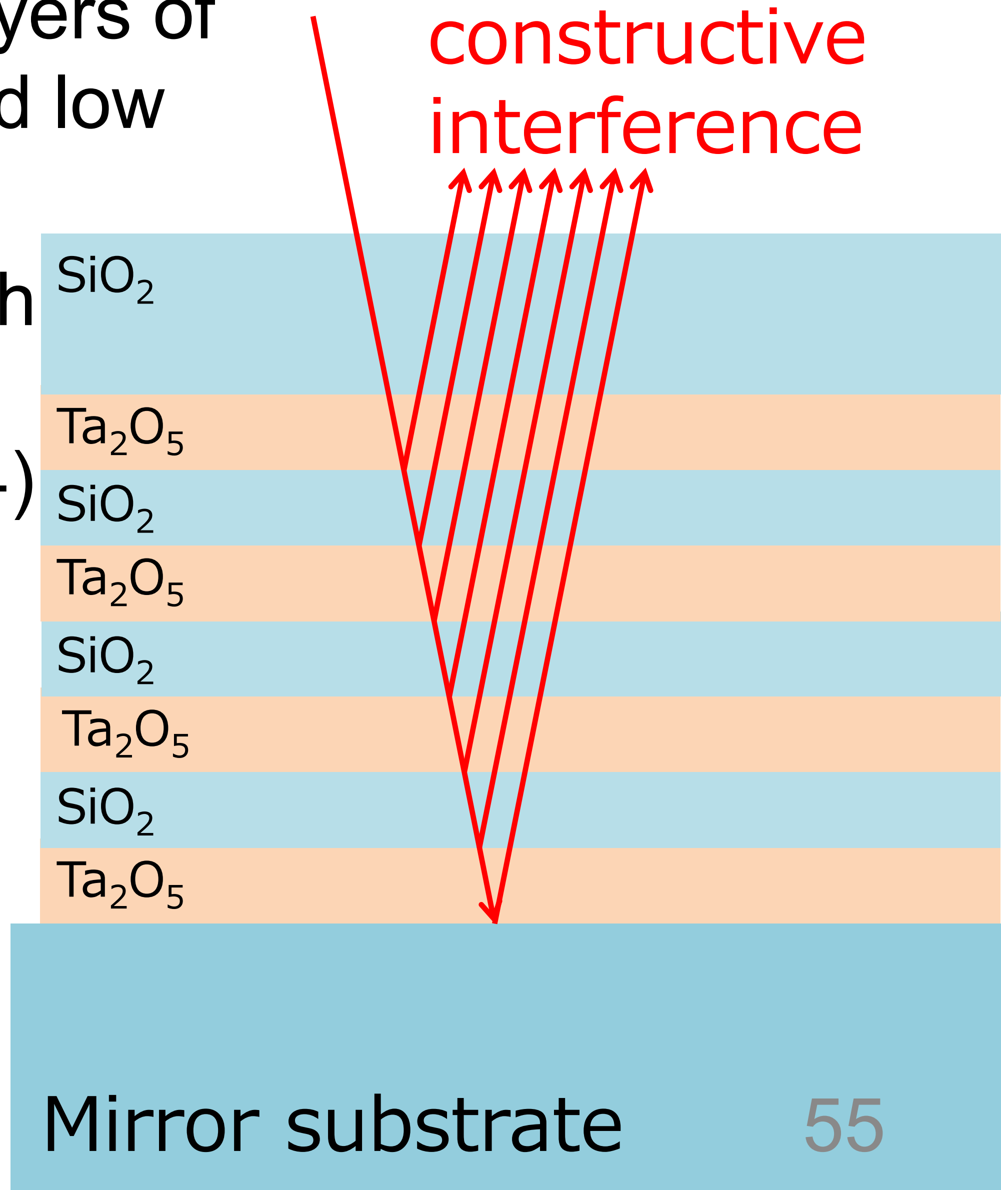


Dielectric Mirror Coating

- λ/4 thick alternating layers of materials with high and low refractive indices
- Active area of research due to low loss angle (φ~1e-4)

AR (anti-reflection) coating

HR (high-reflection) coating



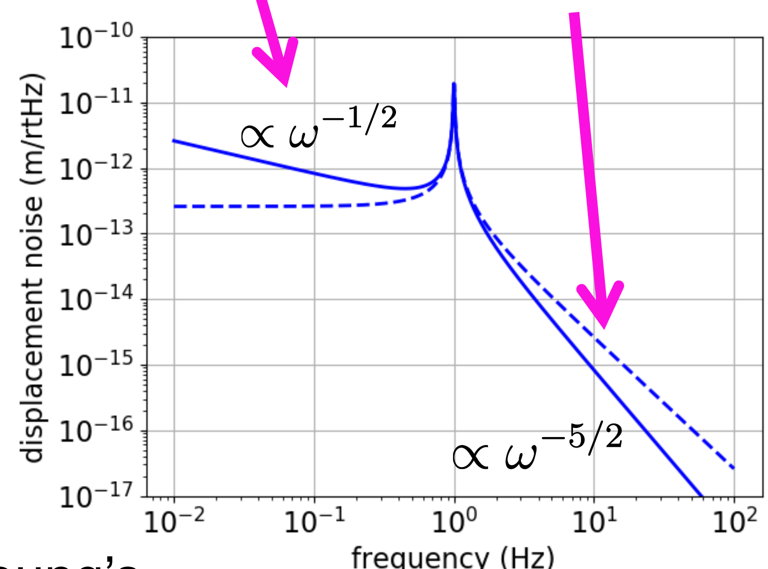
Deriving Thermal Noise Formulae

- Structure damping with $\omega \ll \omega_{\rm m}$
  (for suspension thermal noise we considered $\omega \gg \omega_{\rm m}$)

$$x_{\rm mir}^2(\omega) = \frac{4k_{\rm B}T}{m_{\rm eff}\omega_{\rm m}^2}\frac{\phi}{\omega}$$

Regime for mirror/coating thermal noise

Regime for suspension thermal noise



- $m_{\rm eff}$ is reduced mass
  - function of mirror aspect ratio, mirror radius, beam radius ratio w, Poisson's ratio σ
- $\omega_{\rm m}$ is resonant frequency of vibration mode $\omega_{\rm m} \propto \sqrt{E}$ Young's modulus
- You also have to sum up all the vibration modes

Simple Formulae

- Mirror substrate thermal noise

$$x_{\mathrm{sub}}^2(\omega) = \frac{4k_{\mathrm{B}}T\phi_{\mathrm{sub}}}{\omega}\frac{1-\sigma_{\mathrm{sub}}^2}{\sqrt{\pi}E_{\mathrm{sub}}w}$$

K. Numata+, [PRL **93**, 250602 (2004)]
N. Nakagawa+, [PRD **65**, 102001 (2002)]

- Coating thermal noise

$$x_{\mathrm{coat}}^2(\omega) = \frac{4k_{\mathrm{B}}T\phi_{\mathrm{co}}}{\omega}\frac{2d_{\mathrm{co}}(1+\sigma_{\mathrm{co}})(1-2\sigma_{\mathrm{co}})}{\pi E_{\mathrm{sub}}w^2}$$

- To lower thermal noises
  - lower temperature
  - lower loss angle
  - larger beam size (needs larger mirror)

$$x_{\mathrm{sub/coat}}(\omega) \propto \sqrt{T\phi}$$

$$x_{\mathrm{sub}}(\omega) \propto 1/w^{1/2} \quad x_{\mathrm{coat}}(\omega) \propto 1/w$$

Thermal Noise Numbers

- Calculate mirror thermal noise for fused silica (E = 72 GPa, σ = 0.17, φ=1e-6) w = 5 cm, T = 300 K, at 100 Hz

$$x_{\text{sub}}(\omega) = 6.3 \times 10^{-20} \left(\frac{100\,\text{Hz}}{f} \right)^{1/2} \text{m}/\sqrt{\text{Hz}}$$

- Calculate coating thermal noise for silica/tantala coating (E = 72 Gpa, σ = 0.17, φ = 4e-4) d = 10 um, w = 5 cm, T = 300 K, at 100 Hz

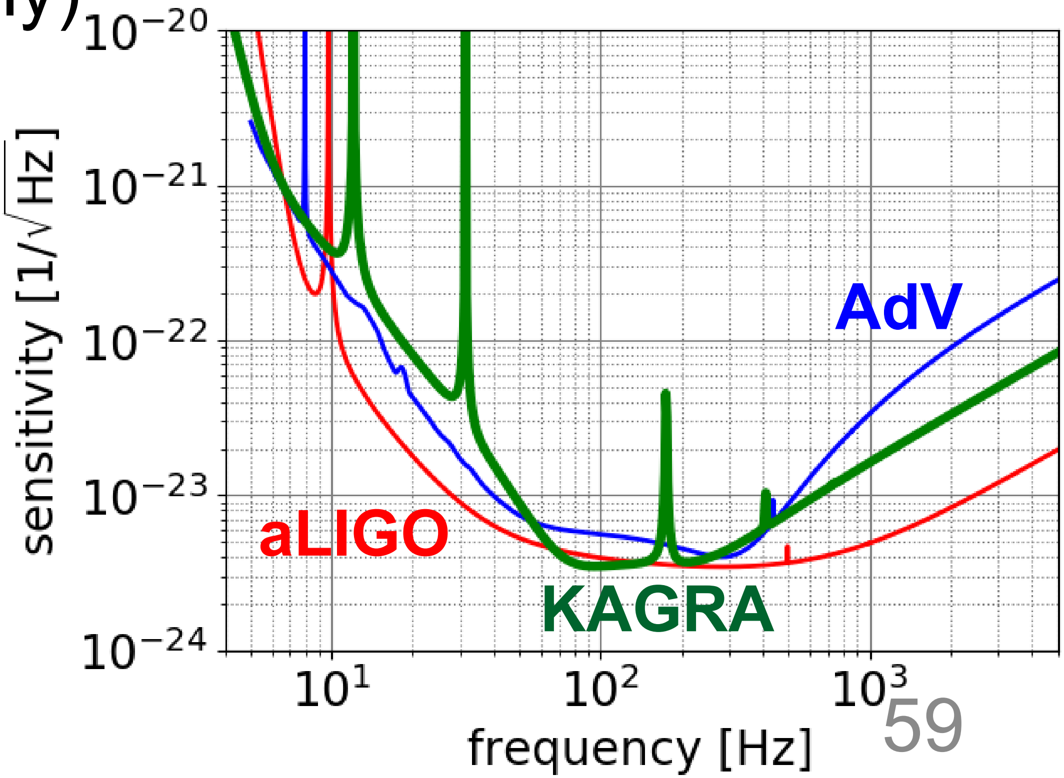
$$x_{\text{coat}}(\omega) = 1.7 \times 10^{-20} \left(\frac{100\,\text{Hz}}{f} \right)^{1/2} \text{m}/\sqrt{\text{Hz}}$$

Sufficient for GW detection!

Mirror Thermal Noises in LVK

- Advanced LIGO (US)
  w= 5.5 / 6.2 cm,
  fused silica,
  295 K, φ_{sub}=~1e-6
- Advanced Virgo (Italy)
  w= 4.9 / 5.8 cm,
  fused silica,
  295 K, φ_{sub}=~1e-6
- KAGRA (Japan)
  w= 3.5 / 3.5 cm,
  sapphire,
  22 K, φ_{sub}=1e-8

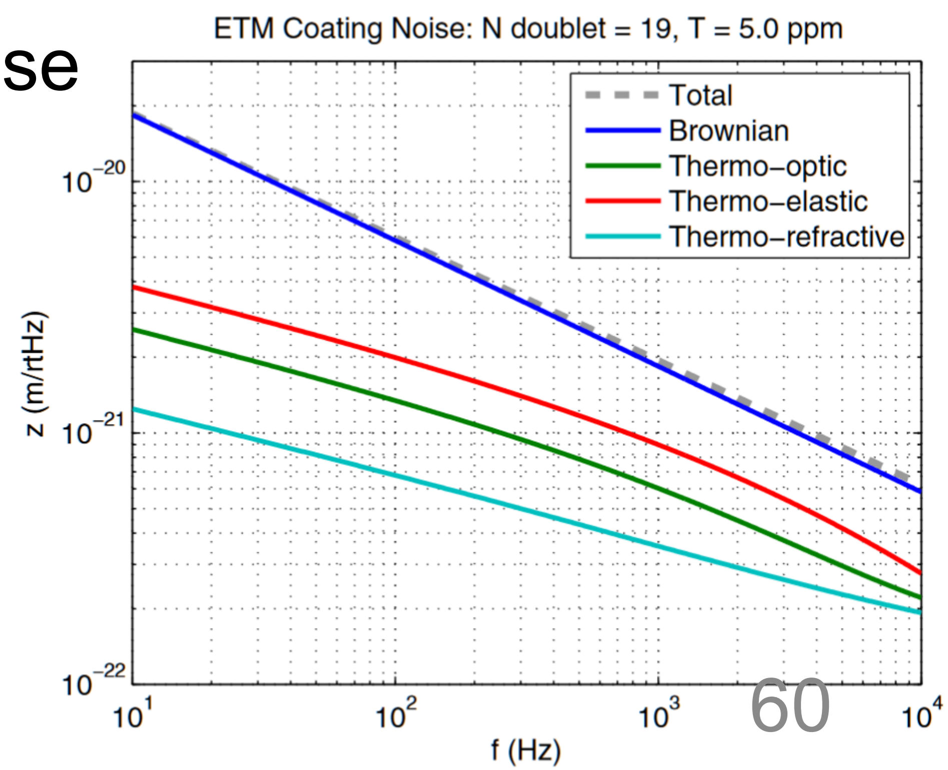
Substrate thermal noise is not significant, but coating thermal noise is troublesome in LVK
φ_{coa}=~3~5e-4



Other Coating Thermal Noises

- Thermal dissipation in the coating leads to temperature fluctuation in the coating, which causes:
  - Thermo-elastic noise
    thermal expansion
  - Thermo-refractive noise
    refractive index change

- These two can cancel



M. Evans+, [PRD 78, 102003 (2009)]

Summary

- Seismic noise
  - Site selection is important
  - Can be attenuated with suspensions
- Newtonian noise
  - Site selection is important
  - Can be cancelled with environment signals
- Thermal noise
  - Fluctuation-dissipation theorem
  - Material selection is important
  - Can be reduced by cooling but not so simple
- All can be reduced by making things bigger

Assignment for Nov 17

- Give an example where bigger is not necessarily better for a gravitational wave detector, and explain the reason.
- You may also answer from the Google Form below https://forms.gle/6AwJ48XcpWQXqMon9

Don't forget to put your name and student#

You may answer in any language

FORM