

Ground state cooling of opto-mechanical oscillators

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- Motivation
- Ground state and phonon number

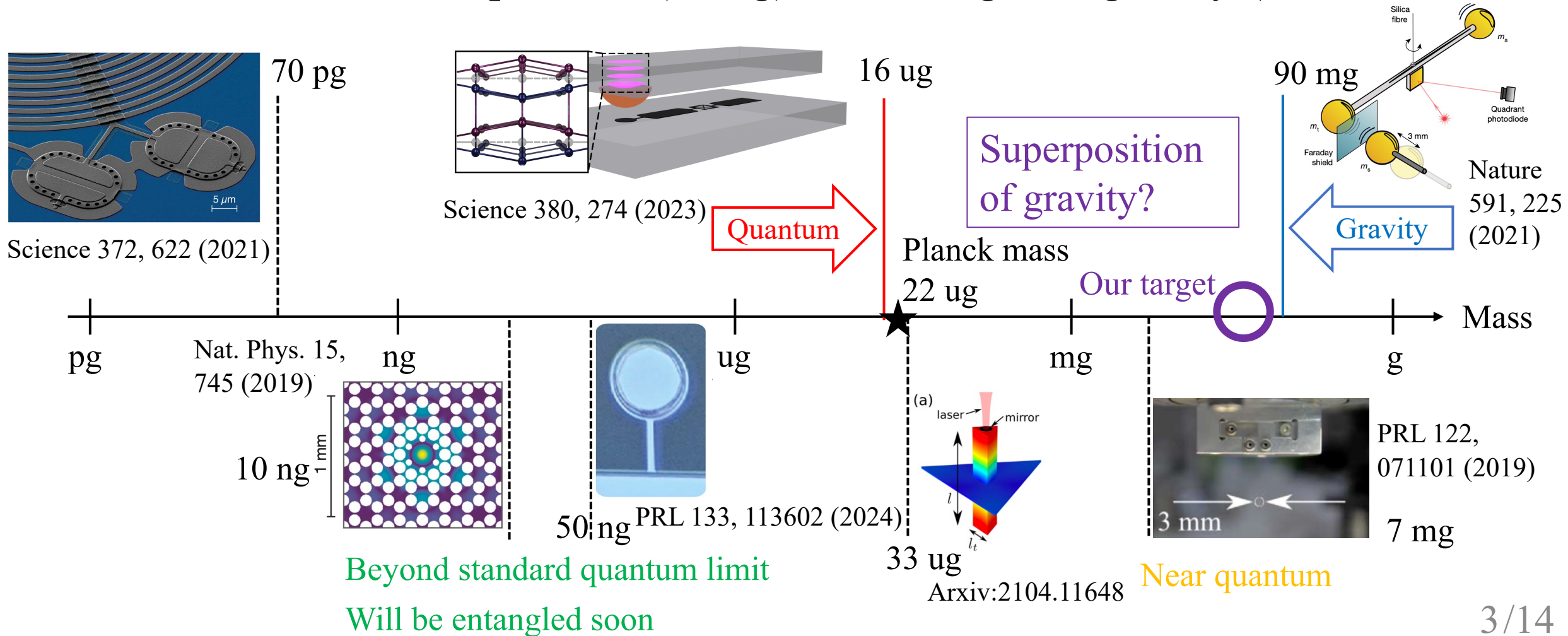
➤ Method

- How to cool down to the ground state
- fQ criteria

➤ Potential oscillators

Motivation

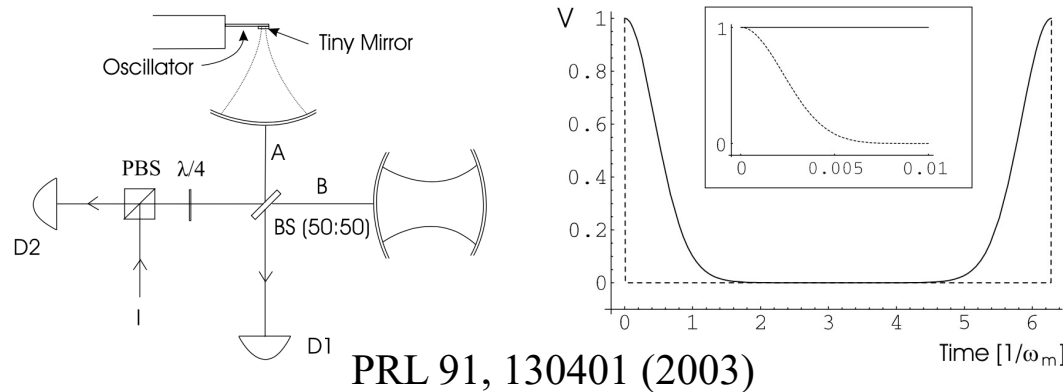
➤ The most massive quantum (16 μg) and the lightest gravity (90 mg)



Motivation

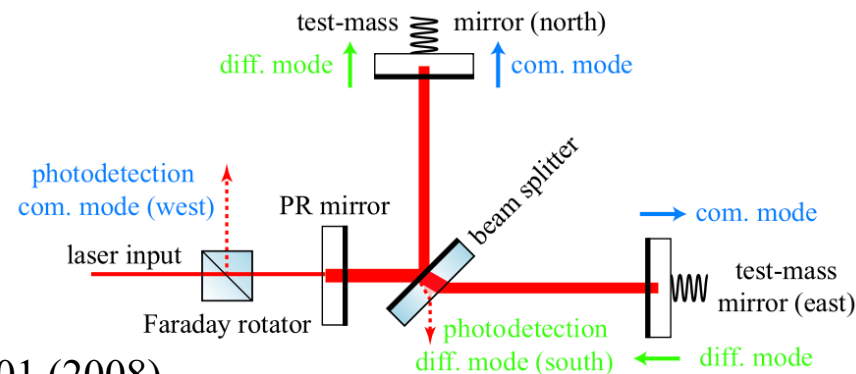
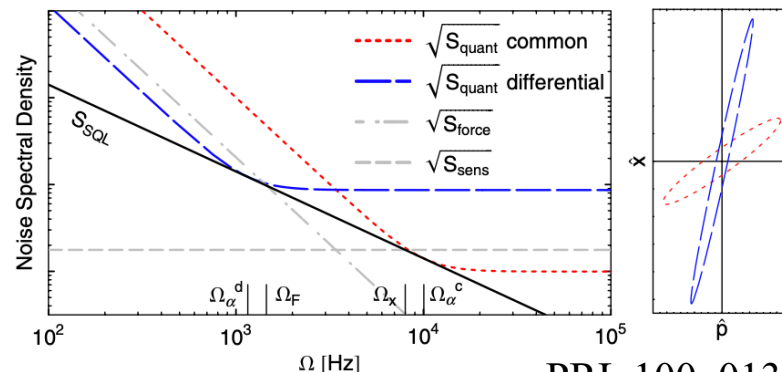
➤ Examples of proposed experiments

- Entanglement between the single photon and the test mass displacement



- ✓ Interference of the single photon will recover after one period
- ✓ Requirement: ground state cooling

- Entanglement between the common mode and the differential mode



- ✓ Logarithmic negativity is not zero
- ✓ Requirement: reaching the SQL

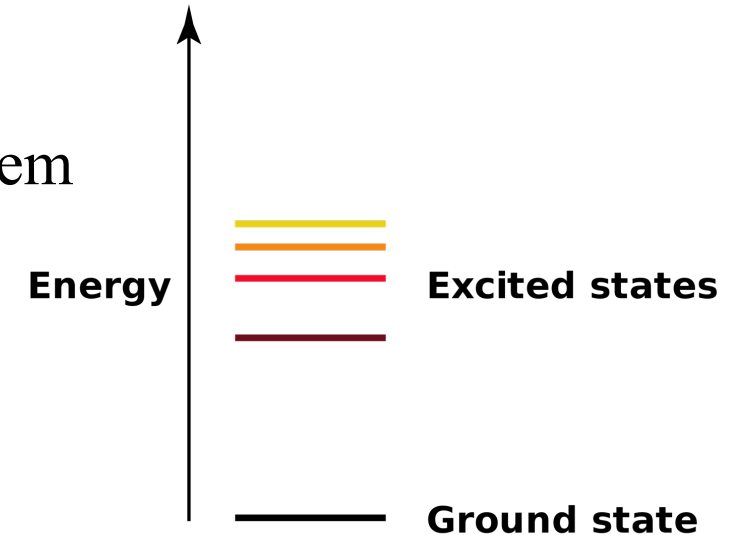
Ground state

➤ General

- Stationary state of the lowest energy
- The energy is known as the zero-point energy of the system

➤ Harmonic oscillator

- Energy: $E_n = \hbar\omega_m \left(n + \frac{1}{2}\right)$ with $n = 0$
- Displacement variance: $\langle x^2 \rangle_{gs} = \frac{\hbar}{2m\omega_m}$
- Total variance: $\langle x^2 \rangle = \frac{\hbar}{m\omega_m} \left(n + \frac{1}{2}\right)$, n is the phonon number
- $n < 1$ means a benchmark of the ground state cooling in the opto-mechanics field
- Purity: $\mu = \frac{1}{2n+1} = \frac{\hbar}{2m\omega_m \langle x^2 \rangle}$ is also a commonly used parameter ($\mu_{max} = 1$)



Wikipedia: “ground state”

Phonon number

➤ Thermal phonon number

- Brownian motion: $\langle x^2 \rangle_{th} = \frac{k_B T}{m \omega_m^2}$
- $\langle x^2 \rangle = \frac{\hbar}{m \omega_m} \left(n_{th} + n_{other} + \frac{1}{2} \right)$
- $n_{th} = \frac{k_B T}{\hbar \omega_m} < 1$ is necessary to reach the ground state

➤ Case studies

- At room temperature ($T = 300$ K), $\frac{\omega_m}{2\pi} > 6$ THz
 - Laser light (~ 300 THz) is originally in the ground state
- At low frequency oscillator ($\frac{\omega_m}{2\pi} = 1$ kHz), $T < 50$ nK
 - We cannot reach such low temperature even with cryogenic system
 - ⇒ Laser cooling with an optical cavity and feedback

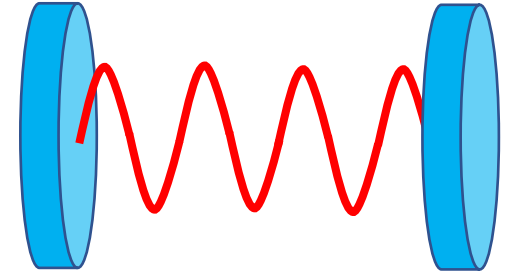
Measurement and feedback

➤ Typical quantum limit of laser interferometer

- Michelson interferometer $\sim 10^{-15}$ m/ $\sqrt{\text{Hz}}$
- Optical cavity $\sim 10^{-18}$ m/ $\sqrt{\text{Hz}}$
 - ✓ Laser wavelength: $\lambda \sim 1$ μm
 - ✓ Input power: $P \sim 10$ mW
 - ✓ Cavity finesse: $\mathcal{F} \sim 10^3$

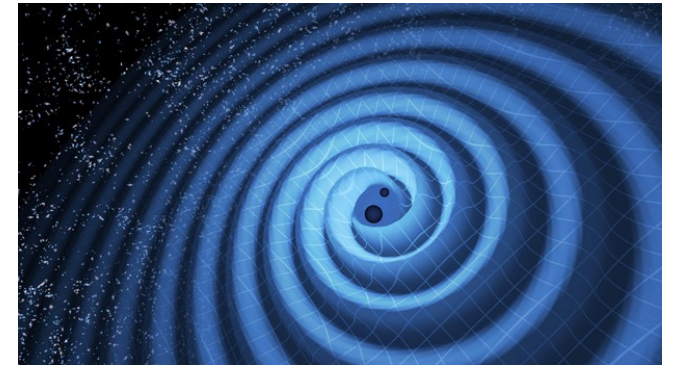
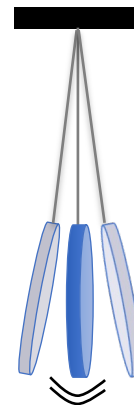
$$S_{\text{shot},x} \sim \frac{\lambda}{\mathcal{F}} \sqrt{\frac{\hbar \omega_l}{P_{\text{in}}}}$$

Relative photon number fluctuation of $1/\sqrt{N} \sim 10^{-8}$ / $\sqrt{\text{Hz}}$ with $P_{\text{in}} \sim 1$ mW



➤ Required displacement sensitivity

- Zero-point fluctuation $\sim 10^{-18}$ m/ $\sqrt{\text{Hz}}$
 - ✓ Mass: $m \sim 1$ mg
 - ✓ Resonant frequency: $\omega_m/2\pi \sim 1$ kHz
- GW detector sensitivity $\sim 10^{-20}$ m/ $\sqrt{\text{Hz}}$



Credit: LIGO

Feedback cooling

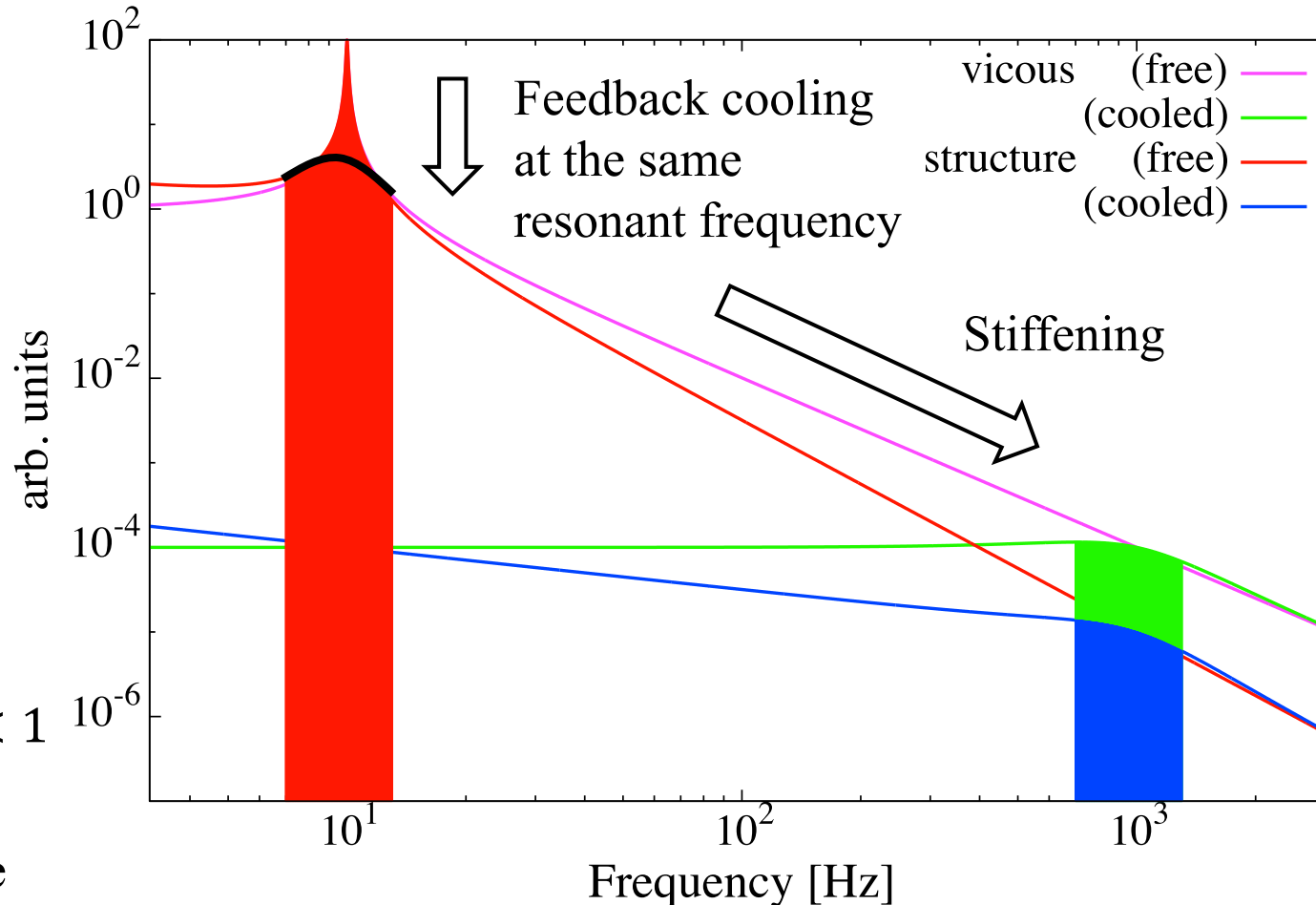
➤ Damping, stiffening, and f^*Q criteria

$$\langle x^2 \rangle_{th} = \frac{k_B T}{m \omega_m^2}$$



$$\begin{aligned} \langle x^2 \rangle_{th} &= \frac{k_B T}{m \omega_m^2} \frac{Q_{eff}}{Q_m} \\ &= \frac{k_B T_{eff}}{m \omega_m^2} \end{aligned}$$

- ✓ $Q_{eff,min} = 1$
- ✓ $n_{th,min} = \frac{k_B T}{\hbar \omega_m Q_m} < 1$
- ✓ $f_m Q_m > 6 \times 10^{12}$
at room temperature



- Viscous

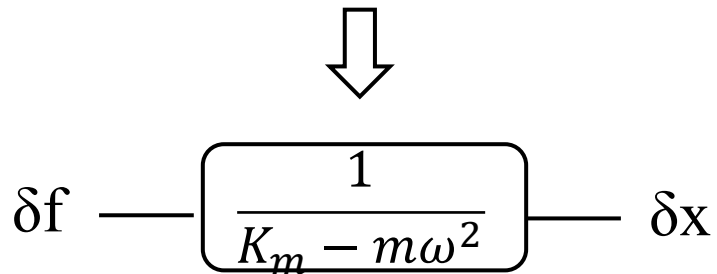
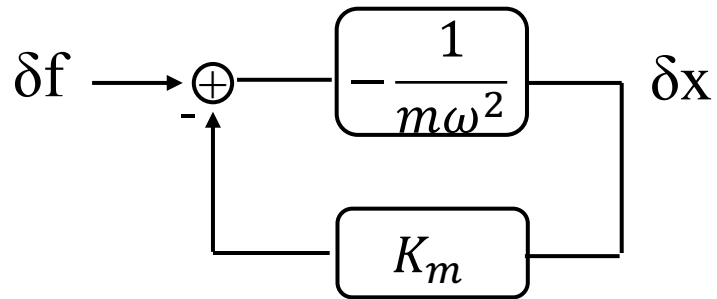
- ✓ $n_{th,min} = \frac{k_B T}{\hbar \omega_m Q_m} \left(\frac{\omega_m}{\omega_{eff}} \right)^2$
- ✓ $f_m Q_m > 6 \times 10^{12} \left(\frac{\omega_m}{\omega_{eff}} \right)^2$

- Structure

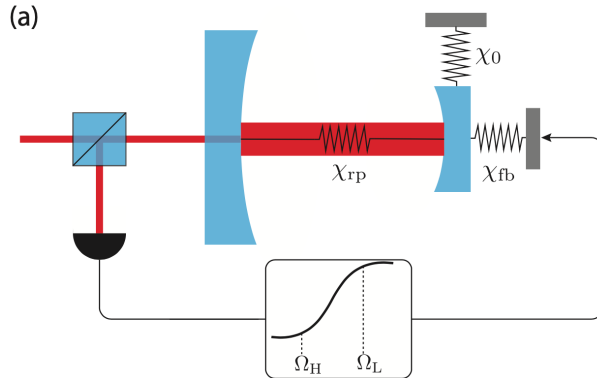
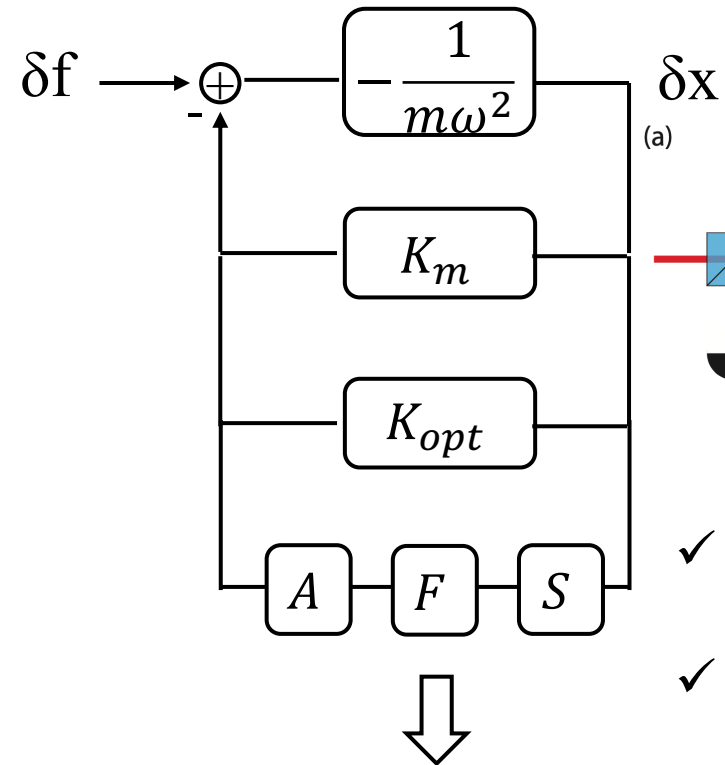
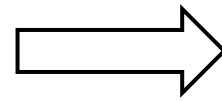
- ✓ $n_{th,min} = \frac{k_B T}{\hbar \omega_m Q_m} \left(\frac{\omega_m}{\omega_{eff}} \right)^3$
- ✓ $f_m Q_m > 6 \times 10^{12} \left(\frac{\omega_m}{\omega_{eff}} \right)^3$

Stiffening

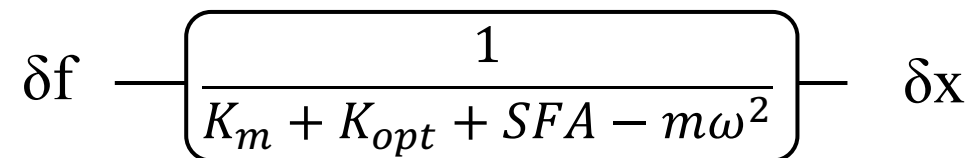
➤ Optical spring and feedback spring



- ✓ Even mechanical spring can be described as feedback

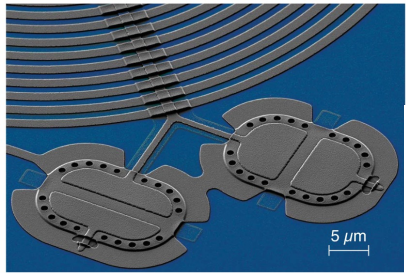


- ✓ Restoring force from the radiation pressure
- ✓ Resonance at UGF



Examples

➤ fQ and temperature in the experiments



70 pg

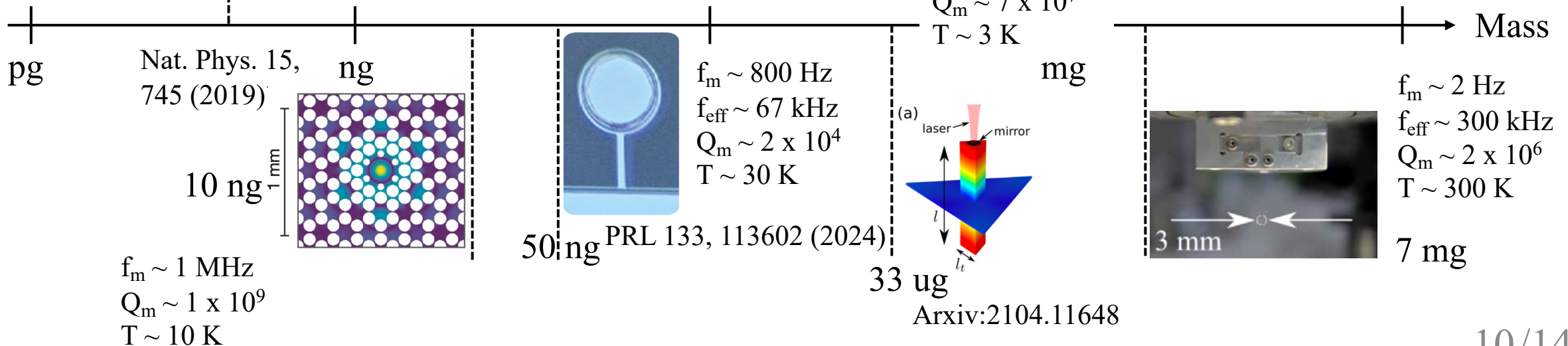
$f_m \sim 10$ MHz

$Q_m \sim 2 \times 10^5$

$T \sim 10$ mK

Science 372, 622 (2021)

$$f_m Q_m > 6 \times 10^{12} \left(\frac{T}{300 \text{ K}} \right) \left(\frac{\omega_m}{\omega_{eff}} \right)^3$$



Sensitivity and SQL

➤ Reaching the SQL is almost equivalent to the ground state cooling

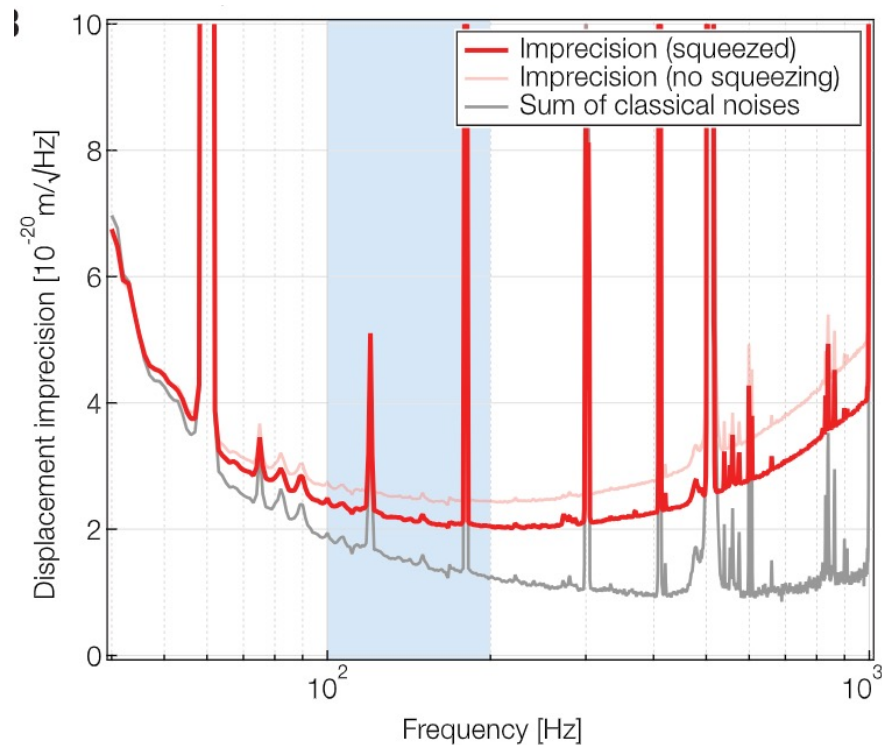
- Thermal noise: $S_{th}(\omega) = \frac{4k_B T m \omega_m^2}{Q_m \omega}$ [N/√Hz] in the structure damping
- SQL: $S_{SQL}(\omega) = 2\hbar m \omega^2$ [N/√Hz]
- Ratio: $\frac{S_{th}(\omega_{eff})}{S_{SQL}(\omega_{eff})} = \frac{2k_B T}{\hbar \omega_m Q_m} \left(\frac{\omega_m}{\omega_{eff}} \right)^3 \sim n_{th,min}$ $\langle x^2 \rangle = \frac{\hbar}{m \omega_m} \left(n_{th} + n_{other} + \frac{1}{2} \right)$

➤ When the sensitivity reaches the SQL at a specific frequency, you only have to increase the resonant frequency up to the SQL frequency

➤ Even the fQ criteria is satisfied, n_{other} should be below unity (the other noises should be below the SQL)

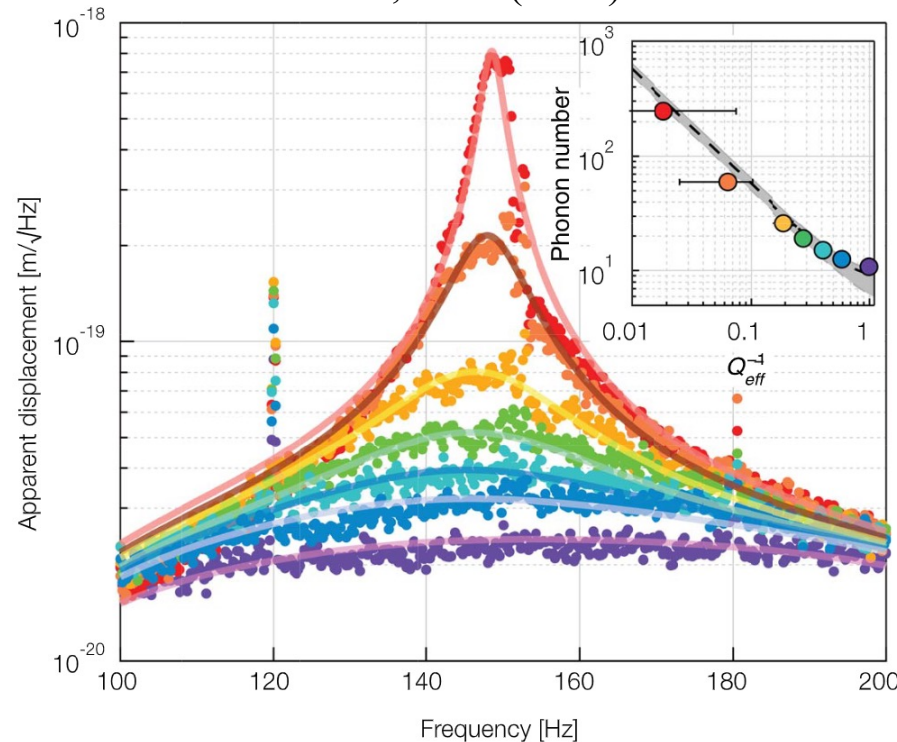
Examples

➤ Cooling of aLIGO test masses



- ✓ Choosing the frequency close to the SQL

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- ✓ Feedback spring without optical spring
- ✓ Tuning the DARM loop gain

$$f_m Q_m > 6 \times 10^{12} \left(\frac{T}{300 \text{ K}} \right) \left(\frac{\omega_m}{\omega_{\text{eff}}} \right)^3$$

$$f_m \sim 1 \text{ Hz}$$

$$f_{\text{eff}} \sim 100 \text{ Hz}$$

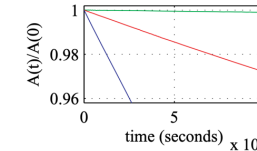
$$Q_m \sim 1 \times 10^9$$

$$T \sim 300 \text{ K}$$

- ✓ Other noises such as the coating thermal noise and the shot noise prevents the ground state cooling

Potential torsion pendulum

➤ Picking up some familiar torsion pendula



CQG 26, 094017 (2009)	f_m	f_{eff}	Q_m	T	$f_m Q_m \left(\frac{300 \text{ K}}{T} \right) \left(\frac{\omega_{eff}}{\omega_m} \right)^3$
Fused silica torsion pendulum for LISA experiment	2 mHz	20 Hz	7×10^5	300 K	1×10^{15}
Our optomechanical experiment	0.1 Hz	1 kHz	3×10^3	300 K	3×10^{14}
TOBA	9 mHz	1 Hz	1×10^8	4 K	1×10^{14}

➤ If we increase the TOBA resonant frequency up to 1 Hz using optical spring or feedback spring with the SQL, we can achieve the ground state cooling

Summary

- Ground state cooling of mechanical oscillators
- Phonon number and fQ criteria
- Potential torsion pendulum experiments