# Research plan II: Ground state cooling of a sub-mg cantilever

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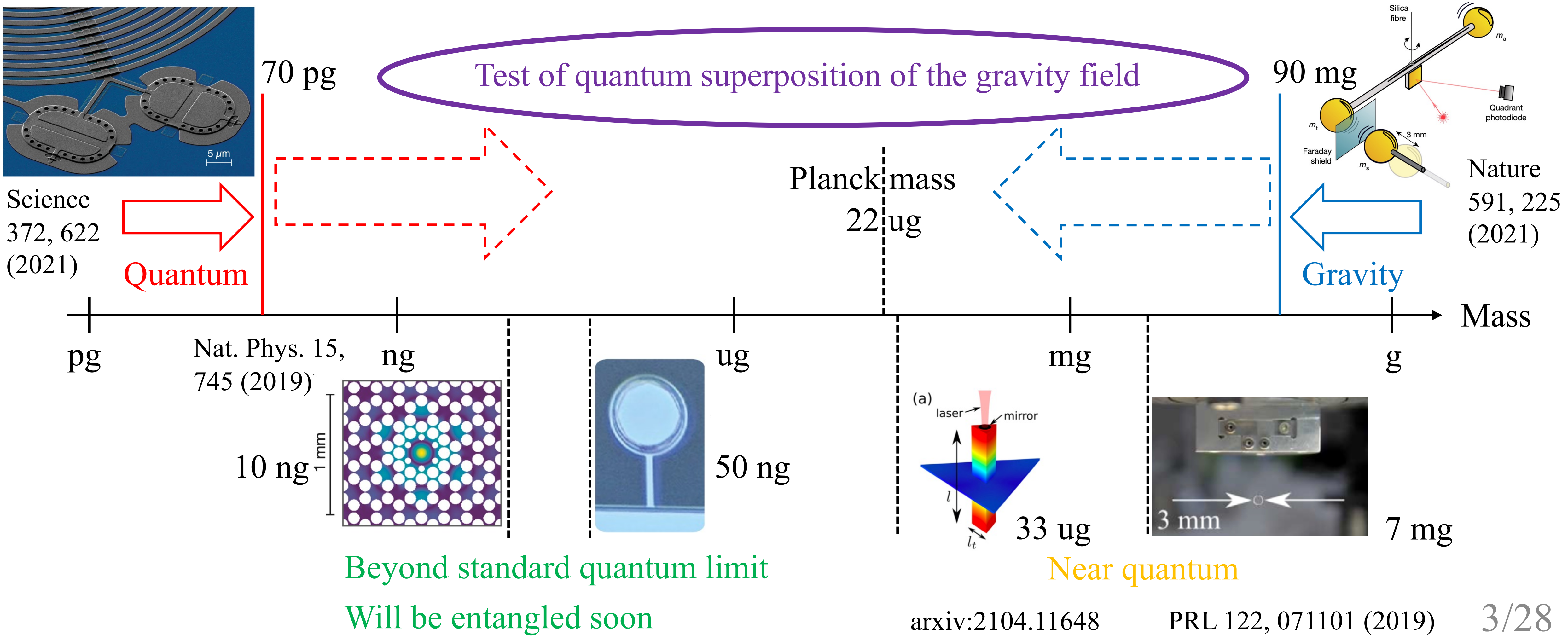
Lab seminar 2023/01/27

# Contents

- Introduction of the ground state cooling
- Method and previous works
- My experimental design

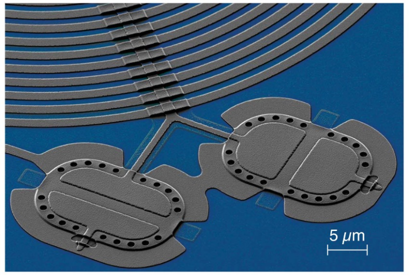
# Background (at my previous talk)

➢ The massive quantum (70 pg) and the lightest gravity (90 mg)



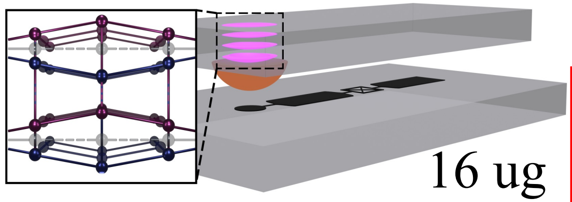
# Background (now)

➢The massive quantum (16 ug) and the lightest gravity (90 mg)




70 pg

Science 372, 622 (2021)



16 ug

Arxiv:2211.00449

✓ Bulk acoustic wave

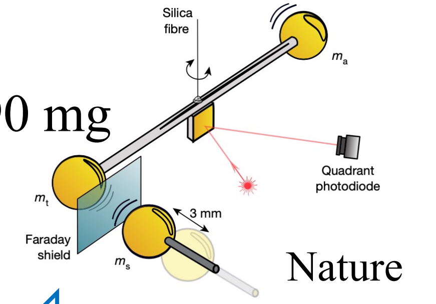
✓ Electro-mrchanics

Quantum

Planck mass

22 ug

90 mg



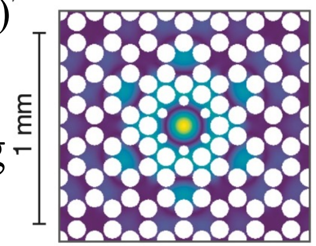

Nature 591, 225 (2021)

Gravity

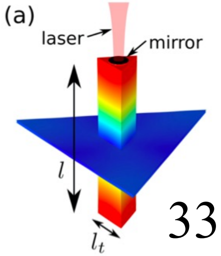
pg ng ug mg g Mass

Nat. Phys. 15, 745 (2019)

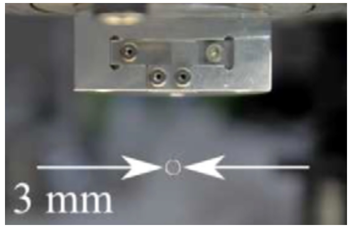
10 ng




50 ng




33 ug




7 mg

Beyond standard quantum limit

Will be entangled soon

Near quantum

arxiv:2104.11648

PRL 122, 071101 (2019)

# Ground state of a mechanical mode

## ➢Energy eigenvalue

Effective mass

Phonon number

$$\frac{m\omega_m^2\langle x^2\rangle}{2}+\frac{\langle p^2\rangle}{2m}=\left(\boxed{n}+\frac{1}{2}\right)\hbar\omega_m$$

Resonant frequency

✓ Assuming the equal contribution of the displacement and momentum (strictly correct in the thermal noise limit)

$$\Longrightarrow \quad m\omega_m^2\boxed{\langle x^2\rangle}=\left(n+\frac{1}{2}\right)\hbar\omega_m$$

Displacement variance

$$\langle x^2\rangle=\int_0^\infty\frac{d\omega}{2\pi}S_x$$

## ➢Ground state

- Absolute ground state: $n=0 \;\rightarrow\; \langle x^2\rangle=\dfrac{\hbar}{2m\omega_m}$ (zero-point fluctuation)
- Conventional ground state in optomechanics: $n<1$
- The entanglement can keep for a long time enough

# Thermal phonon

- Brownian motion

$$m\omega_m^2\langle x^2\rangle_{th} = k_B T = \left(n_{th} + \frac{1}{2}\right)\hbar\omega_m \quad \Longrightarrow \quad n_{th} \approx \frac{k_B T}{\hbar\omega_m}$$

- Huge thermal phonon

$$T = 300\ \mathrm{K},\ \omega_m/2\pi = 340\ \mathrm{Hz} \longrightarrow n_{th} \sim 10^{10}$$

  - ✓ $n_{th} < 1$ at room temperature is achieved by $\frac{\omega_m}{2\pi} > 6$ THz
  - ✓ Laser light (1064 nm ↔ 282 THz) is not thermally driven
  - ✓ Microwave (GHz) should be treated carefully

- Requiring the cooling by many orders of magnitude
- Cryogenic system is not enough

# Damping or cooling

➢ With or without additional mechanical loss

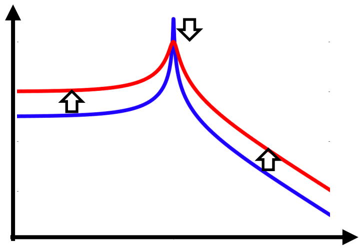
- Original thermal noise variance

$$\langle x^2\rangle_{th} = \int_0^\infty \frac{d\omega}{2\pi}\frac{4k_BT\gamma_m}{m|\omega_m^2 + i\gamma_m\omega - \omega^2|^2}$$

$$= \frac{k_BT}{m\omega_m^2} \qquad \gamma_m = \frac{\omega_m}{Q_m}$$

✓ Independent of the mechanical loss

✓ e.g. Magnet damping



- Cooled to the effective temperature

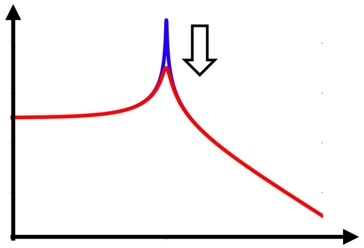
$$\langle x^2\rangle_{th} = \int_0^\infty \frac{d\omega}{2\pi}\frac{4k_BT\gamma_m}{m|\omega_m^2 + i\gamma_{eff}\omega - \omega^2|^2}$$

$$= \frac{k_BT\gamma_m}{m\omega_m^2\gamma_{eff}}$$

$$T_{eff} = T\frac{\gamma_m}{\gamma_{eff}} \ll T$$

✓ Effective temperature

✓ e.g. Active damping via feedback



# Maximum cooling and f-Q criteria

- The mechanical oscillator must keep the "oscillator"
  - Effective Q-value should be above the unity, $Q_{eff} = \frac{\omega_m}{\gamma_{eff}} > 1$
  - Over-damped oscillator ($\gamma_{eff} > \omega_m$) is meaningless

- The ground state cooling requires the f-Q criteria

$$1 > n_{th,eff} = \frac{k_B T \gamma_m}{\hbar \omega_m \gamma_{eff}} > \frac{k_B T \gamma_m}{\hbar \omega_m^2}$$

$$\Rightarrow \boxed{f_m Q_m > 6\ \mathrm{THz} \left( \frac{T}{300\ \mathrm{K}} \right)}$$

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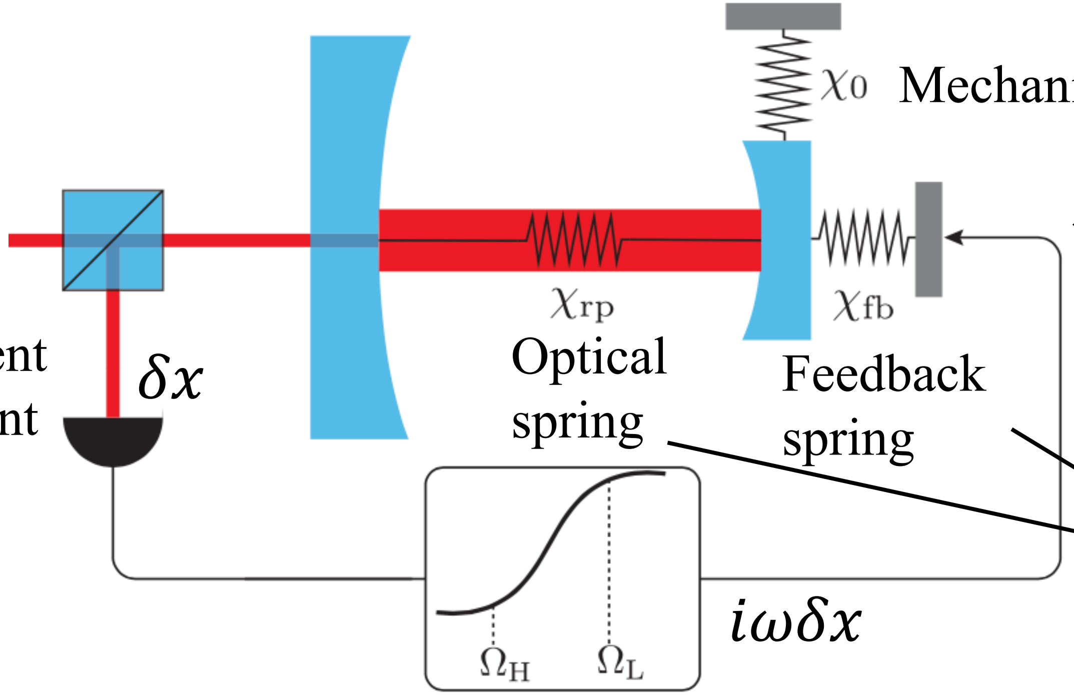
# Feedback cooling

- ➢Feedback is a loss-less damping because information is not lost




✓ Displacement measurement

✓ The viscous force is added to the oscillator

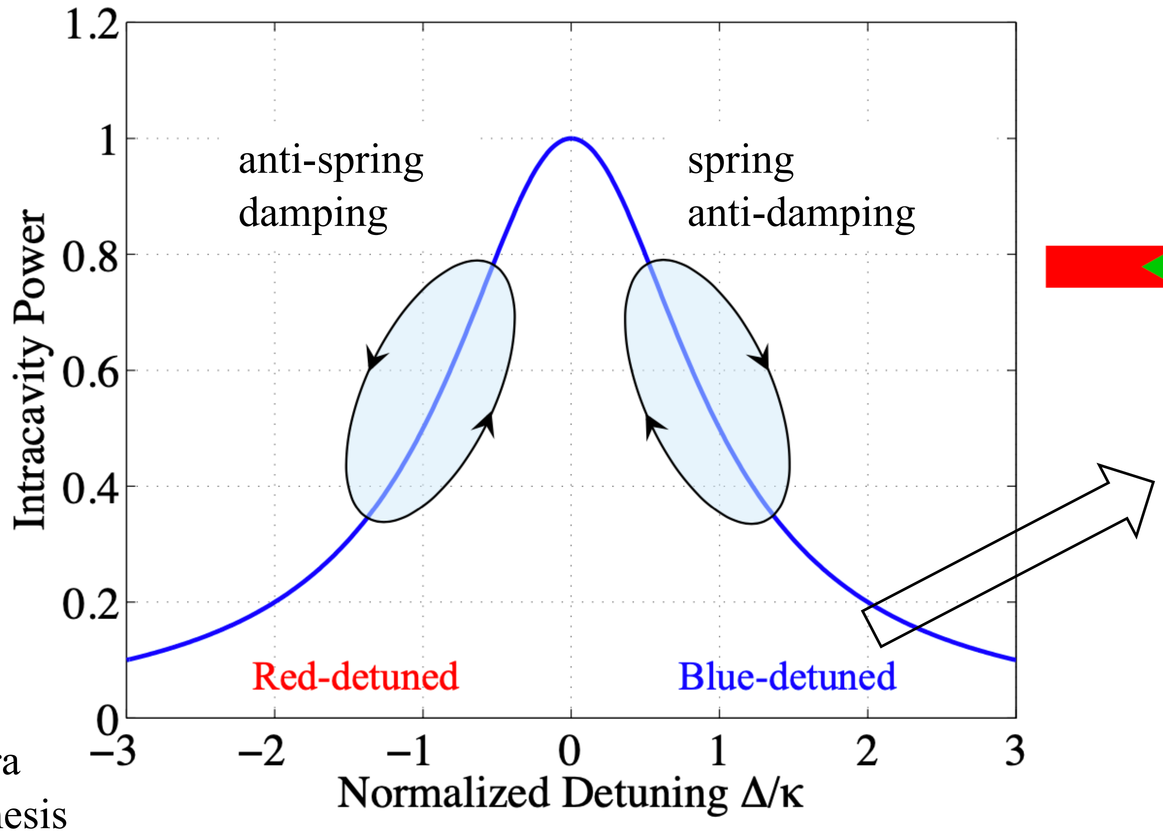
✓ Increasing resonant frequency due to the additional restoring force also leads to cooling

✓ High-pass filter changes the displacement to the velocity

$$n_{th,eff} = \frac{k_B T \gamma_m}{\hbar \omega_m \gamma_{eff}} \Rightarrow \frac{k_B T \gamma_m}{\hbar \omega_{eff} \gamma_{eff}}$$

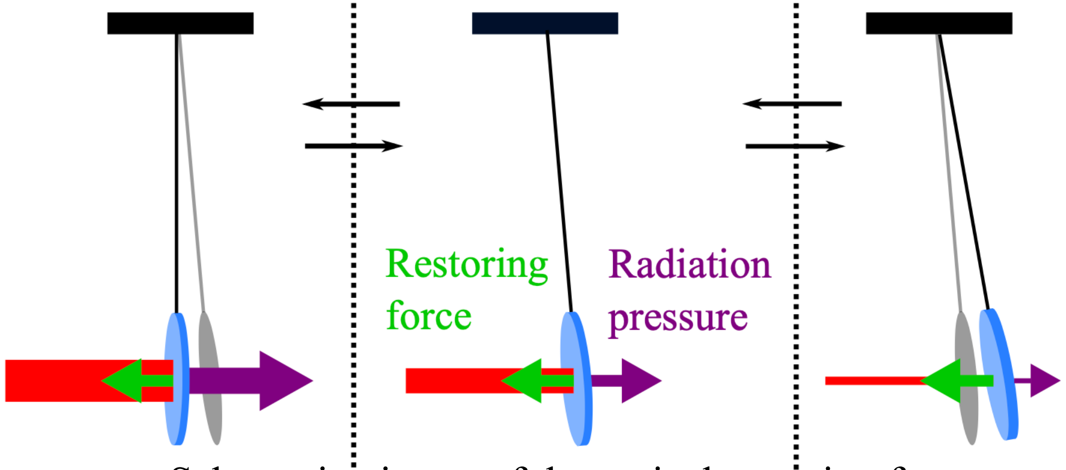
# Optical spring

➢Optical spring and damping

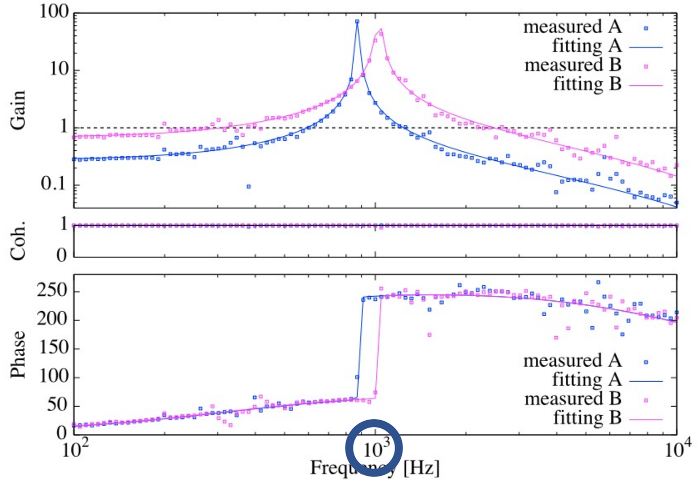



Kuwahara master thesis

Low frequency ⟵ ⟶ High frequency

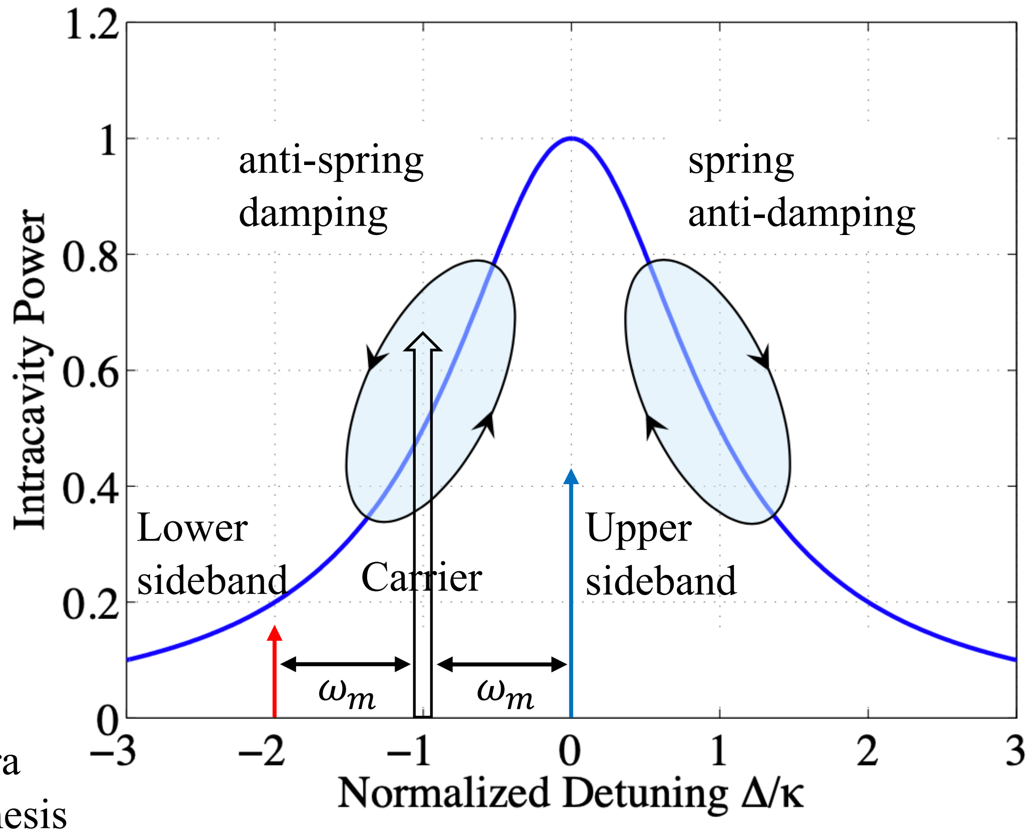



Schematic picture of the optical restoring force



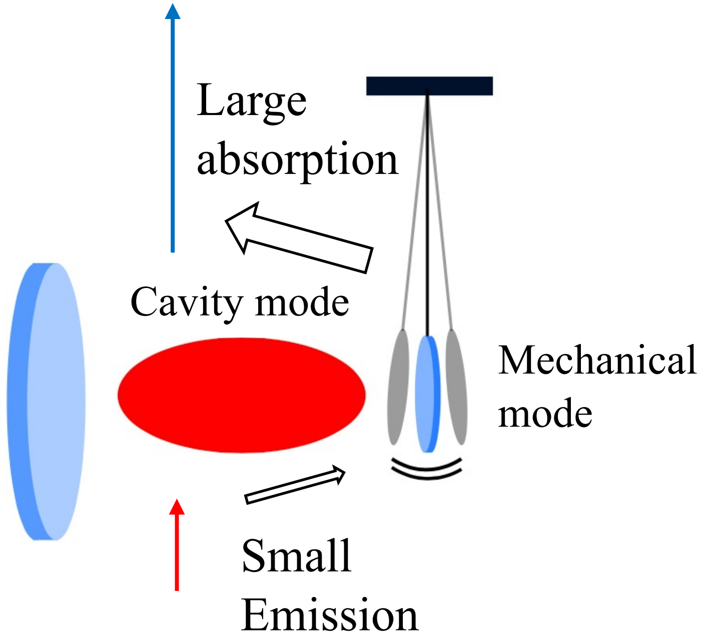
# Sideband cooling

- ➢The upper sideband absorbs the motional energy



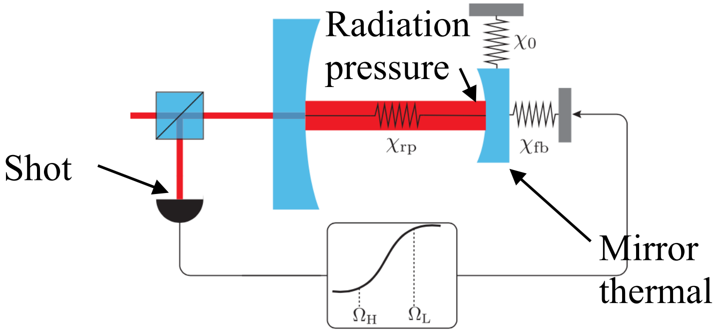
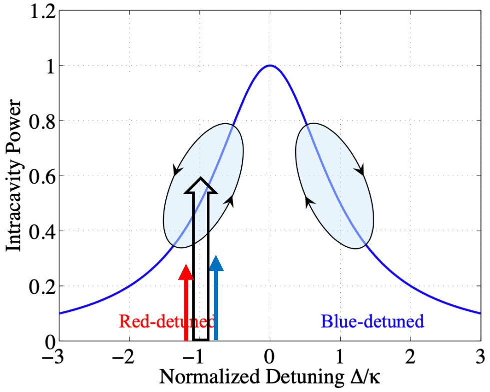

Kuwahara
master thesis

Low frequency ⟵ ⟶ High frequency

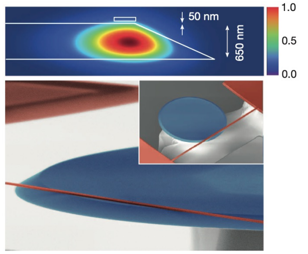
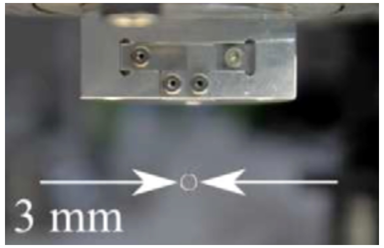
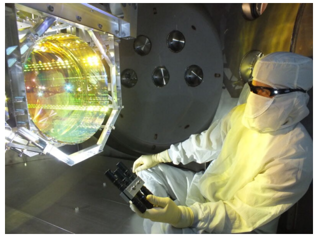
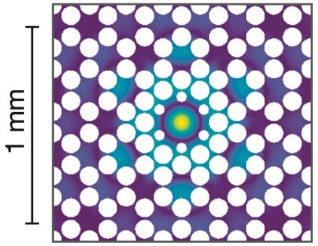
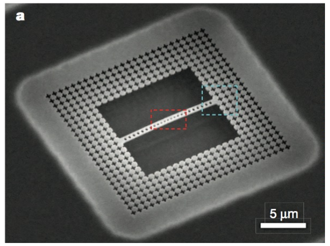
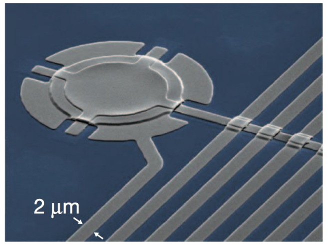
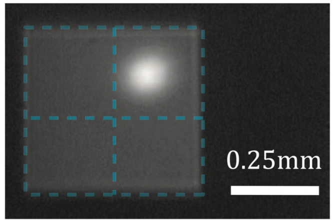
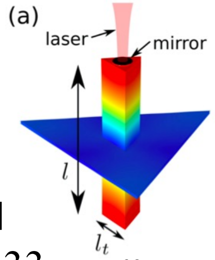



- ✓ The energy absorption leads to the damping

# Feedback v.s. sideband

| | **Feedback cooling** | **Sideband cooling** |
|---|---|---|
| Pros | • Can be used for low-frequency oscillators ($\omega_m \ll \kappa$) | • No feedback of the sensing noises (e.g. shot noise)<br>• Low radiation pressure noise |
| Cons | • Feedback of the sensing noises<br>• Non-negligible radiation pressure noise<br> | • Only for high-frequency oscillators ($\omega_m \gtrsim \kappa$)<br>Small energy absorption due to low-frequency sidebands<br> |

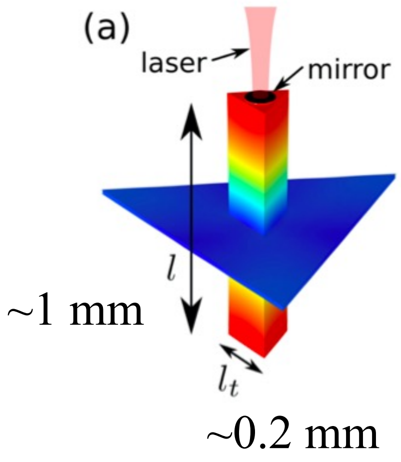
# Feedback v.s. sideband

| | **Feedback cooling** | **Sideband cooling** |
|---|---|---|
| Previous works | [1]   2.9 pg, $n_{tot}$~5.3<br>[2]   8 mg, $n_{tot}$~$O(10^3)$<br>[3]  10 kg, $n_{tot}$~10.8<br>[4]   10 ng, $n_{tot}$ = 0.29<br>[1] Nature 524, 325 (2015) [2] PRL 122, 071101 (2019) [3] Science 372, 1333 (2021) [4] Nature 563, 53 (2018) | [5]   0.31 pg, $n_{tot}$~0.85<br>[6]   48 pg, $n_{tot}$~0.34<br>[7]   7 ng, $n_{tot}$~0.20<br>[8]   33 ug, $n_{tot}$~20<br>[5] Nature 478, 89 (2011) [6] Nature 475, 359 (2011) [7] PRL 116, 063601 (2016) [8] arxiv:2104.11648 |

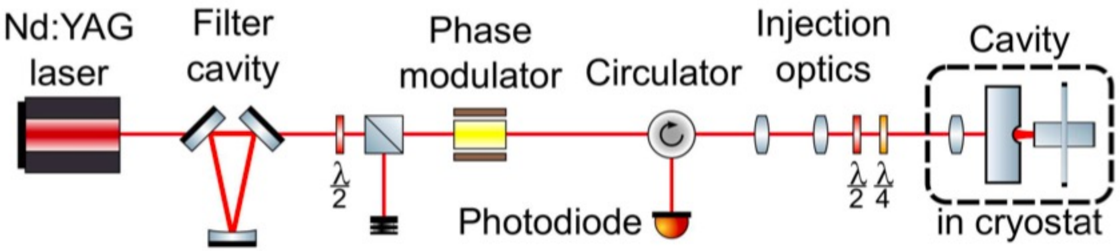
✓ The ground state cooling has been mainly achieved by the sideband cooling

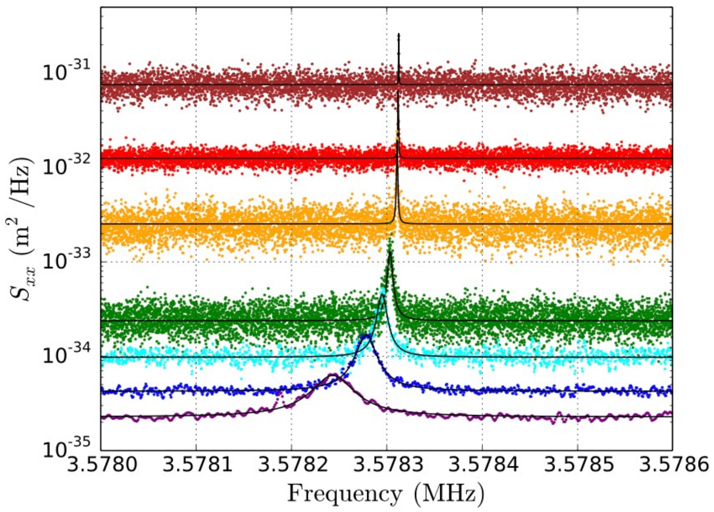
# Sideband cooling of the pillar

## ➢Planck mass scale optomechanics




| Mechanical resonator | | | Optical cavity | | |
|---|---|---|---|---|---|
| Mechanical resonance freq. | $\Omega_m/2\pi$ | 3.58 MHz | Maximum incident power | $P_{max}$ | 25 $\mu$W |
| Mechanical quality factor | $Q$ | $7 \times 10^7$ | Optical finesse | $\mathcal{F}$ | 79,000 |
| Effective mass | $m$ | 33 $\mu$g | Cavity length | $L$ | 58 $\mu$m |
| Cryostat base temperature | $T$ | 50 mK | Cavity bandwidth (HWHM) | $\Omega_{cav}/2\pi$ | 16.3 MHz |
| | | | Detection efficiency | $\eta_{opt} \times \eta_{cav}$ | $0.9 \times 0.5$ |




## ➢Issues

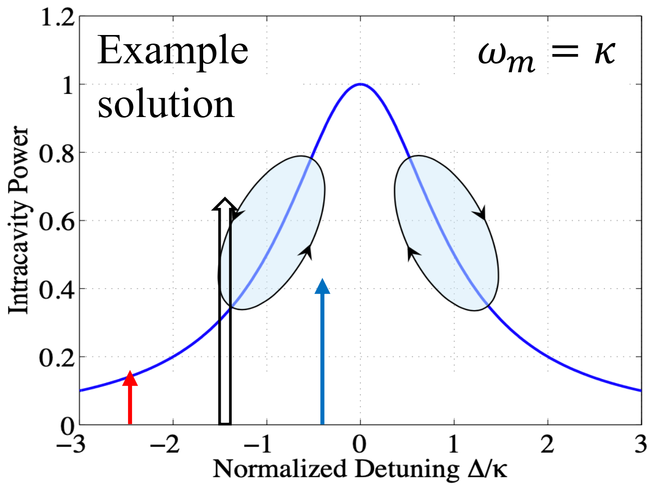
- Instability of the membrane suspending the pillar (probably due to the anti-spring)
- Only the power of 25 uW was injected, the phonon number stooped at ~20
- Ultimately, the back-action limits the phonon number $n_{ba}$~4

# Back-action limit

➢ Back-action (radiation pressure noise) gives additional displacement, or phonon

$$n_{ba} = -\frac{\kappa^2 + (\Delta + \omega_m)^2}{4\omega_m \Delta}$$

$$\geq \frac{\kappa^2}{2\omega_m^2} \frac{1}{1 + \sqrt{1 + \kappa^2/\omega_m^2}} \quad \text{at } \Delta = -\sqrt{\kappa^2 + \omega_m^2}$$




- Very roughly, the same detuning as the cavity line width is the most efficient and gives the smallest back-action noise

$$n_{ba,min} \sim \frac{\kappa^2}{4\omega_m^2} < 1 \; (\omega_m \gtrsim \kappa) \qquad n_{ba,min} \sim \frac{\kappa}{2\omega_m} \gg 1 \; (\omega_m \ll \kappa)$$

- High resonance
- Resolved sideband regime
- Good cavity

$\Longleftrightarrow$

- Low resonance
- Doppler regime
- Bad cavity

# Contents

- ➢Introduction of the ground state cooling
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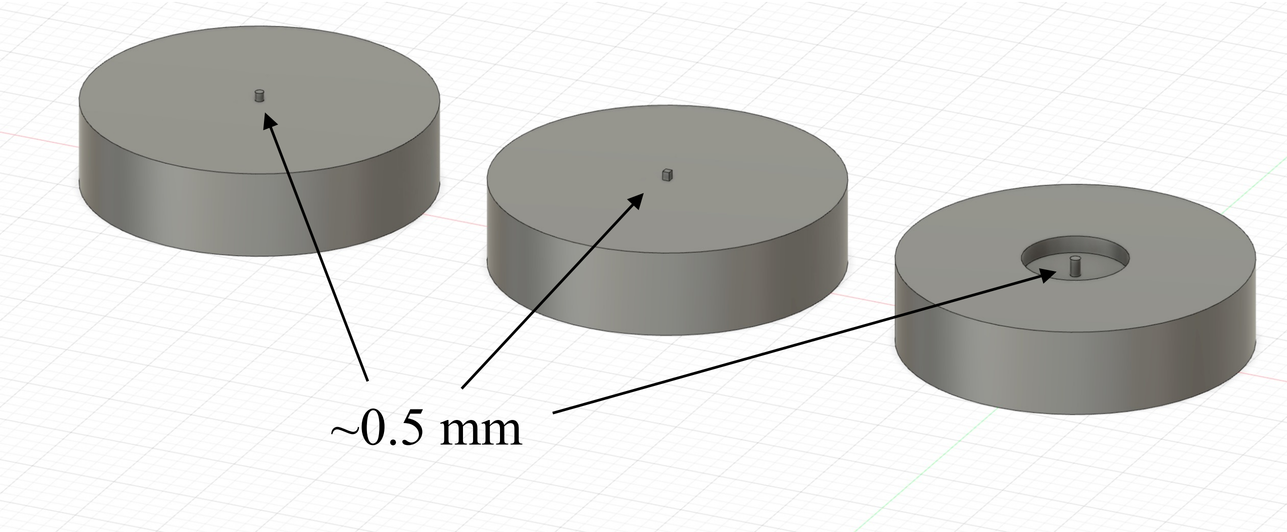
- ➢My experimental design

# Initial plan

➢Similar experiment with sapphire pillar



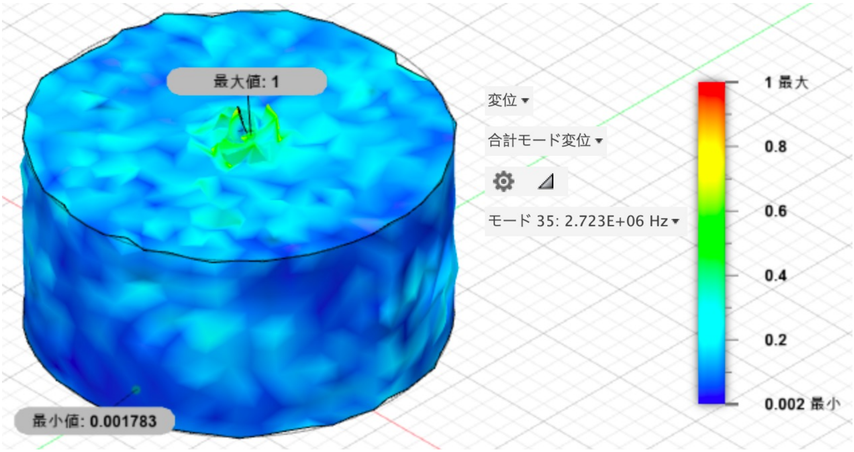

✓ The pillar supported by the bulk is stable

✓ The sapphire is due to the large Young's modulus (high resonance), good for f-Q criteria (???)

$$f_m Q_m > 6\ \mathrm{THz}\left(\frac{T}{300\ \mathrm{K}}\right)$$

➢Issues

- Cannot find the pillar mode, the isolation is necessary to have the mode stand out
- The high resonance is not necessarily good



# Phonon number and damping

- Lower resonance leads to the smaller phonon number

$$n_{th,eff} = \frac{k_B T \gamma_m}{\hbar \omega_m \gamma_{eff}}$$

$$= \frac{k_B T}{\hbar Q_m \gamma_{eff}}$$

$$\boxed{\gamma_{eff} \sim \frac{3\omega_0 \mathcal{F} P_{circ}}{\pi c^2 m \omega_m}}$$

with $\omega_m \sim \kappa, \Delta \sim -\kappa$

✓ Important parameters are

- Low temperature
- High Q-value
- Light mass
- Low resonance
- High finesse
- High intra-cavity power

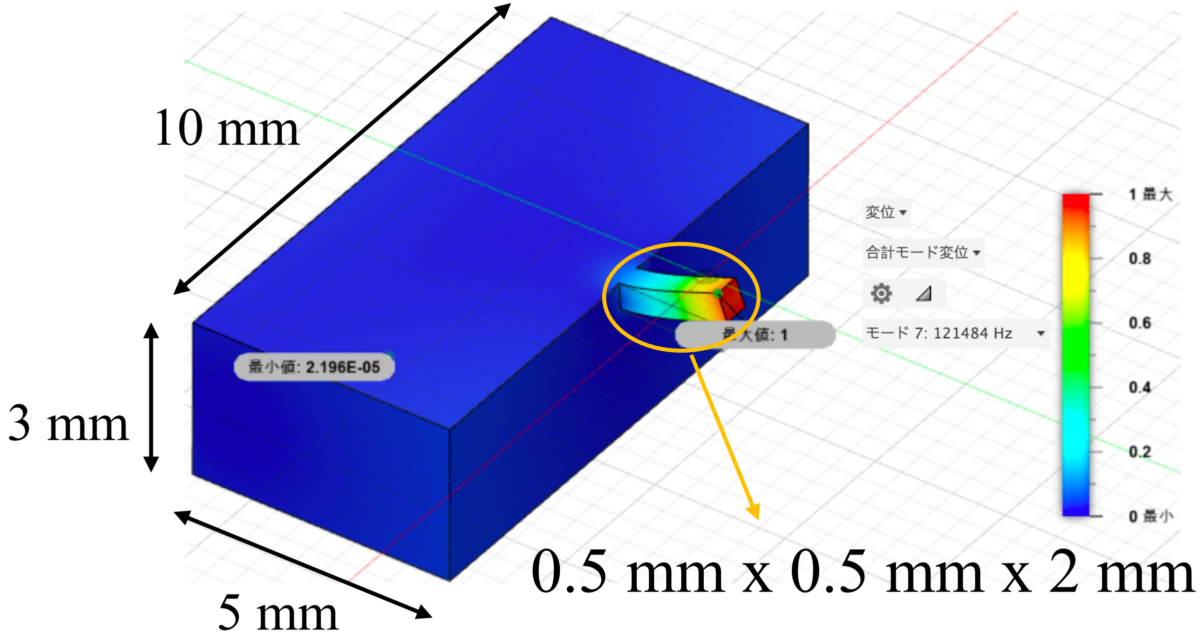
$\left.\begin{array}{l}\text{Light mass}\\ \text{Low resonance}\\ \text{High finesse}\\ \text{High intra-cavity power}\end{array}\right\}$ Large damping

while keeping the f-Q criteria

# Current plan

- Silicon monolithic cantilever
  - High-Q
  - Easy fabrication
  - Relatively low resonance
  - Standing out (the lowest) mode

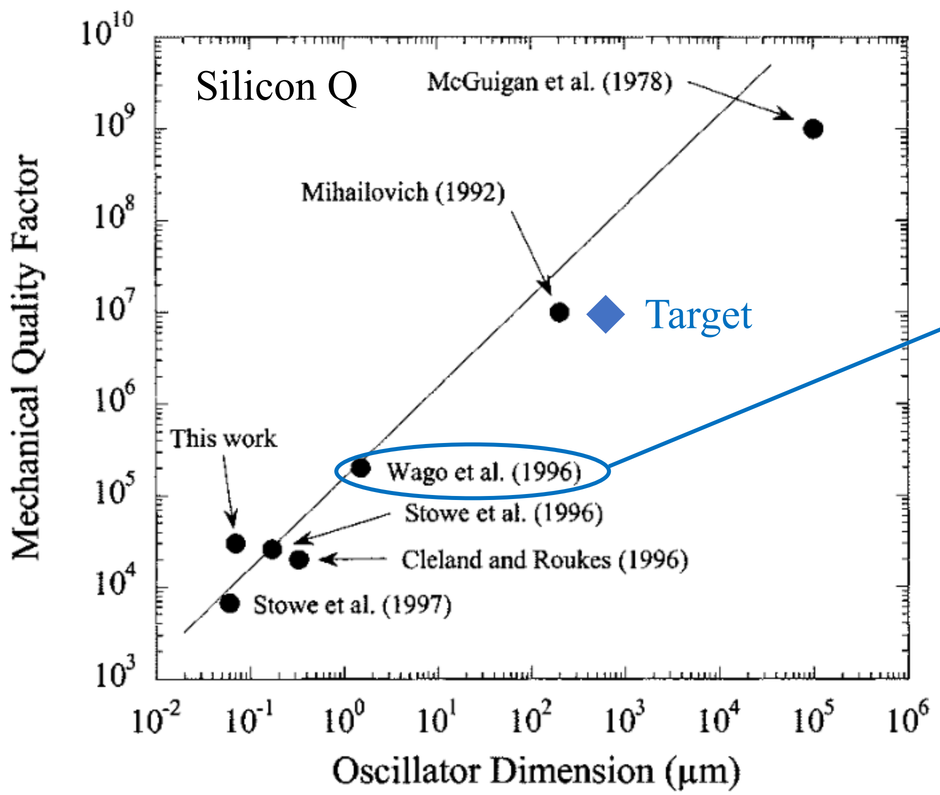
- Mechanical parameters
  - Effective mass: 0.3 mg
  - Resonant frequency: 120 kHz
  - Q-value: $10^7$




✓ The base plate should be as large as possible to reduce the recoil and the cramp loss

# Q-value

➢ Should exceed $10^7$, and possible




J. Microelectromech. Syst. 9, 117 (2000)

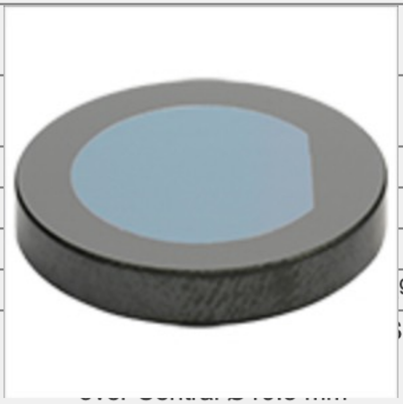
Low-temperature force sensitivity was characterized using a bare silicon cantilever with dimensions of 470 $\mu$m×45 $\mu$m×1.5 $\mu$m, a spring constant of 0.07 N/m, and a resonance frequency of 9.8 kHz. The $Q$ of the cantilever resonance was 40 000 at room temperature in vacuum ($\sim 10^{-5}$ Torr), and improved to 200 000 at 6 K. Figure 2 shows the thermal

- ✓ The Q-value is linearly increased by the mechanical dimension
- ✓ The surface loss is diluted by the bulk volume
- ✓ Sub-mm cantilever can have the Q of $10^7$

# Coating and finesse

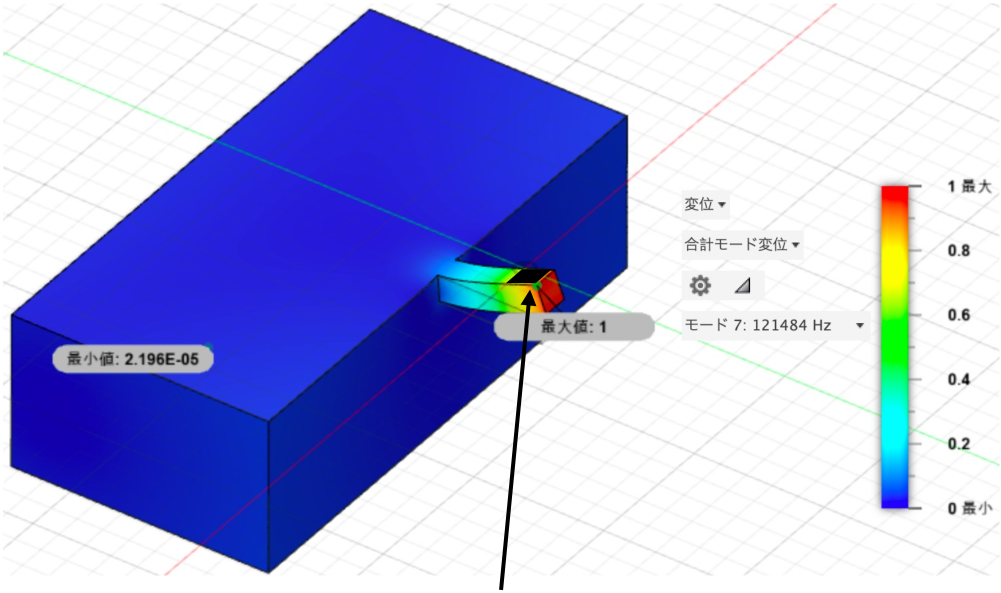
- Crystalline coating
  - The typical optical loss ~ 5 ppm at 1550 nm
  - The input mirror transmissivity ~ 50 ppm, and other losses ~ 50 ppm in total (conservatory assumption), lead to the finesse ~ 63000

| Supermirror Specifications | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Item #s | XM11P8 | XM11R8 | XM23P8 | XM23R8 | XM12P8 | XM12R8 | XM14P8 | XM14R8 |
| Center Wavelength | 1064 nm | | 1397 nm | | 1550 nm | | | |
| Mirror Shape | Plano | Concave | Plano | Concave | Plano | Concave | Plano | Concave |
| Finesse[a] | 100 000 | | 300 000 | | 200 000 | | 300 000 | |
| Crystalline Coating Reflectance[b] | > 99.9969% at 1064 nm<br>0° AOI over Central Ø8 mm | | > 99.999% at 1397 nm<br>0° AOI over Central Ø8 mm | | > 99.9984% at 1550 nm<br>0° AOI over Central Ø8 mm | | > 99.999% at 1550 nm<br>0° AOI over Central Ø8 mm | |
| Crystalline Coating Transmission[b] | [illegible] | | > 3.5 ppm at Center Wavelength<br>0° AOI over Central Ø8 mm | | > 5 ppm at Center Wavelength<br>0° AOI over Central Ø8 mm | | > 5 ppm at Center Wavelength<br>0° AOI over Central Ø8 mm | |
| Crystalline Coating Loss Angle[b,c] | $< 4 \times 10^{-5}$ at 300 K<br>$< 5 \times 10^{-6}$ at 10 K | | | | | | | |
| Damage Threshold[d] CW[e] | 46.2 kW/cm (1064 nm, Ø5.5 mm) | | | | | | | |
| Damage Threshold[d] Pulsed | 5 J/cm$^2$ (1030 nm, 10 ns, 10 Hz, Ø0.240 mm) | | | | | | | |
| AR Coating Reflectance (Back Side[f]) | < 0.1% at Center Wavelength; 0° AOI over Central Ø5 mm | | | | | | | |
| Substrate Material | [illegible] | | Corning 7979 0A IR Fused Silica[g] | | Corning 7980 0A UV Fused Silica[g] | | | |
| Surface Roughness | [illegible] | | Crystalline Coating: < 1.5 Å (RMS)<br>Back Side: < 1 Å (RMS)<br>over Central Ø19.5 mm | | Crystalline Coating: < 1.5 Å (RMS)<br>Back Side: < 1.5 Å (RMS)<br>over Central Ø19.5 mm | | Crystalline Coating: < 1.5 Å (RMS)<br>Back Side: < 1 Å (RMS)<br>over Central Ø19.5 mm | |



# Coating and finesse

➢Can be coated on the small area



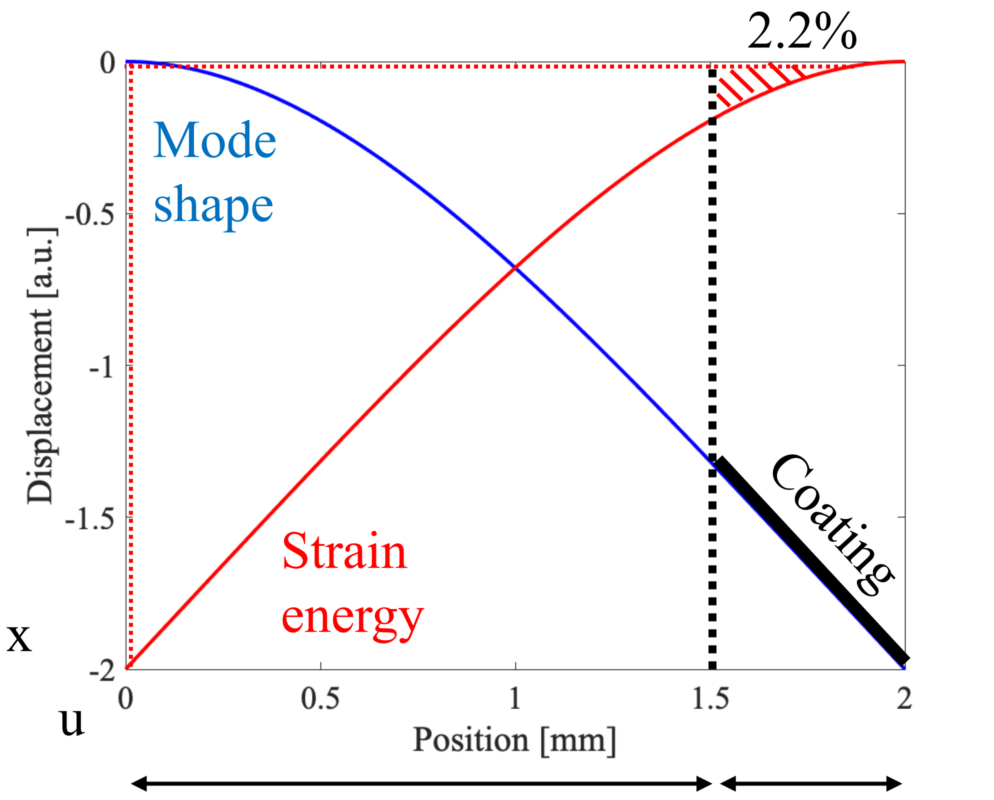

- Coating area: 0.5 mm x 0.5 mm
- Spatial optical power loss < 0.1 ppm with the designed beam radius ~ 80 um

**3.剥離**

コーティングが要求仕様を満たすことが確認できたら、リソグラフィやエッチング技術を用いて形状を精密に限定します。その後コーティングをGaAsウェハより剥離します。当社では、最小Ø20 µmから最大Ø200 mmまでの範囲で、任意の形状のコーティングが可能です。

# Coating and finesse

- Low mechanical loss
  - Below $10^{-5}$ at cryogenic temperature
- Two dilution
  - Thickness:
    Coating ~7 um v.s. substrate 0.5 mm
    $\alpha_t \sim 0.014$
  - Strain energy:
    $$\phi_{total} = C \int \frac{d^2x}{du^2} \phi(u) du$$
    The strain energy ratio is only 2.2% at the quarter edge of the cantilever
- In total, the drop of the Q-value is 3%




$\phi = 10^{-7}$ $\quad \phi = \frac{10^{-7} + 10^{-5}\alpha_t}{1+\alpha_t}$

# Frequency noise

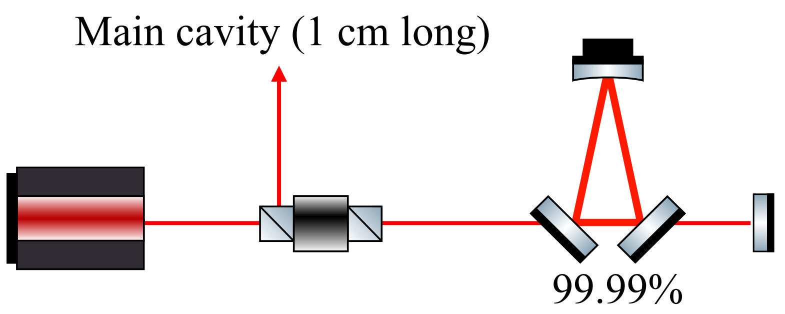
- The sideband from the displacement causes the heating
  - Requirement for the ground state cooling:

$$S_x < \frac{\hbar^2}{k_B T m \gamma_m} \quad \Rightarrow \quad \sqrt{S_x} < \sqrt{\frac{\hbar^2}{10 k_B T m \gamma_m}} = 2.8\times10^{-20}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

$\Leftrightarrow \quad n_x < 0.1$

  - Requirement on the frequency noise

$$\sqrt{S_{freq}} < 5.3\times10^{-4}\ \mathrm{Hz}/\sqrt{\mathrm{Hz}}$$




- Round trip length: 1 m
- Finesse: 30000 (cavity pole 5 kHz)

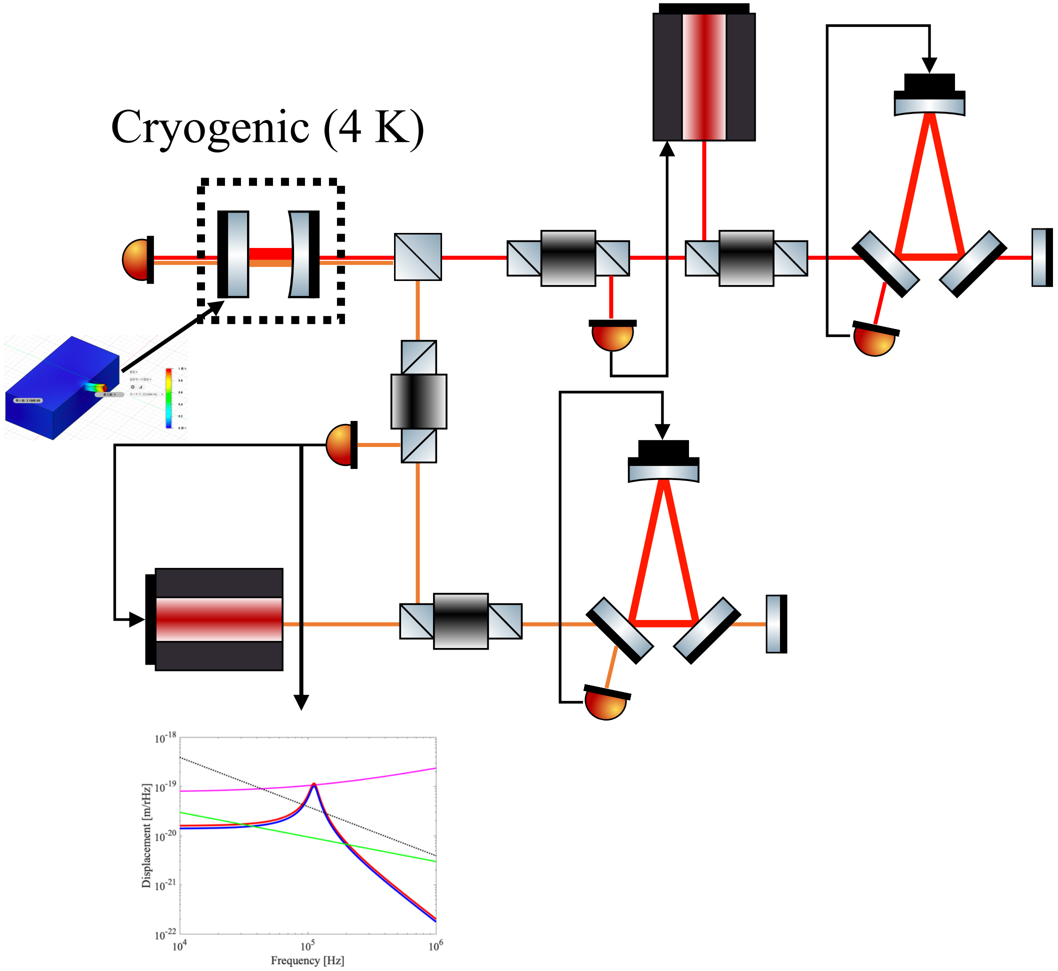
✓ Passive frequency stabilization with the double pass cavity

✓ Mephisto-like laser source: $10^4/f$ Hz/√Hz

✓ Suppressing by $1/20^2$, $2.5\times10^{-4}$ Hz/√Hz at 100 kHz

# Experimental setup

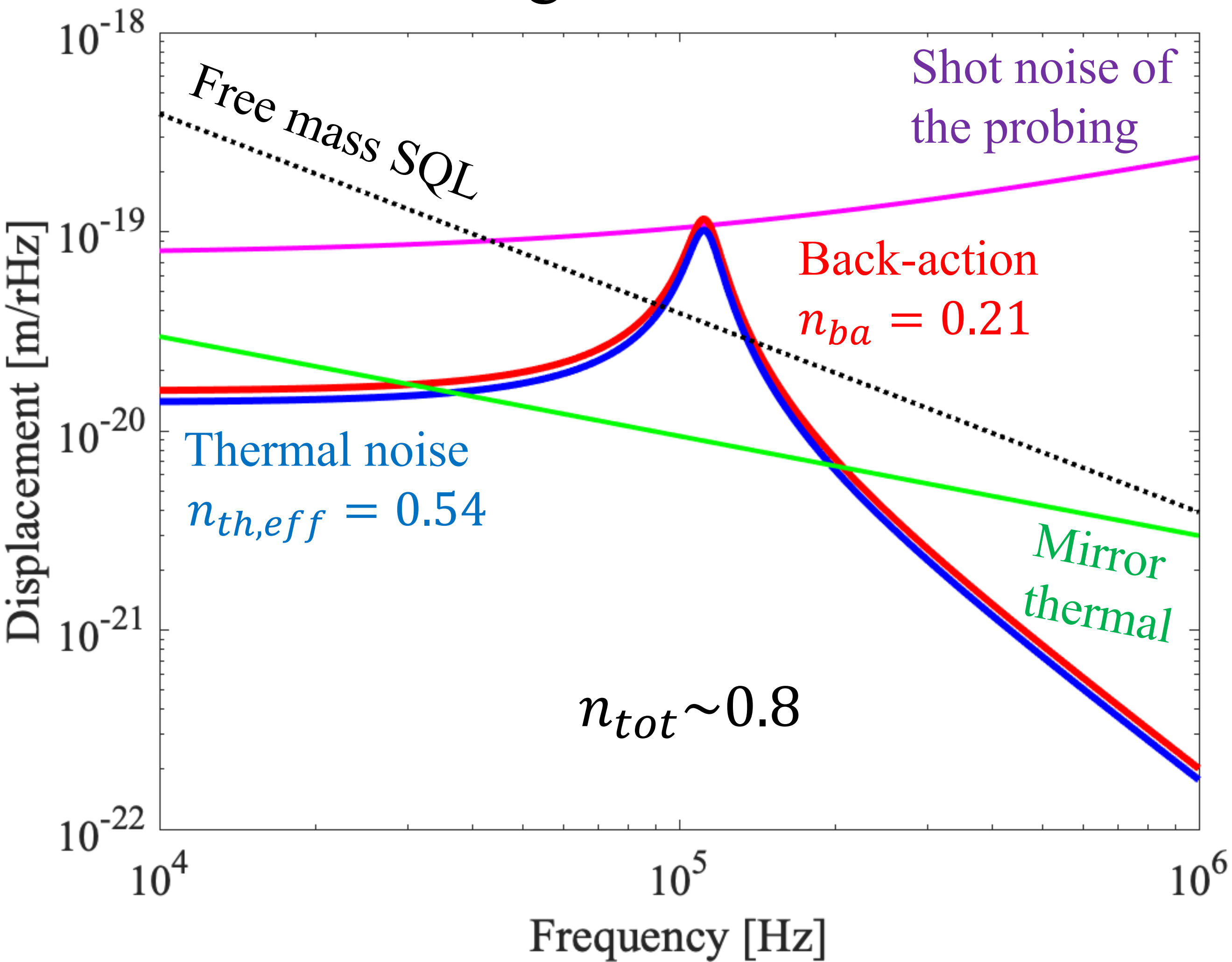
- One laser is used as cooling, and the other is used as probing




✓ Optical parameters
- Cooling laser power: 5 mW
- Cavity length: 1 cm
- Finesse: 63000
- Intra-cavity power: 33 W
- Probing laser power: 0.2 mW

# Goal

➢Ground state cooling




✓ Future steps

- Discussion on the fabrication
- Q measurement
- Discussion on the coating with CMS (Thorlab)
- Finesse and Q measurement
- Laser selection

# Summary

- Ground state cooling is equivalent to suppressing the displacement to the level of the zero-point fluctuation
- The feedback or sideband cooling are necessary
- Proposing sideband cooling of a sub-mg cantilever
- The ground state cooling should be possible