Stochastic fluctuation on vector DM search

Collaborators

Fujita Tomohiro, Jun’ya Kume, Soichiro Morisaki,
Yuta Michimura, Atsushi Nishizawa, Ippei Obata, Koji Nagano
(in the KAGRA Collaboration)

Institute for Cosmic Ray Research
University of Tokyo

Speaker:
Hiromasa Nakatsuka
ICRR, The University of Tokyo
3rd year PhD student

SYRTE-Utokyo DM seminar 2021-06-25

Purpose of this talk

- Our group

Experiment

Y. Michimura, K. Nagano

J. Kume, A. Nishizawa,
S. Morisaki

Theory

F. Tomohiro, I. Obata,
H. Nakatsuka

- What I want to do:
  - To formulate a vector dark matter signal
  - In axion DM, stochastic nature of amplitude weakens the constraints about factor 3. Centers, et.al. (2019)

Q: How large is the stochastic effect on vector DM signal ?

1. Vector DM searches
2. Stochastic effect of scalar DM
3. Stochastic effect of vector DM
4. Conclusion

Vector Dark Matter

Theoretical motivation

- U(1) extension of Standard model, e.g. $U_{B\text{-}L}(1)$ (Baryon-Lepton number)
- As ultralight dark matter (DM):

$$n\lambda^3 = \frac{\rho_{\rm DM}}{m^4 v_{\rm vir}^3} = 8\times 10^3 \frac{\rho_{\rm DM}}{0.4{\rm GeV/cm^3}}\left(\frac{1\ {\rm eV}}{m}\right)^4\left(\frac{220\ {\rm km/sec}}{v_{\rm vir}}\right)^3,$$

DM should be boson for $m \lesssim 1\text{eV}$.

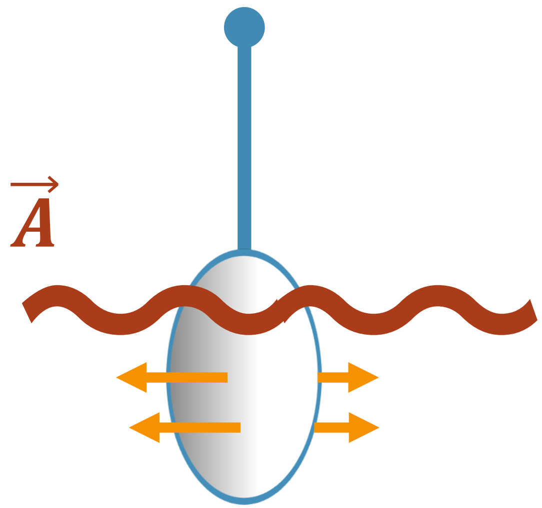
Coupling to visible matter

- $U_D(1)$ gauge field

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu + g_D J_D^\mu A_\mu,$$

- Dark electric force

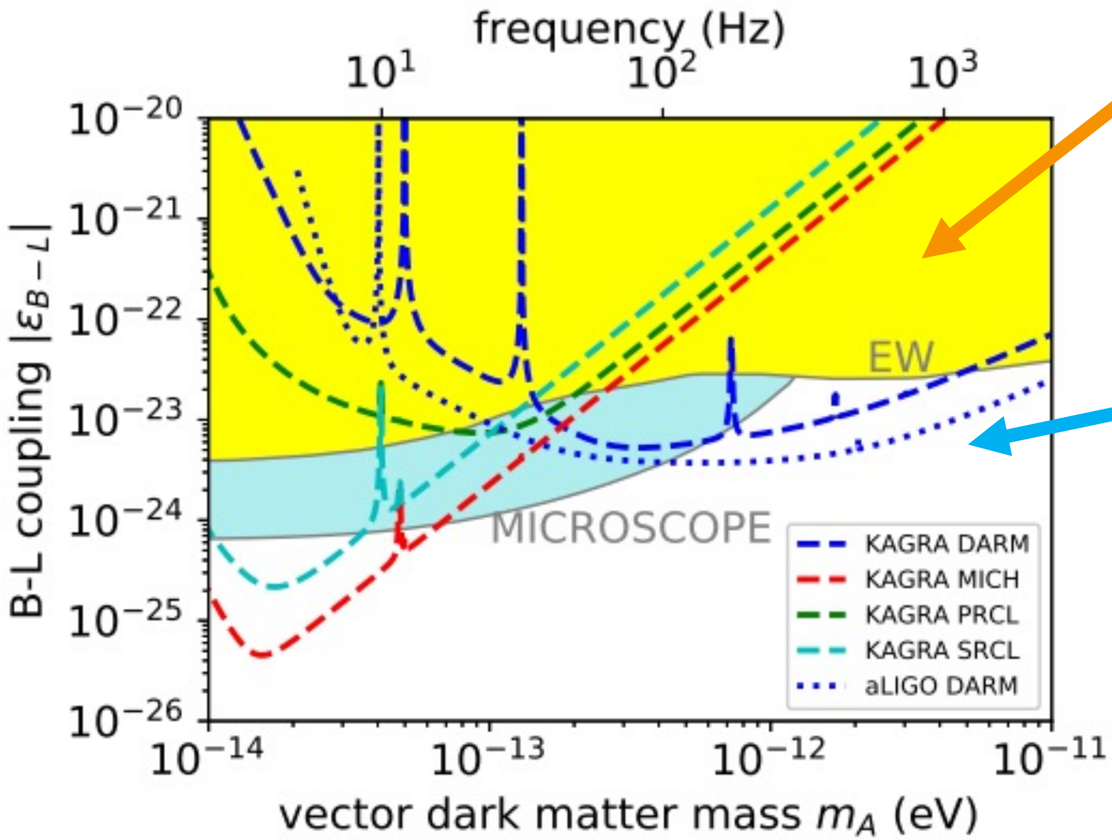
$$\vec{E}_{\rm D} = -g_{\rm D}\frac{\partial \vec{A}}{\partial t},$$



Mirror in detector

Sensitivity on coupling constant

- For $U_{B-L}(1)$



Y. Michimura, et. al. (2020)

Current constraints by
"fifth force measurement"

Sensitivity by
GW interferometer

Phase modulation of Laser induced from displacement of mirrors (KAGRA: design sensitivity)

Comparison btw scalar & vector DM

- The signal of vector DM is same as that of scaler field ?

Criterion	Scalar	Vector
Type	Bosonic field	
Wave form	Periodic oscillation within coherent time	
Amplitude	Stochastic variable: $\phi, A_i \sim \sqrt{\rho_{\mathrm{DM}}/m^2}$	
Degree of freedom	1	3 (x,y,z-direction)
Coupling	$g\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$, $g\phi F_{\mu\nu}F^{\mu\nu}$	$g_D J_D^\mu A_\mu$
Signal dependence	$S \propto \phi(t)$	$S(\vec{A}(t), \vec{n}_{arm})$

1. Vector DM searches
2. Stochastic effect of scalar DM
3. Stochastic effect of vector DM
4. Conclusion

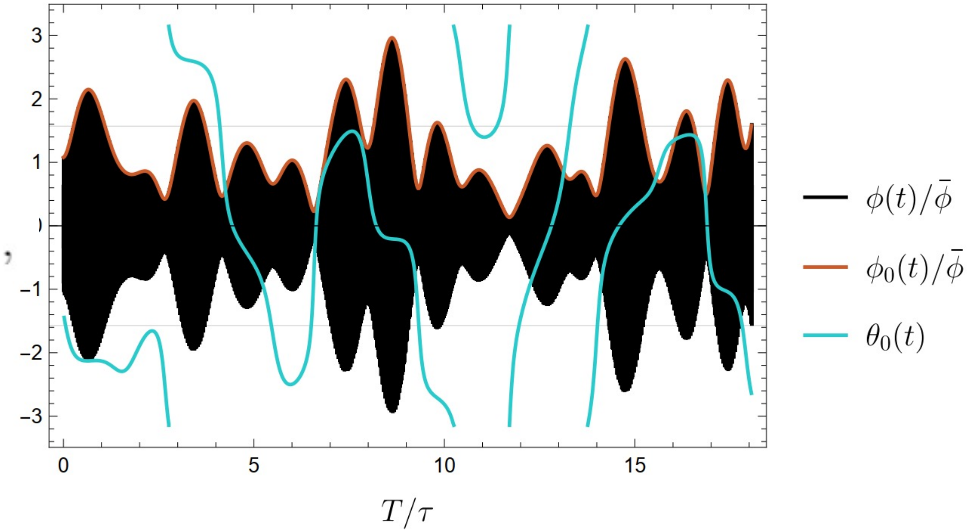
Stochastic effect on scalar DM

Superposition of waves

$$\Phi(t,\vec{x}) = \sum_{i}^{N_\phi} \Phi_i(t,\vec{x}), \quad (i=1,\ \dots,\ N_\phi),$$

$$\Phi_i(t,\vec{x}) = \sigma_\phi N_\phi^{-1/2} \cos\left(m(1+v_i^2/2)t + m\vec{v}_i\cdot\vec{x} + \theta_i\right),$$

$$\sigma_\phi \simeq \sqrt{\frac{2\rho_{\rm DM}}{m^2}}. \qquad \tau \equiv \frac{2\pi}{m(v_{\rm vir}^2 + v_\odot^2)}.$$



Probability distribution

$$\Phi(0,\vec{0}) = \sigma_\phi \alpha_0 {\rm Re}[e^{i\theta_0}].$$

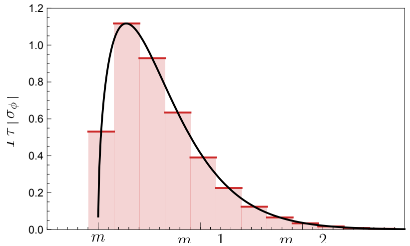
- phase θ_0: Uniform distribution on $[0,2\pi]$
- amplitude α_0: Rayleigh distribution

$$P_{\rm R}(\alpha_0){\rm d}\alpha_0 = 2\alpha_0 \exp(-\alpha_0^2)$$

Stochastic effect on scalar DM

Stochastic effect by phase

- Frequency distribution of DM particles:



$$\Delta N_j = N_\phi \int_{\Delta f} \overline{f}_{\rm SHM}(v) \frac{{\rm d}v}{{\rm d}f} {\rm d}f$$

number of partial waves in frequency bin $\Delta f = 1/T$

- Field value summed in each bin: $\Phi(t) = \sum_j \sigma_\phi \sqrt{\Delta N_j / N_\phi}\, \alpha_j {\rm Re}[e^{i(2\pi f_j t + \theta_j)}]$

$$\sigma_\phi \simeq \sqrt{\frac{2\rho_{\rm DM}}{m^2}}.$$

Since the phase of partial waves are different,
the total amplitude is proportional to $\sqrt{\Delta N_j}$, not to ΔN_j .

Foster, et.al. 2017

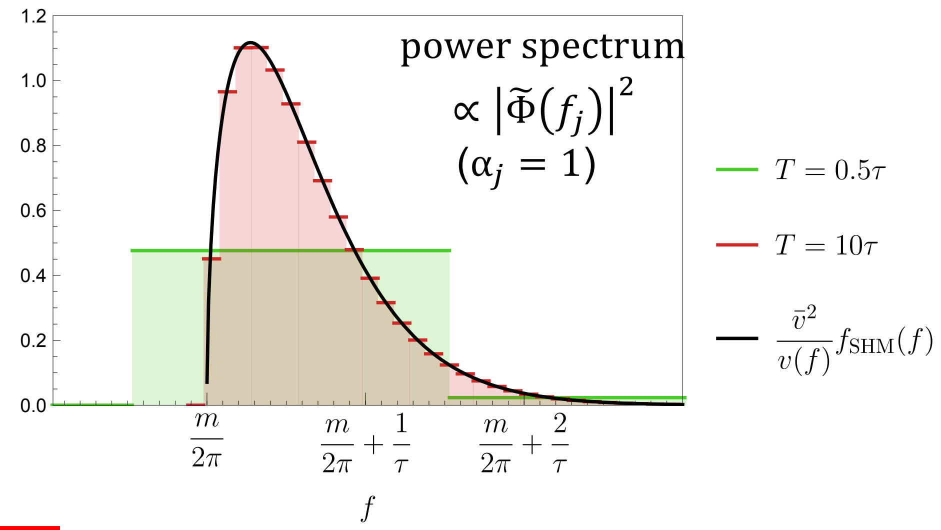
- Power spectrum

$$\tilde{\Phi}(f_j) \equiv \int {\rm d}t e^{2\pi i f t} \Phi(t)$$

$$\left|\tilde{\Phi}(f_j)\right|^2 \propto T^2 \Delta N_j \propto T^2 \int_{f_j - \Delta f/2}^{f_j + \Delta f/2} \overline{f}_{\rm SHM}(v) \frac{{\rm d}v}{{\rm d}f} {\rm d}f$$

Stochastic effect on scalar DM

Stochastic effect by phase



- The power spectrum:

$$\left|\tilde{\Phi}(f_j)\right|^2 \propto T^2 \Delta N_j \propto T^2 \int_{f_j-\Delta f/2}^{f_j+\Delta f/2} \bar{f}_{\rm SHM}(v) \frac{{\rm d}v}{{\rm d}f} {\rm d}f$$

- For $\mathrm{T} < \tau$, one bin ($\Delta f = \mathrm{T}^{-1}$) covers all DMs (Green).

$$\int_{f_j-\Delta f/2}^{f_j+\Delta f/2} \bar{f}_{\rm SHM}(v) \frac{{\rm d}v}{{\rm d}f} {\rm d}f \simeq 1 \quad , \quad \left|\tilde{\Phi}(f_j)\right|^2 \propto T^2$$

- For $\mathrm{T} \gg \tau$, about $N_{\rm bin} = 2\mathrm{T}/\tau$ bins divide distribution (Red).

$$\int_{f_j-\Delta f/2}^{f_j+\Delta f/2} \bar{f}_{\rm SHM}(v) \frac{{\rm d}v}{{\rm d}f} {\rm d}f \simeq \bar{f}_{\rm SHM} \frac{{\rm d}v}{{\rm d}f} \Delta f = \bar{f}_{\rm SHM} \frac{{\rm d}v}{{\rm d}f} T^{-1} \quad , \quad \left|\tilde{\Phi}(f_j)\right|^2 \propto T^1$$

- The signal to noise ratio is (noise power spectrum: $P_{\rm noise} \propto T^1$)

$$\frac{S}{N} \propto \frac{1}{\sqrt{N_{\rm bin}}} \frac{|\tilde{\Phi}|^2}{P_{\rm noise}} \propto \begin{cases} 1\,\frac{T^2}{T} = T^1 & \text{for } \mathrm{T} < \tau \\ \sqrt{T/\tau}\,\frac{T}{T} = \sqrt{T/\tau} & \text{for } \mathrm{T} > \tau \end{cases}$$

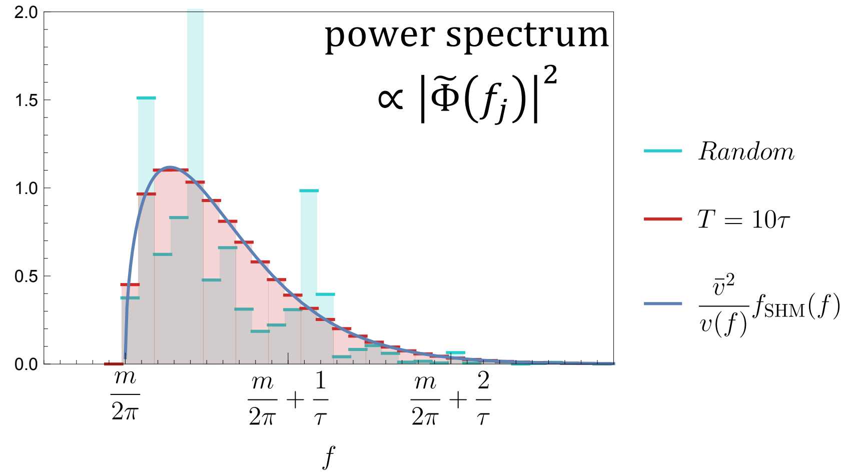
SNR improves differently.

Stochastic effect on scalar DM

Stochastic effect by amplitude

- Power spectrum fluctuates by Rayleigh distribution.

$$\Phi(t) = \sum_j \sigma_\phi \sqrt{\Delta N_j / N_\phi}\, \alpha_j \mathrm{Re}[e^{i(2\pi f_j t + \theta_j)}]$$



- When $\tilde{\Phi}(f_j)$ is accidentally small, the upper limit on coupling constant becomes loose.

$$g_D^{(\mathrm{upper})} \propto \tilde{\Phi}^{-1}$$

- The uncertainty of amplitude loosens constraints.

$$\frac{g_{(\mathrm{stocastic})}(T)}{g_{(\mathrm{deterministic})}(T)} \sim 3 \quad \text{for } \mathrm{T} < \tau \ (N_{\mathrm{bin}} = 1)\text{, 95\% C.L.}$$

Centers, et.al. (2019)

- For $N_{\mathrm{bin}} \gg 1$, randomness is averaged.

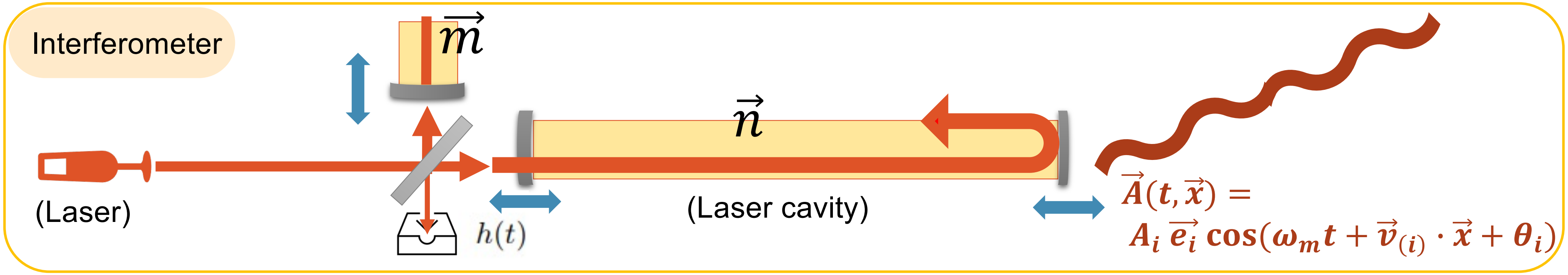
Is this the same for vector DM?

1. Vector DM searches
2. Stochastic effect of scalar DM
3. Stochastic effect of vector DM
4. Conclusion

Stochastic effect on vector DM

Observational signal

- $h(t) = \frac{\varphi(t;\boldsymbol{n}) - \varphi(t;\boldsymbol{m})}{4\pi\nu L}$, : the difference of laser phase btw two arms



1. Common motion (finite-time-traveling effect)

$$h_1 = -\frac{2g_D}{mL}\frac{Q_D}{M}\sin^2(mL/2)\sum_i A_{i,0}(\vec{n}\cdot\vec{e}_i - \vec{m}\cdot\vec{e}_i)\ \sin(\omega_m t + \theta_{i,0})$$

2. Differential motion

$$h_2 = -g_D\frac{Q_D}{M}\sin^2(mL/2)\sum_i A_{i,0}((\vec{n}\cdot\vec{e}_i)(\vec{n}\cdot\vec{v}_{i,0}) - (\vec{m}\cdot\vec{e}_i)(\vec{m}\cdot\vec{v}_{i,0}))\ \cos(\omega_m t + \theta_{i,0})$$

Q: What is the probability distribution of h_1 & h_2 ?

S. Morisaki, et. al. (2020)

Probability distribution of h_1

- Common motion:

$$h_1 = -\frac{2g_D}{mL}\frac{Q_D}{M}\sin^2(mL/2)\sum_i A_{i,0}(\vec{n}\cdot\vec{e}_i - \vec{m}\cdot\vec{e}_i)\ \sin(\omega_m t + \theta_{i,0})$$

A: h_1 follows the Rayleigh distribution like Axion case.

- Interferometer arms are fixed to x-, y-directions. $\vec{n} = \overrightarrow{e_x},\ \vec{m} = \overrightarrow{e_y}$
- Vector DM oscillates coherently ($\mathrm{T} < \tau$). $A_i = A_{i,0}\cos(mt + \theta_{i,0})$

$$|\widetilde{h_1}| = \left|\int \mathrm{dt}\, h_1(t)\right| \propto |A_{x,0}e^{-i\theta_{x,0}} - A_{y,0}e^{-i\theta_{y,0}}|$$

Complex Gaussian variable

- Sum of Gaussian variables also follows Gaussian distribution.

Probability distribution of h_2

- Differential motion:

$$h_2 = -g_D \frac{Q_D}{M} \sin^2(mL/2) \sum_i A_{i,0}((\vec{n}\cdot\vec{e}_i)(\vec{n}\cdot\vec{v}_{i,0}) - (\vec{m}\cdot\vec{e}_i)(\vec{m}\cdot\vec{v}_{i,0}))\ \cos(\omega_m t + \theta_{i,0})$$

A: h_2 follows broader distribution than the Rayleigh distribution.

$$\left|\widetilde{h_2}\right| \propto \left|A_{x,0}(v_{\odot,x} + v_{xx,h})e^{-i\theta_{x,0}} - A_{y,0}(v_{\odot,y} + v_{yy,h})e^{-i\theta_{y,0}}\right|,$$

Solar velocity   Halo velocity

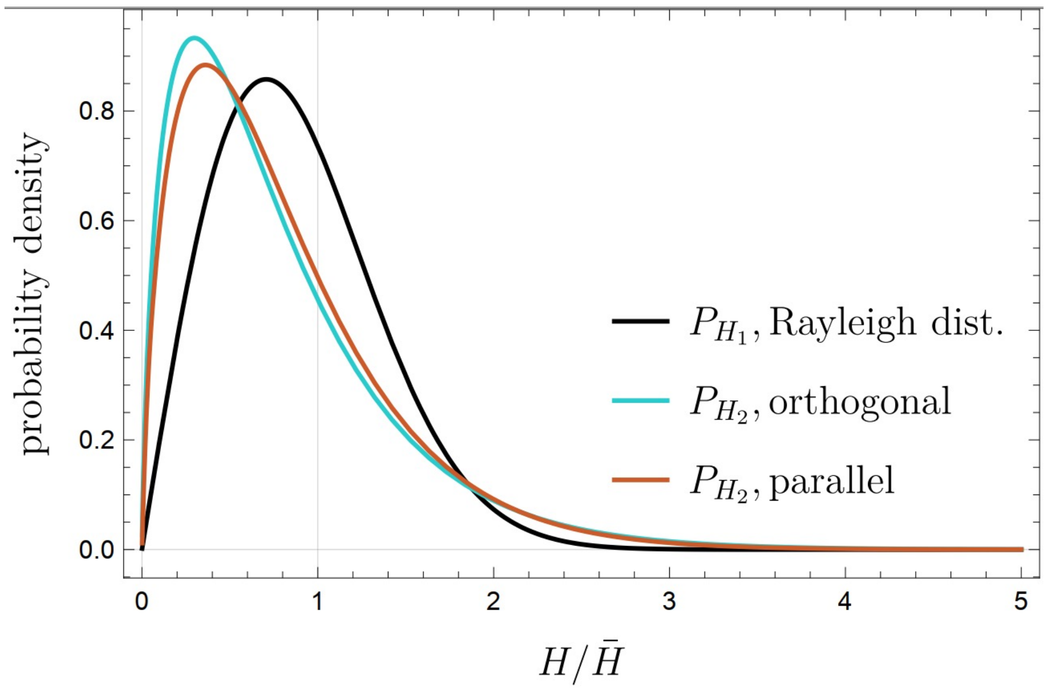
Assumptions:

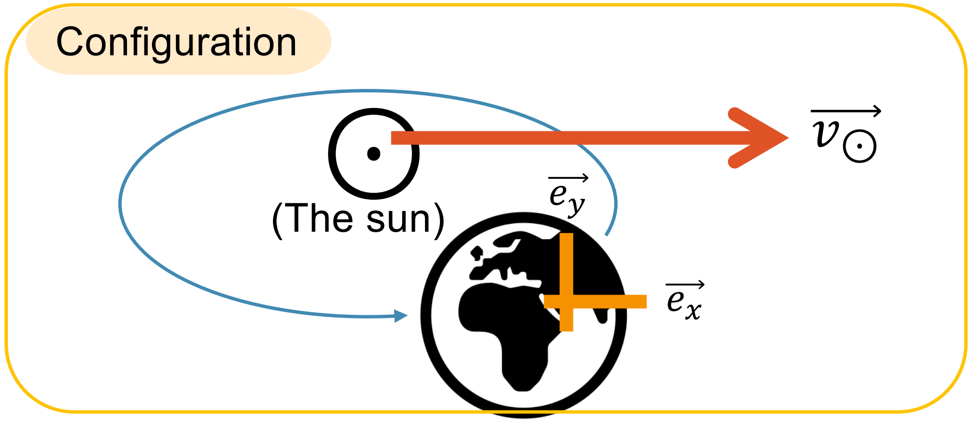
- The change of solar velocity $\vec{v}_\odot$ is ignored ($\tau < 1\mathrm{day}$)
- Halo velocity follows standard halo model:

$$f_{\mathrm{SHM}}(\vec{v}_h)\ \mathrm{d}^3\vec{v}_h = \frac{1}{(\pi v_{\mathrm{vir}}^2)^{3/2}} \exp\left[-\frac{(\vec{v}_h)^2}{v_{\mathrm{vir}}^2}\right]\ \mathrm{d}^3\vec{v}_h,$$

Probability distribution of h_2

We numerically calculate PDF of $\mathrm{H} \equiv |\widetilde{h_2}|$





- The configuration of interferometer arms and solar velocity.

  - Parallel : $v_{\odot,x} = |\vec{v_\odot}|, \quad v_{\odot,y} = 0$
  - Orthogonal : $v_{\odot,x} = v_{\odot,y} = 0$

A: h_2 follows broader distribution than the Rayleigh distribution.

Stochastic effect of vector DM

- To put upper limit of coupling constant for $\mathrm{T} < \tau$

① Derive likelihood of data with signal:

$$d \text{ (experimental data)} = |n \text{ (noise)} + s \text{ (signal)}|$$

$$\mathcal{L}(d|s) \equiv \int \mathrm{d}^2 n \; P_n(n) \delta\left(d - \left|n + s\; e^{i\theta_0}\right|\right)$$
$$= 2de^{-d^2-s^2} I_0(2ds),$$

② Marginalize over the stochastic variable of signal:

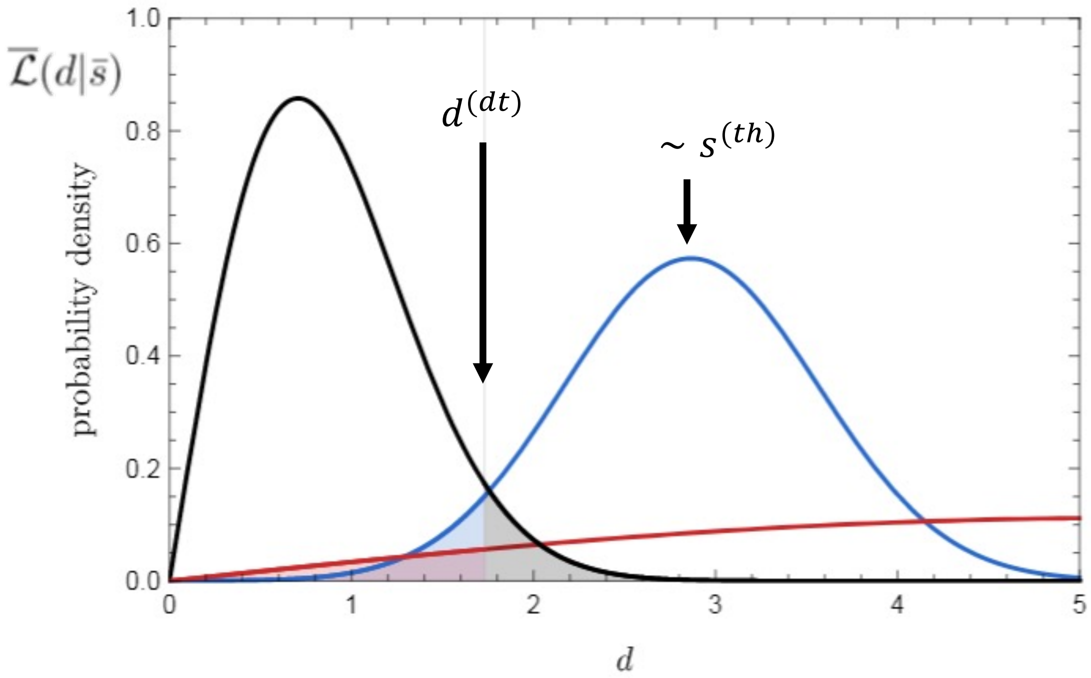
$$\overline{\mathcal{L}}(d|\bar{s}) \equiv \int \mathrm{d}s \; P_s(s) \mathcal{L}\left(d|s\right)$$
$$= \int \mathrm{d}\phi_0 \; P_\phi(\phi_0) \mathcal{L}\left(d\left|\frac{\phi_0}{\phi}\bar{s}\right.\right).$$

$P_s(s)$: Probability distribution of signal, e.g. Rayleigh distribution for scalar field.
$\bar{s}$. : Mean value of signal

③ Determine the upper bound of signal based on frequentist's view.

Stochastic effect of vector DM

③ Determine the upper bound of signal based on frequentist's view.



- With background only hypothesis, we determine null-detection limit:

$$1-\alpha = \int_{d^{(\mathrm{dt})}}^{\infty} \mathrm{d}d\, \overline{\mathcal{L}}(d|s=0)$$

$(\alpha = 0.95)$

- The upper bound of signal is determined so that to keep the false exclusion rate as $1-\beta$:

$$1-\beta = \int_0^{d^{(\mathrm{dt})}} \mathrm{d}d\overline{\mathcal{L}}(d|s^{(\mathrm{th})}).$$

$(\beta = 0.95)$

Stochastic effect of vector DM

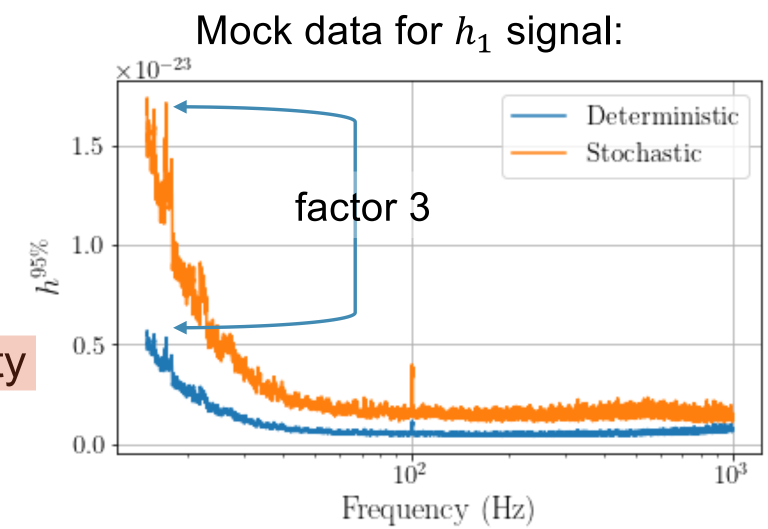
Application on the sensitivity curve

- axion, vector DM(h_1): uncertainty of amplitude

$$\frac{g_{\text{stocastic}}(T)}{g_{\text{deterministic}}(T)} \sim 2.8 \quad (95\%\text{C. L.})$$

- vector DM(h_2): uncertainty of amplitude & velocity

$$\frac{g_{\text{stocastic}}(T)}{g_{\text{deterministic}}(T)} \sim 5 \quad (95\%\text{C. L.})$$



The stochastic effect is about 70% larger for vector DM case !!

- For $T \to \infty$, the stochastic effect of amplitude vanishes since we can sample many realization of field value.

$$(A_i \to \overline{A_i})$$

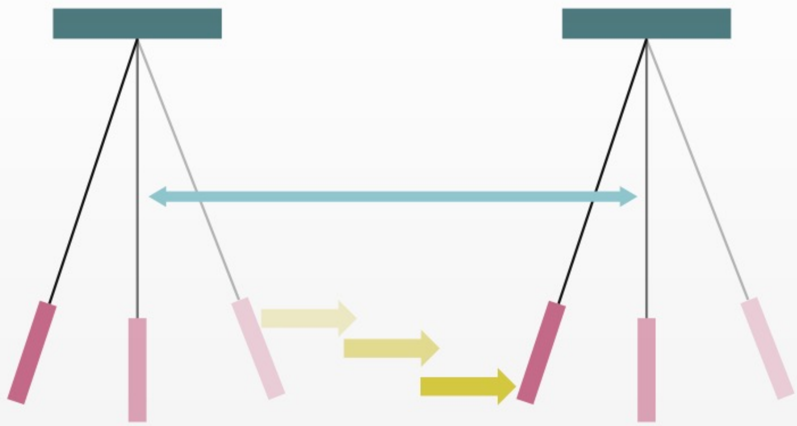
Fig. by Morisaki

Conclusion

- GW interferometer is a promising tool to search for vector DM.
- Stochastic nature of bosonic DM weakens the constraints.
- We estimate the probability distribution of signal of vector DM, which is more significant than that of axion DM.
- Our analysis will be used in vector DM search by KAGRA data in preparation.

Backup

Backup



If round-trip time = period of VDM

$(L_{\mathrm{arm}}/c = \pi/m_A)$

Written by Fujita