Quantum decoherence caused by gravitons

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Based on arXiv: 2007.09838, in progress

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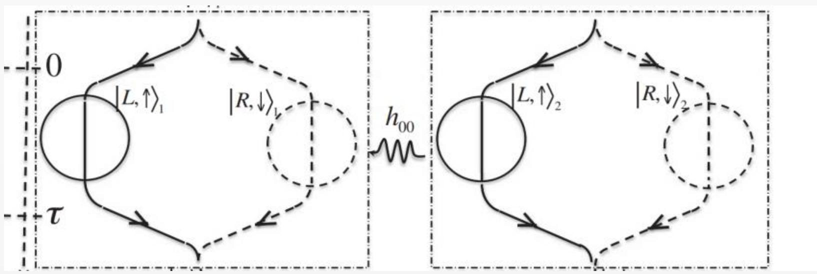
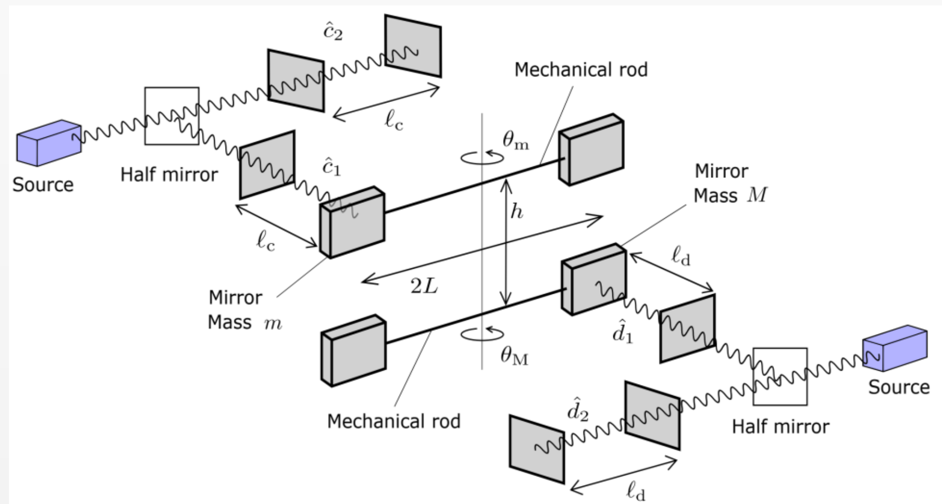
Introduction (1/2)

- Quantum nature of gravity has been investigated.

  E.g.) Newton gravity S. Bose *et al.*('17), A. Matsumura *et al.*('20), …



(from S. Bose *et al.* PRL**119**, 240401(2017))



(from A. Matsumura *et al.* arXiv:2010.05161)

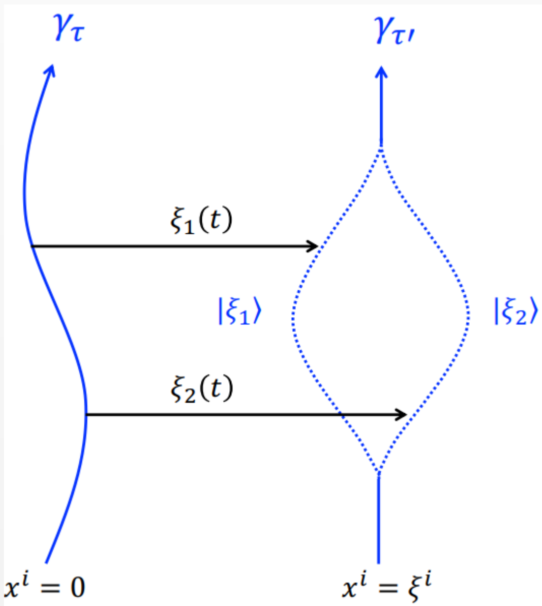
- We want to prove the presence of **gravitons**!

  …dynamical component of gravity

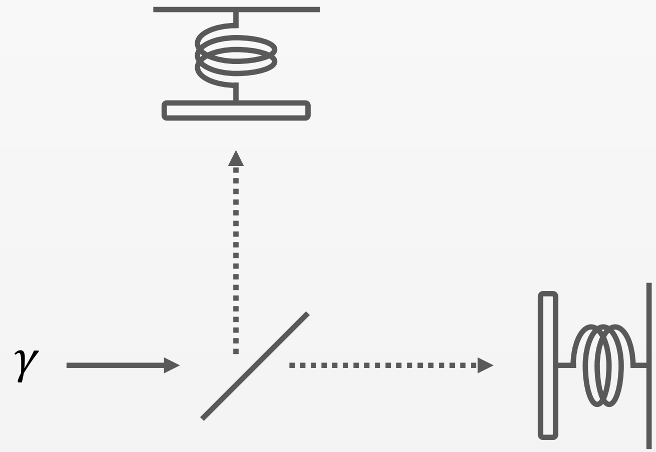
Introduction (2/2)

- We consider the following two systems

(1)



(2)



Two test particles + gravitons

- We analyze (too) simplified toy model in order to find some potentially interesting setups.

Based on arXiv: 2007.09838, in progress

Plan

In collaboration with **S. Kanno** & **J. Soda**

1. We analyze the model (1):

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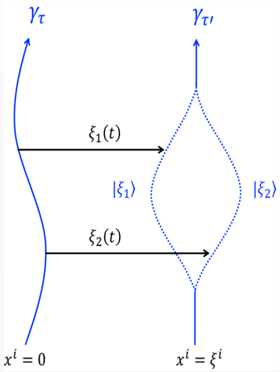
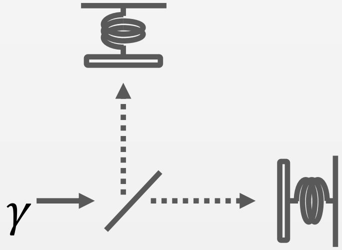


16 pages

- Construct the action
- Compute the decoherence of superpositions of massive particles caused by gravitons

2. We analyze the model (2):

In progress.



4pages

- Compute the decoherence caused by gravitons

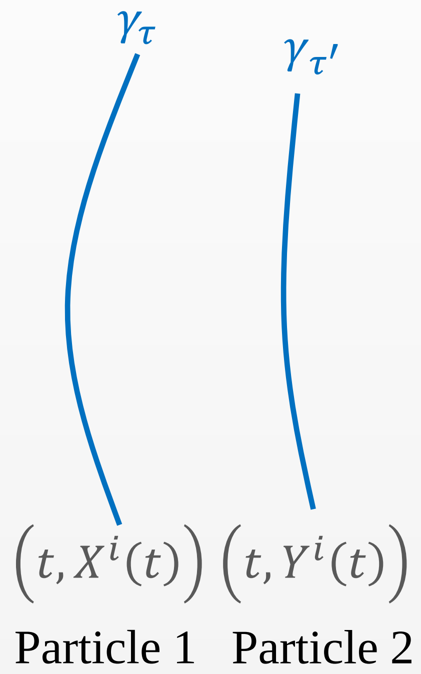
3. Summary

System

- The system: two test particles + gravitons.

$$S = \frac{M_{\rm pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\,R + S_{\rm particle}$$

$$S_{\rm particle} = \frac{M}{2}\int \mathrm{d}t\sqrt{-g_{\mu\nu}(t,X^i)\dot{X}^\mu\dot{X}^\nu} + \frac{m}{2}\int \mathrm{d}t\sqrt{-g_{\mu\nu}(t,Y^i)\dot{Y}^\mu\dot{Y}^\nu}$$



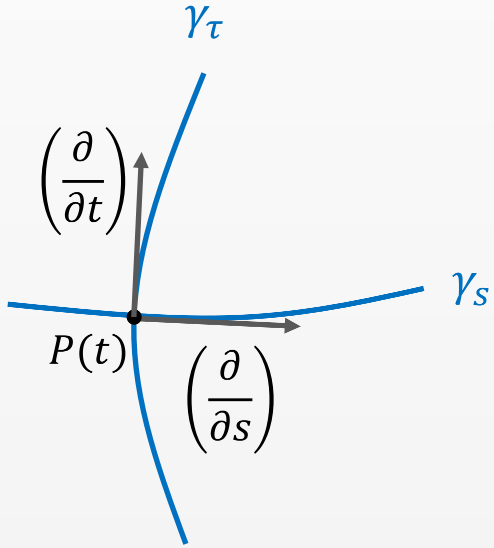
- Suppose that particle 1 freely falls. → γ_τ: time-like geodesics

Interaction term (1/2)

- **Fermi-normal coordinates**: one of the local inertial frames

  - ✓ Christoffel symbols vanish along γ_τ.
  - ✓ given by $x^\mu = (t, s\alpha^i)$,
    s: proper distance along a geodesics γ_s

$$\left(\frac{\partial}{\partial s}\right)_{P(t)} = \alpha^i \left(\frac{\partial}{\partial x^i}\right)_{P(t)}$$



γ_τ: time-like geodesics
γ_s: space-like geodesics

- Action for particles is given by

$$S_{\text{particle}} = \frac{m}{2}\int \mathrm{d}t \sqrt{-g_{\mu\nu}(t,\xi^i)\dot{\xi}^\mu\dot{\xi}^\nu}$$

(t, ξ^i): location of particle 2.

$(t, \mathbf{0})$: location of particle 1 (origin)

Interaction term (2/2)

- Metric can be expanded as

$$g_{\mu\nu}(t, \xi^i) = \eta_{\mu\nu} + \frac{g_{\mu\nu,ij}(t, \mathbf{0})}{2} \xi^i \xi^j + O(\xi^3). \qquad \cdots \quad (☆)$$

$\sim$ (Riemann tensor)

- Then, the action for particles reads

$$S_{\text{particle}} \simeq \frac{m}{2} \int \mathrm{d}t \left[\dot{\xi}^i \dot{\xi}_i - R_{0i0j} \xi^i \xi^j\right] \simeq \frac{m}{2} \int \mathrm{d}t \left[\dot{\xi}^i \dot{\xi}_i + \frac{1}{2} \ddot{h}_{ij} \xi^i \xi^j\right]$$

$$R_{0i0j}(t, \mathbf{0}) = -\left.\frac{\ddot{h}_{ij}(t, \mathbf{0})}{2}\right|_{\text{TT}\, gauge}$$

- The expansion (☆) is valid only for long-wavelength modes $k \leq \xi^{-1}$.
- We therefore introduce the UV cutoff $\Omega_{\text{m}} \sim \xi^{-1}$ and neglect the contributions from short-wavelength modes.

Quantization of h

- Action for graviton in the TT gauge

$$S_{\rm GR}[h] \simeq \frac{M_{\rm pl}^2}{8}\int \mathrm{d}^4x\left[-\frac{1}{2}(\partial h_{ij})^2\right] = \sum_{A=+,\times}\int \mathrm{d}t\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left[\left|\dot{h}^A(\boldsymbol{k},t)\right|^2 - k^2\left|h^A(\boldsymbol{k},\boldsymbol{t})\right|^2\right]$$

$$h_{ij}(t,\boldsymbol{x}) = \frac{2}{M_{\rm pl}}\sum_{A=+,\times}\int\frac{\mathrm{d}^3k}{(2\pi)^3}h^A(\boldsymbol{k},t)e_{ij}^A(\hat{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \qquad e_{ij}^{A*}(\hat{k})e_{ij}^B(\hat{k}) = \delta^{AB}$$

- **Canonical quantization:**

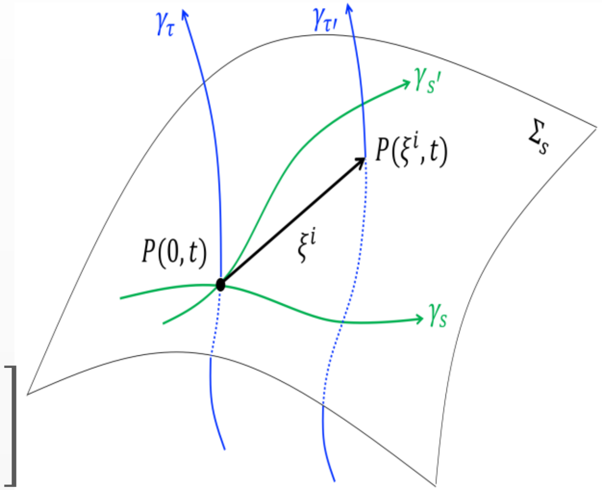
$$\hat{h}^A(\boldsymbol{k},t) = \hat{a}_A(\boldsymbol{k})u_k(t) + \hat{a}_A^\dagger(-\boldsymbol{k})u_k^*(t) \qquad \dot{u}_k(t)u_k^*(t) - u_k(t)\dot{u}_k^*(t) = -i \text{ (Normalization)}$$

$$\left[\hat{a}_A(\boldsymbol{k}),\hat{a}_B^\dagger(\boldsymbol{k}')\right] = \delta_{AB}(2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}') \quad \text{, otherwise } 0$$

- Choice of mode function $u_k(t)$ determines the vacuum state

e.g.) Minkowski vacuum state $|0_{\rm M}\rangle$ $\qquad \hat{a}(\boldsymbol{k})|0_{\rm M}\rangle = 0, \qquad u_k(t) = \frac{1}{\sqrt{2k}}e^{-ikt}$

Concrete setup (1/2)

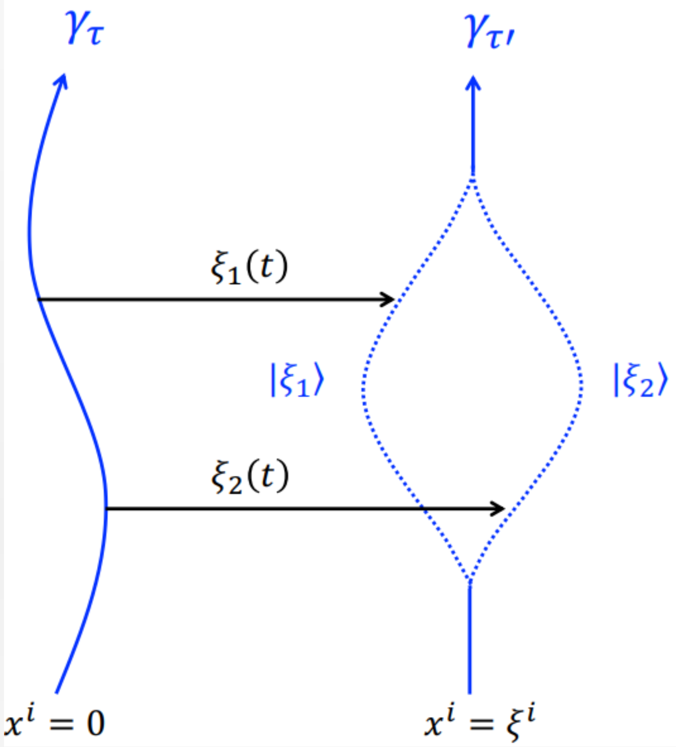


- Action of two test particles + gravitons

$$\hat{S}[h,\xi] = \hat{S}_{\mathrm{GR}}[h] + \frac{m}{2}\int \mathrm{d}t \left[\dot{\hat{\xi}}^i(t)\dot{\hat{\xi}}_i(t) + \frac{1}{2}\ddot{\hat{h}}_{ij}(t,\mathbf{0})\hat{\xi}^i(t)\hat{\xi}^j(t)\right]$$

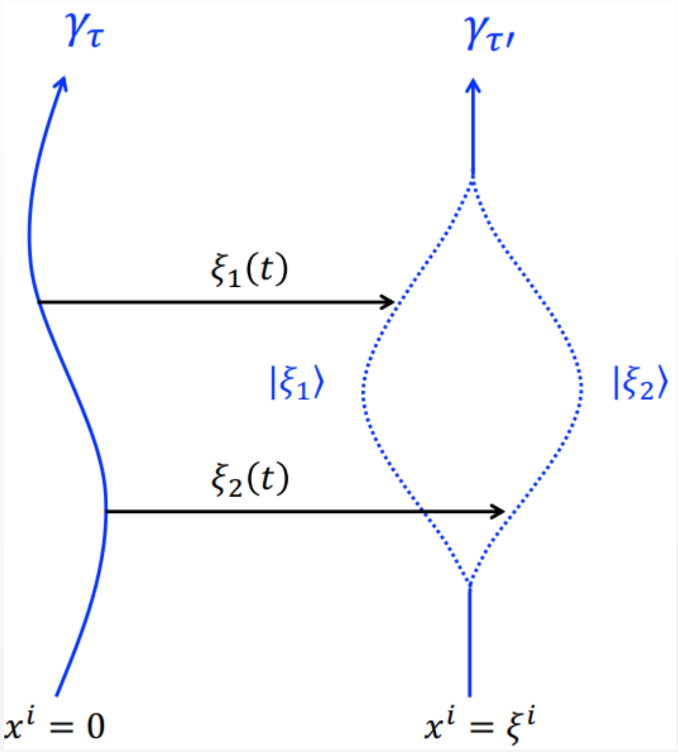
※high frequency modes $k \geq \Omega_{\mathrm{m}}$ is neglected.

- Specifically, we consider the following setup.



- Particle 1=time-like geodesics
- Particle 2=superposed state of a spatially-separated locations

Concrete setup (2/2)



- Superposition is created for $0 < t < t_\mathrm{f}$

$$|\Psi(t_0 < 0)\rangle \to |\Psi(t)\rangle = |\xi_1(t)\rangle + |\xi_2(t)\rangle$$

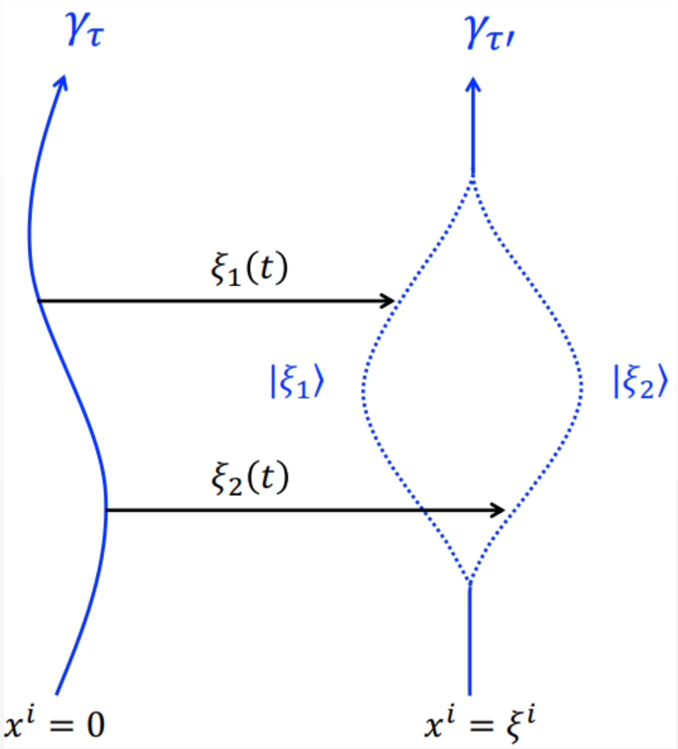
$$|\xi_a(t_0 < 0)\rangle = c_a|\Psi(t_0)\rangle, \qquad c_1 + c_2 = 1.$$

$t_0 < 0$: initial time

$\langle\Psi(t_0)|\Psi(t_0)\rangle = 1$: normalization

- $|\xi_1(t)\rangle$ and $|\xi_2(t)\rangle$ are eigenstates of $\hat{\xi}$: $\quad \hat{\xi}^i|\xi_a(t)\rangle = \xi_a^i(t)|\xi_a(t)\rangle$
- $\xi_1^i(t) \neq \xi_2^i(t)$ only for $0 < t < t_\mathrm{f}$

Influence functional (1/7)



- Coherence between superposed states $\propto |\langle \xi_1(t)|\xi_2(t)\rangle|$

- We want to evaluate how this coherence is lost by gravitons.

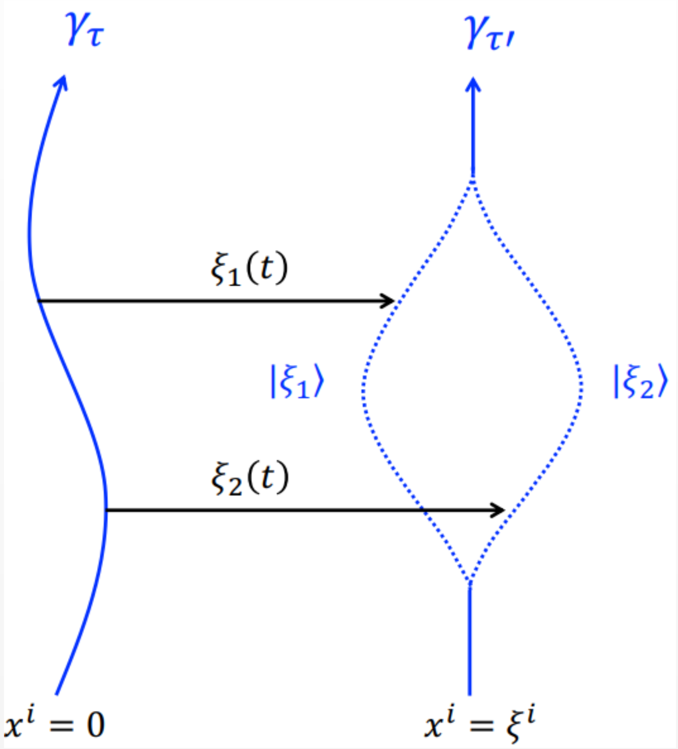
- We compute **an influence functional**

$$\exp[-\Gamma(t_\mathrm{f})] := \left|\frac{\langle \xi_1(t_\mathrm{f})|\xi_2(t_\mathrm{f})\rangle}{\langle \xi_1(t_0)|\xi_2(t_0)\rangle}\right|$$

✓ $\Gamma(t_\mathrm{f}) \ll 1 \to$ Coherence is maintained. ✓ $\Gamma(t_\mathrm{f}) \gtrsim 1 \to$ Decoherence!

- To compute $\Gamma(t_\mathrm{f})$, it is convenient to **make use of the path-integral.**

Influence functional (2/7)



- Computation of $\Gamma(t_{\rm f})$.

$$|\xi_a(t)\rangle = c_a \hat{U}(t, t_0; \xi_a)|\Psi(t_0)\rangle$$

$$\hat{U}(t, t_0; \xi_a) = T\text{exp}\left[-i\int_{t_0}^{t} \text{d}s\, H(s; \xi_a)\right]$$

…Unitary time evolution

✓ Form of the action/Hamiltonian depend on $\xi_a(t)$, and hence the time evolution of the system is also affected by $\xi_a(t)$.

- Path-integral expression of $\hat{U}(t, t_0; \xi_a)$:

$$\hat{U}(t, t_0; \xi_a) = T\text{exp}\left[-i\int_{t_0}^{t} \text{d}s\, H(s; \xi_a)\right] = \int \mathfrak{D}h\; e^{iS[h,\xi_a]}|h(t)\rangle\langle h(t_0)| \qquad \cdots (\bigstar)$$

- We will explain the derivation of eq. (★) and how to utilize it to compute $\Gamma(t_{\rm f})$.

Influence functional (3/7)

- Path-integral expression of the unitary evolution

$$\hat{U}(t,t_0;\xi_a) = T\exp\left[-i\int_{t_0}^{t} \mathrm{ds}\, H(s;\xi_a)\right] = \int \mathfrak{D}h\ e^{iS[h,\xi_a]}|h(t)\rangle\langle h(t_0)|\Psi(t_0)\rangle$$

- Simplest model: quantum mechanics

$$H(p,q) = \frac{p^2}{2m} + V(q) \quad L(q,\dot{q}) = [p\dot{q} - H(p,q)]_{p=m\dot{q}}$$

$$\hat{U}(t,t_0) = \hat{U}(t_N,t_{N-1})\hat{U}(t_{N-1},t_{N-2})\cdots\hat{U}(t_1,t_0)$$

$$= \prod_{0\le j\le N}\int \mathrm{d}q_j\,\langle q_{j+1}|\hat{U}(t_{j+1},t_j)|q_j\rangle|q_N\rangle\langle q_0|$$

$$t_j = t_0 + j\Delta t\ (j = 0\cdots N)$$

$$t_N = t$$

$$\hat{q}|q_j\rangle = q_j|q_j\rangle$$

Influence functional (4/7)

$$\langle q_{j+1}|\hat{U}(t_{j+1},t_j)|q_j\rangle = \langle q_{j+1}|[1-i\hat{H}\Delta t]|q_j\rangle + O(\Delta t^2)$$

$$= \int \frac{\mathrm{d}p_j}{2\pi}\langle q_{j+1}|[1-i\hat{H}\Delta t]|p_j\rangle\langle p_j|q_j\rangle + O(\Delta t^2) = \int \frac{\mathrm{d}p_j}{2\pi} e^{ip_j(q_{j+1}-q_j)} e^{-iH[p_j,q_{j+1}]\Delta t} + O(\Delta t^2)$$

$$= \int \frac{\mathrm{d}p_j}{2\pi} e^{i\left[p_j\left(\frac{q_{j+1}-q_j}{\Delta t}\right)-H[p_j,q_{j+1}]\right]\Delta t} + O(\Delta t^2)$$

$$\therefore \hat{U}(t,t_0) = \prod_{0\le j\le N}\int \mathrm{d}q_j\,\langle q_{j+1}|\hat{U}(t_{j+1},t_j)|q_j\rangle|q_N\rangle\langle q_0|$$

$$= \prod_{0\le j\le N}\int \mathrm{d}q_j \int \frac{\mathrm{d}p_j}{2\pi} e^{i\sum_{j=0}^{N}\left[p_j\left(\frac{q_{j+1}-q_j}{\Delta t}\right)-H[p_j,q_{j+1}]\right]\Delta t}|q_N\rangle\langle q_0| + O(\Delta t^2)$$

- In the limit $N\to\infty$ ($\Leftrightarrow \Delta t\to 0$), we have

$$\hat{U}(t,t_0) = \int \mathfrak{D}q\mathfrak{D}p\,\exp\left[i\int_{t_0}^{t}\mathrm{d}t'\,[p\dot{q}-H(p,q)]\right]|q_t\rangle\langle q_0|$$

Influence functional (5/7)

- We can perform the integration over p !

$$H(p,q) = \frac{p^2}{2m} + V(q)$$

$$\hat{U}(t,t_0) = \int \mathfrak{D}q\mathfrak{D}p \exp\left[i\int_{t_0}^{t} \mathrm{d}t' \, [p\dot{q} - H(p,q)]\right] |q_t\rangle\langle q_0|$$

Quadratic in *p*

$$p\dot{q} - H(p,q) = \frac{1}{2m}(p - m\dot{q})^2 - \frac{m\dot{q}^2}{2} - V(q)$$

$$= \int \mathfrak{D}q \exp\left[i\int_{t_0}^{t} \mathrm{d}t' \, L(q,\dot{q})\right] |q_t\rangle\langle q_0| \; = \int \mathfrak{D}q \, e^{iS} \, |q_t\rangle\langle q_0|$$

- Our case:

$$\hat{U}(t,t_0;\xi_a) = T\exp\left[-i\int_{t_0}^{t} \mathrm{d}s \, H(s;\xi_a)\right] = \int \mathfrak{D}h \; e^{iS[h,\xi_a]}|h(t)\rangle\langle h(t_0)|$$

$$S[h,\xi] = S_{\mathrm{GR}}[h] + \frac{m}{2}\int \mathrm{d}t \left[\dot{\xi}_a^i\dot{\xi}_a^i + \frac{1}{2}\ddot{h}_{ij}\xi_a^i\xi_a^j\right]$$

$$\hat{h}_{ij}|h(t)\rangle = h_{ij}(t)|h(t)\rangle$$

$$\hat{\xi}_a^i|h(t)\rangle = \xi_a^i(t)\,|h(t)\rangle$$

Influence functional (6/7)

- We want to compute $\Gamma(t_{\mathrm{f}})$

$$\exp[-\Gamma(t_{\mathrm{f}})] \coloneqq \left|\frac{\langle \xi_1(t_{\mathrm{f}})|\xi_2(t_{\mathrm{f}})\rangle}{\langle \xi_1(t_0)|\xi_2(t_0)\rangle}\right|$$

- So far, we obtained

$$\hat{U}(t, t_0; \xi_a) = \int \mathfrak{D}h\ e^{iS[h,\xi_a]}|h(t)\rangle\langle h(t_0)|$$

- We have

$$|\xi_a(t)\rangle = c_a\hat{U}(t, t_0; \xi_a)|\Psi(t_0)\rangle = c_a \int \mathfrak{D}h\ e^{iS[h,\xi_a]}|h(t)\rangle \underbrace{\langle h(t_0)|\Psi(t_0)\rangle}_{=:\,\Psi_0[h_+]}$$

$$\therefore \langle \xi_1(t_{\mathrm{f}})|\xi_2(t)\rangle = c_1 c_2 \underbrace{\int \mathfrak{D}h_+ \int \mathfrak{D}h_-\, \delta\big(h_+(t_{\mathrm{f}}) - h_-(t_{\mathrm{f}})\big) e^{i(S[h_+,\xi_2]-S[h_-,\xi_1])}\Psi_0[h_+]\Psi_0^*[h_-]}_{=:\,Z[\xi_1,\xi_2]}$$

$$\therefore \exp[-\Gamma(t_{\mathrm{f}})] = \left|\frac{Z[\xi_1,\xi_2]}{Z[0]}\right|$$

Influence functional (7/7)

$$\exp[-\Gamma(t_f)] = \left|\frac{Z[\xi_1,\xi_2]}{Z[0]}\right|$$

- $Z[J_2, J_1]$: generating functional for gravitons.

$$Z[\xi_1,\xi_2] = \int \mathfrak{D}h_+ \int \mathfrak{D}h_- \, \delta\big(h_+(t_f) - h_-(t_f)\big) e^{i(S[h_+,\xi_2]-S[h_-,\xi_1])} \Psi_0[h_+]\Psi_0^*[h_-]$$

$$S[h,\xi] = S_{GR}[h] + \frac{m}{4}\int \ddot{h}_{ij}\xi^i\xi^j + \frac{m}{2}\int \dot{\xi}^2$$

$$\sim \int \mathfrak{D}h_+ \int \mathfrak{D}h_- \, \delta\big(h_+(t_f) - h_-(t_f)\big) e^{i(S_{GR}[h_+]-S_{GR}[h_-])} \Psi_0[h_+]\Psi_0^*[h_-] \, e^{i\frac{m}{4}\int[\ddot{h}_+\xi_2^2 - \ddot{h}_-\xi_1^2]}$$

"weight function"

$$=: < e^{i\frac{m}{4}\int[\ddot{h}_+\xi_2^2 - \ddot{h}_-\xi_1^2]} >$$

$$\to \frac{i\delta}{\delta(\xi_1^2(t))}\frac{i\delta}{\delta(\xi_1^2(t'))} Z[\xi_1,\xi_2]|_{\xi_1=\xi_2=0} = \frac{m^2}{8} < \ddot{h}_-(t)\ddot{h}_-(t') >$$ etc.

- Generating functional can be written in terms of cumulants as

$$Z[\xi_1,\xi_2] = \exp\left[\int dt \left(-i\xi_1^2(t)\right) \int dt' \left(-i\xi_1^2(t')\right) \frac{m^2}{16} < \ddot{h}_-(t)\ddot{h}_-(t') >_c + \cdots\right]$$

Decoherence rate (1/2)

$$\exp[-\Gamma(t_\mathrm{f})] = \left|\frac{Z[\xi_1,\xi_2]}{Z[0]}\right|$$

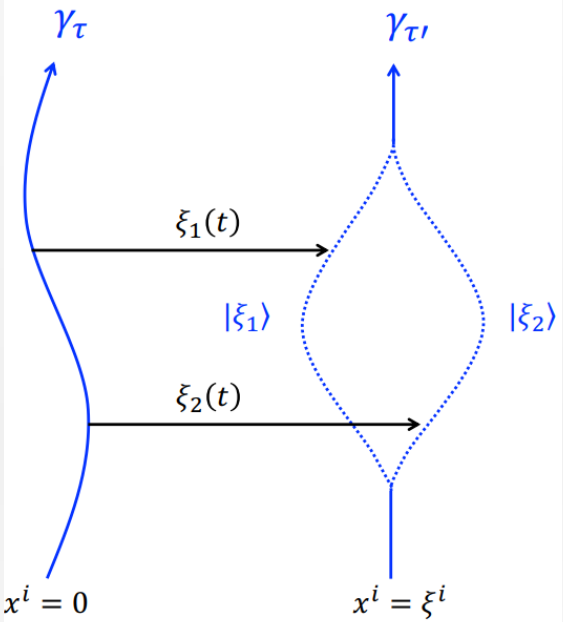
- We can obtain $\Gamma(t_\mathrm{f})$ by computing the graviton 2-point functions and specifying the particle trajectories $\xi_1^i(t)$ and $\xi_2^i(t)$.

$$\Gamma(t_\mathrm{f}) = \frac{m^2}{32}\int_{t_0}^{t_\mathrm{f}} \mathrm{d}t\, \Delta\big(\xi^i\xi^j\big)(t) \int_{t_0}^{t_\mathrm{f}} \mathrm{d}t'\, \Delta\big(\xi^k\xi^\ell\big)(t') < \{\hat{h}_{ij}(t,\mathbf{0}), \hat{h}_{k\ell}(t',\mathbf{0})\} >_\mathrm{c}$$

Anti-commutator symbol

$$\{\hat{A},\hat{B}\} := \frac{1}{2}\big(\hat{A}\hat{B} + \hat{B}\hat{A}\big)$$

$$\Delta\big(\xi^i\xi^j\big)(t) := \xi_1^i(t)\xi_1^j(t) - \xi_2^i(t)\xi_2^j(t)$$

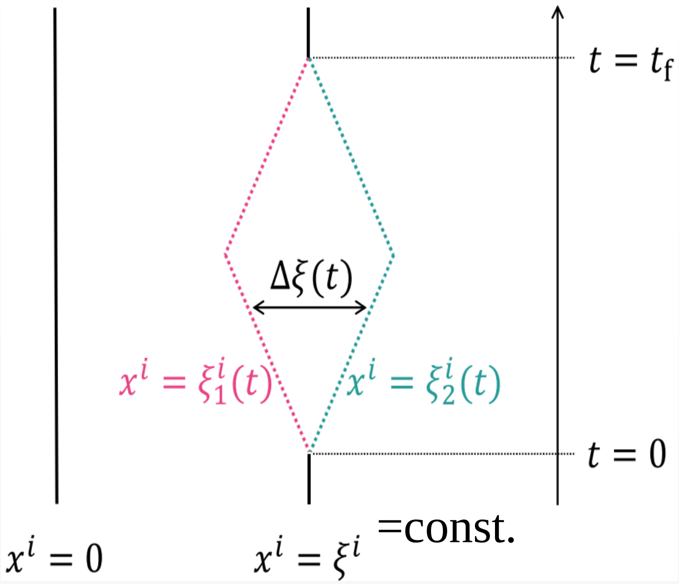


Decoherence rate (2/2)

- We consider the simplest model

$$\Delta\xi(t) = \begin{cases} 2vt & (0 < t < t_f/2) \\ 2v(t_\mathrm{f} - t) & (t_\mathrm{f}/2 < t < t_\mathrm{f}) \end{cases}$$

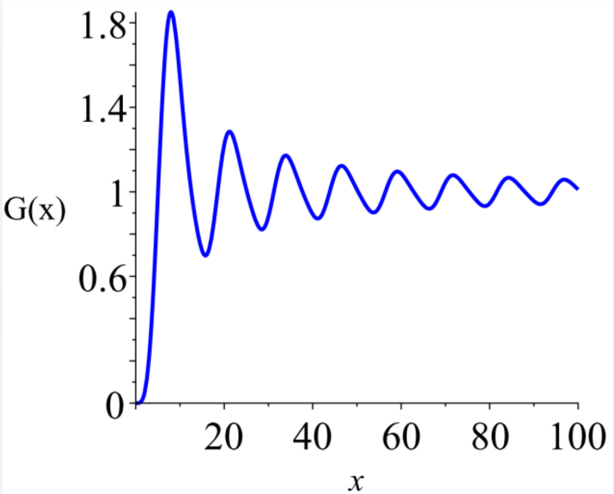
$$(0 < v < 1)$$



- In this setup, we have

$$\Gamma(t_\mathrm{f}) = \frac{2}{5\pi^2}\left(\frac{mv}{M_\mathrm{pl}}\right)^2 (\Omega_\mathrm{m}\xi)^2 G(\Omega_\mathrm{m} t_\mathrm{f}) \sim \frac{2}{5\pi^2}\left(\frac{mv}{M_\mathrm{pl}}\right)^2$$

- $m \gg M_\mathrm{pl}$ is required to realize the graviton-induced decoherence.

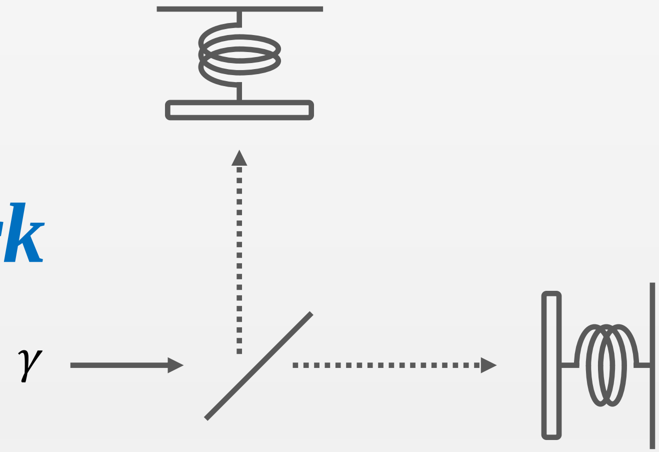


We should consider different setup.

- So far, we have considered spatial superpositions of particles.
  - ✓ $m \gg M_{\rm pl}$ is required to realize the graviton-induced decoherence.
  - ✓ It will be extremely difficult to realize such a setup, however.

- Let us briefly consider another setup (maybe more realistic?)

On-going work



Summary

- We develop the method to compute the graviton-induced decoherence of superpositions of massive particles.

- We focus on two simple toy models. It seems extremely difficult to detect the zero-point fluctuation of gravitons, as expected.

- We will focus on **the decoherence caused by primordial GWs.**

    - What is the detectable value of $\Gamma(t)$?
    - Find similar setup and parameter search.
    - More precise analysis. (opto-mech)