Gravity-induced entanglement in optomechanical systems

Akira Matsumura (Kyushu University)

in collaboration with Kazuhiro Yamamoto (Kyushu University)

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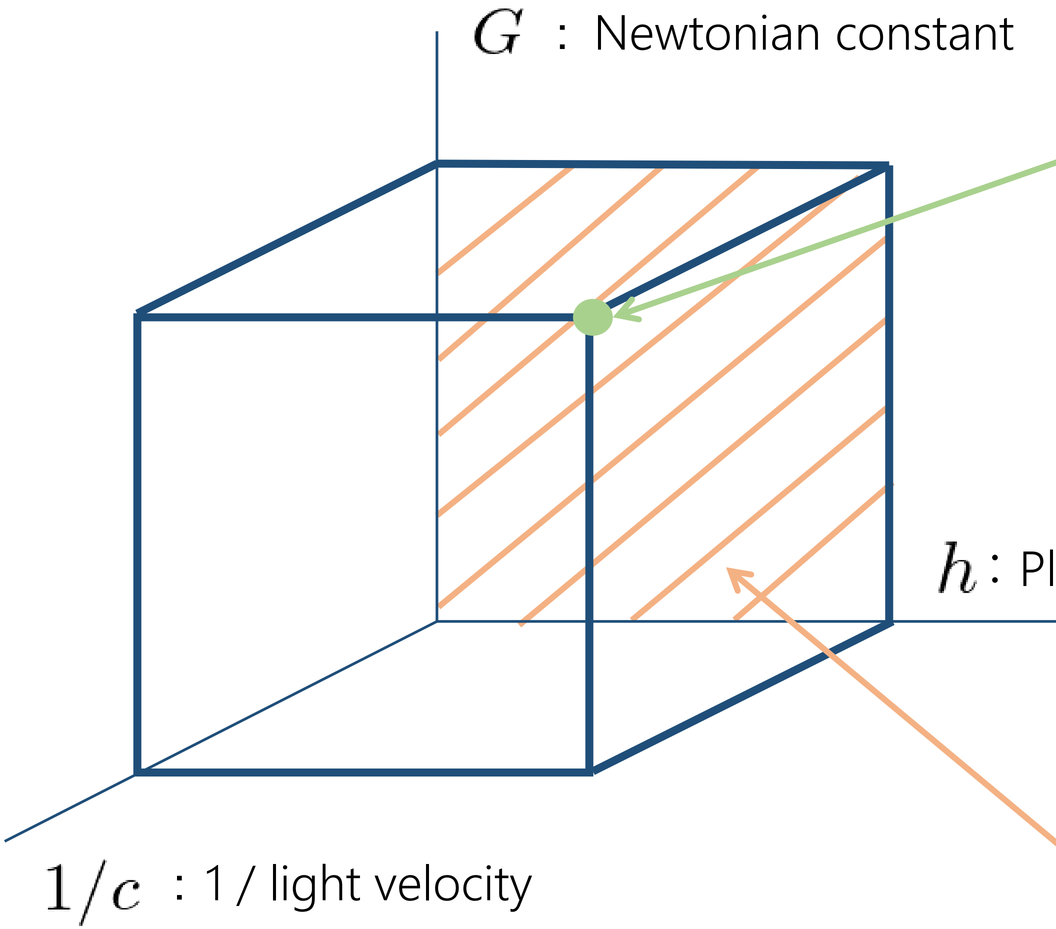
Introduction

☐ Quantum nature of gravity



Quantum gravity
Unification of Quantum mechanics
and General relativity

No widely accepted theory and
experimental evidences

↓

Gravity should be quantized ?

$\cup$

Non-relativistic gravity is described in
quantum mechanics ?

Test of quantumness of non-relativistic gravity by quantum entanglement was proposed.

Bose (2017), Marletto (2017)

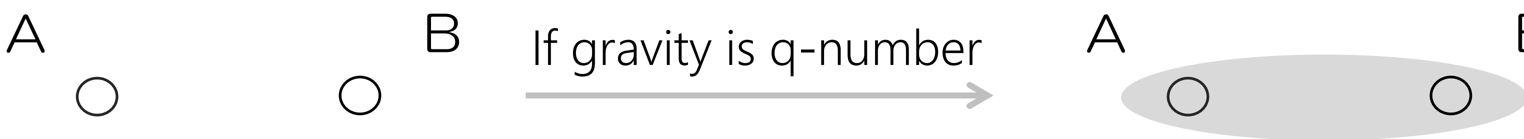
☐Gravity-induced entanglement

Quantum entanglement : Nonlocal correlation known in quantum mechanics

Entangled state

$$|\Psi\rangle_{AB} \neq |\psi\rangle_A |\chi\rangle_B$$

- Interaction given by q-numbers or operators generates
- Not generated by Classical evolution (local hidden variable theories, LOCC)



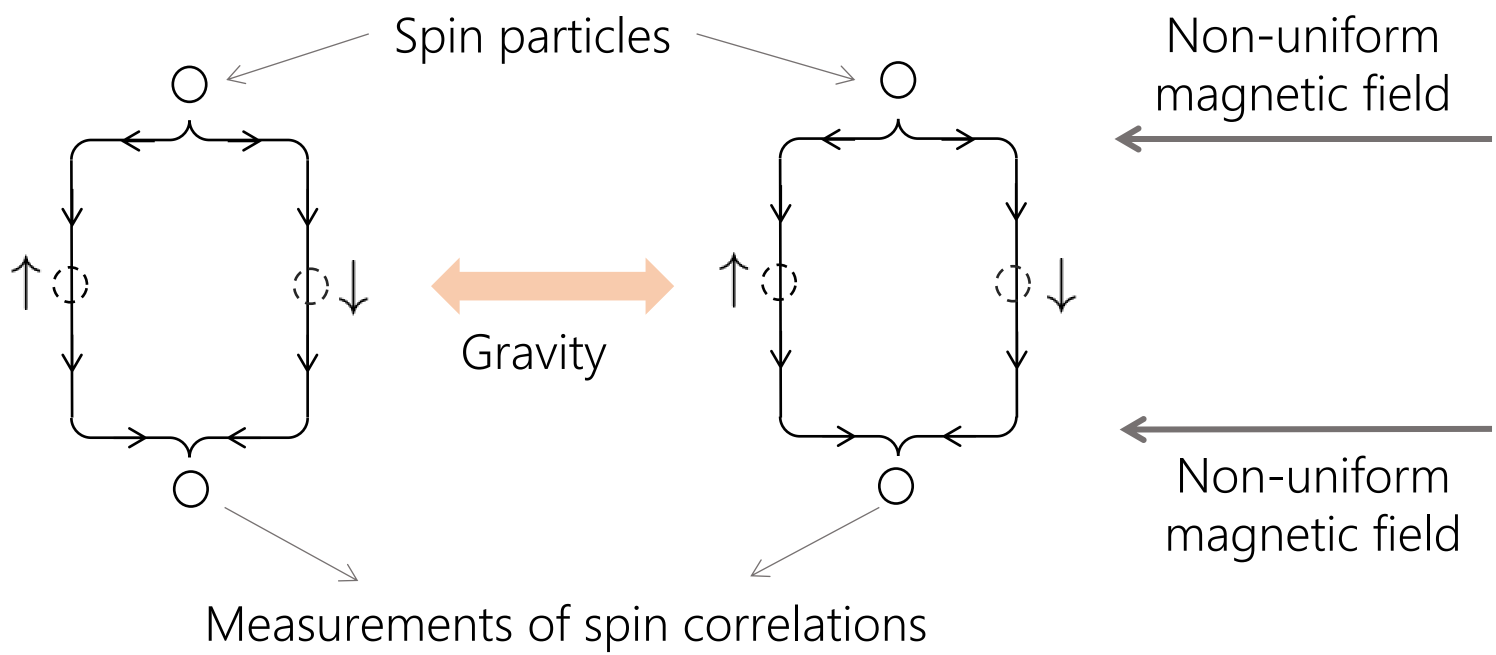
$\neq |\psi\rangle_A |\chi\rangle_B$

Quantum entanglement can be generated

Gravity-induced entanglement can be the evidence of quantumness of gravity

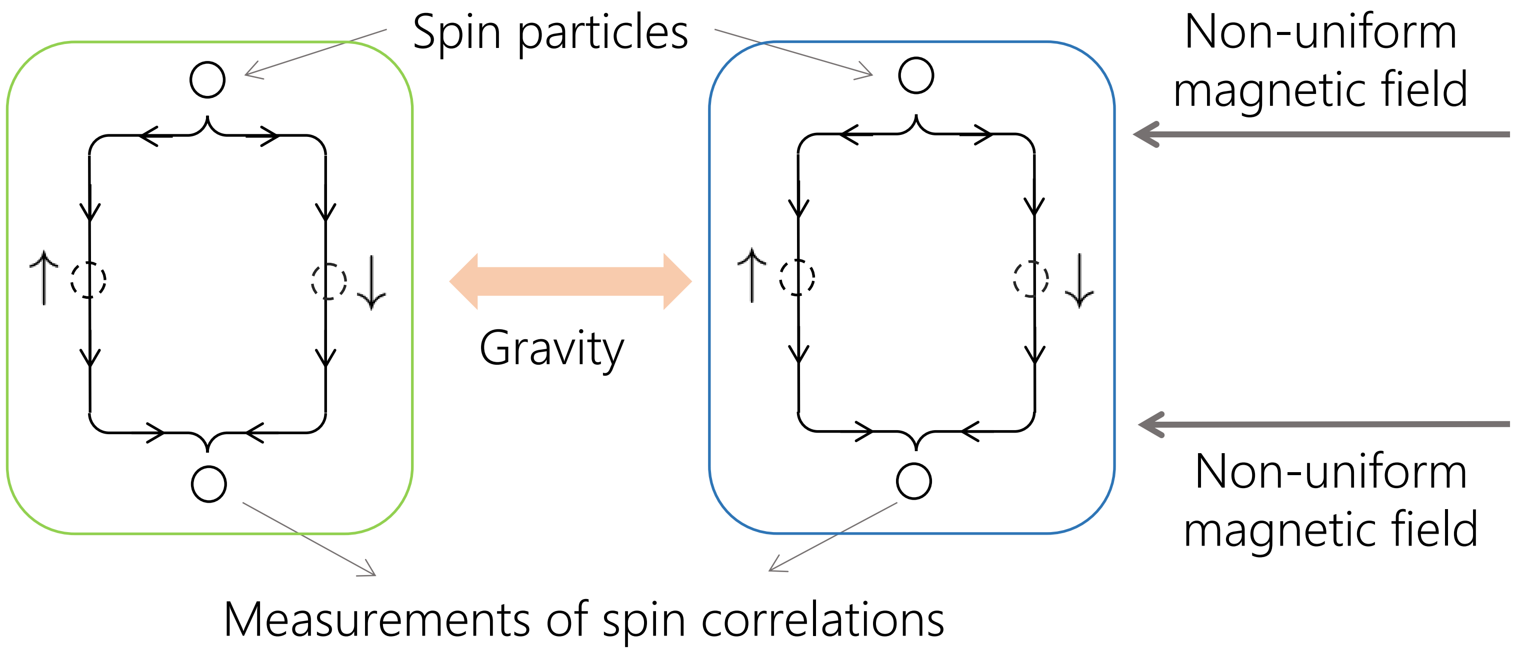
Gravity-induced entanglement in matter-wave interferometers

☐Setup for two matter-wave interferometers   Bose (2017), Marletto (2017)

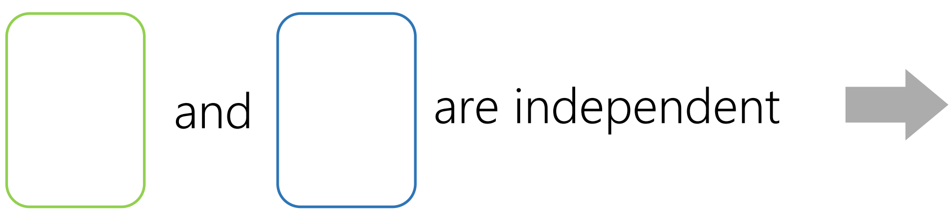


Gravity-induced entanglement in matter-wave interferometers

☐ Setup for two matter-wave interferometers   Bose (2017), Marletto (2017)



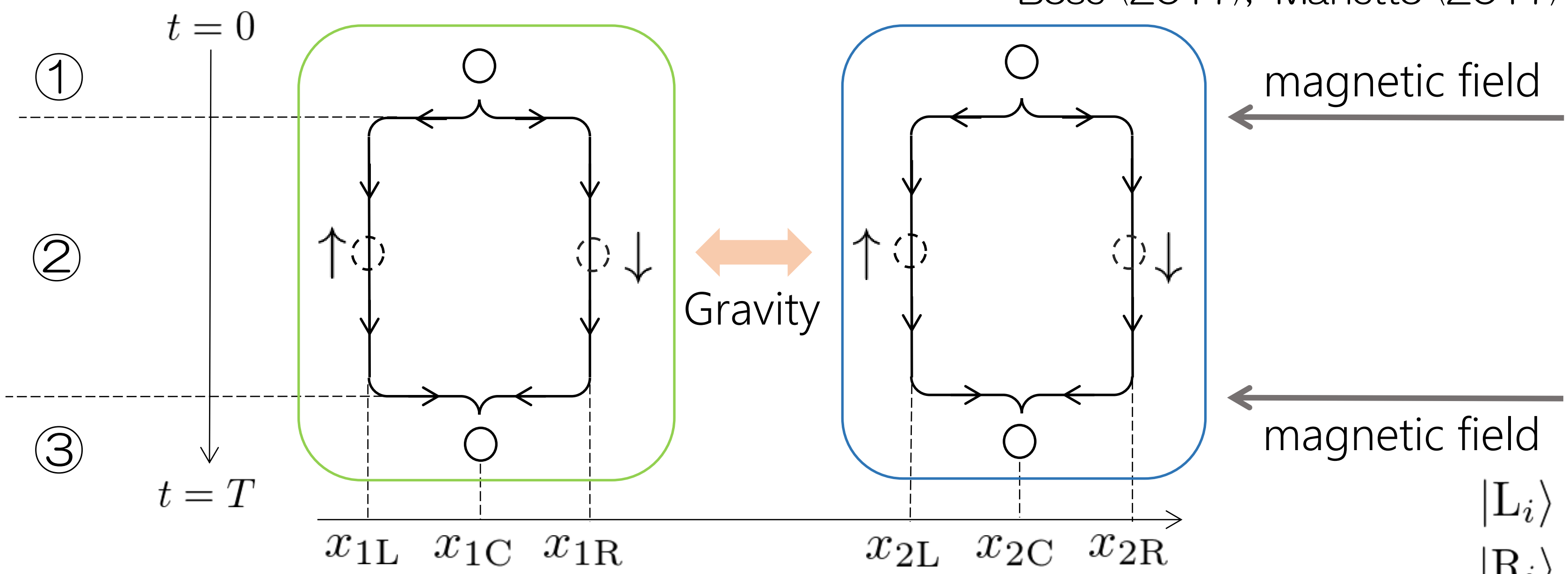
If no gravitational interactions



How to generate quantum entanglement by gravity ?

□Time evolution

Bose (2017), Marletto (2017)



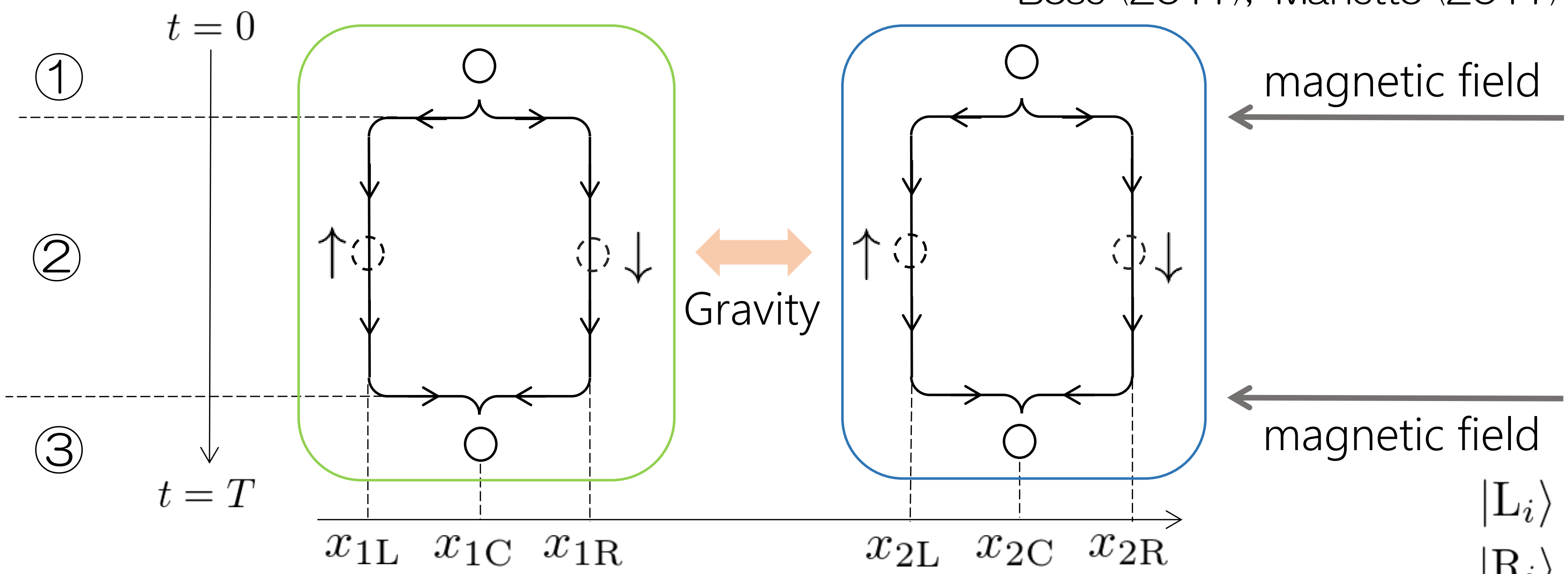
$$|\mathrm{L}_i\rangle = |x_{i\mathrm{L}}\rangle\,|{\uparrow_i}\rangle$$

$$|\mathrm{R}_i\rangle = |x_{i\mathrm{R}}\rangle\,|{\downarrow_i}\rangle$$

① $|x_{1\mathrm{C}}\rangle\left(|{\uparrow_1}\rangle + |{\downarrow_1}\rangle\right)|x_{2\mathrm{C}}\rangle\left(|{\uparrow_2}\rangle + |{\downarrow_2}\rangle\right)$ — magnetic field → $\left(|\mathrm{L}_1\rangle + |\mathrm{R}_1\rangle\right)\left(|\mathrm{L}_2\rangle + |\mathrm{R}_2\rangle\right)$

□Time evolution

Bose (2017), Marletto (2017)



$$|\mathrm{L}_i\rangle = |x_{i\mathrm{L}}\rangle |\uparrow_i\rangle$$
$$|\mathrm{R}_i\rangle = |x_{i\mathrm{R}}\rangle |\downarrow_i\rangle$$

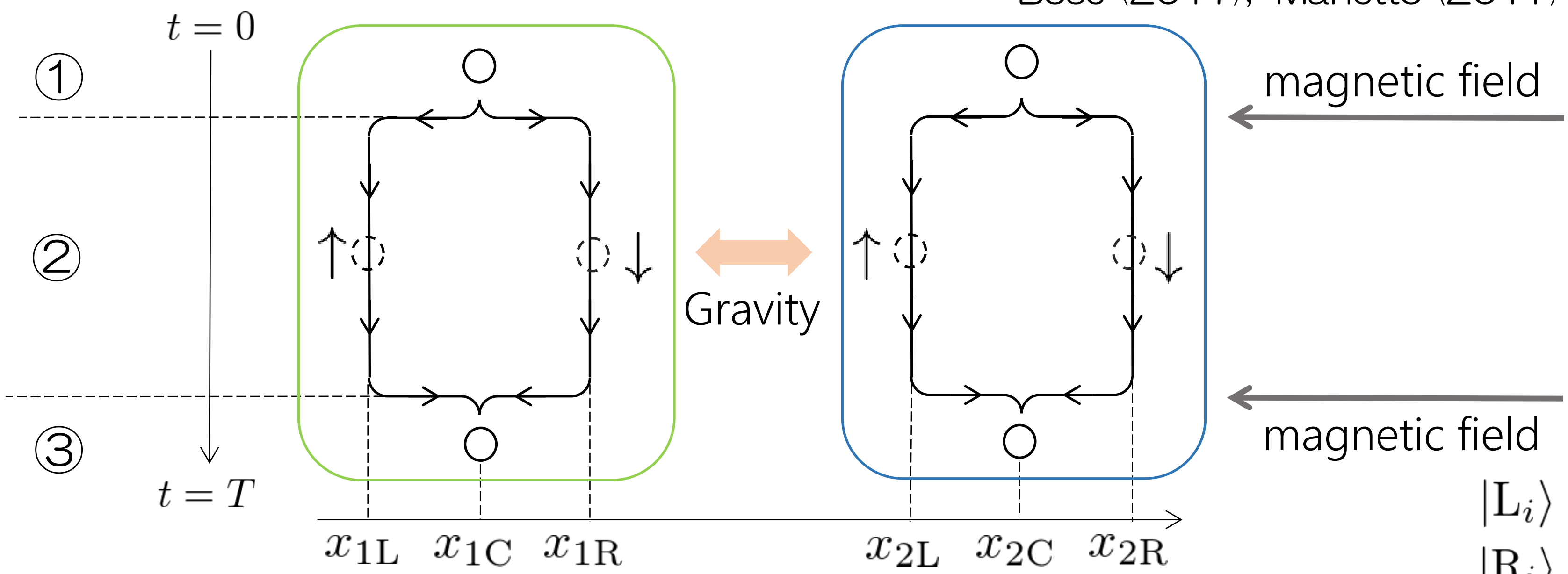
① $|x_{1\mathrm{C}}\rangle (|\uparrow_1\rangle + |\downarrow_1\rangle)\, |x_{2\mathrm{C}}\rangle (|\uparrow_2\rangle + |\downarrow_2\rangle)$ — magnetic field → $(|\mathrm{L}_1\rangle + |\mathrm{R}_1\rangle)(|\mathrm{L}_2\rangle + |\mathrm{R}_2\rangle)$

② $(|\mathrm{L}_1\rangle + |\mathrm{R}_1\rangle)(|\mathrm{L}_2\rangle + |\mathrm{R}_2\rangle)$ $\quad \phi = VT/\hbar \quad V = -\dfrac{Gm_1m_2}{|x_1 - x_2|}$

— Gravity → $e^{i\phi_1} |\mathrm{L}_1\rangle |\mathrm{L}_2\rangle + e^{i\phi_2} |\mathrm{R}_1\rangle |\mathrm{L}_2\rangle + e^{i\phi_3} |\mathrm{L}_1\rangle |\mathrm{R}_2\rangle + e^{i\phi_1} |\mathrm{R}_1\rangle |\mathrm{R}_2\rangle$

☐Time evolution

Bose (2017), Marletto (2017)



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③ $e^{i\phi_1}|\mathrm{L}_1\rangle|\mathrm{L}_2\rangle + e^{i\phi_2}|\mathrm{R}_1\rangle|\mathrm{L}_2\rangle + e^{i\phi_3}|\mathrm{L}_1\rangle|\mathrm{R}_2\rangle + e^{i\phi_1}|\mathrm{R}_1\rangle|\mathrm{R}_2\rangle$

— magnetic field → $|x_{1\mathrm{C}}\rangle|x_{2\mathrm{C}}\rangle\left(e^{i\phi_1}|{\uparrow_1}\rangle|{\uparrow_2}\rangle + e^{i\phi_2}|{\downarrow_1}\rangle|{\uparrow_2}\rangle + e^{i\phi_3}|{\uparrow_1}\rangle|{\downarrow_2}\rangle + e^{i\phi_1}|{\downarrow_1}\rangle|{\downarrow_2}\rangle\right)$

□Generation of quantum entanglement

$$e^{i\phi_1}|\uparrow_1\rangle|\uparrow_2\rangle + e^{i\phi_2}|\downarrow_1\rangle|\uparrow_2\rangle + e^{i\phi_3}|\uparrow_1\rangle|\downarrow_2\rangle + e^{i\phi_1}|\downarrow_1\rangle|\downarrow_2\rangle$$

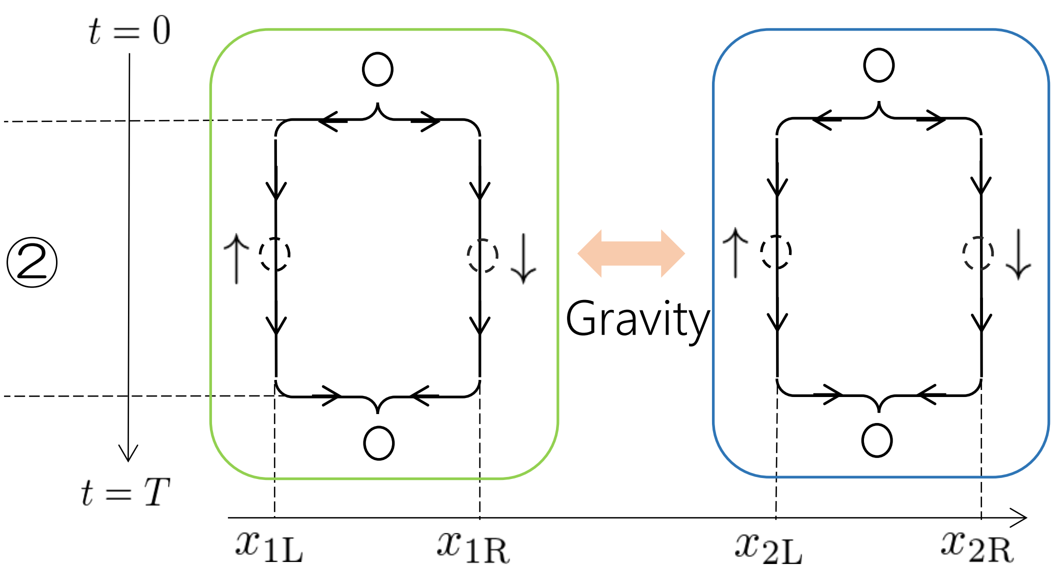
$$\phi = VT/\hbar \quad V = -\frac{Gm_1m_2}{|x_1 - x_2|}$$

$$\sim(|\uparrow_1\rangle + e^{i\phi_2}|\downarrow_1\rangle)|\uparrow_2\rangle + (|\uparrow_1\rangle + |\downarrow_1\rangle)|\downarrow_2\rangle$$

$|x_{1R} - x_{2L}| \ll$ other distances

$\phi_2 \neq 2n\pi$ then, entangled state $\neq |\psi_1\rangle|\chi_2\rangle$

typically $\phi \sim \frac{ct}{d}\frac{m_1m_2}{M_{pl}^2} \sim 100\left(\frac{t}{1\mathrm{s}}\right)\left(\frac{10^{-6}\mathrm{m}}{d}\right)\left(\frac{m}{10^{-11}\mathrm{g}}\right)^2$



② $(|L_1\rangle + |R_1\rangle)(|L_2\rangle + |R_2\rangle)$

Gravity

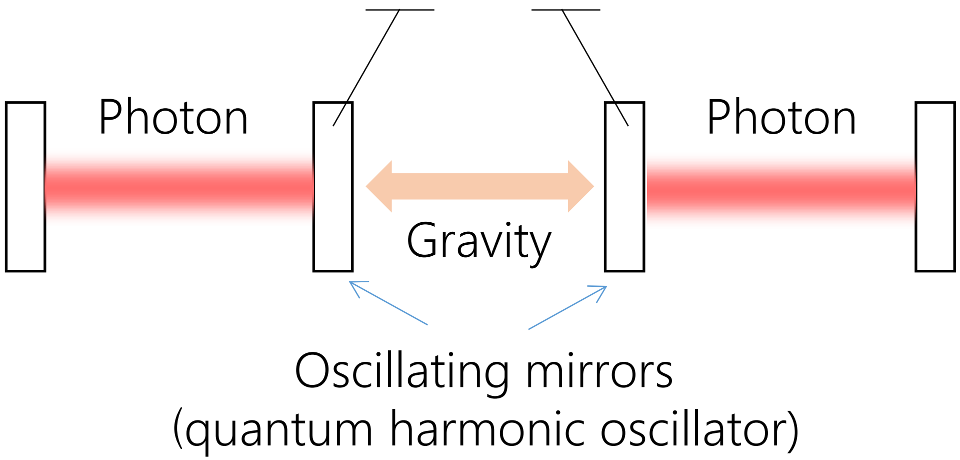
$$e^{i\phi_1}|L_1\rangle|L_2\rangle + e^{i\phi_2}|L_1\rangle|R_2\rangle + e^{i\phi_3}|R_1\rangle|L_2\rangle + e^{i\phi_1}|R_1\rangle|R_2\rangle$$

The value of the interaction changes ∵ q-number

➡ Gravity described by q-number generates quantum entanglement

Gravity-induced entanglement in optomechanical systems

☐Basic model for optomechanical systems



For optomechanical interactions,

Balushi (2018) : perturbative analysis

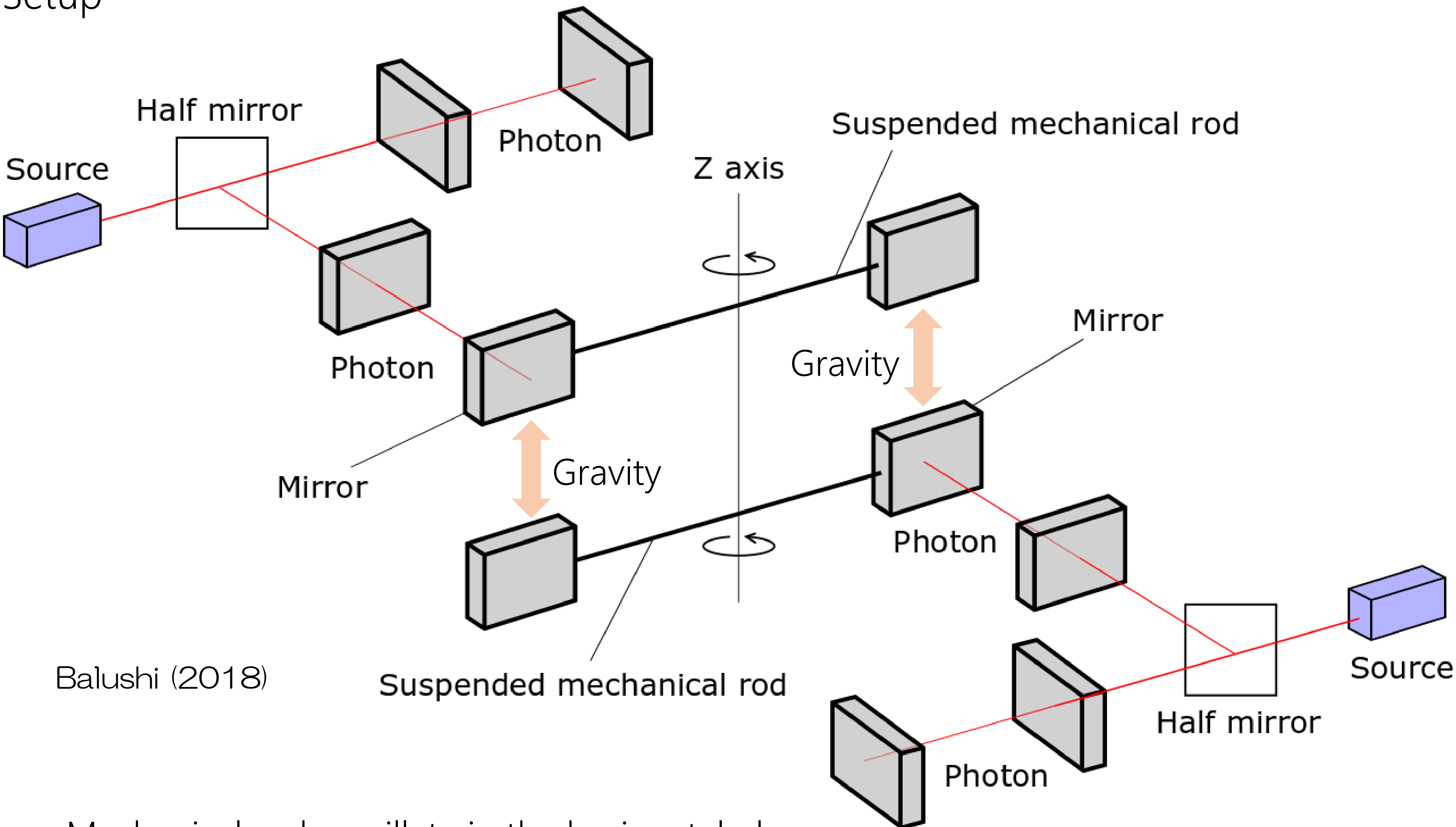
Miao (2020) : linearized analysis (dissipative system)

Entanglement of oscillators or photons

Our research

- non-perturbative and nonlinear analysis for optomechanical couplings
- mechanism of gravity-induced entanglement of photons
- comparison with quantum decoherence of photons

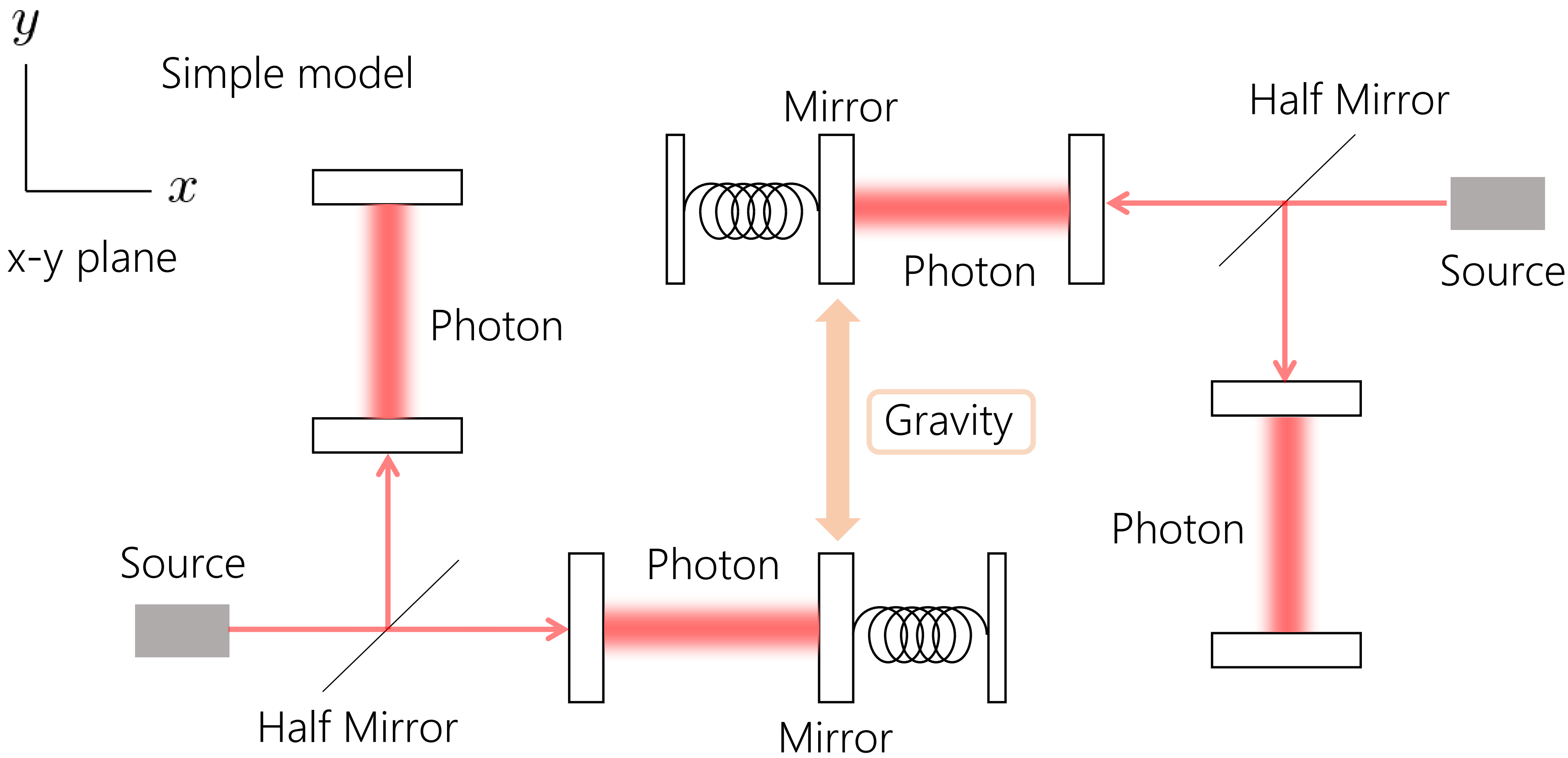
□Setup



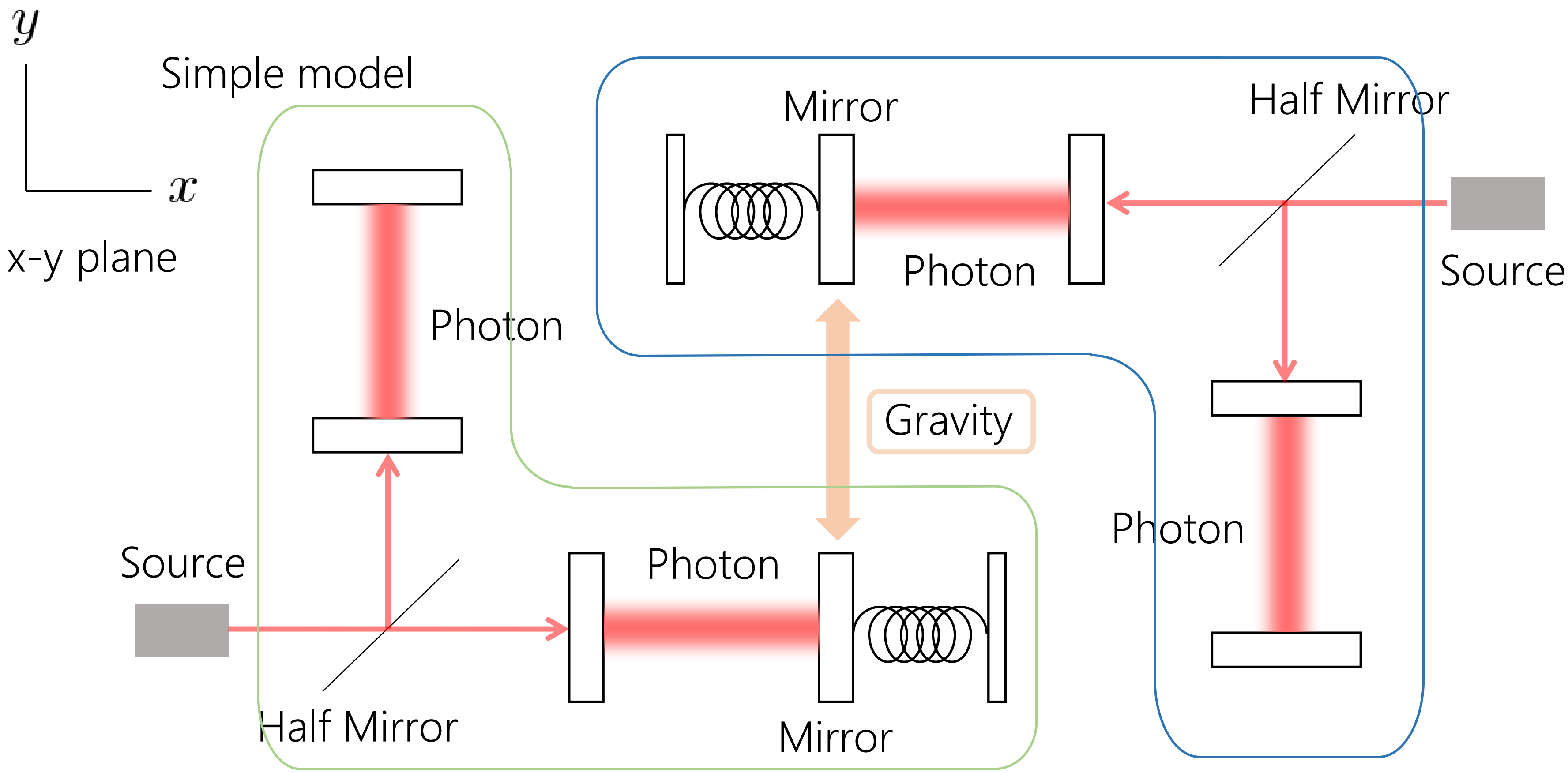
Mechanical rods oscillate in the horizontal plane

Gravity acts among mirrors attached to the rods

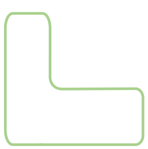
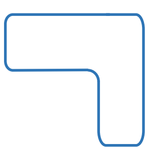
□Analysis for a simple model



□Analysis for a simple model



If no gravitational interactions

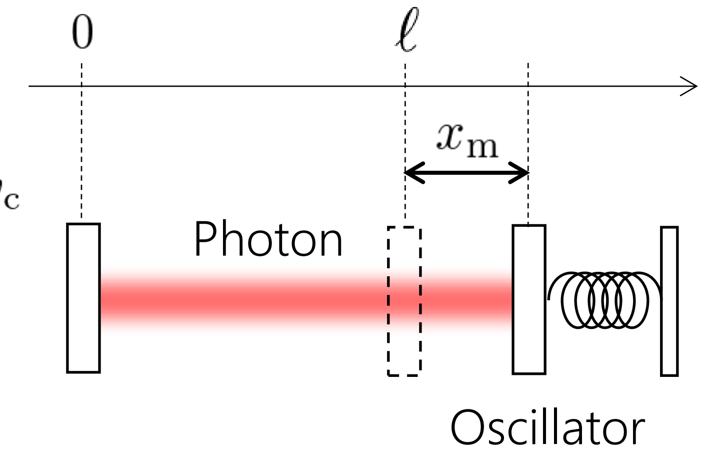
 and  are independent → How to generate quantum entanglement by gravity ?

In the following, I explain the Hamiltonian of our simple model.

□Optomechanical Hamiltonian

Frequency of photon

Light velocity c



$$\omega_{\mathrm{c}} = \frac{n\pi c}{\ell} \xrightarrow[\ell \to \ell + x_{\mathrm{m}}]{} \omega(x_{\mathrm{m}}) = \frac{n\pi c}{\ell + x_{\mathrm{m}}} \sim \omega_{\mathrm{c}} - \frac{x_{\mathrm{m}}}{\ell}\omega_{\mathrm{c}}$$

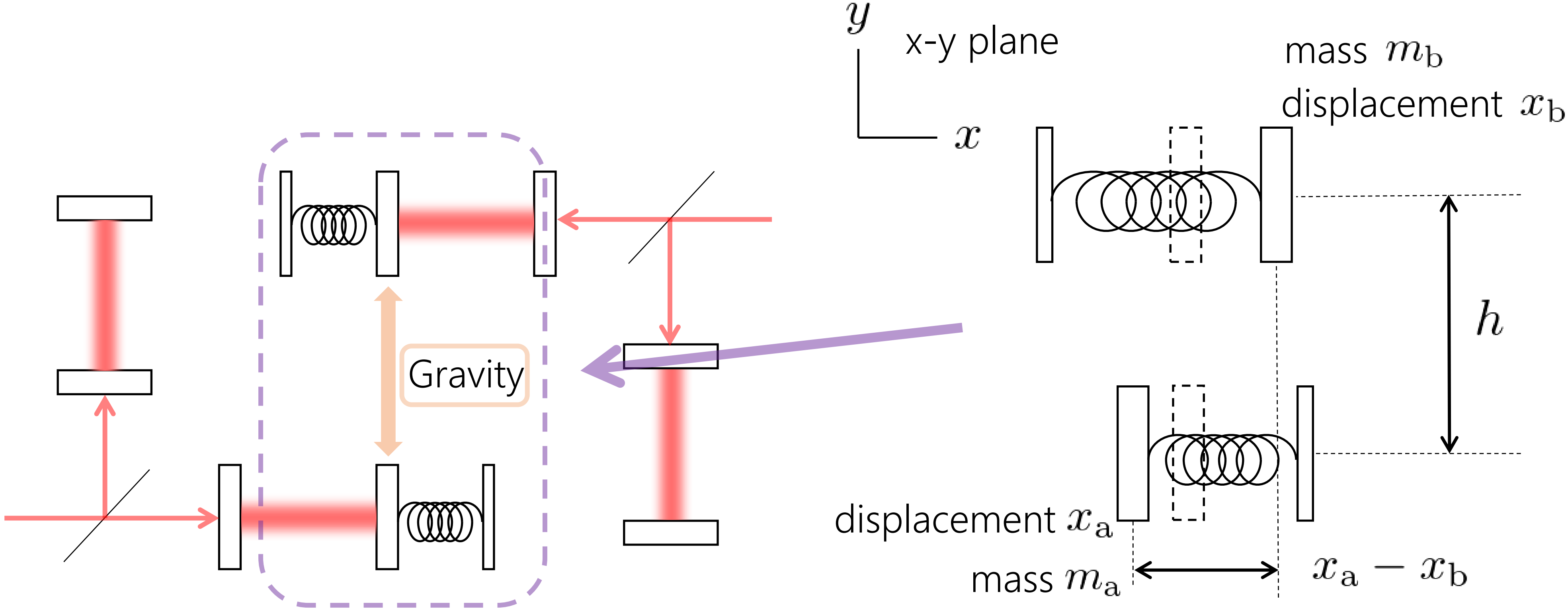
Hamiltonian of photon

$$\hat{H}_{\mathrm{c}} = \hbar\omega_{\mathrm{c}}\hat{c}^{\dagger}\hat{c} \longrightarrow \hat{H}_{\mathrm{c}}(\hat{x}_{\mathrm{m}}) = \hbar\omega(\hat{x}_{\mathrm{m}})\hat{c}^{\dagger}\hat{c} \sim \hbar\omega_{\mathrm{c}}\hat{c}^{\dagger}\hat{c} - \frac{\hbar\omega_{\mathrm{c}}}{\ell}\hat{x}_{\mathrm{m}}\hat{c}^{\dagger}\hat{c}$$

Optomechanical Hamiltonian

$$\hat{H}_{\mathrm{opt.mech.}} = \underbrace{\hbar\omega_{\mathrm{c}}\hat{c}^{\dagger}\hat{c}}_{\text{photon}} + \underbrace{\frac{\hat{p}_{\mathrm{m}}^2}{2m} + \frac{1}{2}m\Omega_{\mathrm{m}}^2\hat{x}_{\mathrm{m}}^2}_{\text{oscillator}} - \underbrace{\frac{\hbar\omega_{\mathrm{c}}}{\ell}\hat{x}_{\mathrm{m}}\hat{c}^{\dagger}\hat{c}}_{\text{photon-oscillator}}$$

□Gravitational interaction : Newtonian gravity



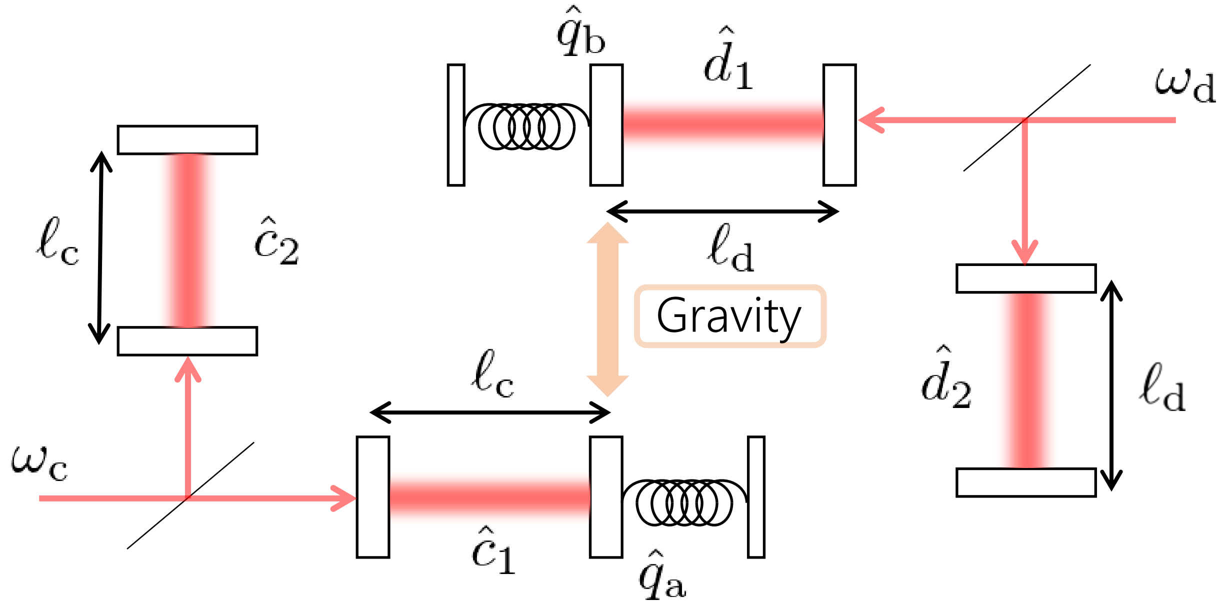
Newtonian potential

$$-\frac{G m_{\mathrm{a}} m_{\mathrm{b}}}{\sqrt{h^2 + (\hat{x}_{\mathrm{a}} - \hat{x}_{\mathrm{b}})^2}} \sim \mathrm{const} + \underline{\frac{G m_{\mathrm{a}} m_{\mathrm{b}}}{2h^3}(\hat{x}_{\mathrm{a}} - \hat{x}_{\mathrm{b}})^2}$$

oscillator-oscillator (cross term)

□Total Hamiltonian

$$\hat{H} = \hat{H}_{\rm a} + \hat{H}_{\rm b} + \hat{H}_{\rm a,b} + \hat{H}_{\rm c} + \hat{H}_{\rm d} + \hat{H}_{\rm a,c_1} + \hat{H}_{\rm b,d_1}$$



Harmonic oscillators (dimensionless, quadratic)

$$\begin{cases} \hat{H}_{\rm a} = \dfrac{\hbar\omega_{\rm a}}{2}(\hat{p}_{\rm a}^2 + \hat{q}_{\rm a}^2) & \omega_{\rm a}^2 = \Omega_{\rm a}^2 + \dfrac{Gm_{\rm b}}{h^3} \\ \hat{H}_{\rm b} = \dfrac{\hbar\omega_{\rm b}}{2}(\hat{p}_{\rm b}^2 + \hat{q}_{\rm b}^2) & \omega_{\rm b}^2 = \Omega_{\rm b}^2 + \dfrac{Gm_{\rm a}}{h^3} \end{cases}$$

oscillator-oscillator
(quadratic, couple by gravity)

$$\hat{H}_{\rm a,b} = -\hbar\gamma\hat{q}_{\rm a}\hat{q}_{\rm b} \quad \gamma = \frac{G}{h^3}\sqrt{\frac{m_{\rm a}m_{\rm b}}{\omega_{\rm a}\omega_{\rm b}}}$$

Photons (quadratic)

$$\begin{cases} \hat{H}_{\rm c} = \hbar\omega_{\rm c}(\hat{c}_1^\dagger\hat{c}_1 + \hat{c}_2^\dagger\hat{c}_2) \\ \hat{H}_{\rm d} = \hbar\omega_{\rm d}(\hat{d}_1^\dagger\hat{d}_1 + \hat{d}_2^\dagger\hat{d}_2) \end{cases}$$

photon-oscillator (third order, nonlinear)

$$\begin{cases} \hat{H}_{\rm a,c_1} = -\hbar\lambda_{\rm a}\omega_{\rm a}\hat{c}_1^\dagger\hat{c}_1\hat{q}_{\rm a} & \lambda_{\rm a} = \dfrac{\omega_{\rm c}}{\omega_{\rm a}\ell_{\rm c}}\sqrt{\dfrac{\hbar}{2m_{\rm a}\omega_{\rm a}}} \\ \hat{H}_{\rm b,d_1} = -\hbar\lambda_{\rm b}\omega_{\rm b}\hat{d}_1^\dagger\hat{d}_1\hat{q}_{\rm b} & \lambda_{\rm b} = \dfrac{\omega_{\rm d}}{\omega_{\rm b}\ell_{\rm d}}\sqrt{\dfrac{\hbar}{2m_{\rm b}\omega_{\rm b}}} \end{cases}$$

□Conservation of photon numbers and diagonalization

photon-oscillator (third order, nonlinear)

$$\begin{cases} \hat{H}_{\mathrm{a,c_1}} = -\hbar\lambda_{\mathrm{a}}\omega_{\mathrm{a}}\hat{c}_1^{\dagger}\hat{c}_1\hat{q}_{\mathrm{a}} \\ \hat{H}_{\mathrm{b,d_1}} = -\hbar\lambda_{\mathrm{b}}\omega_{\mathrm{b}}\hat{d}_1^{\dagger}\hat{d}_1\hat{q}_{\mathrm{b}} \end{cases}$$

Conservation of photon numbers

$$[\hat{N}_{\mathrm{ph}}, \hat{H}_{\mathrm{a,c_1}}] = [\hat{N}_{\mathrm{ph}}, \hat{H}_{\mathrm{b,d_1}}] = 0$$

$$\hat{N}_{\mathrm{ph}} = \hat{c}_1^{\dagger}\hat{c}_1, \hat{d}_1^{\dagger}\hat{d}_1, \hat{c}_2^{\dagger}\hat{c}_2, \hat{d}_2^{\dagger}\hat{d}_2$$

➡ Hamiltonian is block diagonalizable

For an eigenstate of photon numbers $\hat{c}_1^{\dagger}\hat{c}_1 |n, n'\rangle_{\mathrm{c}} = n |n, n'\rangle_{\mathrm{c}}$ $\hat{c}_2^{\dagger}\hat{c}_2 |n, n'\rangle_{\mathrm{c}} = n' |n, n'\rangle_{\mathrm{c}}$

as well as the system d

$$\hat{H} |n, n'\rangle_{\mathrm{c}} |m, m'\rangle_{\mathrm{d}} |\psi\rangle_{\mathrm{a,b}}$$

$$= |n, n'\rangle_{\mathrm{c}} |m, m'\rangle_{\mathrm{d}} (E_{\mathrm{ph}} + \hat{H}_{\mathrm{a}} + \hat{H}_{\mathrm{b}} + \hat{H}_{\mathrm{a,b}} - \hbar\lambda_{\mathrm{a}}\omega_{\mathrm{a}}n\hat{q}_{\mathrm{a}} - \hbar\lambda_{\mathrm{b}}\omega_{\mathrm{b}}m\hat{q}_{\mathrm{b}}) |\psi\rangle_{\mathrm{a,b}}$$

Linear order for oscillators

$$E_{\mathrm{ph}} = \hbar\omega_{\mathrm{c}}(n + n') + \hbar\omega_{\mathrm{d}}(m + m')$$

$\hat{H}_{\mathrm{a}} + \hat{H}_{\mathrm{b}} + \hat{H}_{\mathrm{a,b}}$ Harmonic oscillators+gravity

quadratic Hamiltonian

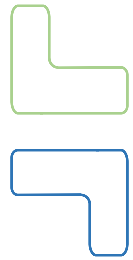
The part of oscillators (operators) is
at most quadratic order
one can solve this system

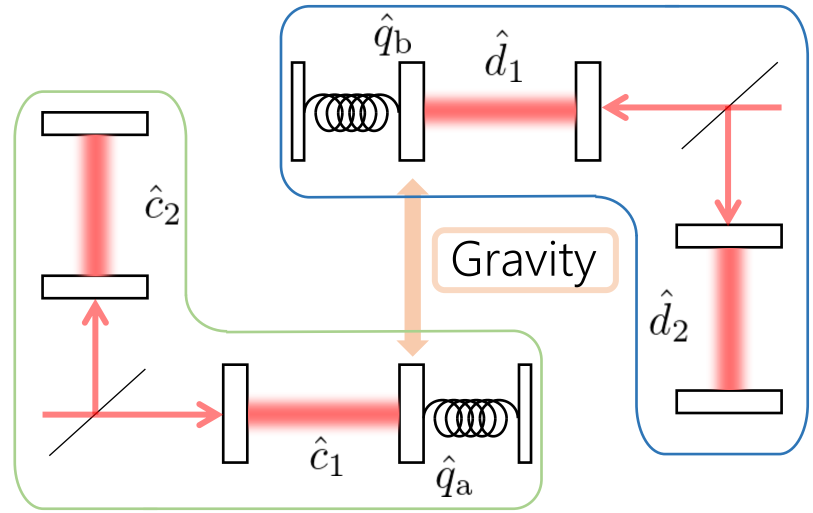
□Entanglement between two photons

Initial state   Superposition of a single photon   Ground state of an oscillator

$$|\Psi_{\rm in}\rangle = \frac{1}{\sqrt{2}}(|0,1\rangle_{\rm c} + |1,0\rangle_{\rm c}) \otimes |0\rangle_{\rm a}$$

$$\otimes \frac{1}{\sqrt{2}}(|0,1\rangle_{\rm d} + |1,0\rangle_{\rm d}) \otimes |0\rangle_{\rm b}$$





$|0,1\rangle_{\rm c} = \hat{c}_2^\dagger |0\rangle_{\rm c}$ $|1,0\rangle_{\rm c} = \hat{c}_1^\dagger |0\rangle_{\rm c}$

as well as the system d

Negativity

$$\mathcal{N} = \sum_{\lambda_i < 0} |\lambda_i|$$

λ_i : eigenvalues of partial transposed matrix $\rho_{\rm AB}^{\rm T_A}$

$\mathcal{N} > 0 \implies \rho_{\rm AB}$: entangled

Horodecki (1996), Peres (1996), Vidal(2002)

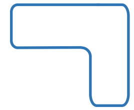
In general, it is difficult to compute the negativity, however,

$$\rho_{\rm c,d}(t) = {\rm Tr}_{\rm a,b}\Big[|\Psi(t)\rangle\langle\Psi(t)|\Big]$$

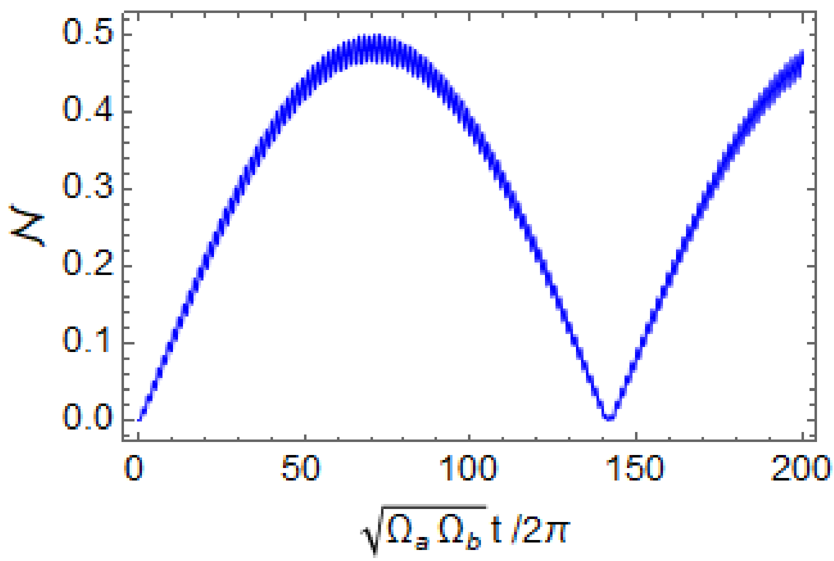
4×4 matrix ∵ photon numbers are conserved

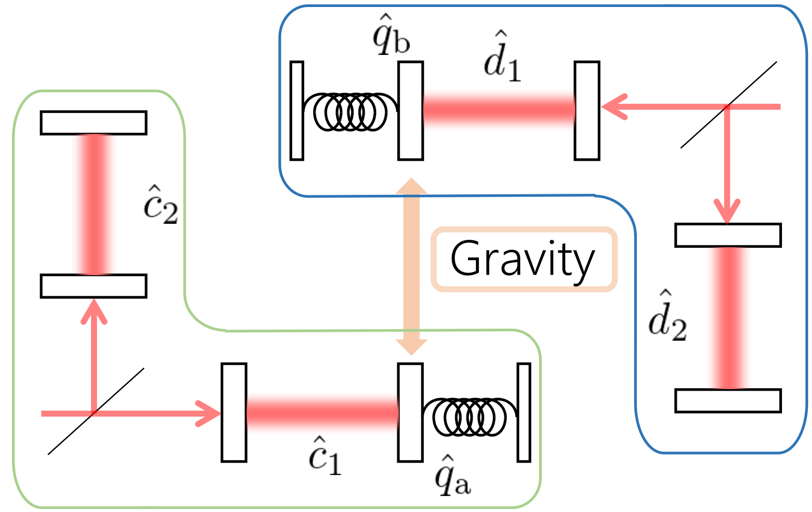
easy to compute

□Negativity between single photons

Negativity between each photon in  and 

small couplings $\lambda_{\rm a}, \lambda_{\rm b}, \gamma/\sqrt{\omega_{\rm a}\omega_{\rm b}} \ll 1$





- Negativity approaches 0.5 → maximum for a 4×4 matrix
- The time scale realizing the maximum $\quad t_{\rm s} = \dfrac{\pi}{\gamma\lambda_{\rm a}\lambda_{\rm b}}\left(1 - \dfrac{\gamma^2}{\omega_{\rm a}\omega_{\rm b}}\right) \quad \gamma \propto G$

□Generating mechanism of entanglement

Potential of the oscillators

$$\gamma \propto G$$

$$\hat{\boldsymbol{q}} = \begin{bmatrix} \hat{q}_a \\ \hat{q}_b \end{bmatrix} \quad \mathcal{M} = \begin{bmatrix} \omega_a & -\gamma \\ -\gamma & \omega_b \end{bmatrix} \quad \hat{\boldsymbol{j}} = \begin{bmatrix} \lambda_{\mathrm{a}}\omega_{\mathrm{a}}\, \hat{c}_1^\dagger \hat{c}_1 \\ \lambda_{\mathrm{b}}\omega_{\mathrm{b}}\, \hat{d}_1^\dagger \hat{d}_1 \end{bmatrix}$$

$$\hat{V}_{\mathrm{osc}}/\hbar = \frac{1}{2}\hat{\boldsymbol{q}}^{\mathrm{T}}\mathcal{M}\hat{\boldsymbol{q}} - \hat{\boldsymbol{j}}^{\mathrm{T}}\hat{\boldsymbol{q}}$$

Harmonic oscillator + gravity + photon-oscillator

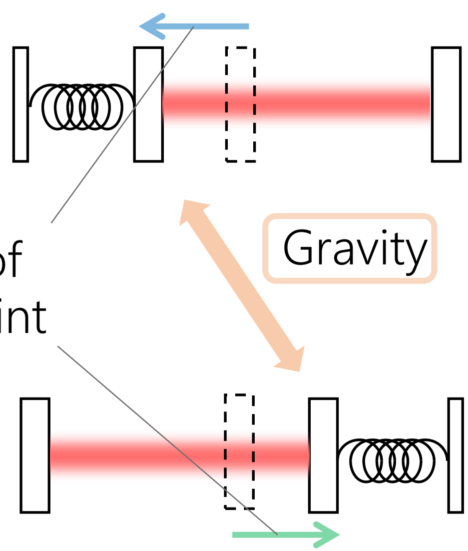
$$= \frac{1}{2}(\hat{\boldsymbol{q}} - \mathcal{M}^{-1}\hat{\boldsymbol{j}})^{\mathrm{T}}\mathcal{M}(\hat{\boldsymbol{q}} - \mathcal{M}^{-1}\hat{\boldsymbol{j}}) - \frac{1}{2}\hat{\boldsymbol{j}}^{\mathrm{T}}\mathcal{M}^{-1}\hat{\boldsymbol{j}} \supset \underline{\frac{\pi}{t_{\mathrm{s}}}\hat{c}_1^\dagger\hat{c}_1\hat{d}_1^\dagger\hat{d}_1} \qquad t_{\mathrm{s}}^{-1} \propto G$$

photon-photon

$$e^{-i\pi\frac{t}{t_{\mathrm{s}}}\hat{c}_1^\dagger\hat{c}_1\hat{d}_1^\dagger\hat{d}_1}(|0,1\rangle_{\mathrm{c}} + |1,0\rangle_{\mathrm{c}})(|0,1\rangle_{\mathrm{d}} + |1,0\rangle_{\mathrm{d}})$$

$$\xrightarrow{t = t_{\mathrm{s}}} (|0,1\rangle_{\mathrm{c}} + |1,0\rangle_{\mathrm{c}})|0,1\rangle_{\mathrm{d}} + (|0,1\rangle_{\mathrm{c}} - |1,0\rangle_{\mathrm{c}})|1,0\rangle_{\mathrm{d}}$$

maximal entanglement



displacement of equilibrium point

equilibrium point changes by radiation pressure
it depends on separated photon states because of gravity

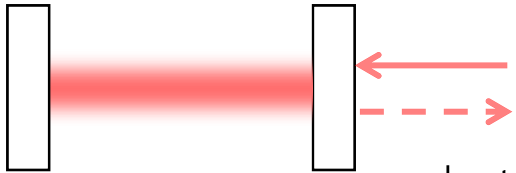
↓

frequency of each photon changes
(phase of photon state changes)

↓

Quantum entanglement generates

□Decoherence for photons



photon leak

coupling

$$\hat{B}^\dagger\,\hat{c} + \hat{B}\,\hat{c}^\dagger \qquad \hat{B} = i\hbar\int_0^\infty d\omega\, g(\omega)\hat{b}(\omega)$$

$$\hat{B}^\dagger\hat{c}\,|1\rangle_{\rm c}\,|0\rangle_{\rm B} = |0\rangle_{\rm c}\,\hat{B}^\dagger\,|0\rangle_{\rm B}$$

conversion between
a photon and an external field

For simplicity, we stimulate this phenomenon by GKSL equation

Gorini(1976), Lindblad(1976), Breuer(2007)

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[\hat{H},\rho(t)] + \kappa_{\rm c}\sum_{i=1,2}[2\hat{c}_i\rho\hat{c}_i^\dagger - \{\hat{c}_i^\dagger\hat{c}_i,\rho\}] + \kappa_{\rm c}\sum_{i=1,2}[2\hat{d}_i\rho\hat{d}_i^\dagger - \{\hat{d}_i^\dagger\hat{d}_i,\rho\}]$$

von Neumann eq
(Schrödinger eq)

photon leak and amplitude damping

$\kappa_{\rm c}$ dissipation rate

more appropriately, we need the quantum master eq
or quantum Brownian motion eq

In this analysis, the negativity between single photons is given as

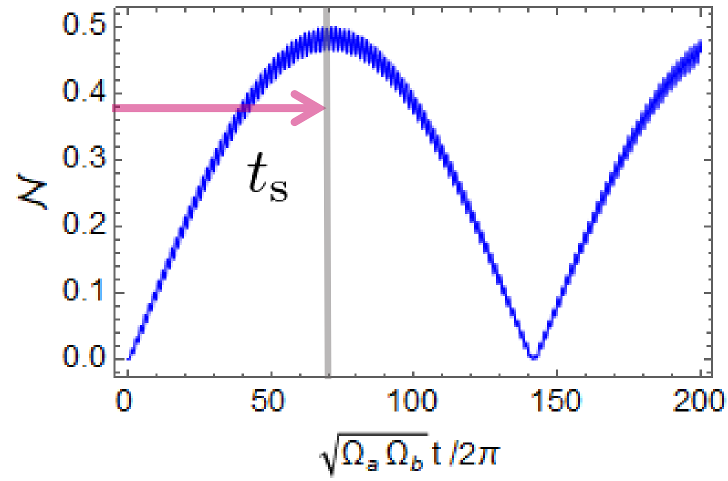
$$\mathcal{N} \longrightarrow e^{-2\kappa_{\rm c}t}\mathcal{N}$$

the negativity typically remains
until a single photon decays

□Entanglement generation vs Decoherence

Required condition for generation of entanglement

$$\kappa_c t_s < 1 \qquad t_s = \frac{\pi}{\gamma \lambda_a \lambda_b}\left(1 - \frac{\gamma^2}{\omega_a \omega_b}\right)$$



$\mathcal{N} > 0 \Rightarrow \rho_{AB}$: entangled

without gravity,

$\Omega_a\ \Omega_b$: frequency of oscillators

$\Lambda_a\ \Lambda_b$: optomechanical couplings

$$\frac{\kappa_c}{\sqrt{\Omega_a \Omega_b}} < \frac{\sqrt{g_a g_b}\Lambda_a \Lambda_b}{1 + g_a + g_b}$$

$$g_a = \frac{G m_b}{h^3 \Omega_a^2} \qquad g_b = \frac{G m_a}{h^3 \Omega_b^2}$$

parameters characterizing the contribution of gravity

$\Lambda_a \sim \Lambda_b \sim O(10^{-1})$ $\qquad g_a \sim g_b \sim O(10^{-1})$ small but extremely large

$$\Rightarrow \frac{\kappa_c}{\sqrt{\Omega_a \Omega_b}} < O(10^{-3})$$

review paper Aspelmeyer(2014)

$O(10^{-2})$ It seems to be difficult in present experiments

Summary

☐We investigated the gravity-induced entanglement in optomechanical systems.

☐We solved the nonlinear dynamics nonperturbatively by using the conservation law of photon numbers.

☐We found the maximal generation of entanglement between single photons.

☐We clarified the generation mechanism of entanglement of photons.

☐From the comparison with decoherence time of photons, we give the condition for the dissipation rate to generate the entanglement sufficiently.