Review of birefringence study in KAGRA

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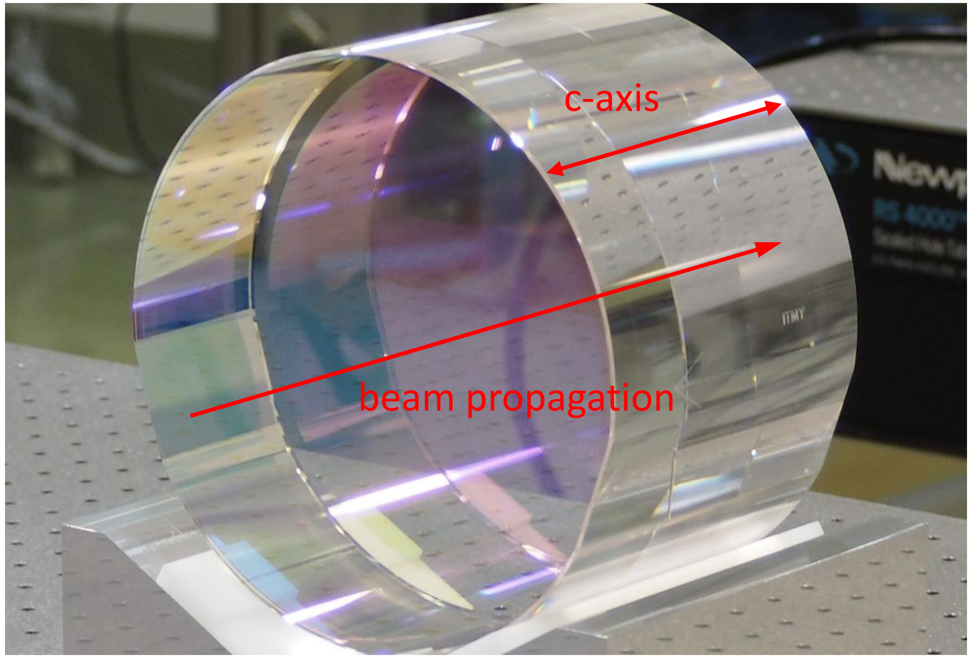
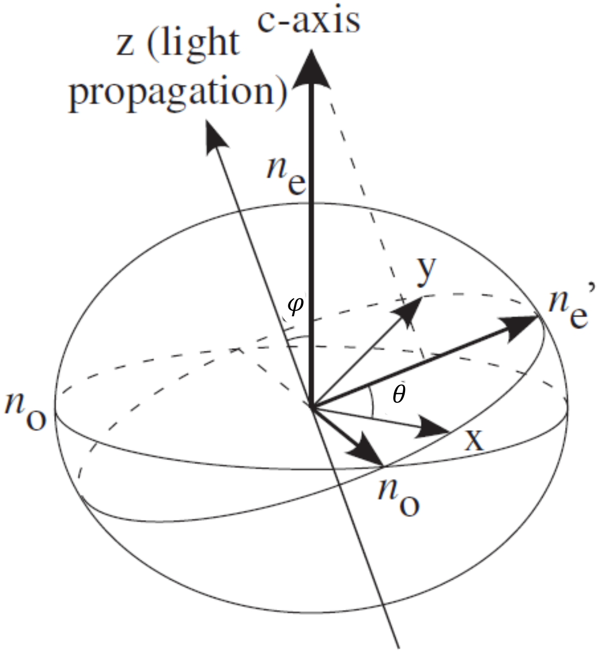
Sapphire test masses for KAGRA

In KAGRA, the four test masses are sapphire.
Other optics are fused silica, e.g., BS, IMC, PRMs and SRMs.

- High thermal conductivity
- Low absorption at cryogenic temperature
- Good optical properties at 1064nm
- However, sapphire is birefringent.

$n_o = 1.754 \quad n_e = 1.747 \quad \Delta n = 0.007$

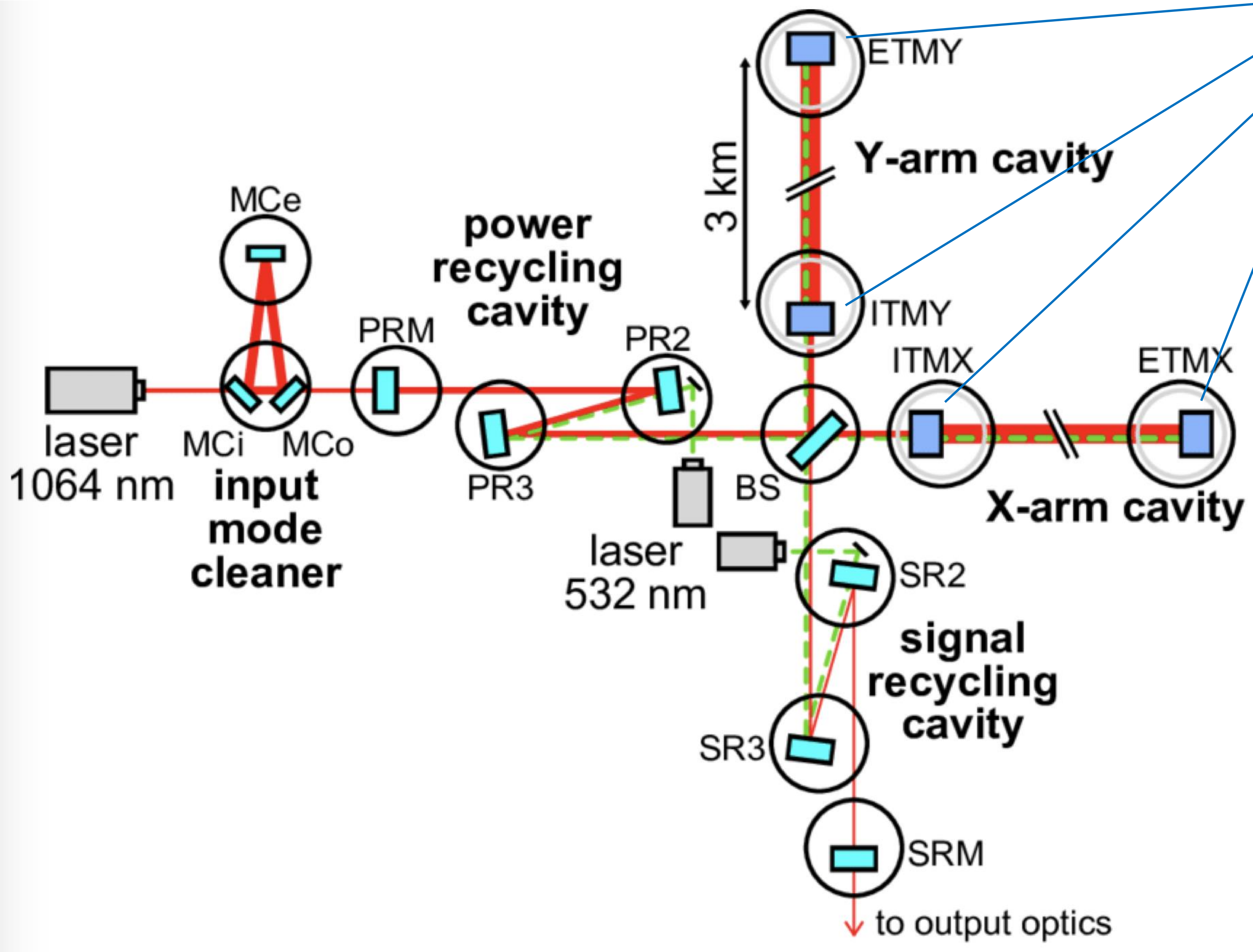




The beam propagation is aligned with the c-axis (no birefringence).

But the inhomogeneity of the substrate (mainly ITMs) will create birefringence.

Birefringence issues in the interferometer



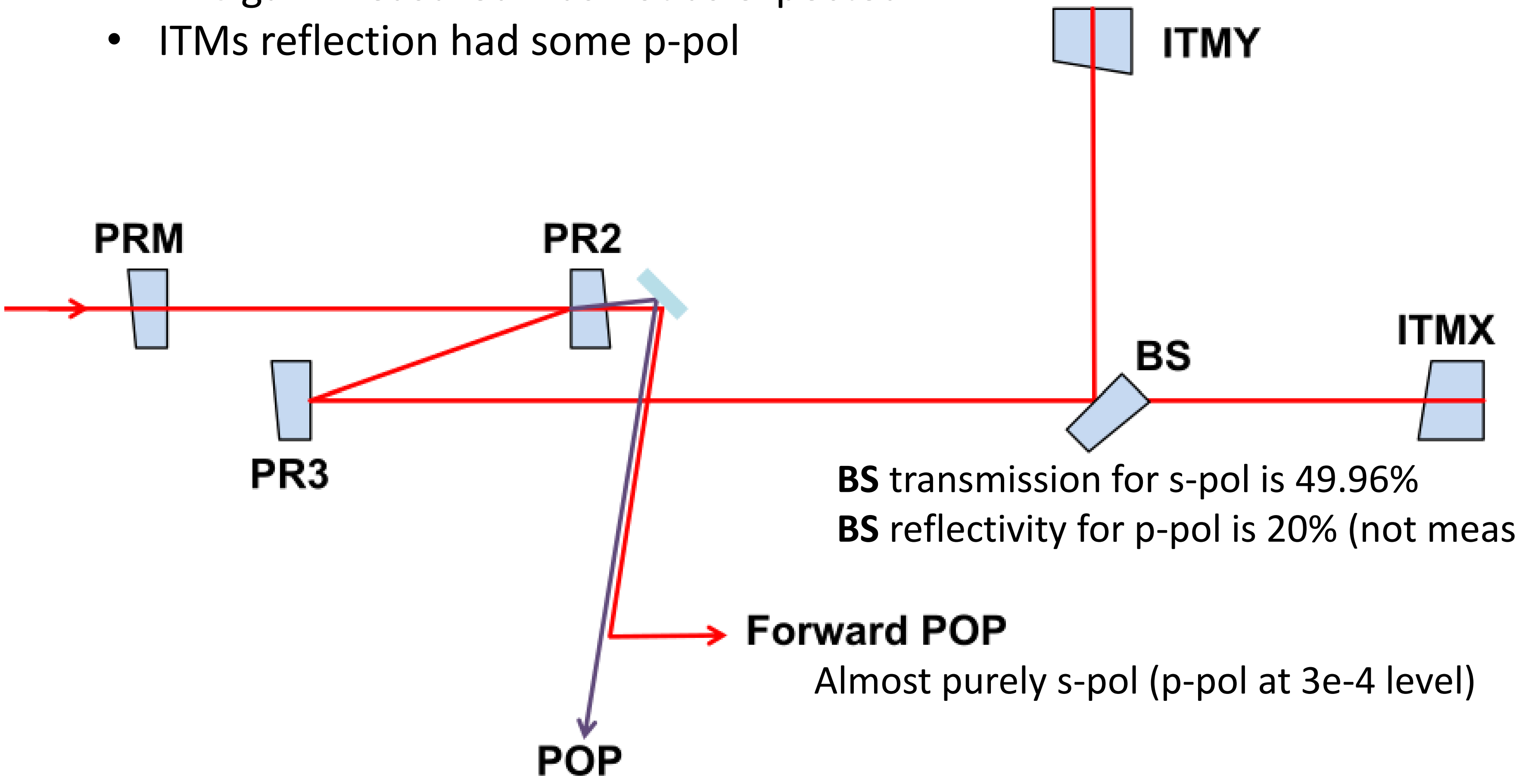
Sapphire test masses

Birefringence mainly comes from ITM substrates.

- Loss in beam power (reduce sensitivity)
- The p-pol can have a bad consequence to length control and alignment control system.
- The amount of birefringence is not balanced between the arms, which leads to additional laser intensity/frequency noises

Current situation JGW-G2012374

- PRC gain measured was not as expected
- ITMs reflection had some p-pol



BS transmission for s-pol is 49.96%
BS reflectivity for p-pol is 20% (not measured)

Forward POP: Almost purely s-pol (p-pol at 3e-4 level)

POP:
6.1 % p-pol from ITMX single bounce
10.8 % p-pol from ITMY single bounce

A portion of input light in s-pol is converted to p-pol.

P-pol image observed in 2019

Analytical description for birefringence

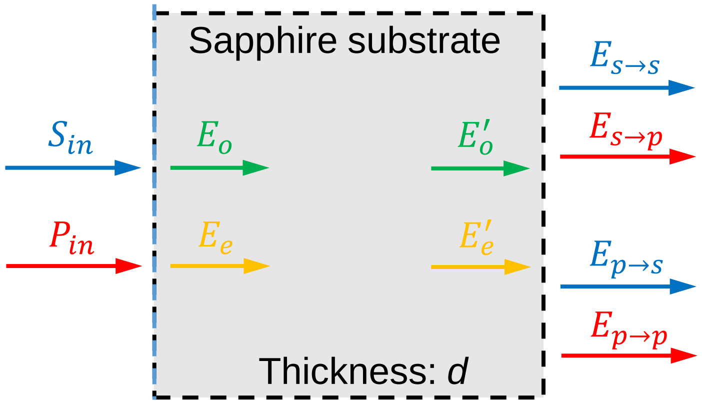
JGW-T2113368
JGW-T1910380

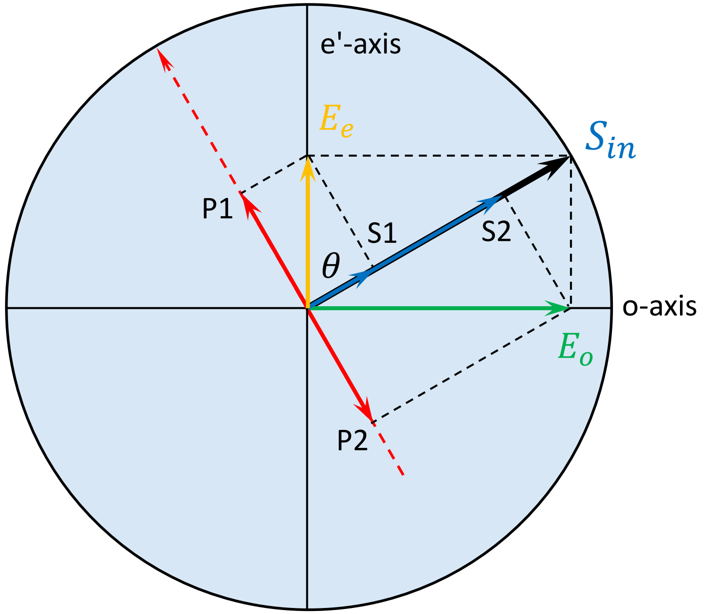
One-way phase

$$\alpha_e = 2\pi \frac{dn'_e}{\lambda} \qquad \alpha_o = 2\pi \frac{dn_o}{\lambda}$$

$$\alpha_- = \frac{\alpha_e - \alpha_o}{2} \qquad \alpha_+ = \frac{\alpha_e + \alpha_o}{2}$$

Birefringence coupling in ITM substrate





Considering the incident beam is pure s-pol. It is projected to e-axis and o-axis. After a propagation of thickness d, the two fields are

$$E_e = S_{in} \cos\phi \, e^{i\alpha_e} \qquad E_o = S_{in} \sin\phi \, e^{i\alpha_o}$$

The two fields are then projected back to s-pol axis and p-pol axis:

$$E_{s\to s} = \cos\theta E_e + \sin\theta E_o = S_{in}\left(\cos^2\theta e^{i\alpha_e} + \sin^2\theta e^{i\alpha_o}\right)$$
$$= S_{in} e^{i\alpha_+}\left(\cos^2\theta \cdot e^{i\alpha_-} + \sin^2\theta \cdot e^{-i\alpha_-}\right)$$

$$E_{s\to p} = \sin\theta E_e - \cos\theta \, E_o = S_{in}\left(\sin\theta\cos\theta \, e^{i\alpha_e} - \sin\theta\cos\theta \, e^{i\alpha_o}\right)$$
$$= S_{in} e^{i\alpha_+} \cdot i \sin 2\theta \sin\alpha_-$$

Similarly, if the input beam is pure p-pol,

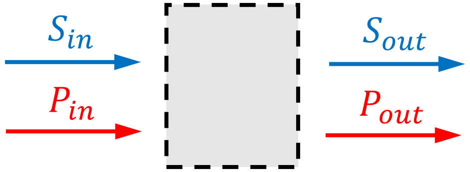
$$E_{p\to s} = P_{in} e^{i\alpha_+} \cdot i \sin 2\theta \sin\alpha_-$$
$$E_{p\to p} = P_{in} e^{i\alpha_+}\left(\sin^2\theta \cdot e^{i\alpha_-} + \cos^2\theta \cdot e^{-i\alpha_-}\right)$$

$$\boldsymbol{M}(\boldsymbol{\theta}, \boldsymbol{\alpha}_-) = e^{i\alpha_+} \begin{pmatrix} \cos^2\theta \cdot e^{i\alpha_-} + \sin^2\theta \cdot e^{-i\alpha_-} & i \sin 2\theta \sin\alpha_- \\ i \sin 2\theta \sin\alpha_- & \sin^2\theta \cdot e^{i\alpha_-} + \cos^2\theta \cdot e^{-i\alpha_-} \end{pmatrix}$$

Transmission of a linearly polarized light through a birefringent medium:

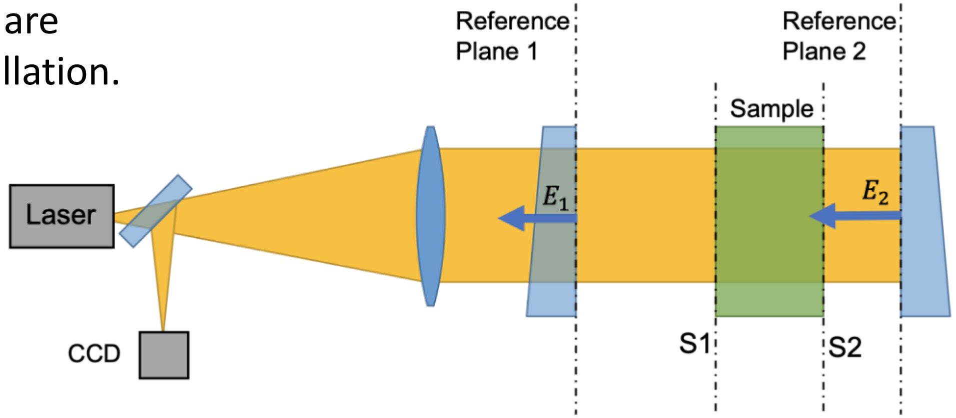
Input beam $\boldsymbol{V} = \begin{pmatrix} S_{in} \\ P_{in} \end{pmatrix}$ Output beam $\boldsymbol{V}' = \boldsymbol{MV}$

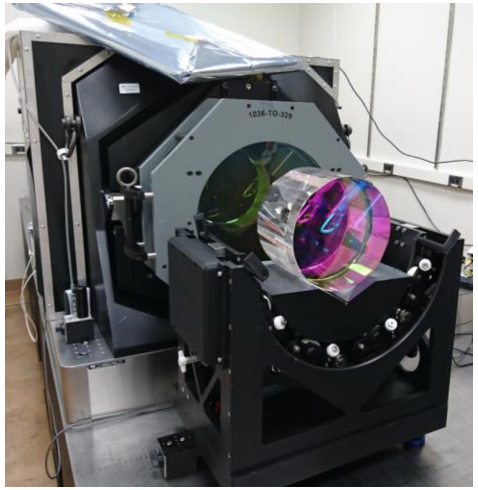
M is the Jones matrix of the substrate



TWE measurements with Fizeau interferometer

Transmission wavefront error (TWE) maps of TMs are measured with Fizeau interferometer before installation.

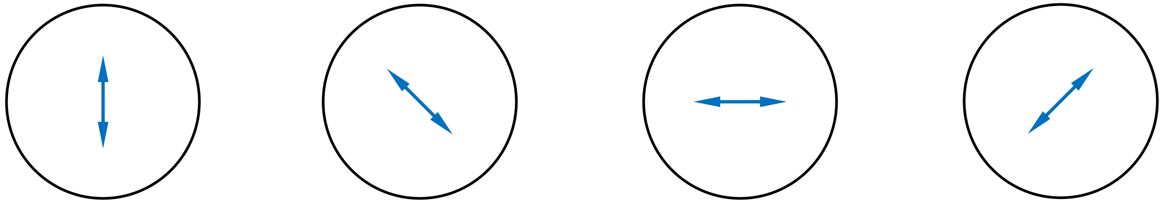




If we take 4 measurements by rotating the polarization of the input beam

$$
\begin{aligned}
P_{out}(\theta + 0) &= 1 + A^2 + 2A\cos 2\theta \sin 2\alpha_- \\
P_{out}(\theta + 45) &= 1 + A^2 - 2A\sin 2\theta \sin 2\alpha_- \\
P_{out}(\theta + 90) &= 1 + A^2 - 2A\cos 2\theta \sin 2\alpha_- \\
P_{out}(\theta + 135) &= 1 + A^2 + 2A\sin 2\theta \sin 2\alpha_-
\end{aligned}
$$

Measure TWE with different input polarization directions

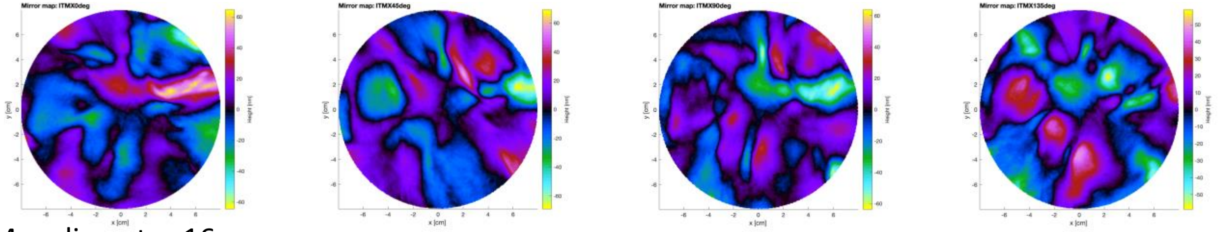


By combining several TWE maps with different orientation of the beam polarization, we can extract θ and α_: JGW-T1910380

$$\theta = -\frac{1}{2}\tan^{-1}\frac{TWE(45) - TWE(135)}{TWE(0) - TWE(90)}$$

$$\alpha_- = \frac{2\pi}{\lambda}\cdot\frac{TWE(0) - TWE(90)}{\cos 2\theta}$$

Most area < 30nm



Map diameter:16cm

$TWE(0)$ $TWE(45)$ $TWE(90)$ $TWE(135)$

Characterization of TWE maps

TWE maps are measured before birefringence problem is realized.

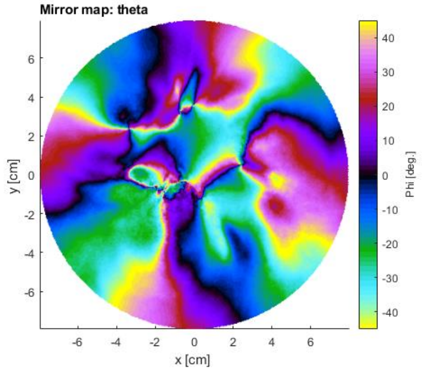
- The setup was not optimized for birefringence study.
- We hope to rotate the input polarization rather than the mirror.
- We don't know how much errors are there in the TWE maps.
  - Piston: unknown
  - Tilt: unknown
  - Curvature: can be removed
  - Astigmatism: can be removed

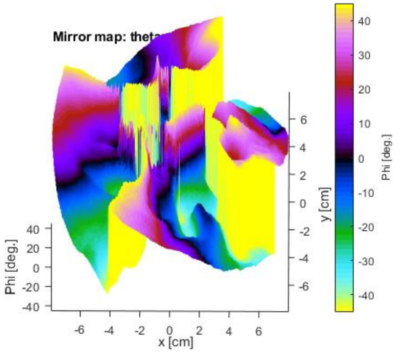
Our method is easily affected by TWE measurement errors.

$$\theta = -\frac{1}{2}\tan^{-1}\frac{TWE(45) - TWE(135)}{TWE(0) - TWE(90)}$$

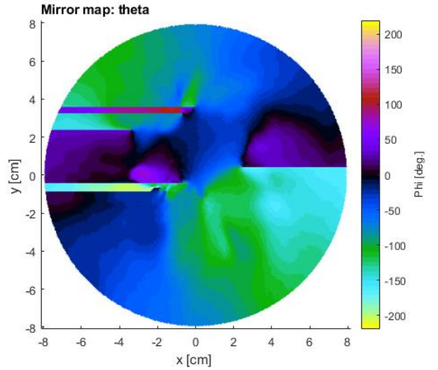
Errors are amplified when denominator is zero.

Theta needs to be unwrapped.



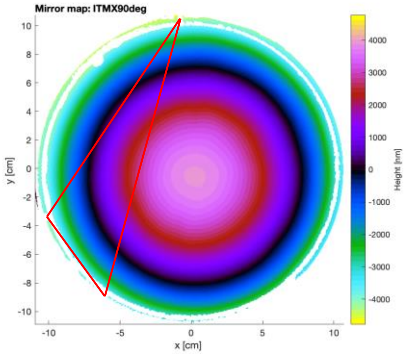


unwrap

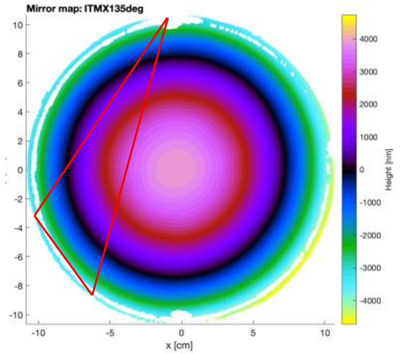


Rotation angle calibration

90° (82°)

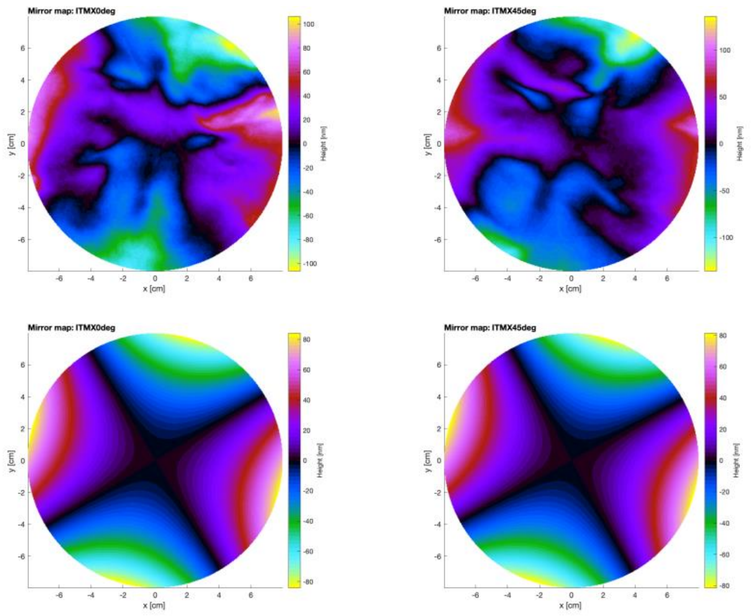


135° (118°)



Astigmatism calibration

$TWE(0)$ $TWE(45)$

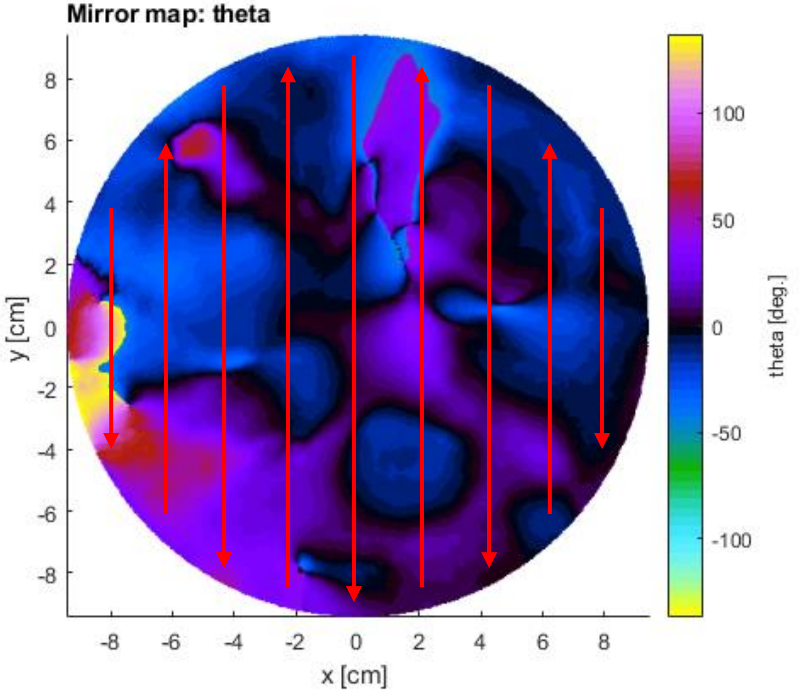


Gravity induced deformation of ITMX + astigmatism of reference sphere

Parameter scan for piston

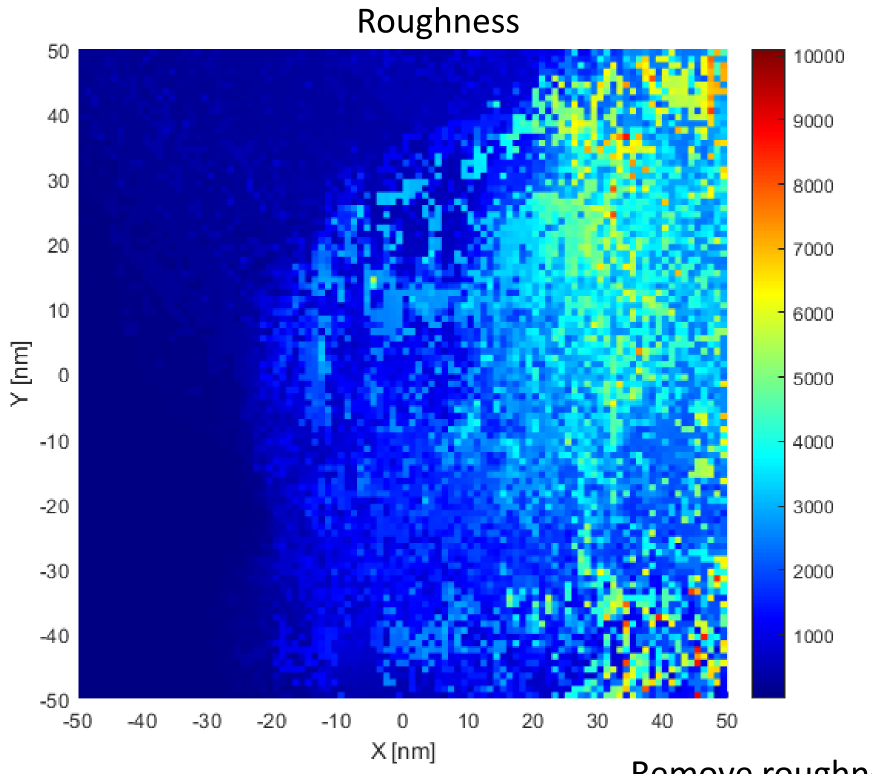
$$\theta = -\frac{1}{2}\tan^{-1}\frac{TWE(45) - TWE(135) + X}{TWE(0) - TWE(90) + Y}$$

Add offsets, calculate theta and unwrap theta.



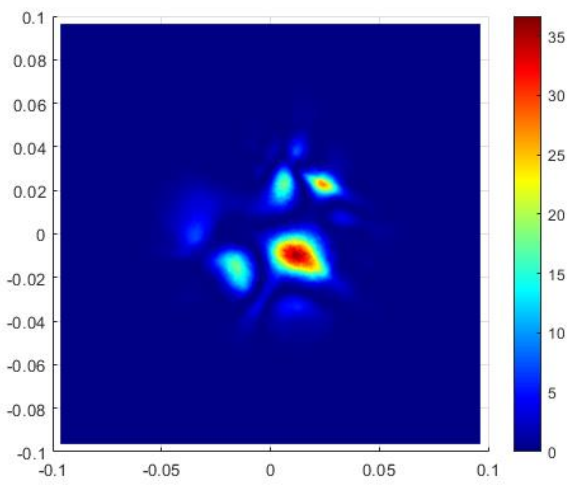
Expand the map to one-dimension and calculate the “roughness”.

$$\mathcal{R} = \sum_{k=1}^{n-1}(x_{k+1} - x_k)^2$$

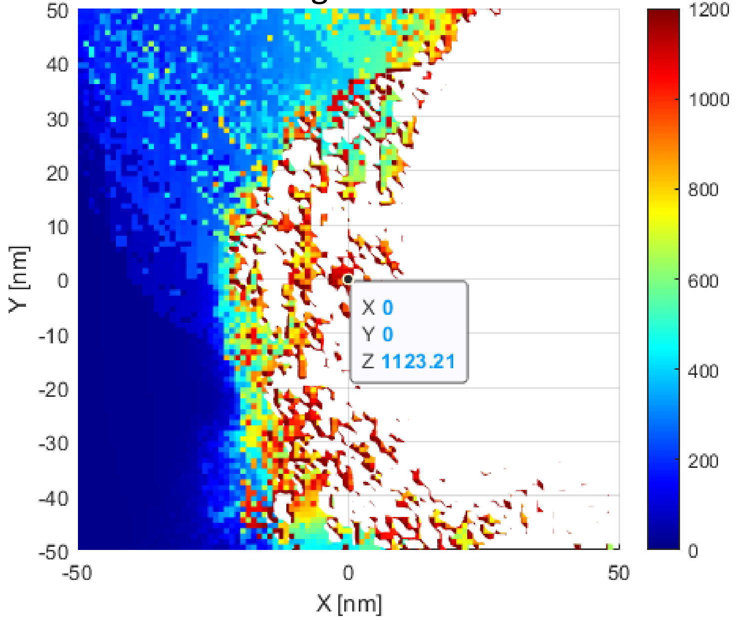


ITMX

Beam shape of p-pol



Remove roughness above 1200

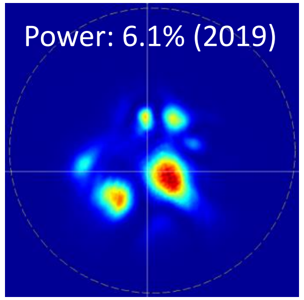


Parameter scan for piston

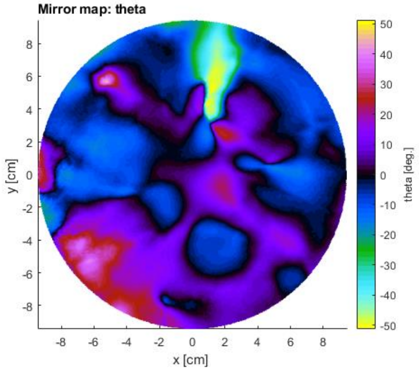
We need to perform more accurate on-site measurements for beam power and shape.
This will help us to calibrate our current ITMs.

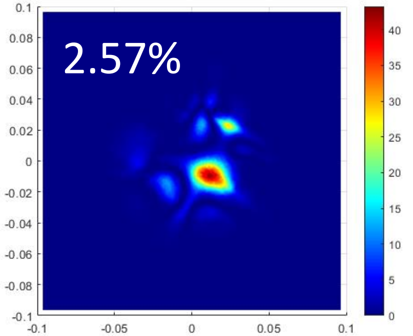
ITMX

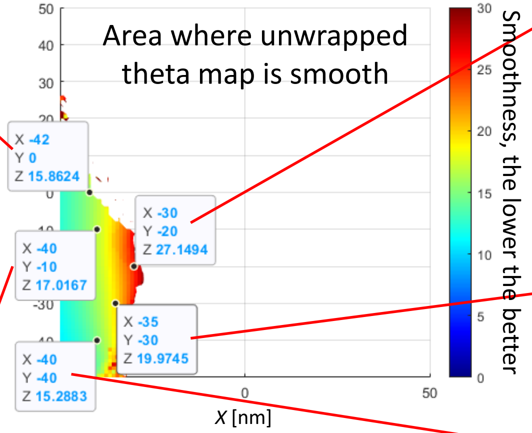
Measured single bounce p-pol shape (March 2022)



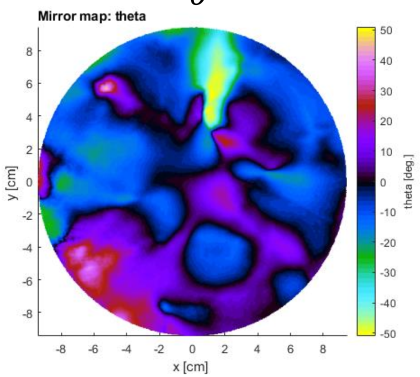
$$\theta = -\frac{1}{2}\tan^{-1}\frac{TWE(45) - TWE(135) + X}{TWE(0) - TWE(90) + Y}$$

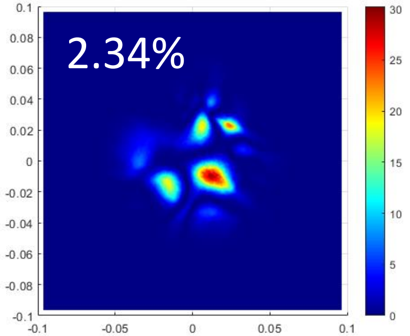




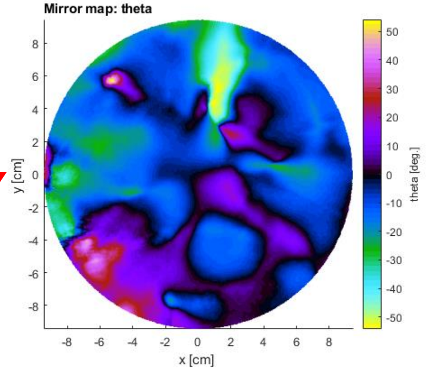


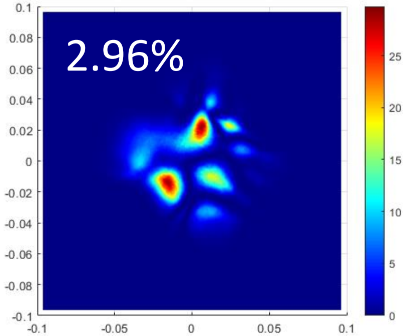
θ

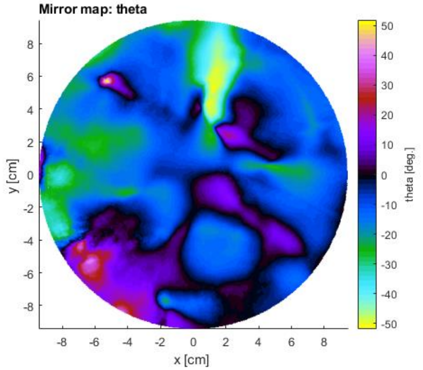


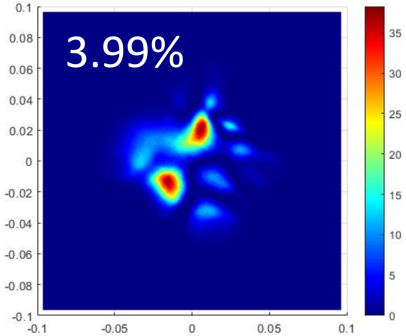


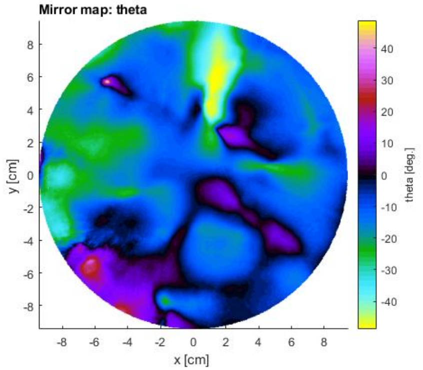
Calculated single bounce p-pol shape

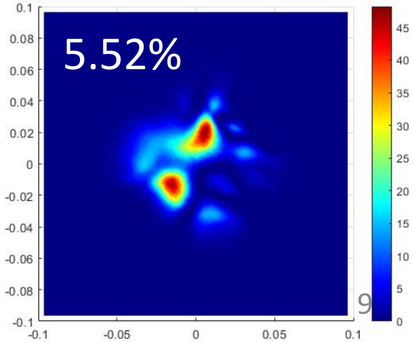




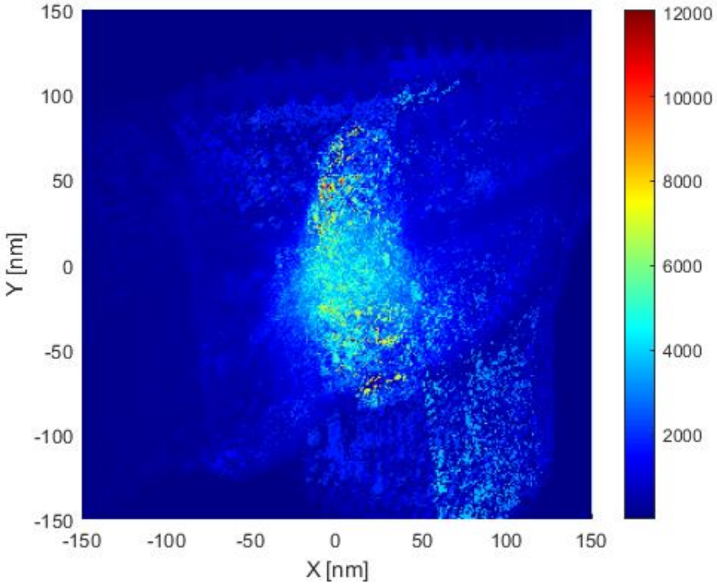




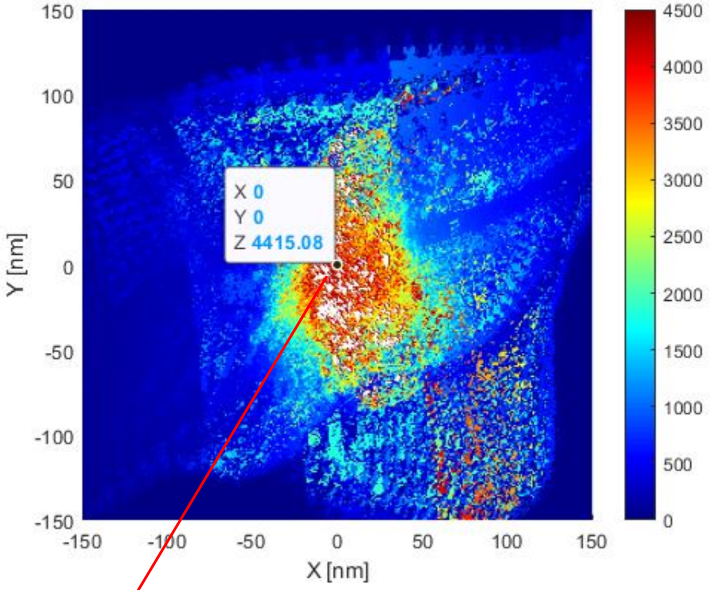




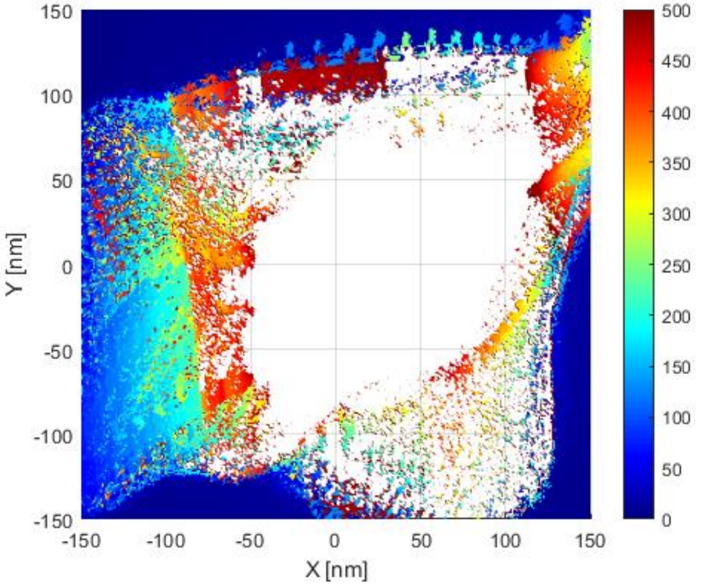
ITMY



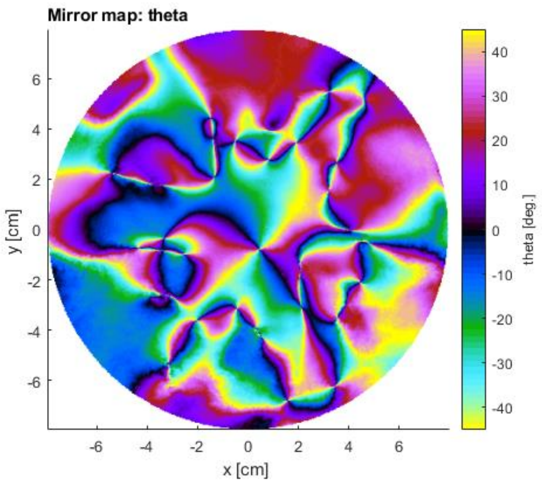
Remove roughness above 4500

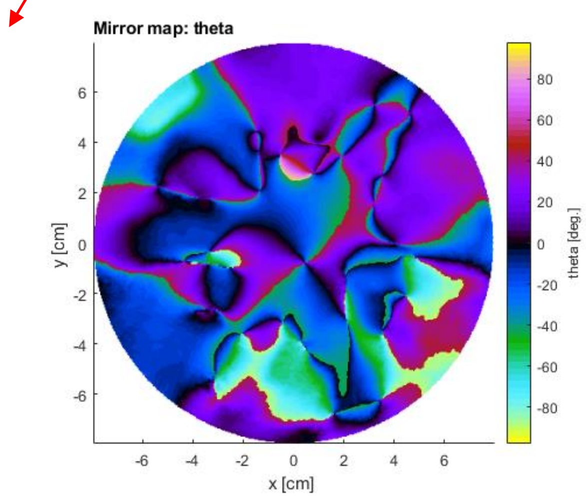


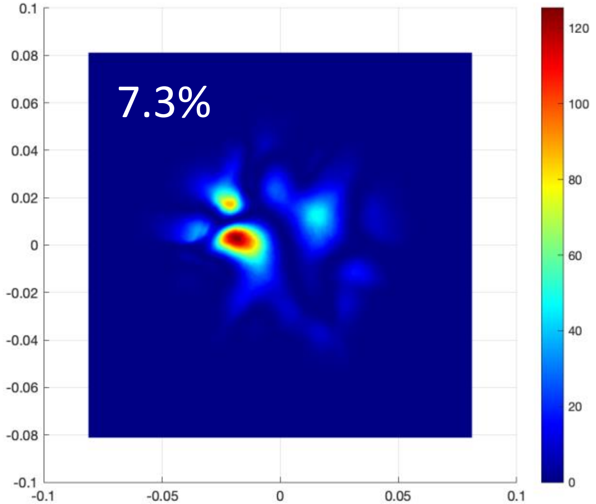
Remove roughness above 500

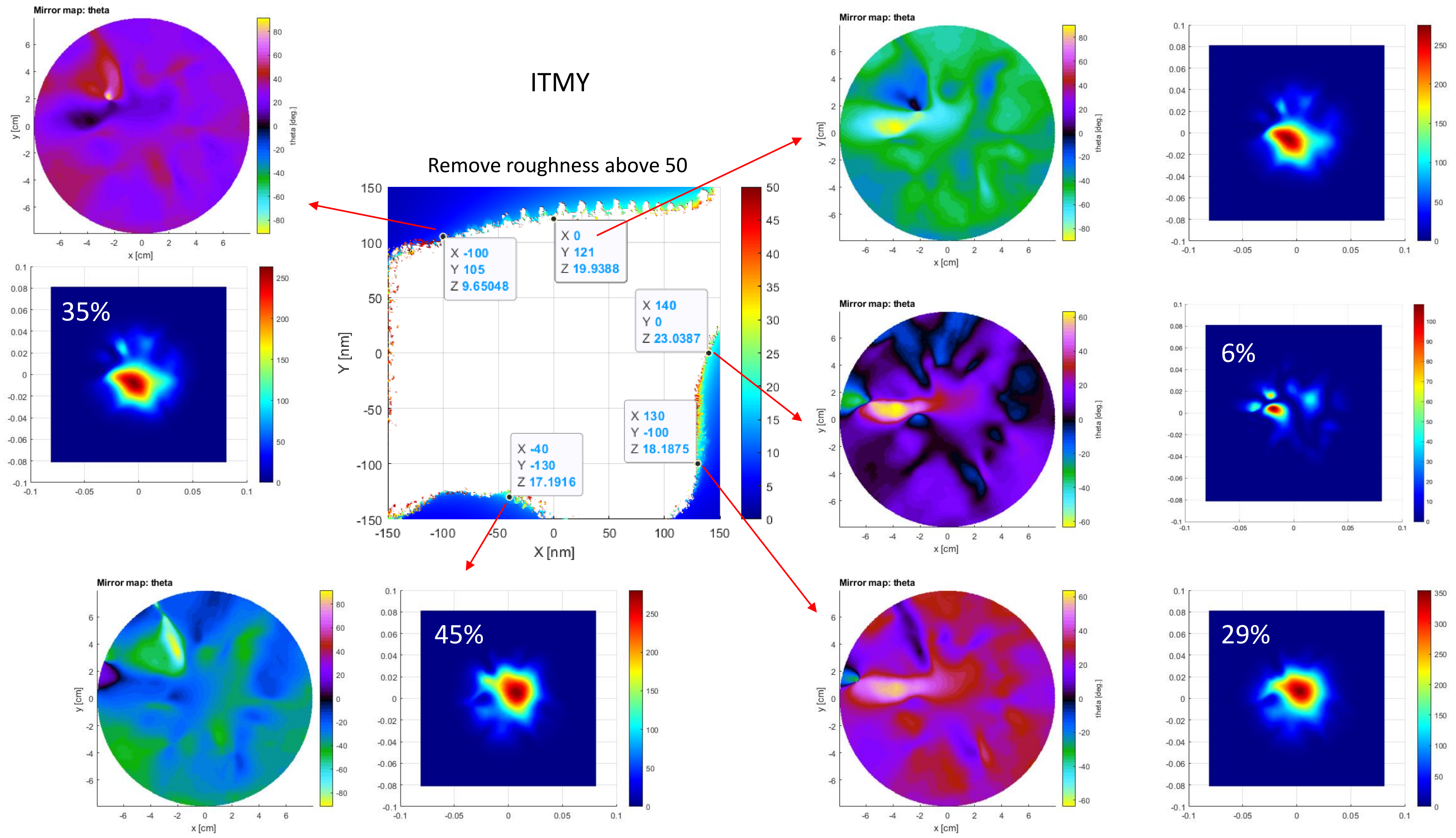


θ





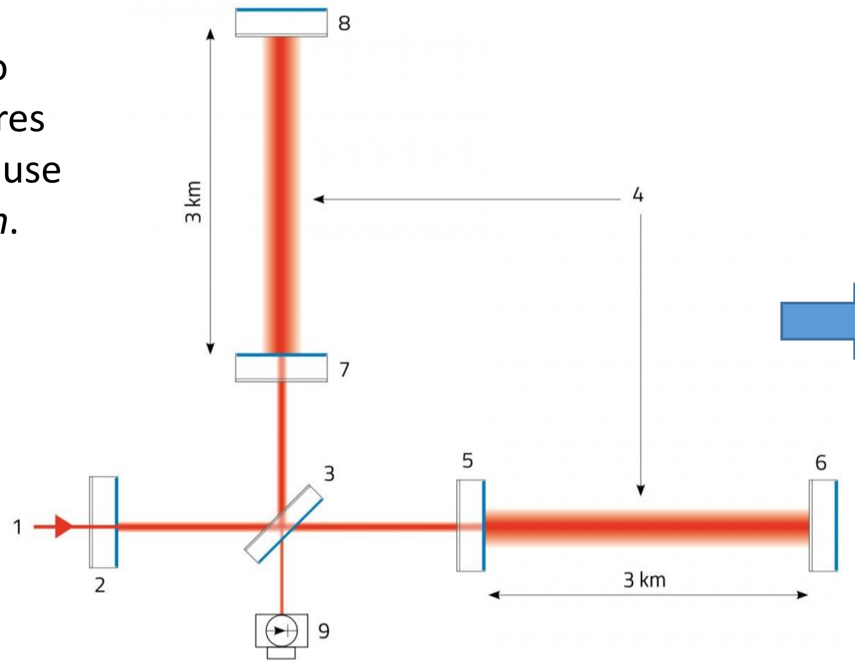


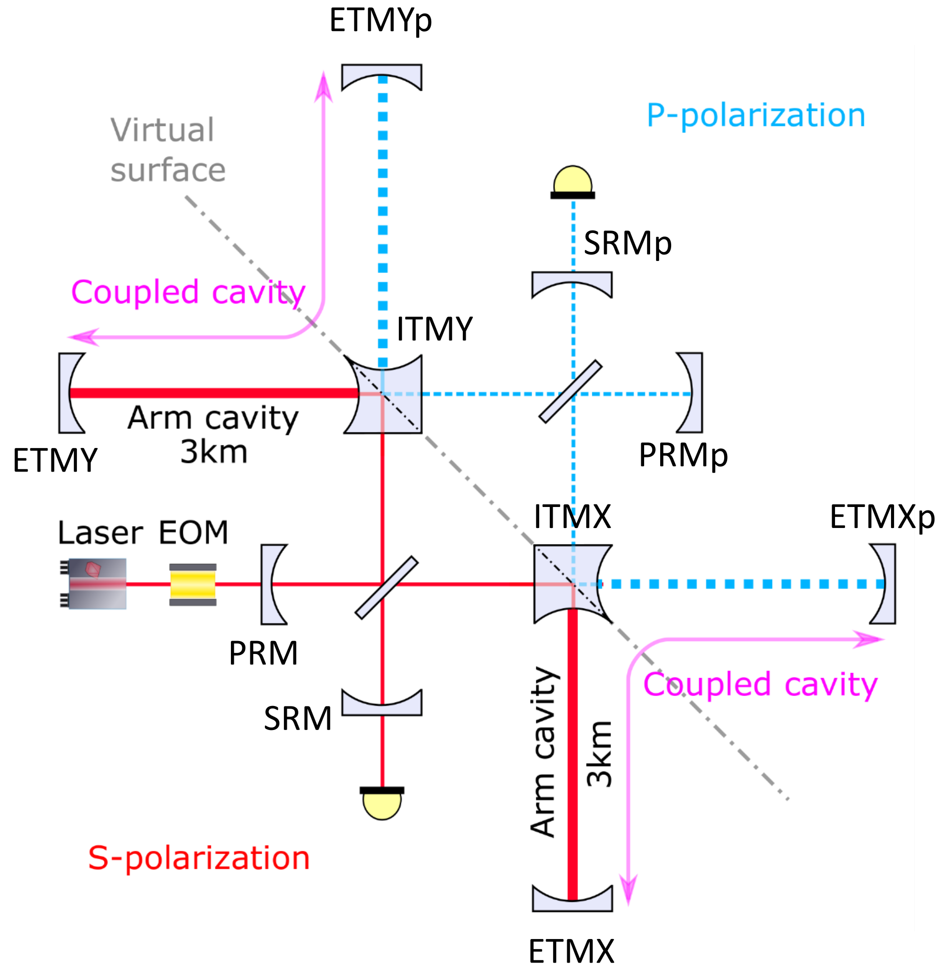

ITMY
Remove roughness above 50
Mirror map: theta
theta [deg.]
x [cm]
y [cm]
X [nm]
Y [nm]
X -100
Y 105
Z 9.65048
X 0
Y 121
Z 19.9388
X 140
Y 0
Z 23.0387
X 130
Y -100
Z 18.1875
X -40
Y -130
Z 17.1916
35%
6%
45%
29%

Modeling birefringence

Two-world approach + Mirror maps

Finesse is considering to add birefringence features but for now we have to use the *two-world approach*.





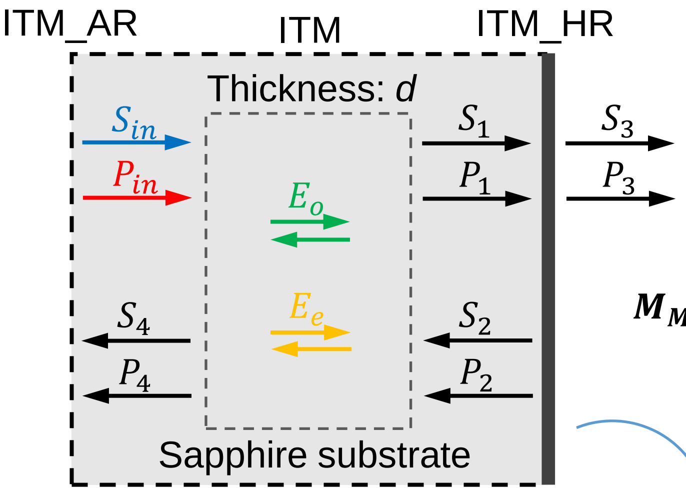
The key of the two-world approach is to define a birefringent component that defines the coupling between s-pol and p-pol light.

Motivation of simulation:

To understand the influence of the birefringence of sapphire mirrors in the cryogenic interferometer and to find ways of mitigating the birefringence effect.

- Help with the commissioning
- Resonance condition and gain in PRC
- Signal degeneracy in alignment control
- scattering effects of both beams in s-polarization and p-polarization.
- Imbalance at the AS port.

A model representing birefringence couplings



We think a Mach–Zehnder interferometer can best describe the birefringent object.

Jones matrix of the MZ model with $\varphi_1 = \alpha_-$, $\varphi_2 = \pi - \alpha_-$

$$\boldsymbol{M_{MZ}} = \begin{pmatrix} t^2 \cdot e^{i\alpha_-} + r^2 \cdot e^{-i\alpha_-} & 2irt \sin\alpha_- \\ 2irt \sin\alpha_- & r^2 \cdot e^{i\alpha_-} + t^2 \cdot e^{-i\alpha_-} \end{pmatrix}$$

Jones matrix of the substrate

$$\boldsymbol{M} = \begin{pmatrix} \cos^2\theta \cdot e^{i\alpha_-} + \sin^2\theta \cdot e^{-i\alpha_-} & 2i \sin\theta \cos\theta \sin\alpha_- \\ 2i \sin\theta \cos\theta \sin\alpha_- & \sin^2\theta \cdot e^{i\alpha_-} + \cos^2\theta \cdot e^{-i\alpha_-} \end{pmatrix}$$

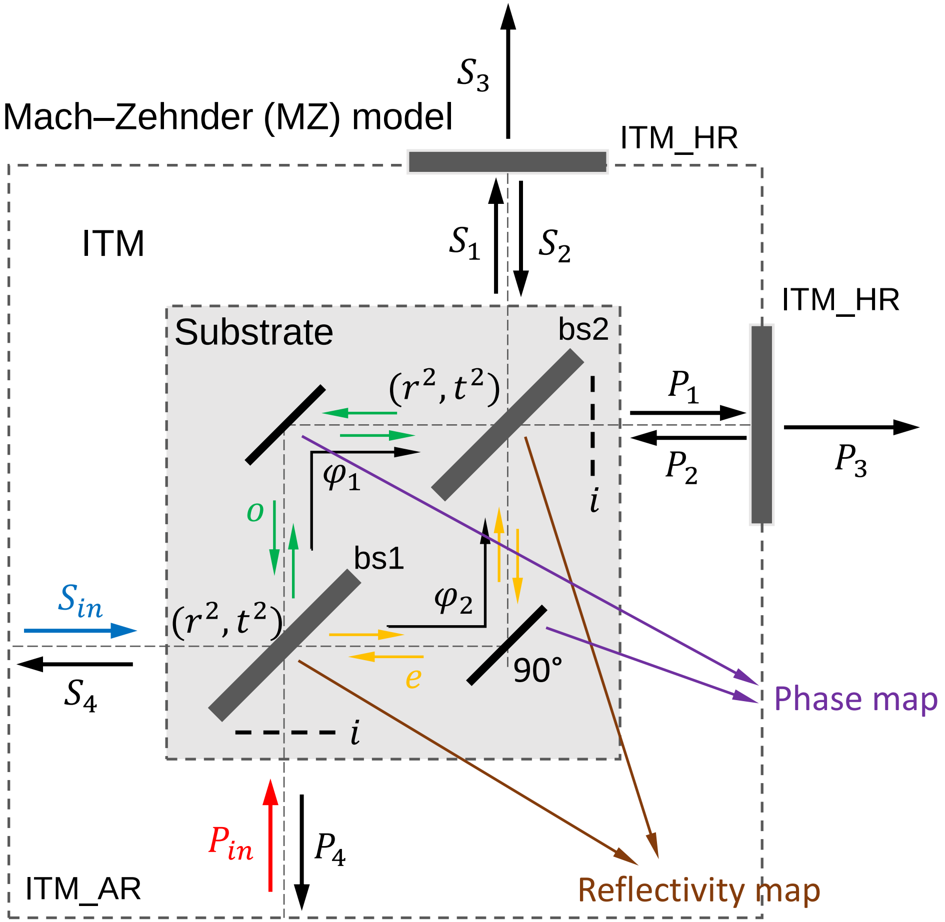
If we compare the two matrices, it is obvious that

$$r = \sin\theta\,,\quad t = \cos\theta$$
$$\varphi_1 = \alpha_-\,,\quad \varphi_2 = \pi - \alpha_-$$

We need to apply reflectivity maps to the two beamsplitters and phase maps to the two steering mirrors in the MZ model.

We are using FINESSE for birefringence simulation, as FINESSE is capable simulating mirror defects using realistic mirror maps.

The Mach–Zehnder (MZ) model



Intuitive understanding of the MZ model:

- Reflectivity and transmissivity of the first beamsplitter represent projections of the input linearly polarized light along o-axis and e-axis.
- The o-light and e-light accumulate different phase delays along their paths, combined and projected back to s/p axis at the second beamsplitter.

Future simplified model

$$\boldsymbol{M}(\boldsymbol{\theta},\boldsymbol{\alpha_-}) = e^{i\alpha_+}\begin{pmatrix}\cos^2\theta\cdot e^{i\alpha_-}+\sin^2\theta\cdot e^{-i\alpha_-} & i\sin 2\theta\sin\alpha_- \\ i\sin 2\theta\sin\alpha_- & \sin^2\theta\cdot e^{i\alpha_-}+\cos^2\theta\cdot e^{-i\alpha_-}\end{pmatrix}$$

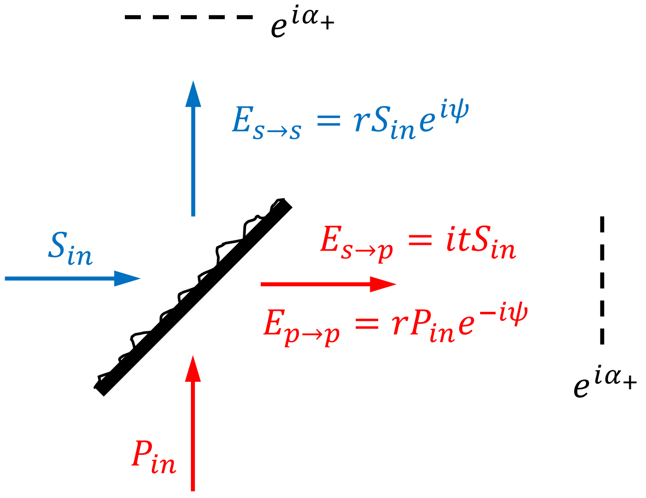
$$= e^{i\alpha_+}\begin{pmatrix}\cos\alpha_- + i\cos 2\theta\sin\alpha_- & i\sin 2\theta\sin\alpha_- \\ i\sin 2\theta\sin\alpha_- & \cos\alpha_- - i\cos 2\theta\sin\alpha_-\end{pmatrix}$$

$$= e^{i\alpha_+}\begin{pmatrix}e^{i\psi}\sqrt{\cos^2\alpha_- + \cos^2 2\theta\sin^2\alpha_-} & i\sin 2\theta\sin\alpha_- \\ i\sin 2\theta\sin\alpha_- & e^{-i\psi}\sqrt{\cos^2\alpha_- + \cos^2 2\theta\sin^2\alpha_-}\end{pmatrix}$$

$$= e^{i\alpha_+}\begin{pmatrix}re^{i\psi} & it \\ it & re^{-i\psi}\end{pmatrix}$$

where $r = \sqrt{\cos^2\alpha_- + \cos^2 2\theta\sin^2\alpha_-}$ $\quad t = \sin 2\theta\sin\alpha_-$ $\quad \tan\psi = \cos 2\theta\tan\alpha_-$

It is obvious that $r^2 + t^2 = 1$



$$E_{s\to s} = rE_{in}e^{i(\alpha_+ + \psi)}$$

$$E_{s\to p} = itE_{in}e^{i\alpha_+}$$

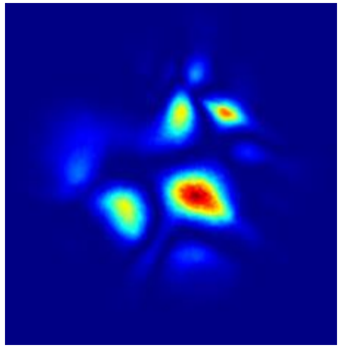
Parameter that we need are $\theta,\ \alpha_+,\ \alpha_-$.

Single arm cavity reflection

S-pol $\xrightarrow{TWE(\phi)}$ S-pol TEM00, S-pol HOMs, P-pol TEM00, P-pol HOMs

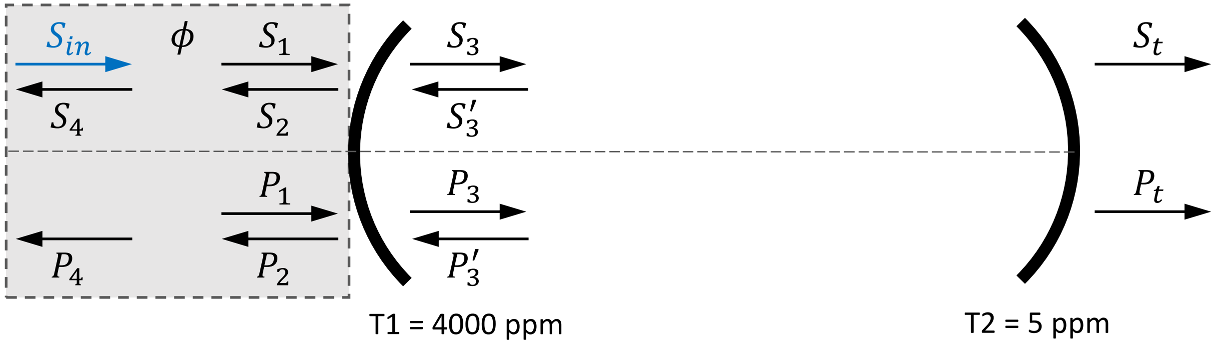
S-pol HOMs, P-pol HOMs → HOM magnitude in S and P are the same!

Mode content of p-pol



TEM00: <10%
1st mode: ~20%
2nd mode: ~10%
Higher order: >60%

The p-pol power reflected by the cavity is smaller than the single bounce reflection power by the ITM due to Lawrence effect.



Supposing mode matched

Round-trip single bounce p-pol power: ρ_{rt}

One way p-pol TEM00 percent: β

Lawrence effect (mode healing effect)

- The effect of TWE(ϕ) at reflection is strongly cancelled for TEM00.
- In other words, at cavity reflection, HOMs introduced by ITM distortion are strongly suppressed.

$$r_{\mathrm{cav}} = 1 - \frac{2T_1}{T_1 + T_2 + \mathcal{L}} \qquad r_{\mathrm{cav}}^2 = 0.937$$

$$r_{\mathrm{LE}} = 1 - \frac{T_1}{T_1 + T_2 + \mathcal{L}} \qquad r_{\mathrm{LE}}^2 = 0.00022$$

Reflected power:

$$P_4 \approx \rho_{\mathrm{rt}}\beta r_{\mathrm{cav}}^2 + \rho_{\mathrm{rt}}(1-\beta) r_{\mathrm{LE}}^2$$

$$S_4 \approx r_{\mathrm{cav}}^2 - \rho_{\mathrm{rt}}\beta r_{\mathrm{cav}}^2 - \rho_{\mathrm{rt}}(1-\beta) r_{\mathrm{LE}}^2$$

Parameters

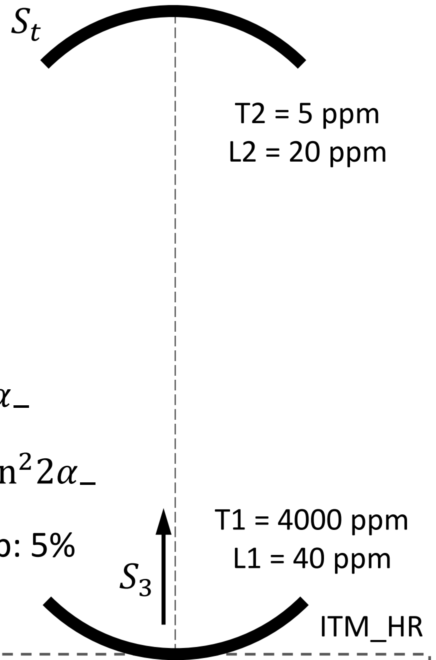
$r^2 = t^2 = 0.5$

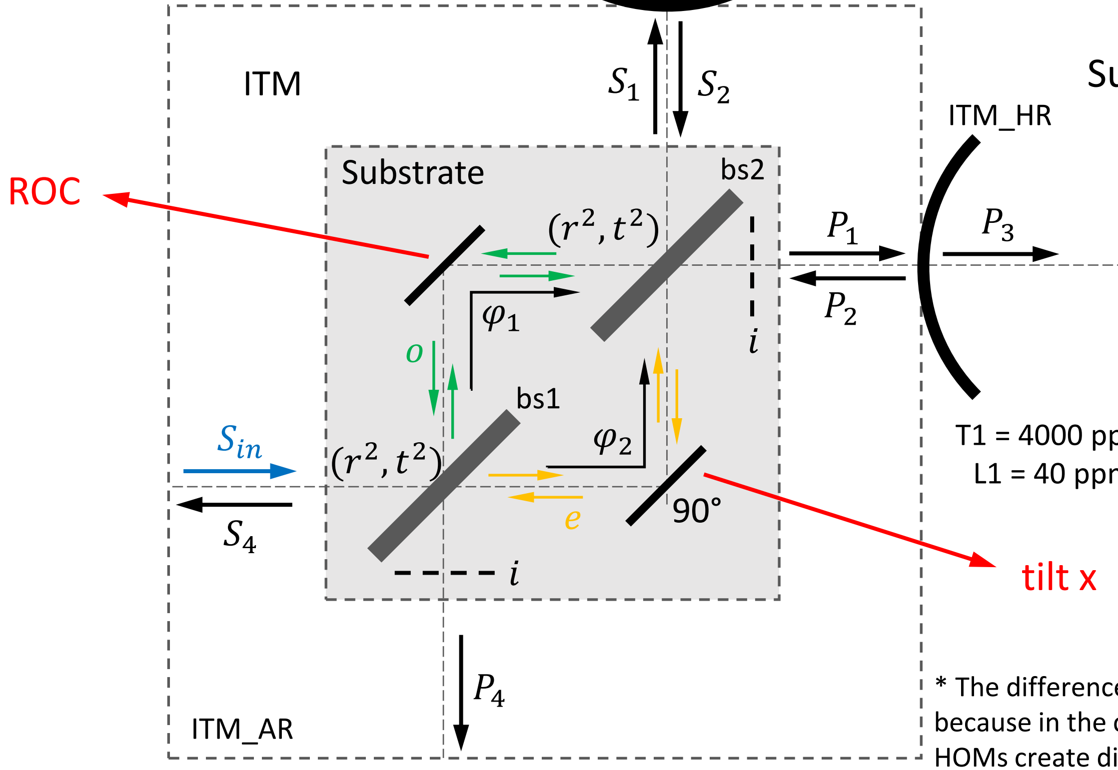
$$\boldsymbol{M} = \begin{pmatrix} \cos\alpha_- & i\sin\alpha_- \\ i\sin\alpha_- & \cos\alpha_- \end{pmatrix}$$

One-way p-pol power: $\rho = \sin^2\alpha_-$

Round trip p-pol power: $\rho_{\mathrm{rt}} = \sin^2 2\alpha_-$

One-way: 1.26%* <---> Round-trip: 5%



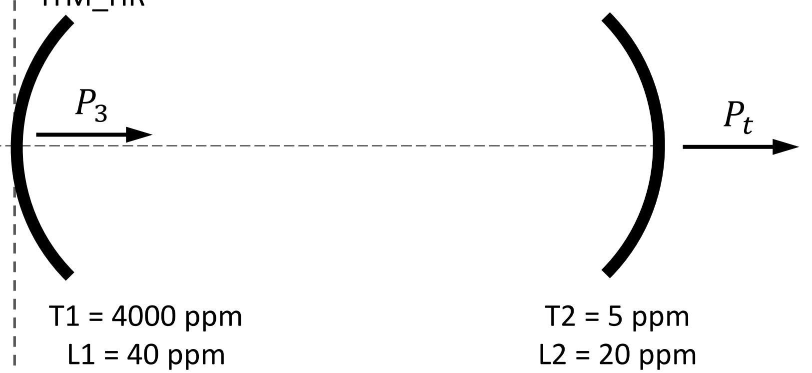


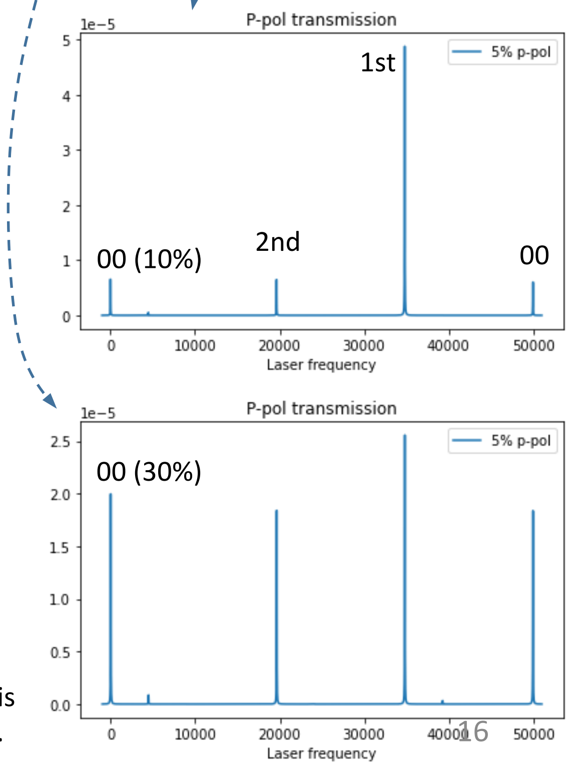
Fixed parameter testing without maps

Parameters of three models to create different mode contests for p-pol

ROC and tilt x to create different modes		P-pol power Round-trip P4	P-pol power One-way P1	P-pol mode content (one-way)		
				TEM00	TEM01	TEM02
ROC (m)	-51702.6	5%	1.34%	10%	79.6%	10.4%
Tilt x (radian)	1.01e-06					
ROC (m)	-28449.5	5%	1.38%	30%	40.9%	29.1%
Tilt x (radian)	7.34e-07					
ROC (m)	-22159.6	5%	1.36%	50%	1.4%	48.6%
Tilt x (radian)	1.35e-07					

Suppose the mode matching is good





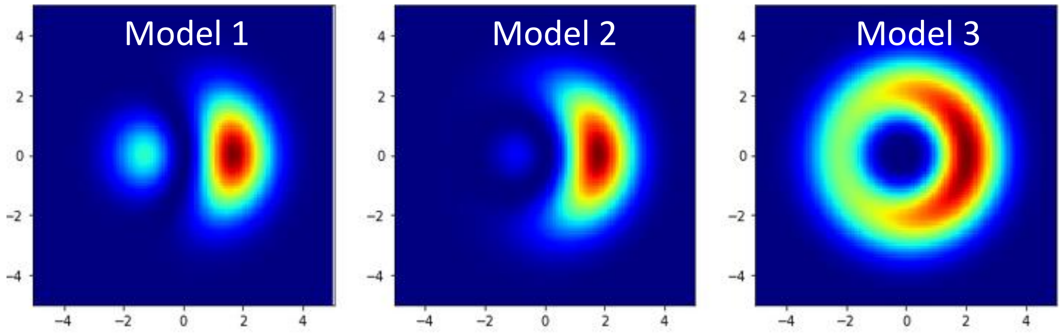
* The difference between 1.26% (calculation) and ~1.36 (simulation) is because in the calculation we don't consider the HOM effect of p-pol. HOMs create different birefringence power from the TEM00 mode.

Simulation without mirror map

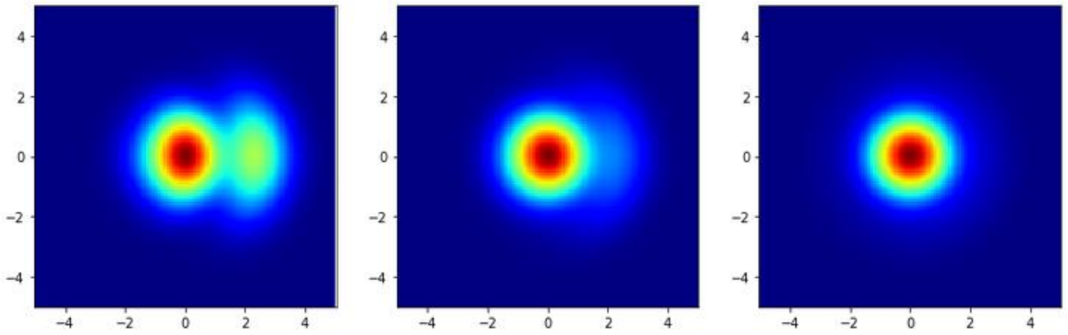
Fixed parameter simulation without realistic mirror map

ROC and tilt x to create different modes	P-pol power Round-trip P4	P-pol mode content (one-way)		
		TEM00	TEM01	TEM02
Model 1	5%	10%	79.6%	10.4%
Model 2	5%	30%	40.9%	29.1%
Model 3	5%	50%	1.4%	48.6%

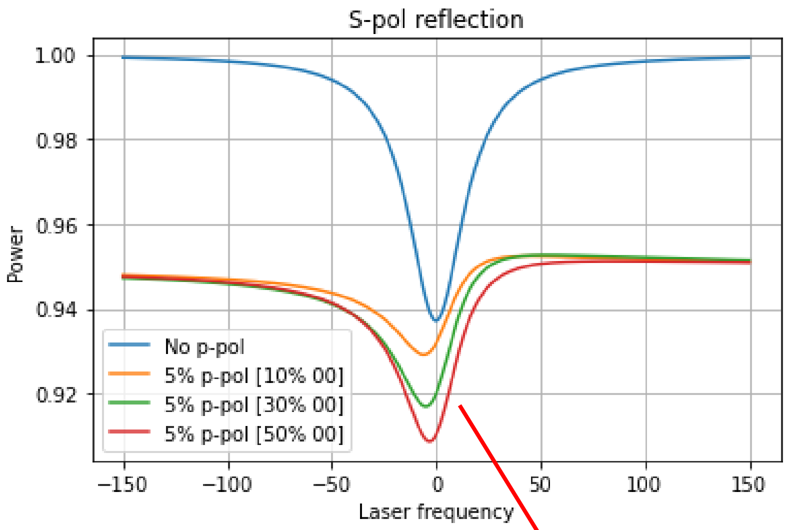
P-pol single bounce reflection from ITM

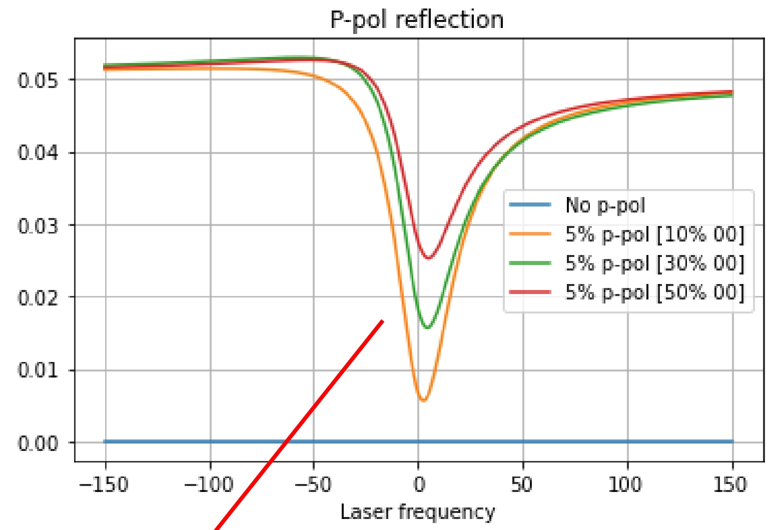


P-pol reflected by arm cavity



Arm cavity round-trip loss simulation





Reflected power in
s-pol and p-pol [W]

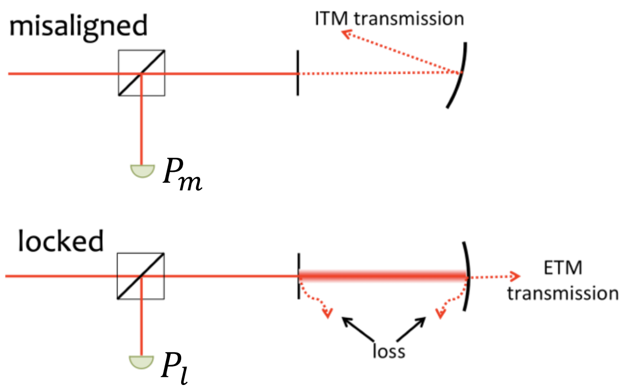
P-pol content	Calculation		Simulation		Round-trip loss (ppm)	
	S-pol	P-pol	S-pol	P-pol	S + P	Only S
Model 1	0.9325	0.0047	0.9318	0.0068	61.5	19.2
Model 2	0.9231	0.0141	0.9199	0.0184	61.9	31.8
Model 3	0.9137	0.0234	0.9102	0.0277	62.2	42.0

Input power is assumed to be 1 W.

We suppose the cavity has 60 ppm round-trip loss.

Round-trip loss measurement process

JGW-T2011633, LIGO-G1501547, LIGO-T1700117



$$R_1 + T_1 + \mathcal{L}_1 = 1$$

$$R_2 + T_2 + \mathcal{L}_2 = 1$$

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 \qquad \text{Round-trip loss}$$

Round-trip loss can be derived from the equation:

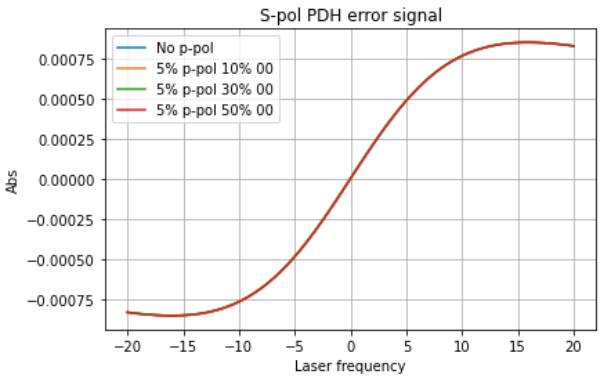
$$\mathcal{L} = \frac{T_1}{4\eta}\left(1 - \frac{P_l}{P_m} + T_1\right)$$

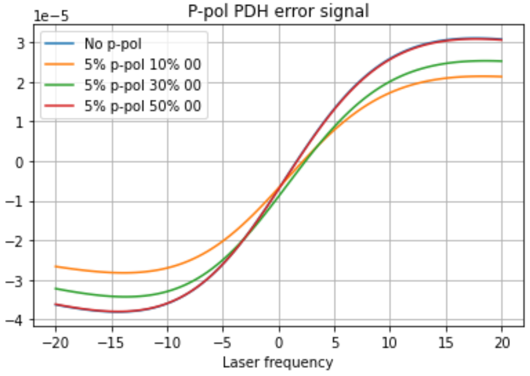
η : mode matching ratio

- When there is birefringence, the beam power reflected by the cavity will change.
- The round-trip loss measurement is not accurate anymore if we only measure s-pol beam.
- We also need to consider BS reflectivity imbalance for s-pol and p-pol.

Simulation for single arm alignment sensing signal

PDH error signal when scanning arm cavity

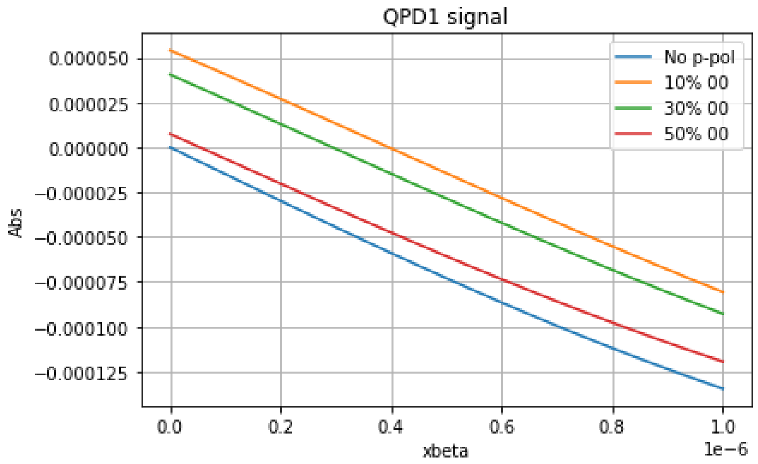


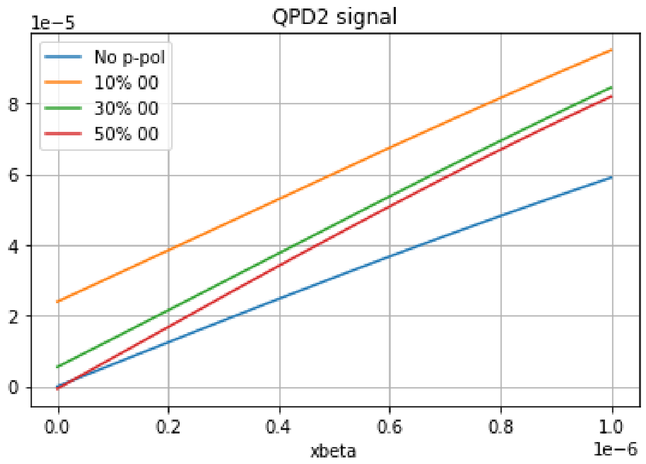


Fixed parameter simulation without realistic mirror map

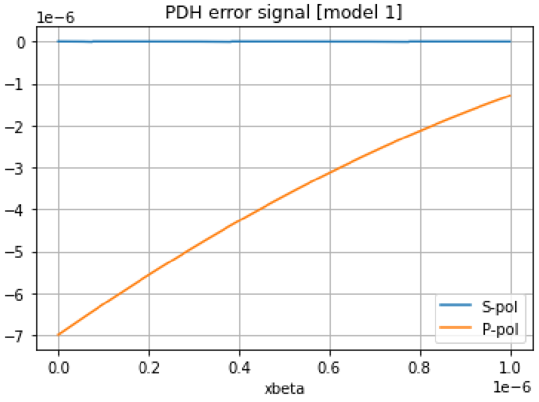
ROC and tilt x to create different modes	P-pol power Round-trip P4	P-pol mode content (one-way)		
		TEM00	TEM01	TEM02
Model 1	5%	10%	79.6%	10.4%
Model 2	5%	30%	40.9%	29.1%
Model 3	5%	50%	1.4%	48.6%

S-pol ASC error signal of single arm cavity





When tuning alignment of the cavity, the cavity is locked via s-pol PDH error signal.

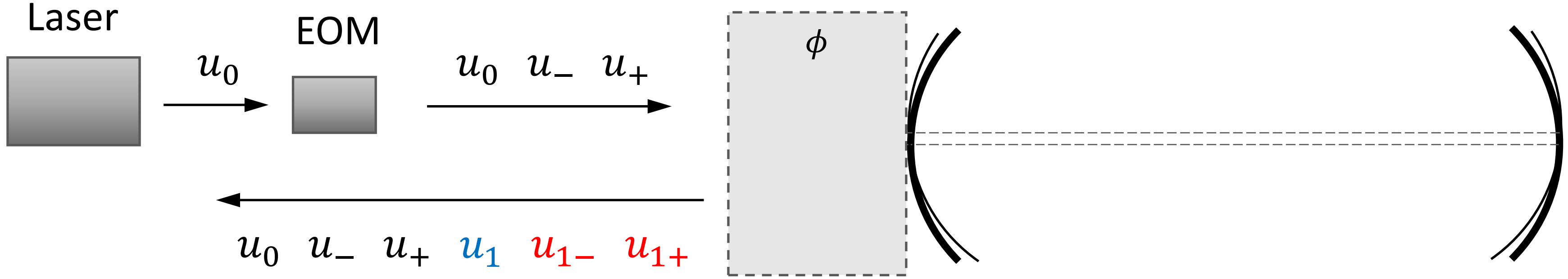


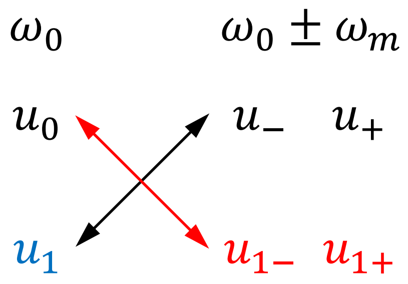
Some simple calculations

Some assumptions:

- RF sidebands power: 2%
- Single bounce birefringence power: 5% (1st order mode content 20%)

$$S_{QPD} \sim u_-^* u_1 + u_+ u_1^*$$





$$u_1 = u_{1a} + u_{1b}$$

u_{1a}: due to misalignment $\left(\frac{\Delta x}{w_0}\right)^2 \approx 1\%$ (1.6mm axis shift)

u_{1b}: due to birefringence: 5%*20%*0.00022 (can be omitted)

$$S_{QPD} \sim \underbrace{u_-^* u_1 + u_+ u_1^*}_{2\%*1\%} + \underbrace{u_0^* u_{1-} + u_0 u_{1+}^*}_{97\%*2\%*5\%*20\%}$$

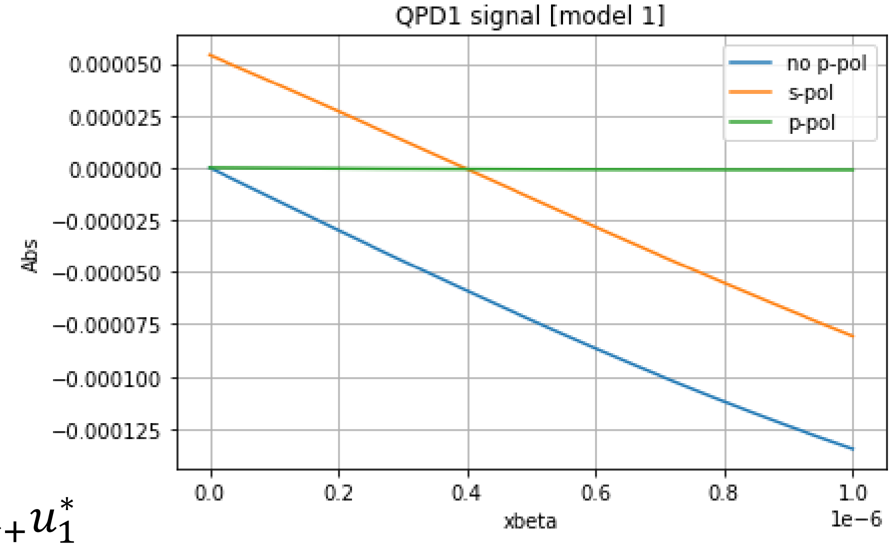
The ASC offset error mainly comes from the beat between the fundamental mode and 1st mode from RF sidebands due to birefringence.

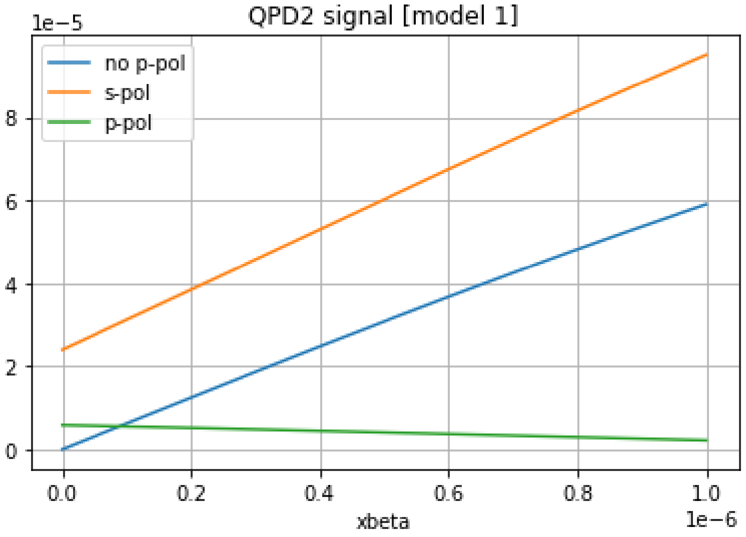
Can we improve the ASC signal by combining s + p signal?

Gouy phase

QPD1: 40° from ITM

QPD2: 130° from ITM





$$S_{QPD} \sim u_-^* u_1 + u_+ u_1^*$$

It is obvious $(u_-^* u_1)_{s+p} \neq (u_-^* u_1)_s + (u_-^* u_1)_p$

How can we improve the ASC signal when there is birefringence?

- New ASC control scheme?
- Compensating the offset introduced by birefringence?
  - Digital signal processing?
  - Mapping the offset?
  - Monitoring the mode contents of p-pol?

The latest measurements (August 2022)

klog #21759

	S-pol power Camera	P-pol power Camera	S-pol power Calibrated	P-pol power Calibrated	P-pol percentage
X unlocked 1	6750.0	859.6	13500.0	1074.5	7.4%
X unlocked 2	6800.0	886.5	13600.0	1108.1	7.5%
X locked 1	5125.0	177.6	10250.0	222.0	2.1%
X locked 2	4933.3	292.7	9866.6	365.9	3.6%

Cavity round-trip loss (ppm)

	90% mode matching	95% mode matching
Measurement 1	317.2	300.5
Only S-pol	271.9	257.6
Measurement 2	342.6	324.5
Only S-pol	309.5	293.1

	S-pol power Camera	P-pol power Camera	S-pol power Calibrated	P-pol power Calibrated	P-pol percentage
Y unlocked 1	7250.0	484.9	14500.0	2424.5	14.3%
Y unlocked 2	7163.74	468.12	14327.48	2340.6	14.0%
Y locked 1	7276.1	126.0	14552.2	630	4.1%
Y locked 2	7337.88	114.29	14675.76	571.45	3.7%

Cavity round-trip loss (ppm)

	90% mode matching	95% mode matching
Measurement 1	118.8	112.6
Only S-pol	0.44	0.42
Measurement 2	99.2	93.9
Only S-pol	-22	-21.3

Single bounce from ITM (arm cavity unlocked)

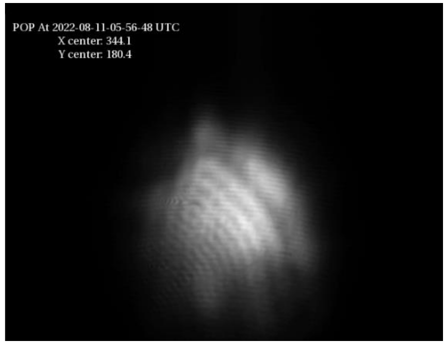
ITMY

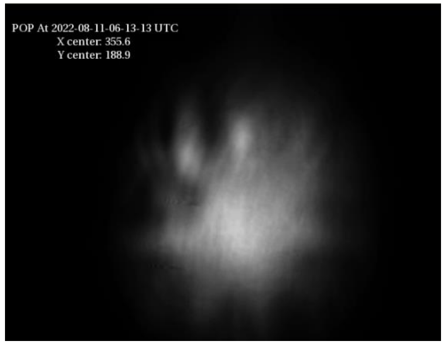
ITMX

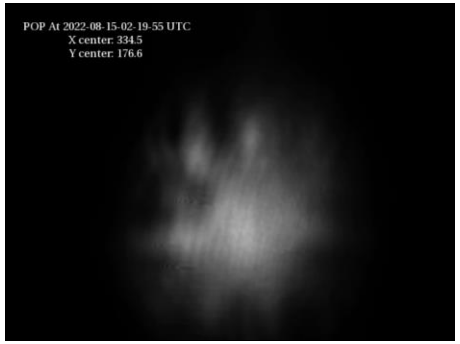
Measurement 1

Measurement 2

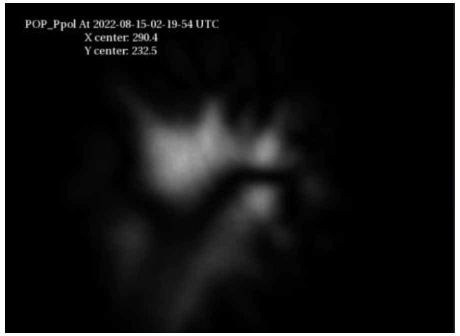
S-pol





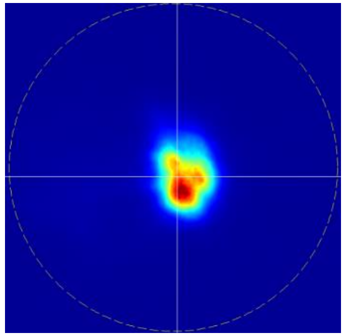


P-pol



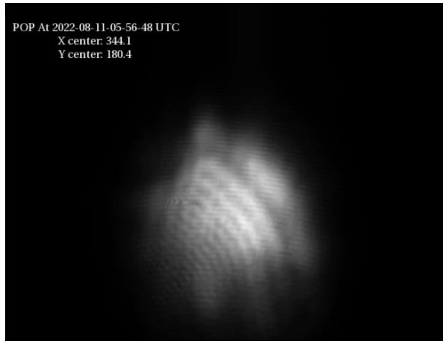
Single bounce from ITMX

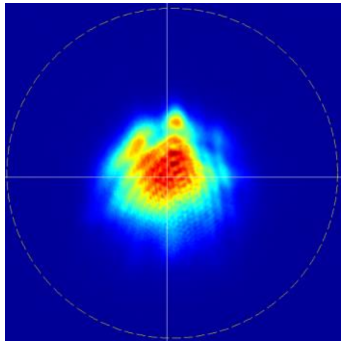
Total

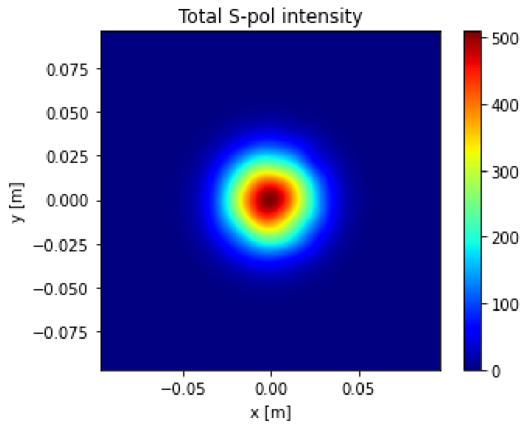


ITMX

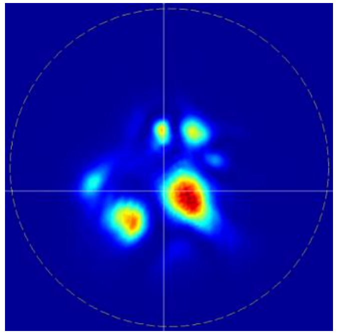
S-pol

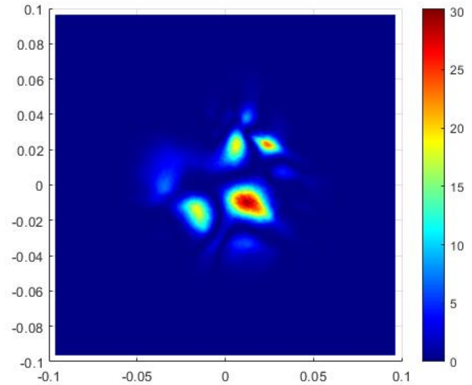






P-pol



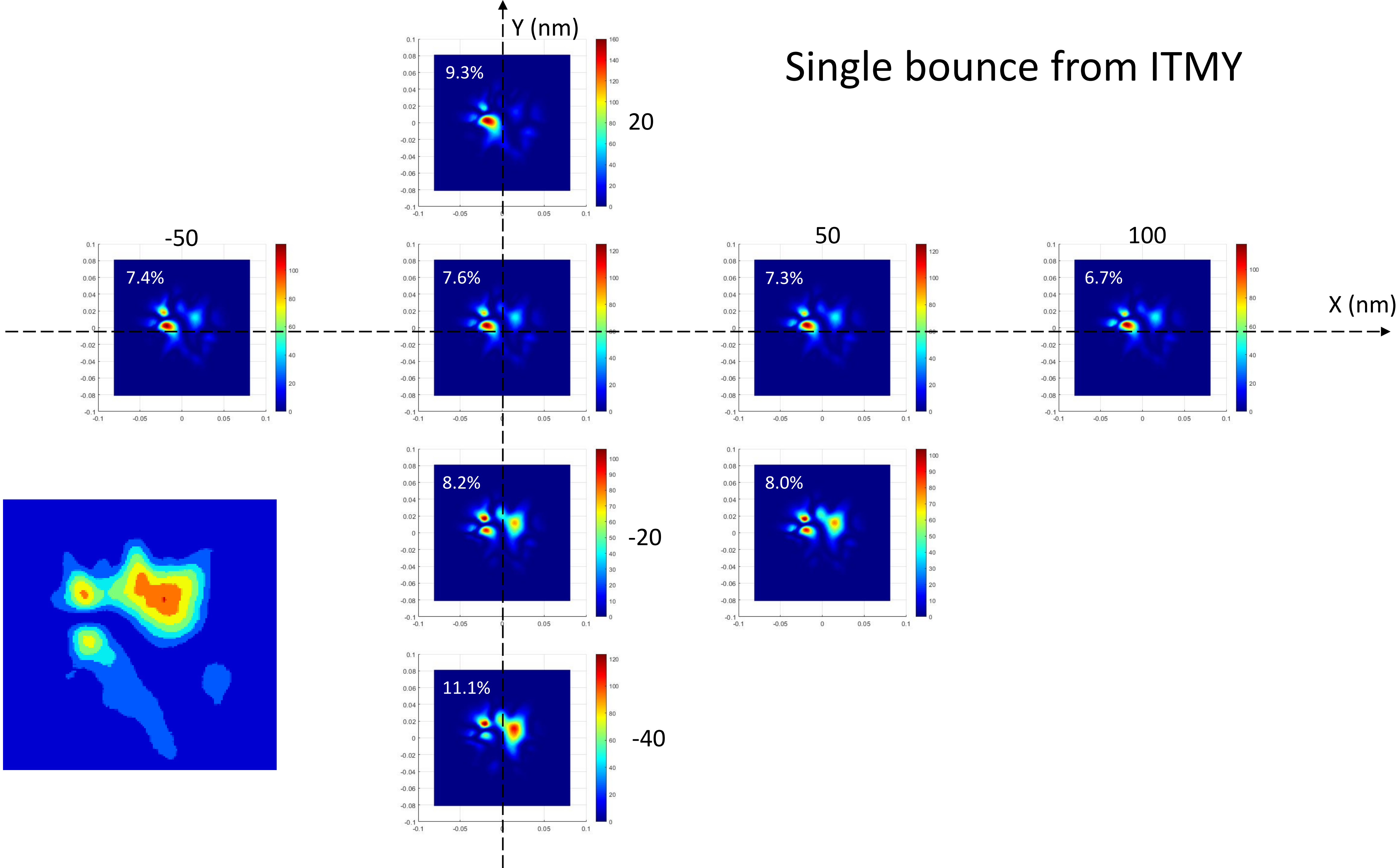


Measurement (August)

Measurement (March)

Simulation

Single bounce from ITMY



Thank you!

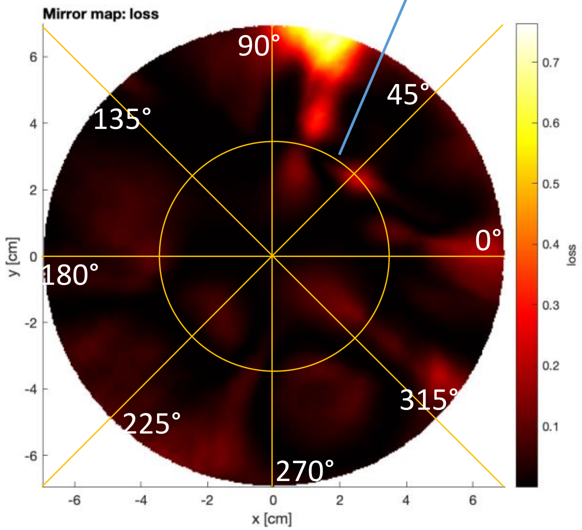
S-pol loss estimation

$$M(\theta,\alpha_-) = \begin{pmatrix} \cos^2\theta \cdot e^{i\alpha_-} + \sin^2\theta \cdot e^{-i\alpha_-} & i\sin 2\theta \sin\alpha_- \\ i\sin 2\theta \sin\alpha_- & \sin^2\theta \cdot e^{i\alpha_-} + \cos^2\theta \cdot e^{-i\alpha_-} \end{pmatrix}$$

$$loss = \sin^2 2\theta \sin^2\alpha_-$$

ITMX

3.5cm beam radius

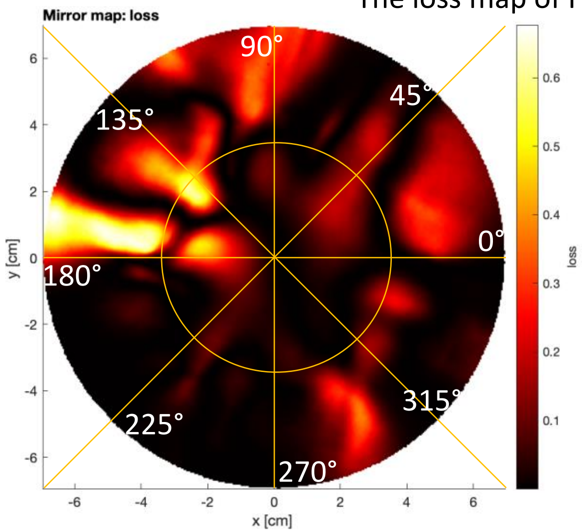


Loss estimation 2.5%
(2.3%~2.7% with 0.5cm decenter)
Measurement 6.1%

We have installed some polarization optics in KAGRA and are going to take more precise measurements for birefringence loss.

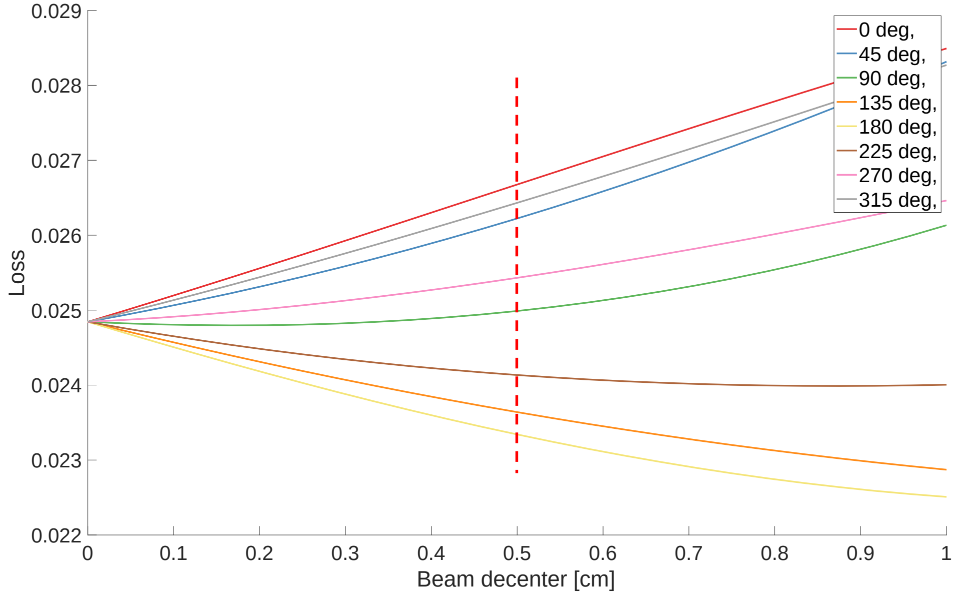
ITMY

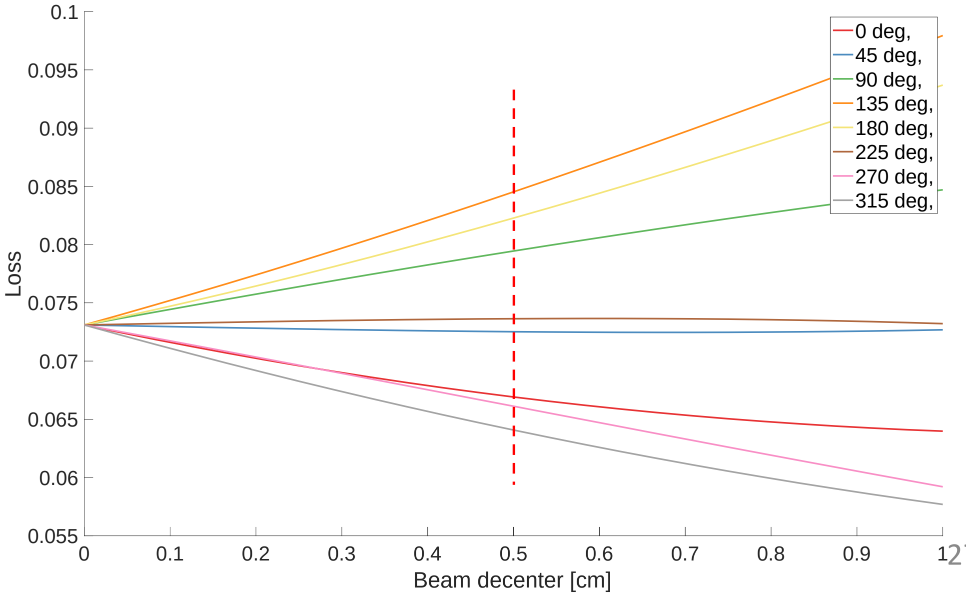
The loss map of ITMY is not well calibrated yet.



Loss estimation 7.3%
(6.5%~8.5% with 0.5cm decenter)
Measurement 10.8%

Birefringence loss can be slightly reduced by about 1% by introducing 0.5cm beam decenter.



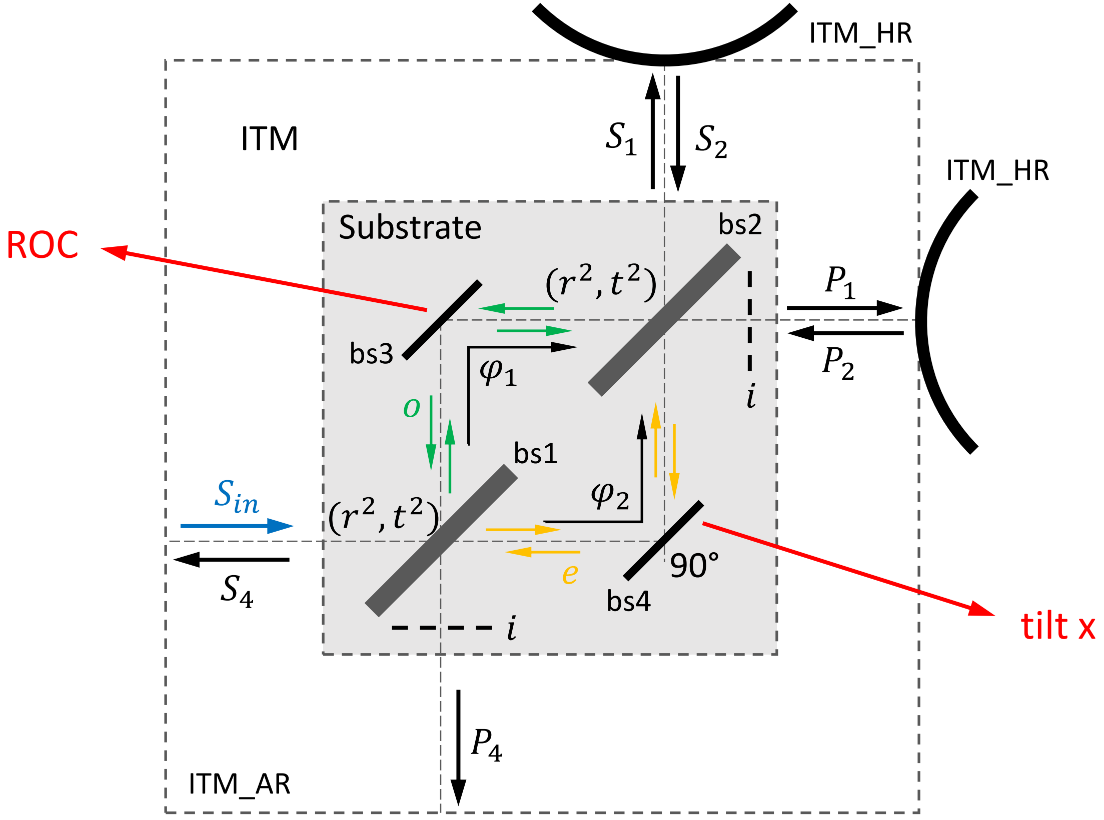


Simulation with fixed parameters

- Add ROC to bs3
- Add tilt to bs4

- Measure p-pol power for P4

ROC m	P-pol power Finesse 2	P-pol power Finesse 3
-22020	5%	2.57%
-24890	4%	2.04%
-29060	3%	1.52%
-35970	2%	1.01%
-51400	1%	0.50%



ROC and tilt x to create different modes		P-pol power Finesse 2	P-pol power Finesse 3
ROC (m)	-51702.6	5%	4.63%
Tilt x (radian)	1.01e-06		
ROC (m)	-28449.5	5%	3.66%
Tilt x (radian)	7.34e-07		
ROC (m)	-22159.6	5%	2.60%
Tilt x (radian)	1.35e-07		