An auto-alignment strategy despite the existence of ITM birefringence

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Ando lab seminar

20 Jan 2023

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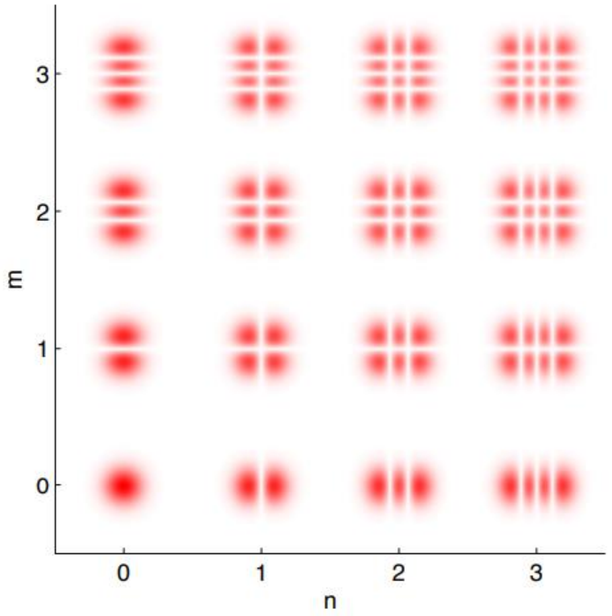
- Description of cavity misalignments
- Detecting misalignments with quadrant photodetectors (QPDs)
- Influence of ITM birefringence on auto-alignment system
- A tabletop experiment

Beam distortions and higher-order mode (HOM)

- Beam distortion: any deviation of a wavefront from the ideal Gaussian wavefront
- Modal model: the shape of a field is not given as a function of x, y coordinates but by a sum of Gaussian modes of different orders. In other words, beam distortions can be described by HOMs.

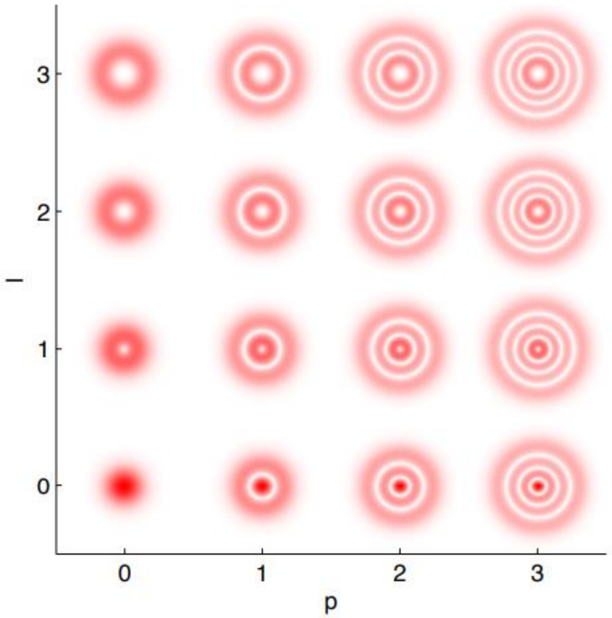
Hermite-Gauss (HG_{nm})

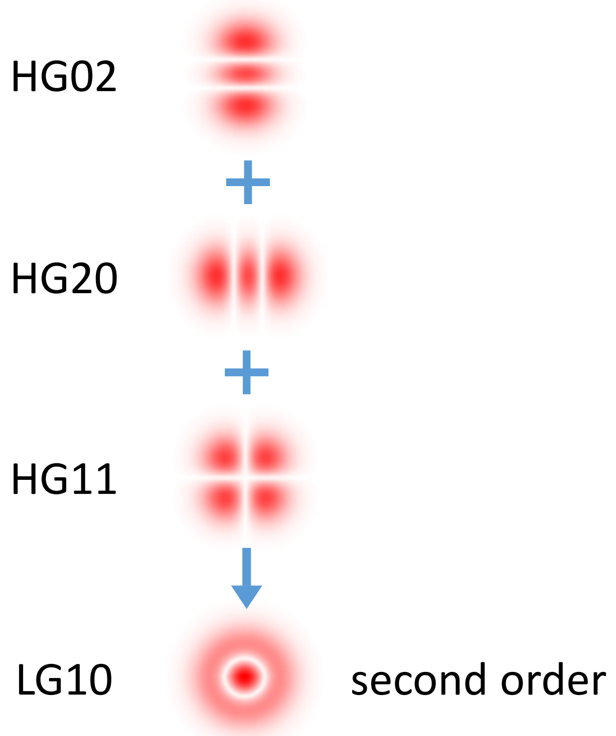
Order: $n+m$



Laguerre-Gauss (LG_{pl})

$2p+|l|$

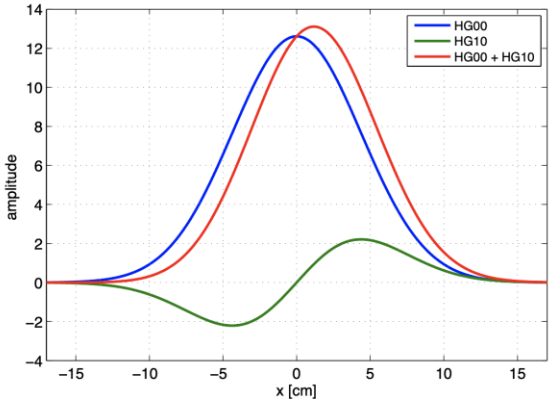


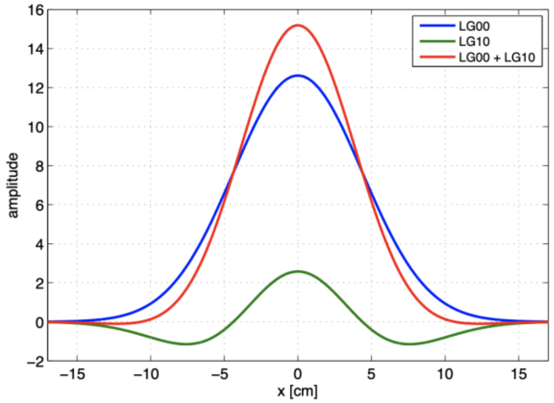


Beam distortions and higher-order mode (HOM)

- Simple distortions: Misalignment and mode-mismatch

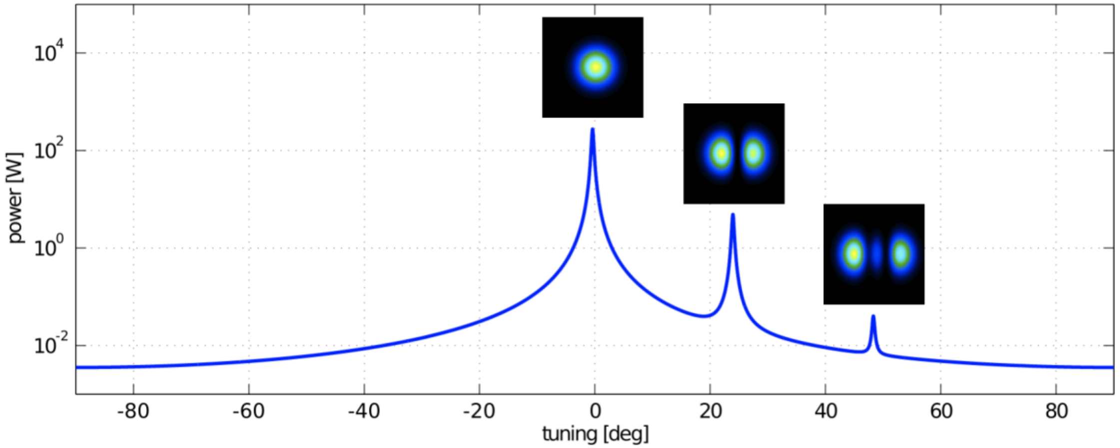
Misalignment:
Mismatch of optical axis between two beams



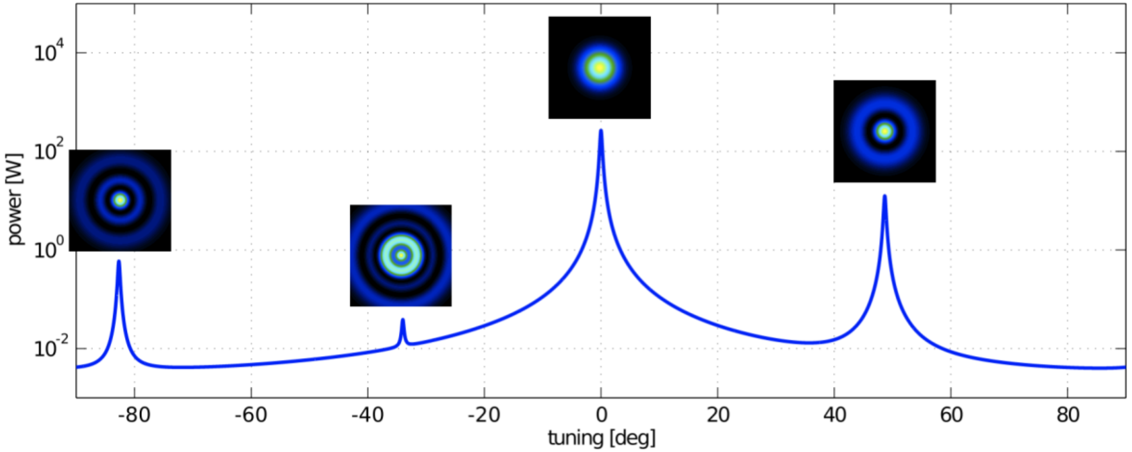


Mode-mismatch:
Mismatch of waist size or position between two beams

Scan of a misaligned cavity



Scan of a mode-mismatched cavity

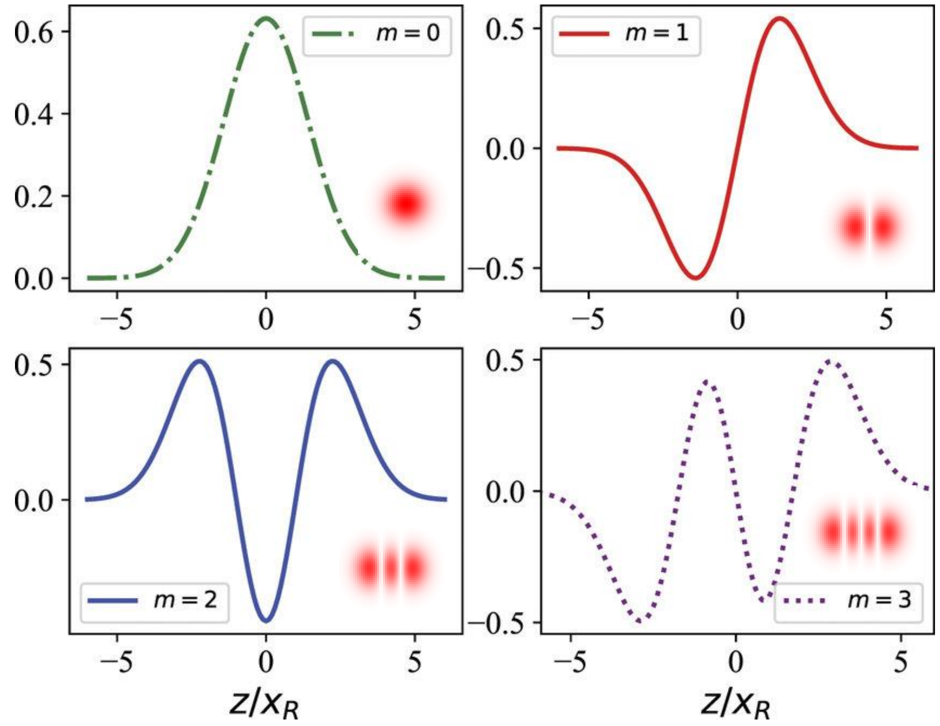


Hermite-Gauss (HG_{nm}) mode

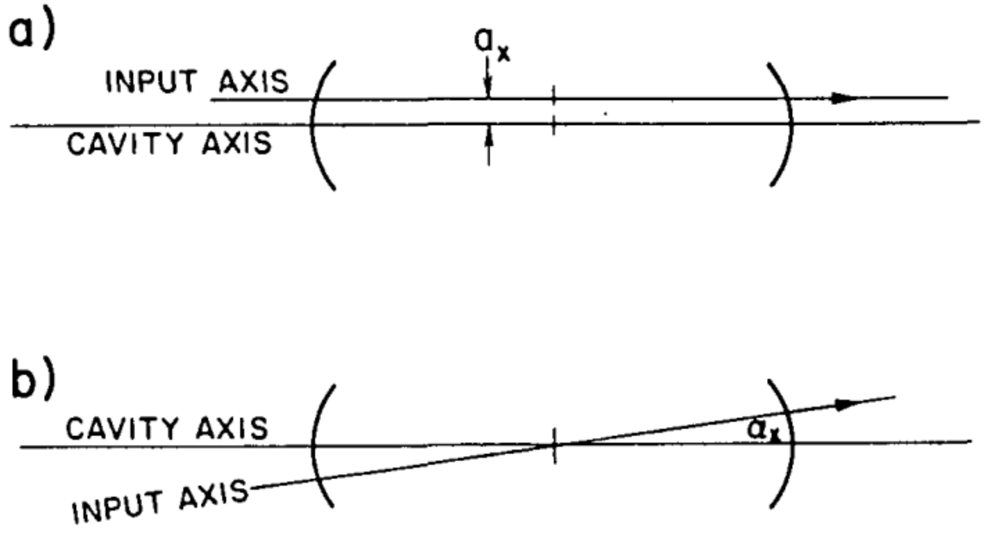
$$
\begin{aligned}
u_{nm}(x,y,z) =& u_n(x,z)\cdot u_m(y,z) \\
=& (2^{n+m-1} n! m! \pi)^{-1/2} \frac{1}{w(z)} \exp\left[i(n+m+1)\Psi(z)\right] \\
& \cdot H_n\left(\frac{\sqrt{2}x}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) \exp\left[-i\frac{k(x^2+y^2)}{2R_C(z)} - \frac{x^2+y^2}{w^2(z)}\right]
\end{aligned}
$$

with

$$
u_n(x,z) = \left(\frac{2}{\pi}\right)^{1/4} \left[\frac{\exp\left[i(2n+1)\Psi(z)\right]}{2^n n! w(z)}\right]^{1/2} H_n\left(\frac{\sqrt{2}x}{w(z)}\right) \exp\left[-i\frac{kx^2}{2R_C(z)} - \frac{x^2}{w^2(z)}\right]
$$



Cavity misalignments



At beam waist position where $z = 0$

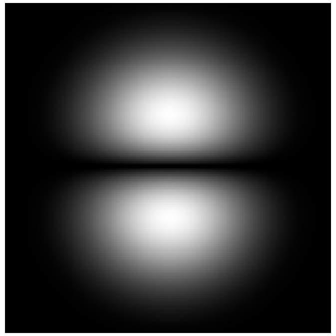
Fundamental mode $$u_0(x) = \left(\frac{2}{\pi w_0^2}\right)^{1/4} \exp\left(-\frac{x^2}{w_0^2}\right)$$

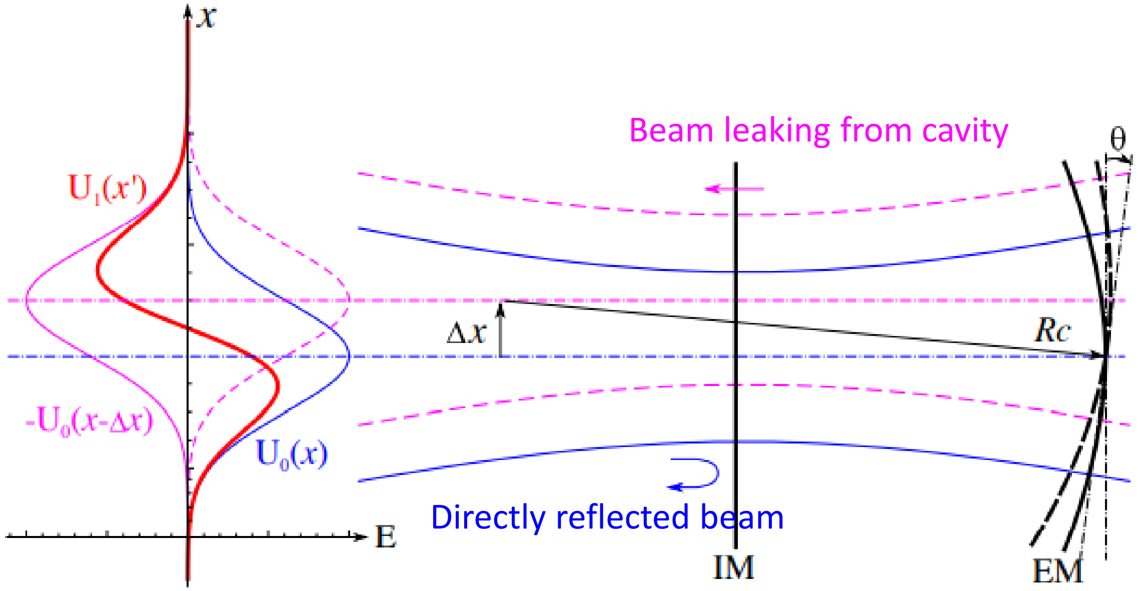
First order mode $$u_1(x) = \left(\frac{2}{\pi w_0^2}\right)^{1/4} \frac{2x}{w_0} \exp\left(-\frac{x^2}{w_0^2}\right)$$

Effect of cavity axis translation

$$
\begin{aligned}
u_0(x-\Delta x) &= (\frac{2}{\pi w_0^2})^{1/4} \exp\left[-\frac{(x-\Delta x)^2}{w_0^2}\right] \\
&\approx (\frac{2}{\pi w_0^2})^{1/4} \exp\left(-\frac{x^2}{w_0^2} + \frac{2x\Delta x}{w_0^2}\right) \\
&\approx (\frac{2}{\pi w_0^2})^{1/4} \left(1 + \frac{2x\Delta x}{w_0^2}\right) \exp\left(-\frac{x^2}{w_0^2}\right) \\
&= u_0(x) + \frac{\Delta x}{w_0} u_1(x)
\end{aligned}
$$

- The offset between the cavity axis and the input beam axis will result in a first order mode.
- The first order mode has the same phase as the input beam.

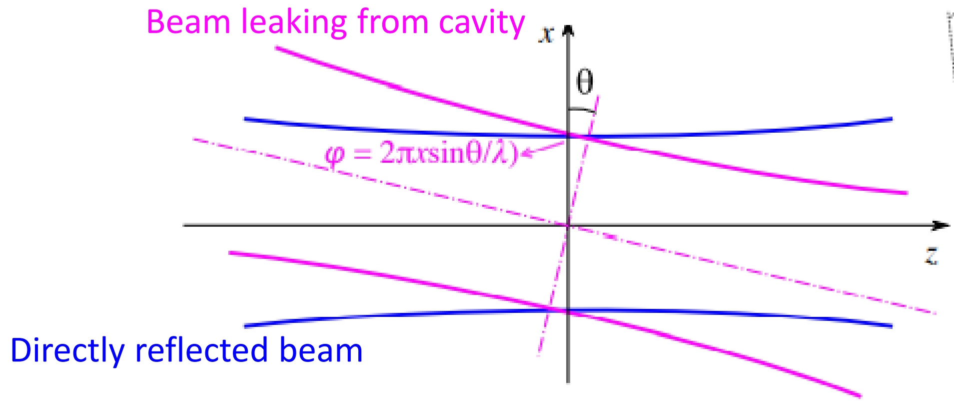


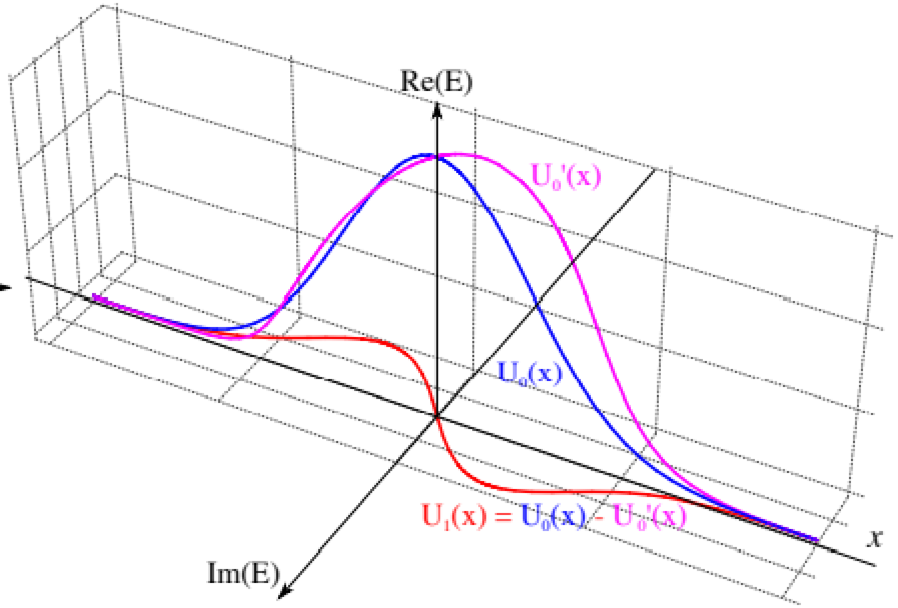


Effect of cavity axis tilt

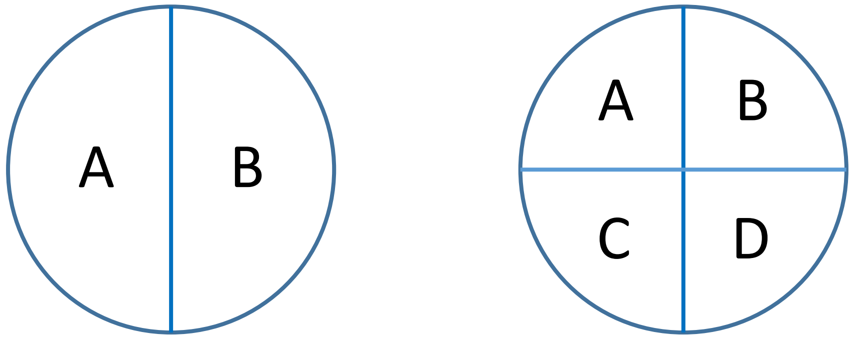
$$
\begin{aligned}
u(x)_{\text{tilt}} &= u_0(x)\exp(i\varphi) = u_0(x)\exp\left(\frac{i2\pi x\sin\theta}{\lambda}\right) \\
&= u_0(x)\left[\cos\left(\frac{2\pi x\sin\theta}{\lambda}\right) + i\sin\left(\frac{2\pi x\sin\theta}{\lambda}\right)\right] \\
&\approx u_0(x)\left(1 + \frac{i\pi w_0\theta}{\lambda}\frac{2x}{w_0}\right) \\
&= u_0(x) + \frac{i\pi w_0\theta}{\lambda}u_1(x) \\
&= u_0(x) + i\frac{\theta}{\theta_{\text{div}}}u_1(x)
\end{aligned}
$$

- The tilt between the cavity axis and the input beam axis will result in a first order mode.
- The resultant first order mode has a phase shift of 90◦ relative to the original beam.

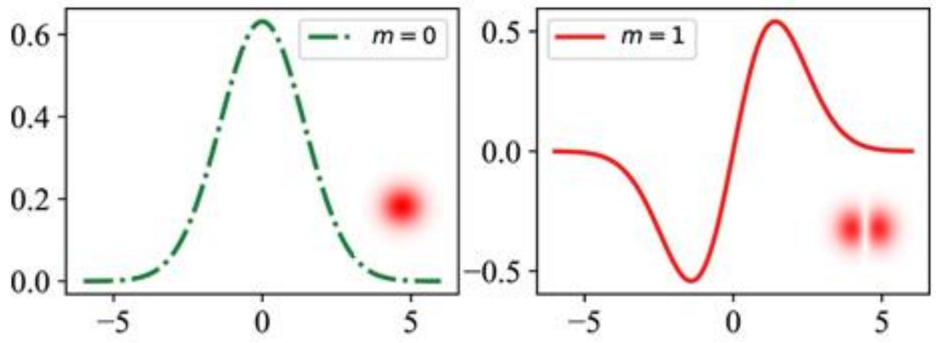


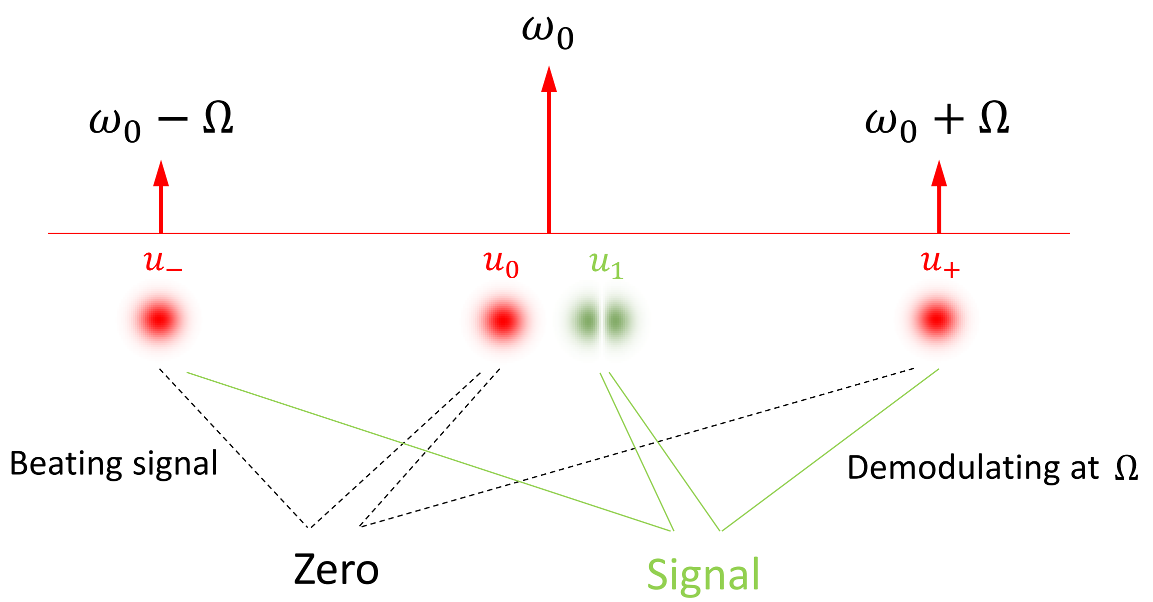


Detecting misalignments with split photodetector



$$S_{\mathrm{QPD}} = \int_{-\infty}^{0} EE^* \mathrm{d}x - \int_{0}^{+\infty} EE^* \mathrm{d}x$$
$$= \begin{cases} 0, & a_i a_j \text{ is even,} \\ -2\int_0^{+\infty} E_0^2 u_0^2 \sum_{i=0}^{3}\sum_{j=0}^{3} \mathrm{Re}(a_i a_j^* e^{-i\varphi_{\mathrm{dem}}})\mathrm{d}x, & a_i a_j \text{ is odd,} \end{cases}$$



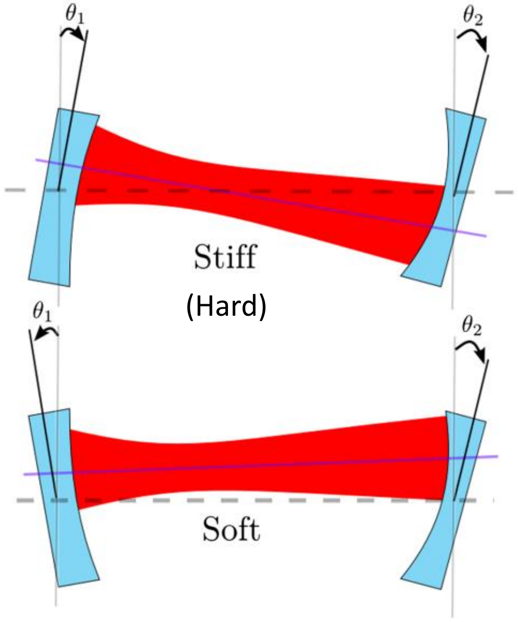


Auto-alignment control system

Error signal

$$S'_{\text{QPD, DC}} = -E_0^2 m\omega_0 \left(\frac{\Delta x}{\omega_0} \sin\psi + \frac{\theta}{\theta_{\text{div}}} \cos\psi \right) \cos\varphi_{\text{dem}}$$

Two angular modes of a Fabry-Perot cavity



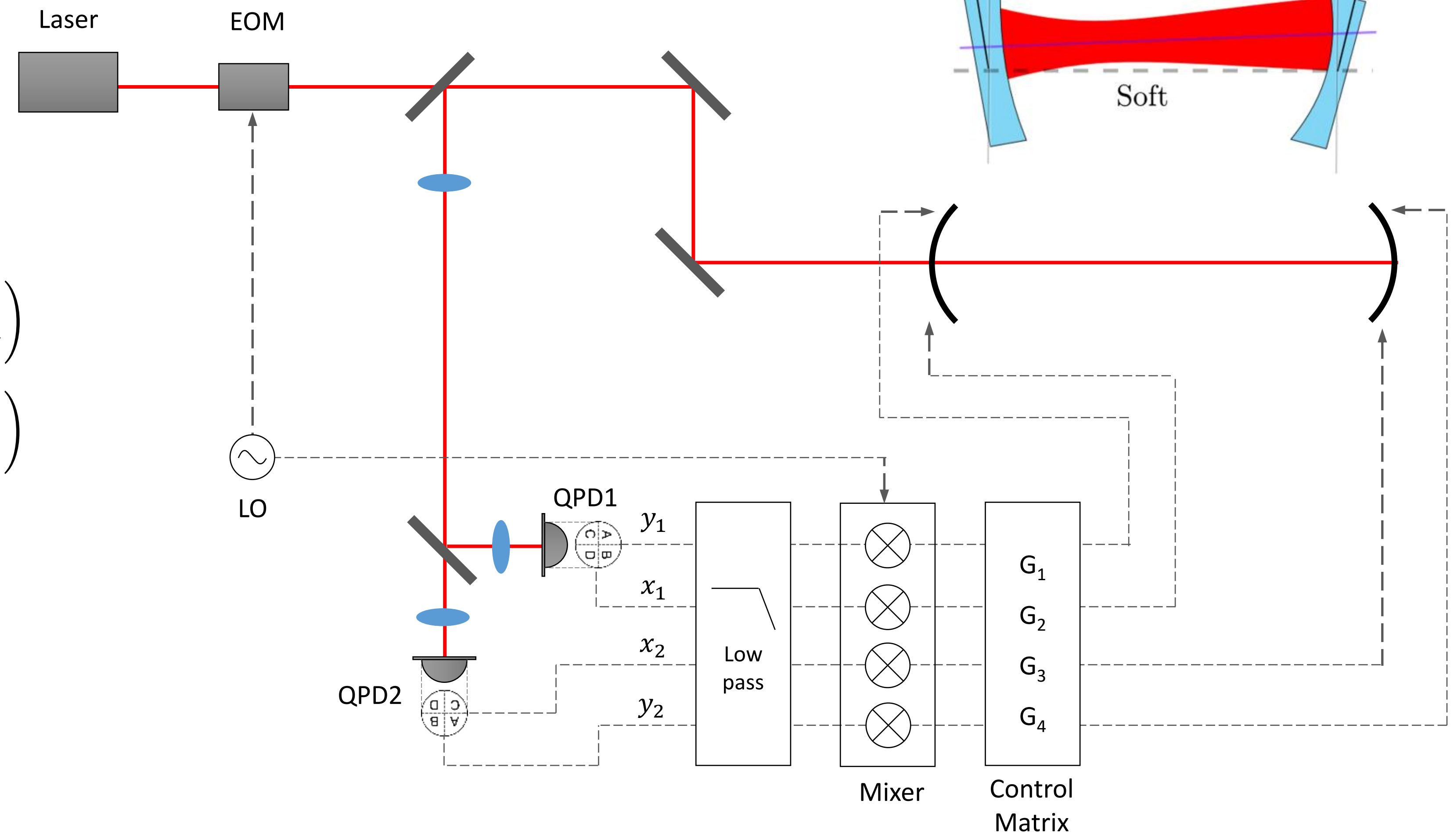
We use two QPDs as a group:

$$S'_{\text{QPD1}} = -E_0^2 m\omega_0 \left(\frac{\Delta x}{\omega_0} \sin\psi_1 + \frac{\theta}{\theta_{\text{div}}} \cos\psi_1 \right)$$

$$S'_{\text{QPD2}} = -E_0^2 m\omega_0 \left(\frac{\Delta x}{\omega_0} \sin\psi_2 + \frac{\theta}{\theta_{\text{div}}} \cos\psi_2 \right)$$

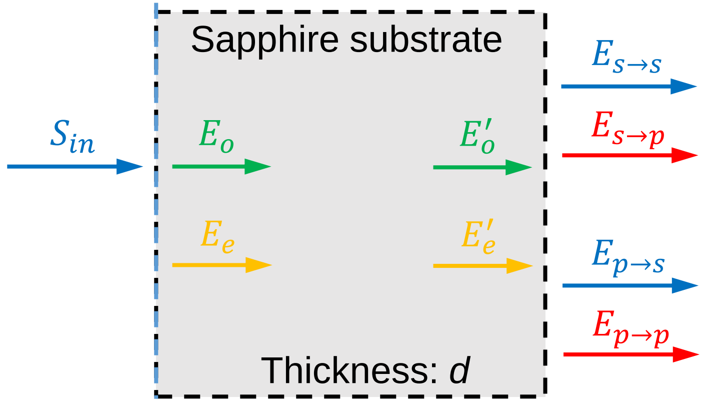
If we choose $\psi_1 = 0$ and $\psi_2 = 90°$,

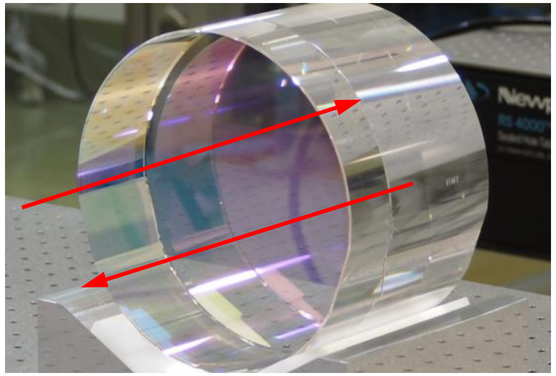
- QPD1 will only see cavity axis tilt
- QPD2 will only see cavity axis offset

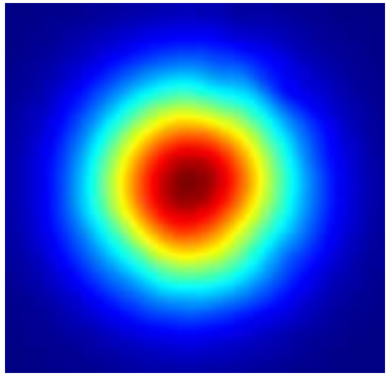


Beam distortion due to birefringence

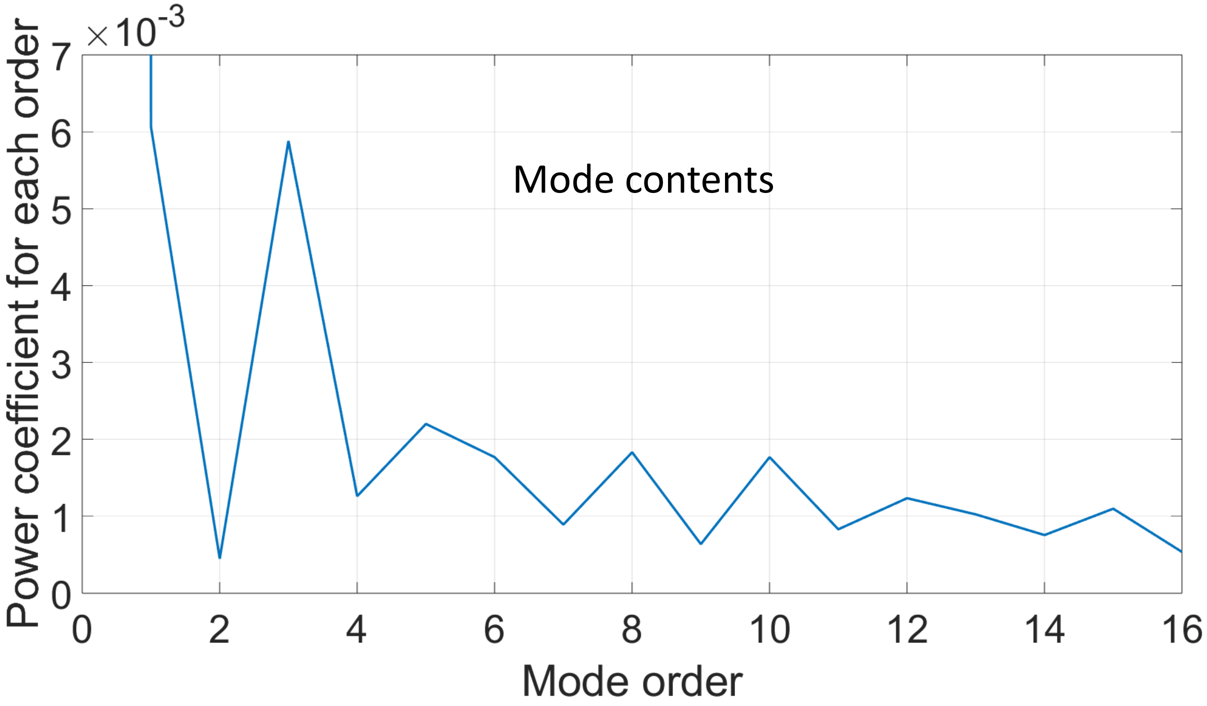
Birefringence coupling in ITM substrate

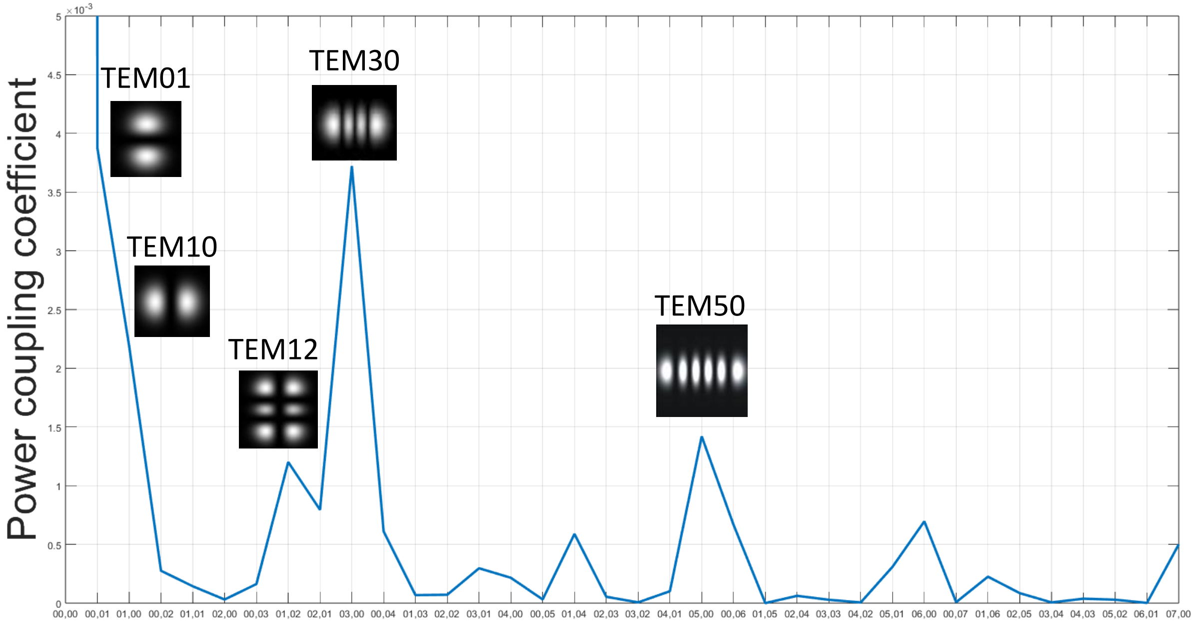




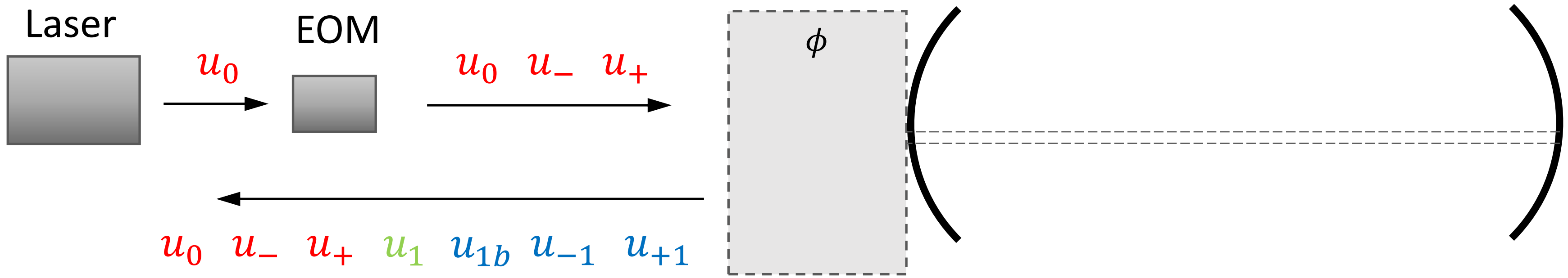


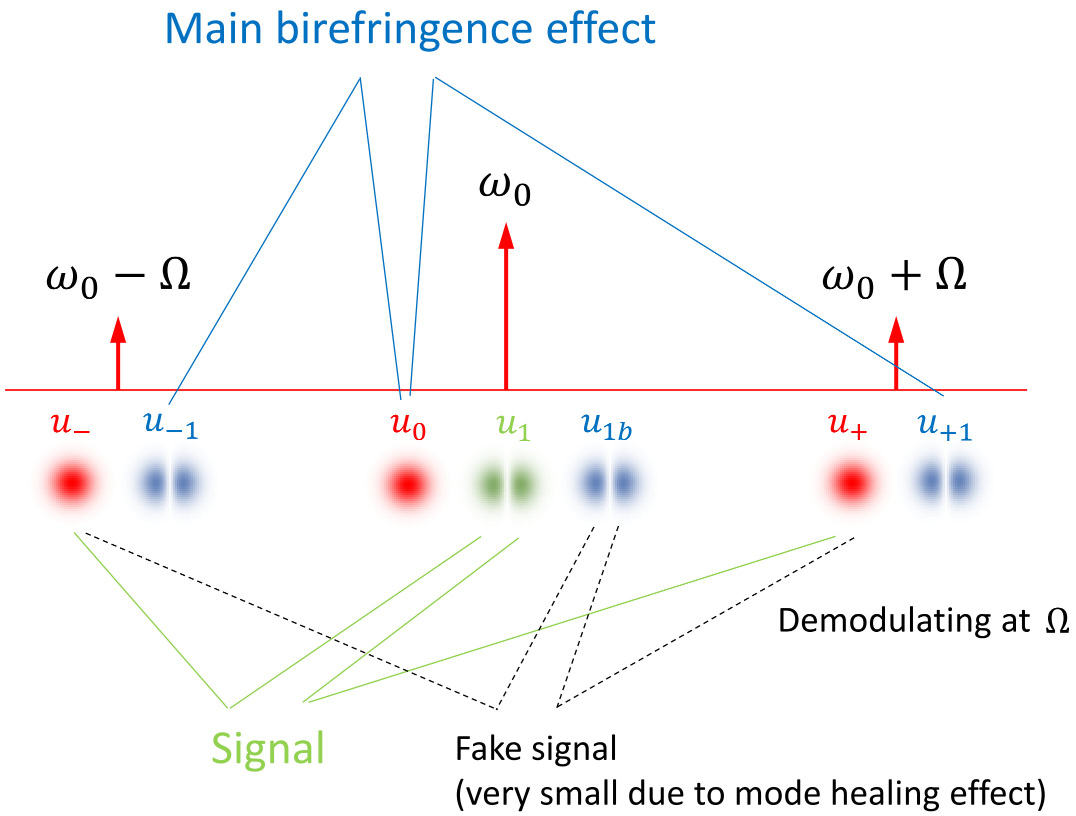
Distorted beam after a round-trip propagation in the ITMX substrate

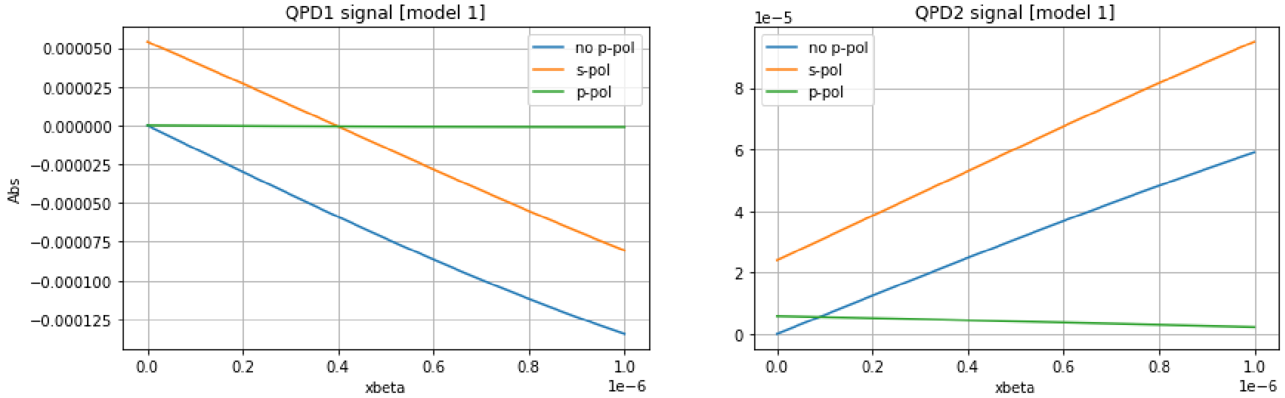




Alignment error signal when there is birefringence

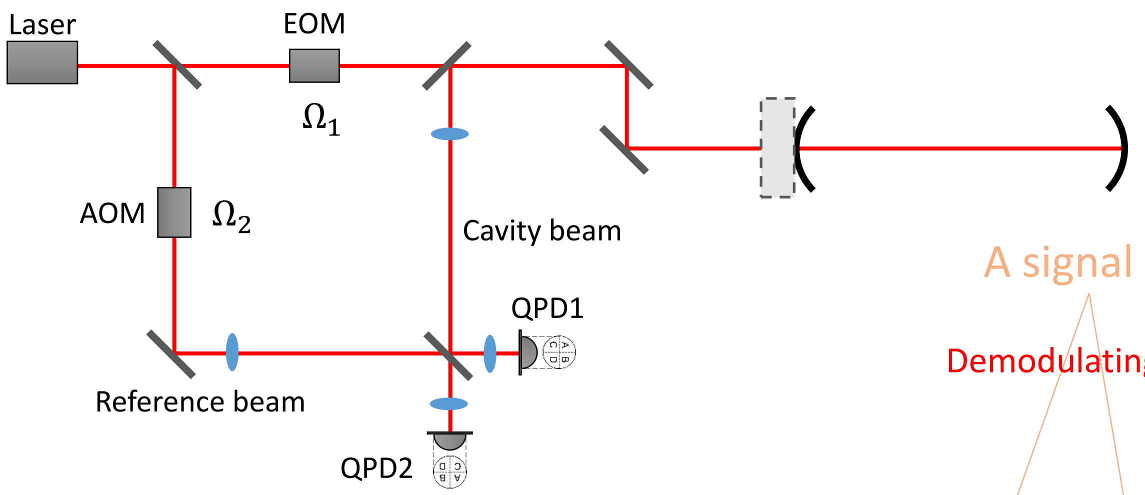


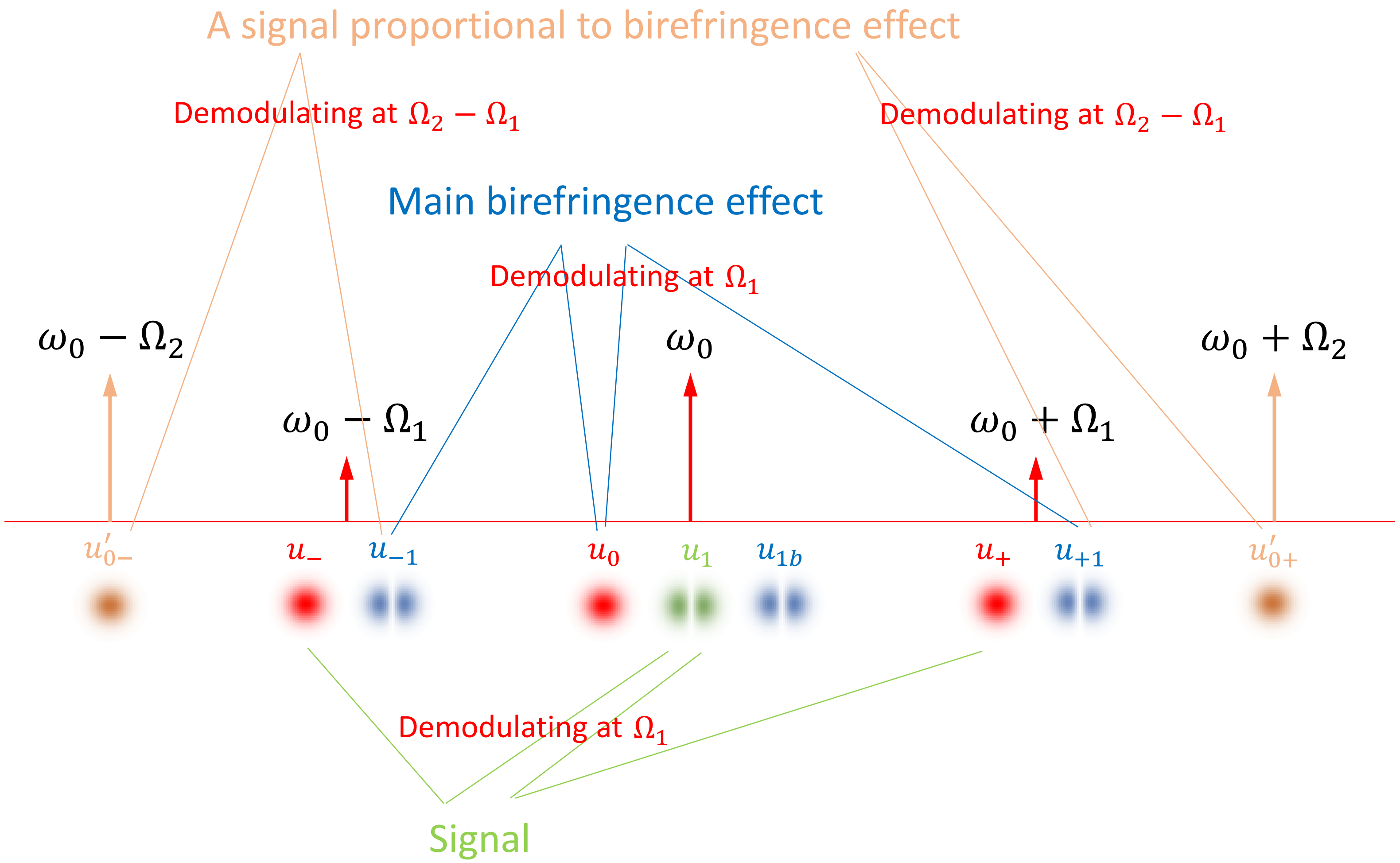




The ASC offset error mainly comes from the beat between the fundamental mode and 1st mode from RF sidebands due to birefringence.

A scheme to extract birefringence effect: heterodyne scheme





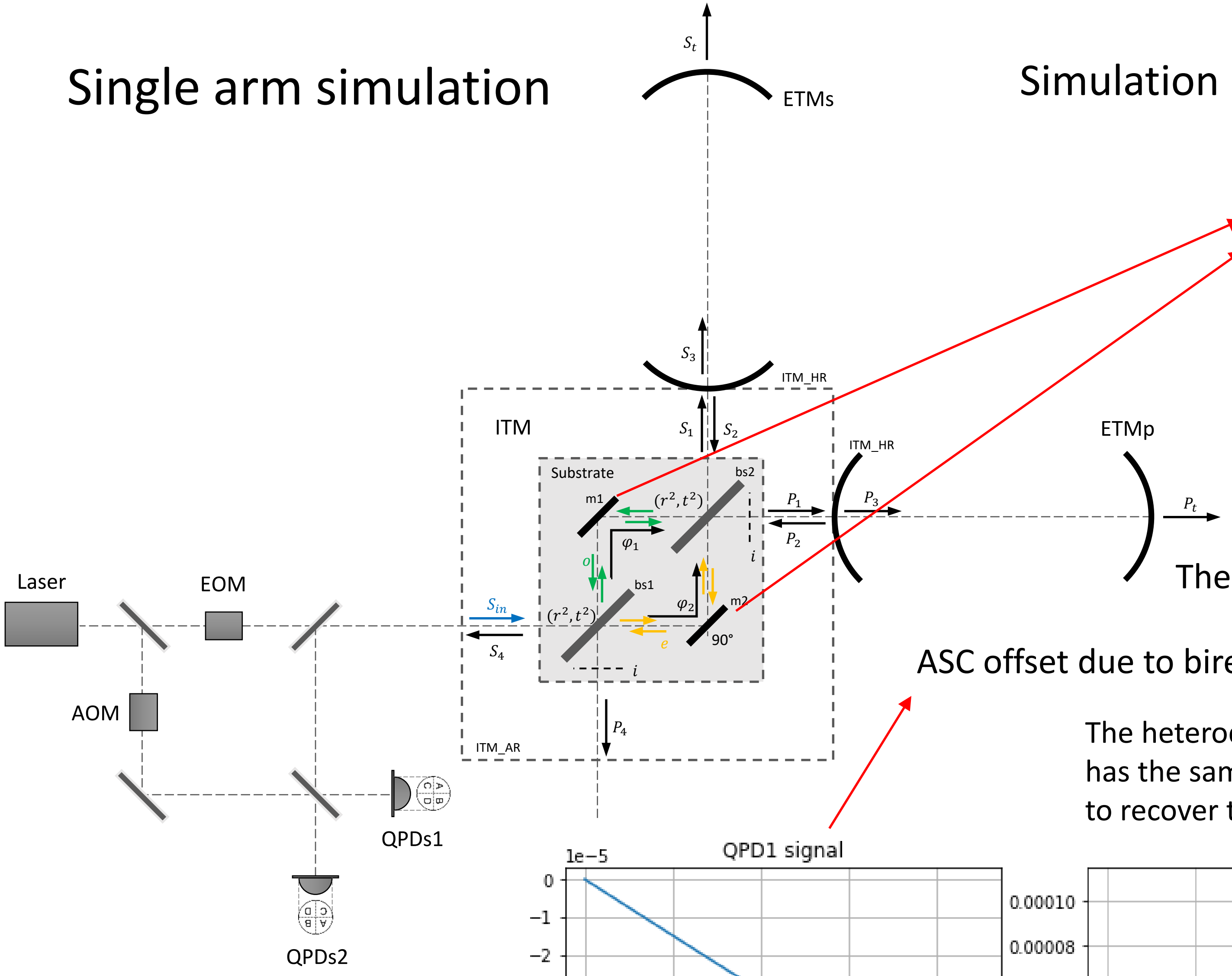
We pick up a portion of the beam from the laser, modulated by the AOM.

This beam is used as a reference beam with a frequency at $\omega_0 \pm \Omega_2$.

The alignment between the reference beam and the cavity beam should be controlled. We also need to demodulate the beam at Ω_2.

Single arm simulation

Simulation of the heterodyne signal with MZ model



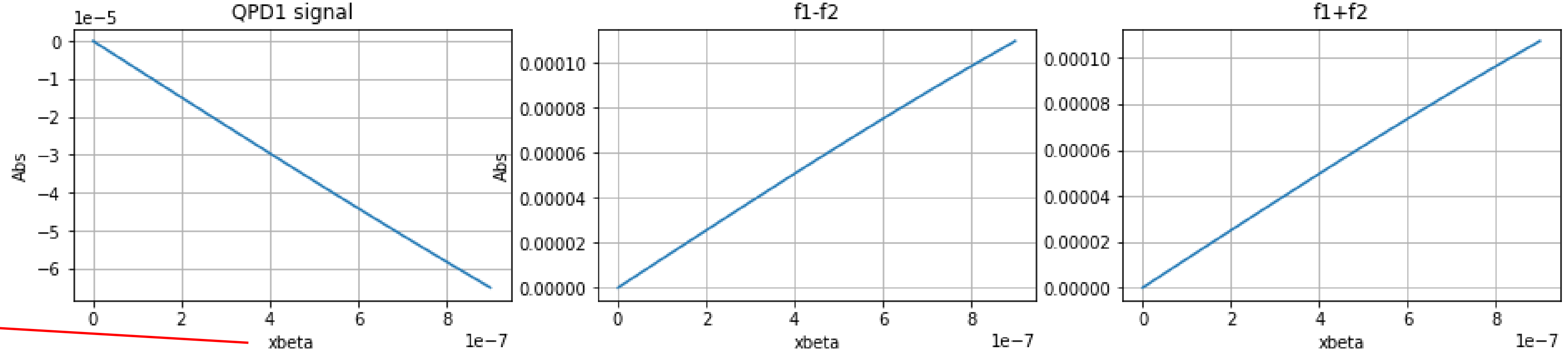
ROC and tilt of m1 and m2 can be tuned to control the mode contents generated by birefringence.

The 1st order mode content (controlled by m1 and m2 tilt) due to ITM birefringence is the reason for the ASC offset.

The cavity is perfectly aligned.

ASC offset due to birefringence

The heterodyne signal (beat between oscillator beam and RF sidebands) has the same trend of ASC offset (the gain is different), which can be used to recover the ASC signal.

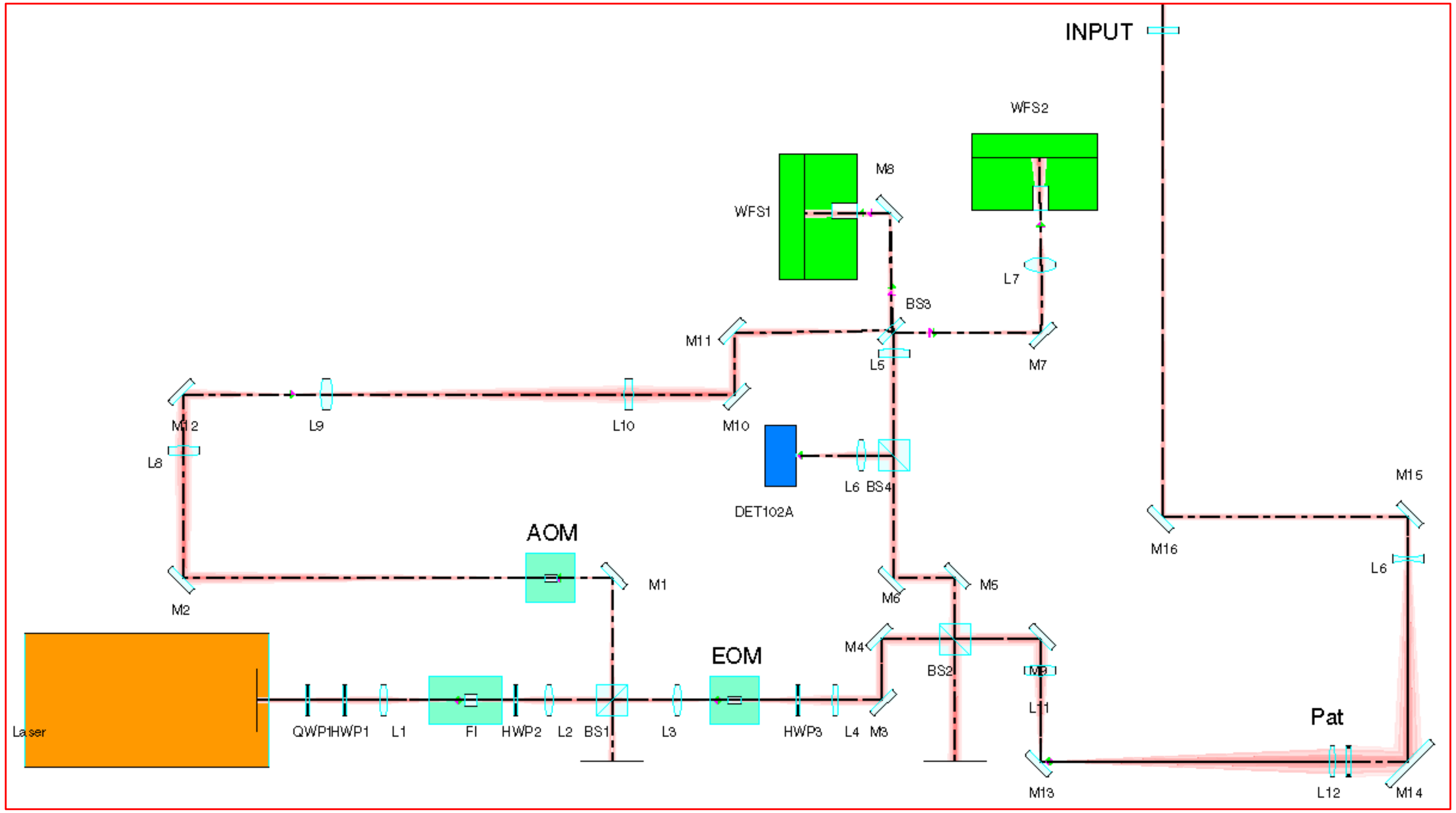


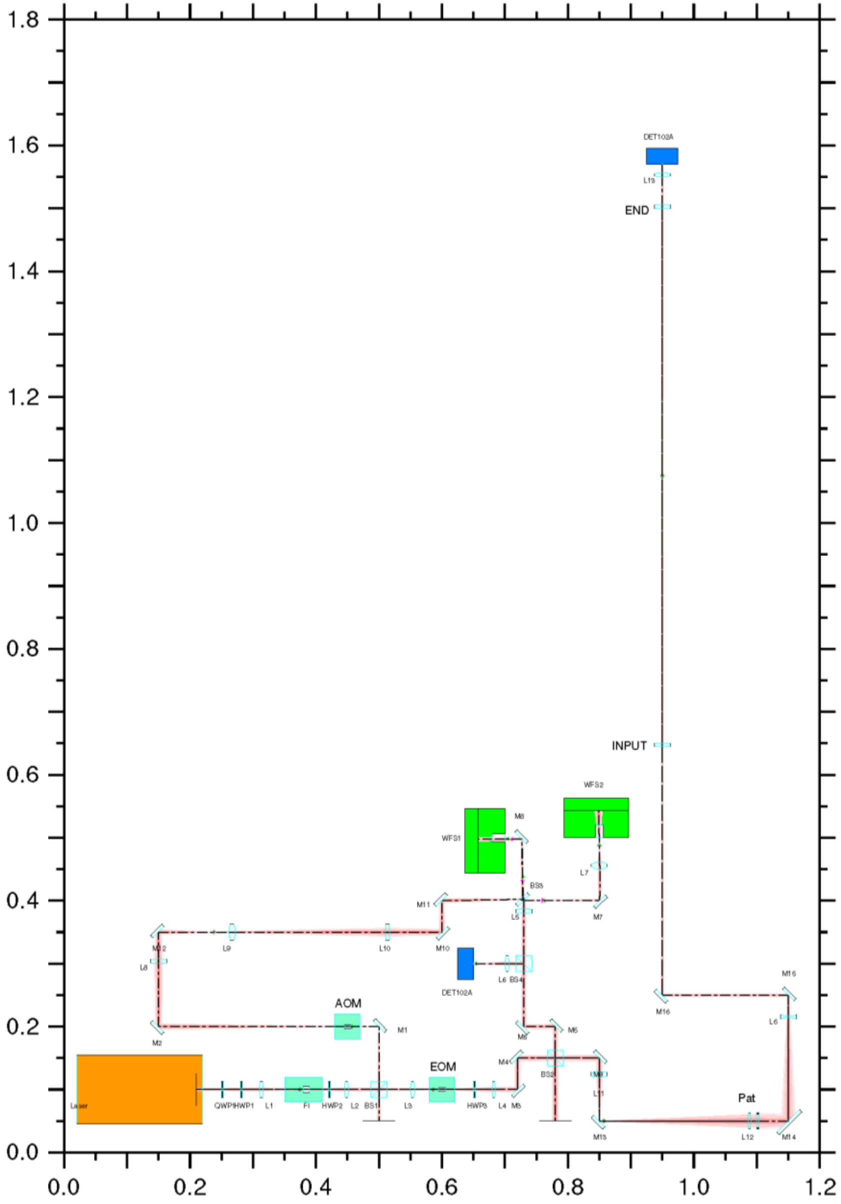
m1 and m2 tilt

A tabletop experiment (NAOJ)

Optical design is ready

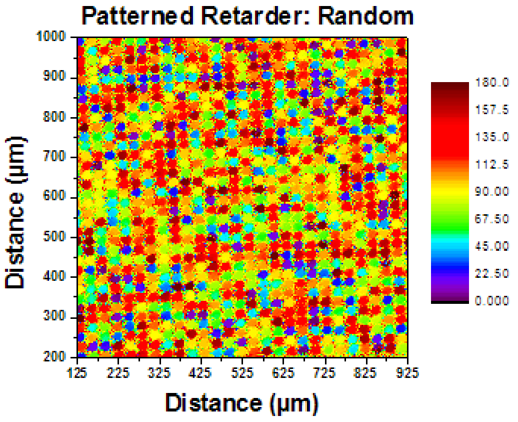
- Cavity length: 80 cm
- EOM: 88 MHz
- AOM: 40 MHz

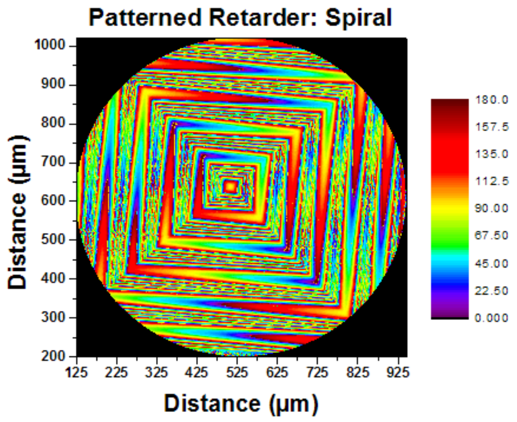




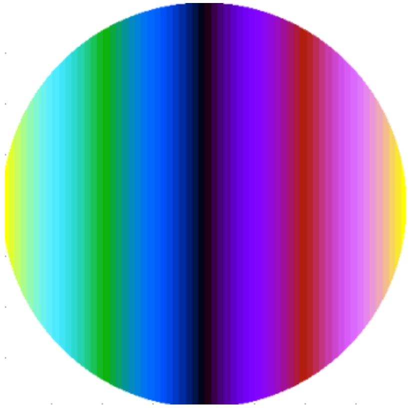
Polarization plate

Custom Capability	Custom Specification
Patterned Retarder Size	Ø100 µm to Ø2"
Patterned Retarder Shape	Any
Microretarder Size	≥Ø30 µm
Microretarder Shape	Round or Square
Retardance Range @ 632.8 nm	50 to 550 nm
Substrate	N-BK7, UV Fused Silica, or Other Glass
Substrate Size	Ø5 mm to Ø2"
AR Coating	-A: 350 - 700 nm -B: 650 - 1050 nm -C: 1050 - 1700 nm





We are going to use customized polarization plate from Thorlabs.

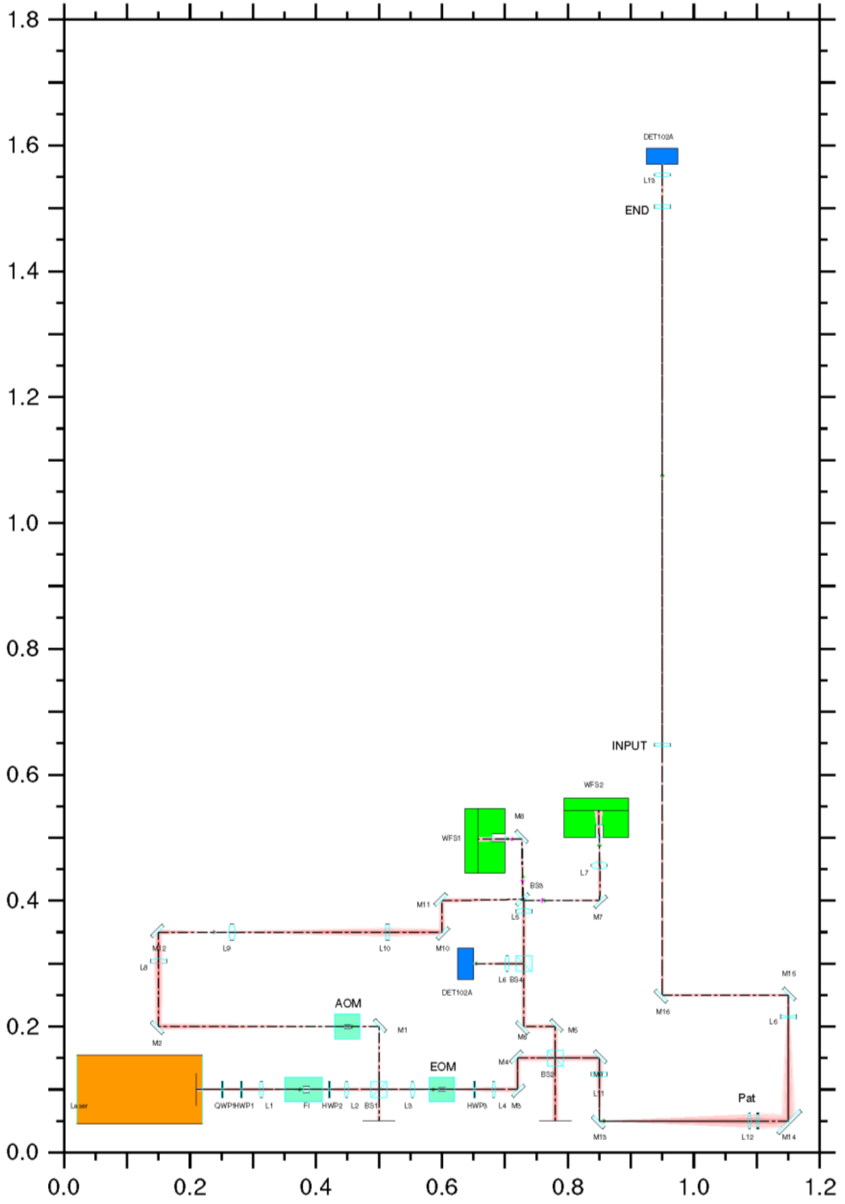


We have ordered a quarter wave plate and a half wave plate with gradient of fast axis.

Tabletop experiment status

- All optics are ready now. The installation has started.

- RFQPDs modification on-going.
  - KAGRA RFQPDs only have 2 demodulation frequencies.
  - Our tabletop setup demodulations are 88 MHz, 48 MHz and 40 MHz.
  - We are going to make the circuit resonant at 88 MHz and 44 MHz.



Thank you for listening!