Application of Modern Control for Interferometric Gravitational Wave Detectors

D1 Hiroki Fujimoto

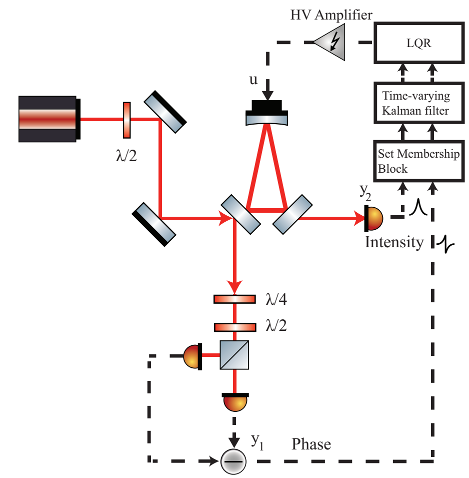
2022/7/15 @Ando Lab Seminar

Paper

◆ Doctoral thesis by D. Shütte in Leibniz University Hannover (AEI)

“Modern Control Approaches for Next-Generation Interferometric Gravitational Wave Detectors (2016)”

- On application of modern control for gravitational wave detectors and quantum optical experiments

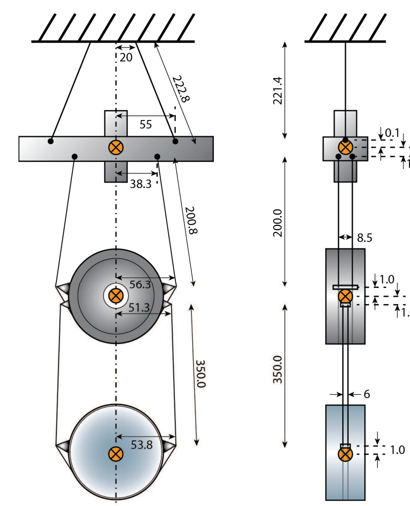


■ Application for a three-mirror ring cavity

- Cavity locking with linear LQG control
- Autolocking with non-linear control

■ Application for a suspension system

- Design of active damping for triple pendulum suspension with $\mathcal{H}_2$ controller synthesis technique



Contents

- Introduction of modern control
  - State-space representation
  - State feedback
  - State observer (Kalman-filter)
- Application for three-mirror ring cavity
  - Cavity locking with linear LQG control
  - Autolocking with non-linear control
- Application for suspension damping system
  - System identification of triple pendulum suspension
  - Designing optimal filters with $\mathcal{H}_2$ controller synthesis
- Advantage/disadvantage of modern control
- Summary

Contents

Transfer function

Equation of motion of a pendulum:

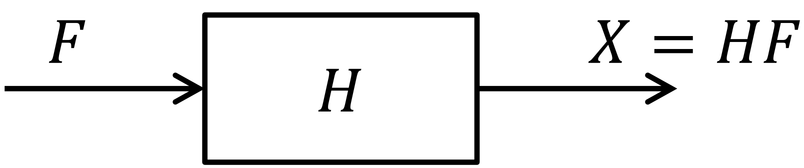
$$m\ddot{x} = -m\omega_0^2 x - 2m\gamma\dot{x} + f$$

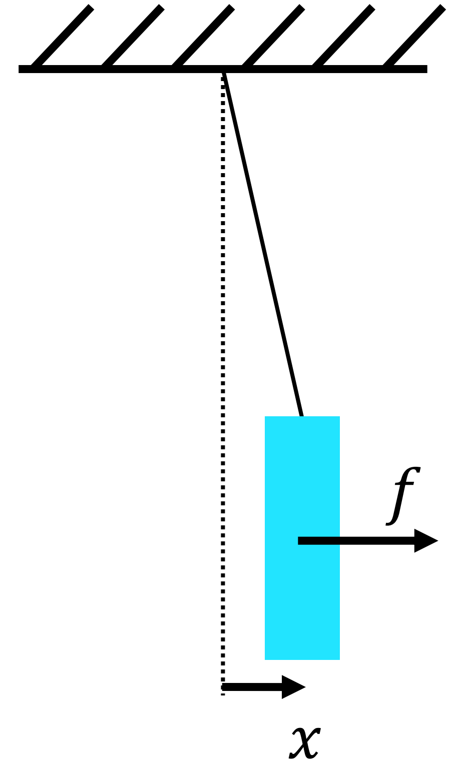
⬇ Fourier transform

$$-m\omega^2 X = -m\omega_0^2 X - i2m\gamma\omega X + F$$

$$\Rightarrow \; \underline{H(\omega) = \frac{X}{F} = \frac{1}{m(\omega_0^2 - \omega^2 + i2\gamma\omega)}}$$

Transfer function





Classical control considers only input/output in frequency domain

⇒ Modern control treats all internal states $(x, \dot{x})$ in time domain

State-space model

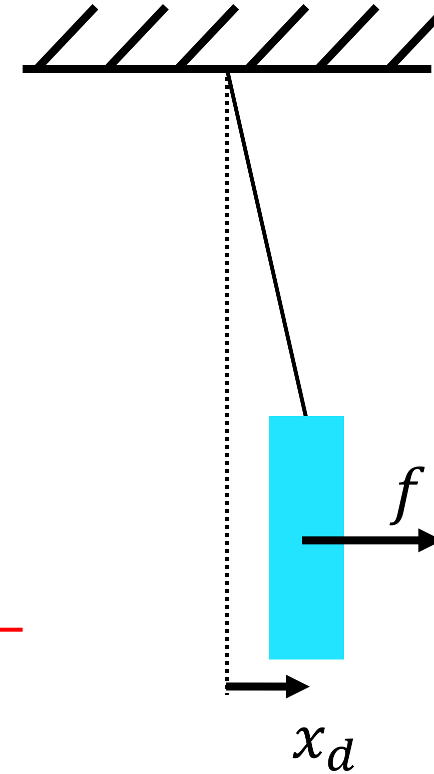
Rewrite EOM with $x_1 \equiv x_d, x_2 \equiv \dot{x}_d$

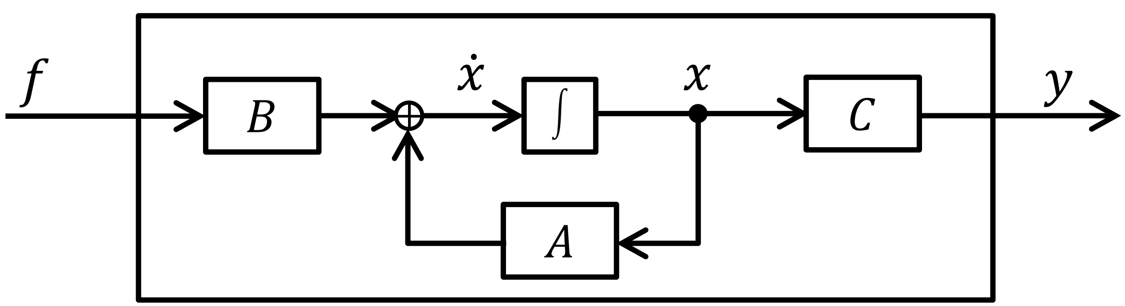
$$\dot{x}_1 = x_2$$
$$m\dot{x}_2 = -m\omega_0^2 x_1 - 2m\gamma x_2 + f$$

Let the internal state be $x \equiv [x_1, x_2]^T$

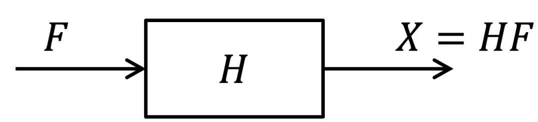
$$\Rightarrow \begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\gamma \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} f \equiv Ax + Bf \\ y = [1\ 0]\ x \equiv Cx \quad \text{(output)} \end{cases}$$

State-space representation





Classical control:



We can treat internal information x, not only input/output

Generalized state-space model

Any linear time-invariant system can be represented as

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$x \in \mathbb{R}^n$: internal states

$u \in \mathbb{R}^m$: inputs

$y \in \mathbb{R}^p$: outputs

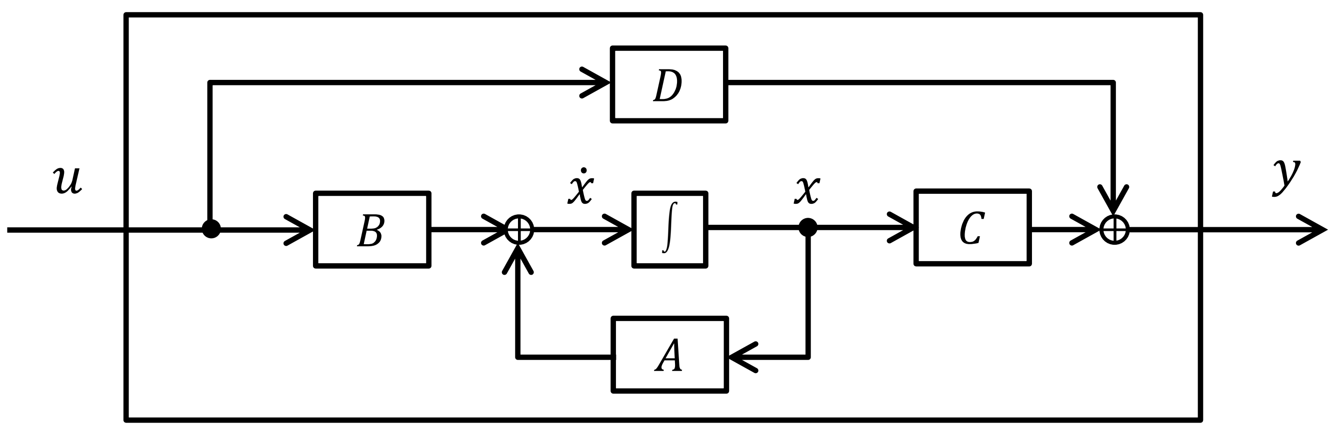
Multiple-input multipule-output system

$A \in \mathbb{R}^{n\times n}$: system matrix

$B \in \mathbb{R}^{n\times m}$: input matrix

$C \in \mathbb{R}^{p\times n}$: output matrix

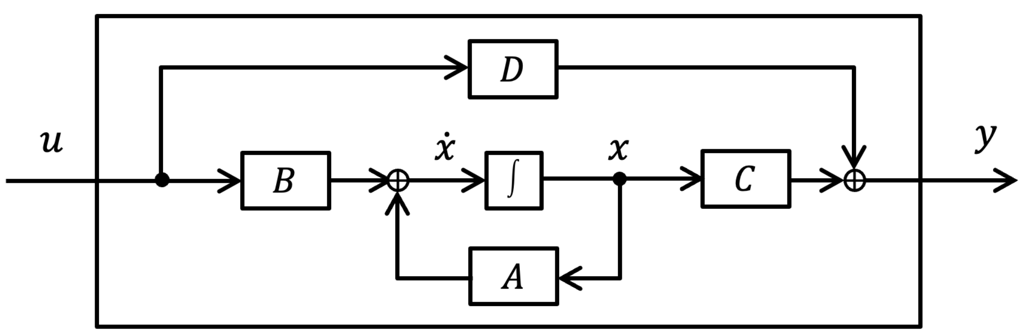
$D \in \mathbb{R}^{p\times m}$: direct feedthrough



Advantages of state-space model

- What are the advantages of state-space model?
  - ➢Able to deal with multiple-input multiple-output(MIMO) system
  - ➢Feedback control with internal state x (State feedback)
  - ➢Able to obtain optimal filters for feedback control mathematically
    (with no need for professional tuning technique)

Disadvantages are mentioned in the last of this seminar



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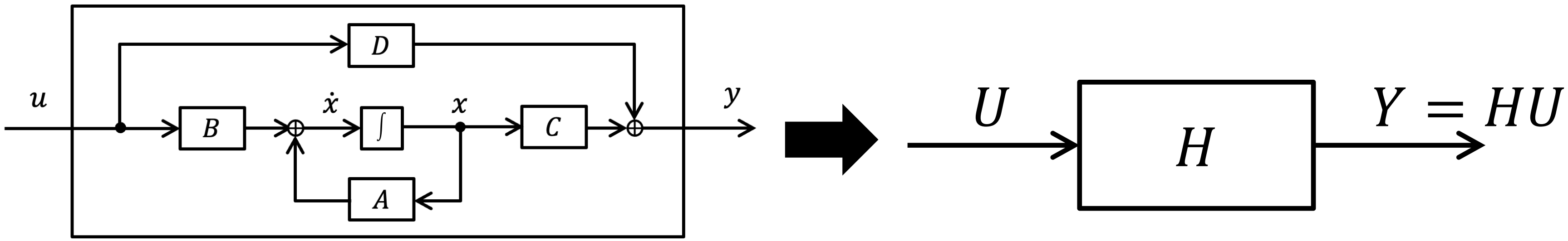
System poles

- State-space ⇒ Transfer function



$$H(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

$$= \frac{1}{\prod_{i=1}^{n}(s - \lambda_i)} C\, adj(sI - A)B + D$$

$adj(M)$: adjugate matrix of M

λ_i: Eigenvalue of system matrix A

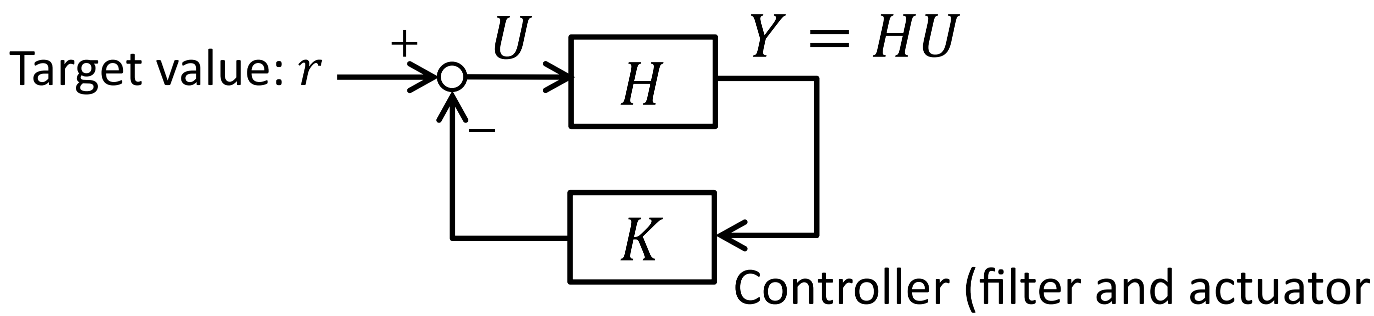
➡ Eigenvalues of A = poles of the system $H(s)$

(if the system is controllable and observable)

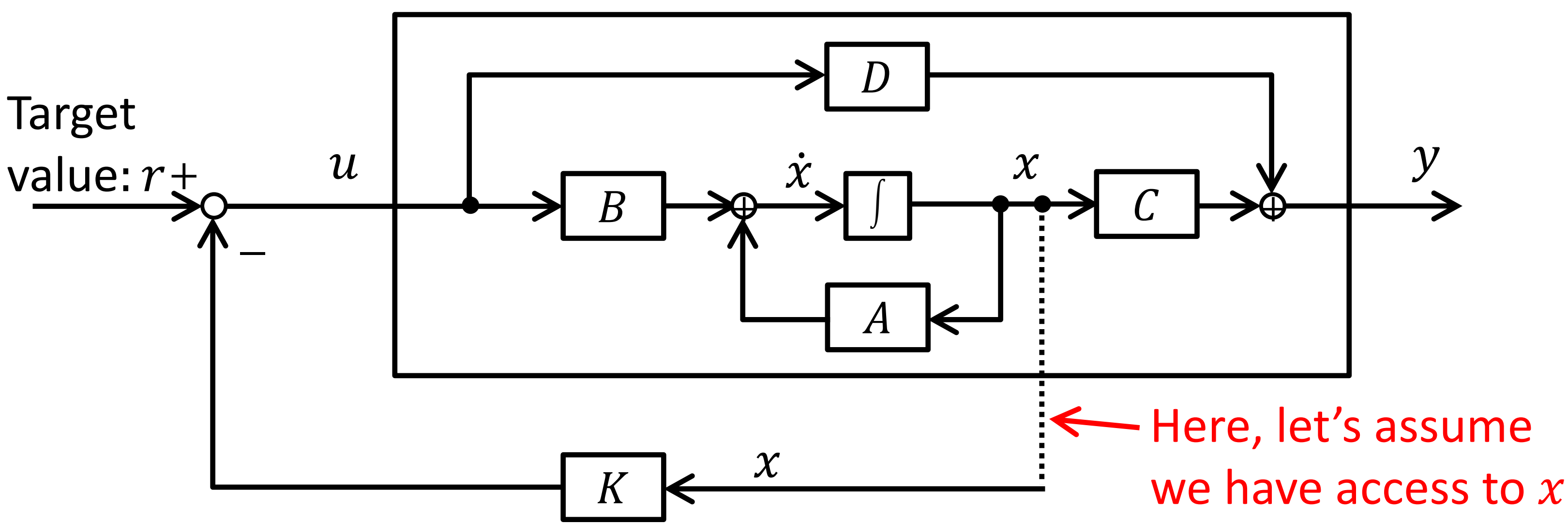
- Transfer function ⇒ State-space transform is also possible (infinite number of expressions)

Static state feedback

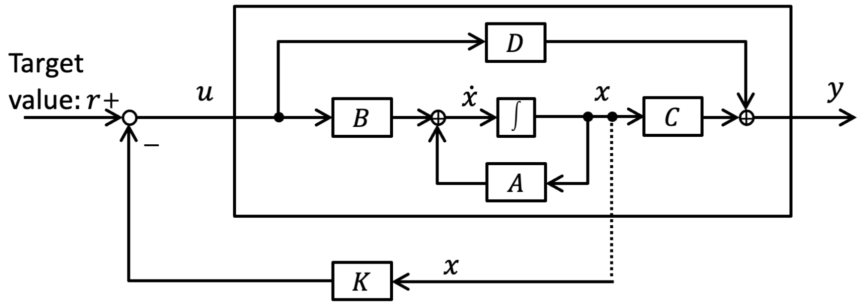
- Classical control
  - Use output Y for feedback control



- State feedback
  - Use internal state x for feedback control



Static state feedback



- Closed-loop system with state feedback

w/o state feedback

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$u = -Kx$

w/ state feedback

$$\begin{cases} \dot{x} = (A - BK)x \\ y = (C - DK)x \end{cases}$$

System matrix: $A \Rightarrow (A - BK)$

Theorem:
Any arrangement of eigenvalues of $(A - BK)$ can be obtained by choosing K (if the system is controllable)

We can arbitrarily place closed-loop poles (eigenvalues)!!

Optimization problem

- Practically, it's impossible to arrange poles arbitrarily due to limitation of control energy

Trade-off
- Performance of closed-loop system
- ↕
- Control energy

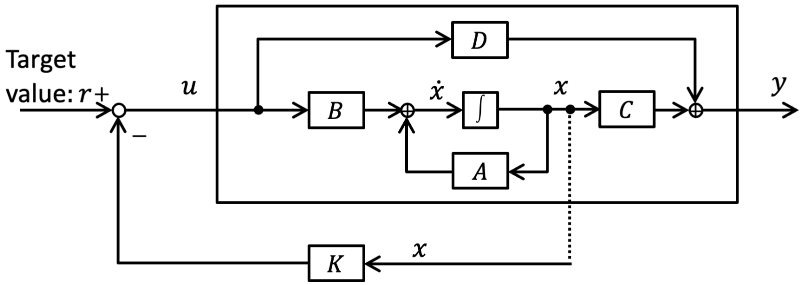
- To optimize the controller K, quadratic cost function J is used:

$$J = \int_0^\infty (\underbrace{x^T Q x}_{\text{Error}} + \underbrace{u^T R u}_{\text{Control energy}})\, dt$$

Q, R: Matrix of design parameters

Optimized K (that minimizes J) can be obtained by solving Riccati eq.

$$A^T[S - SB(B^T SB + R)^{-1} BS]A - S + Q = 0.$$



We can obtain optimized K mathematically!

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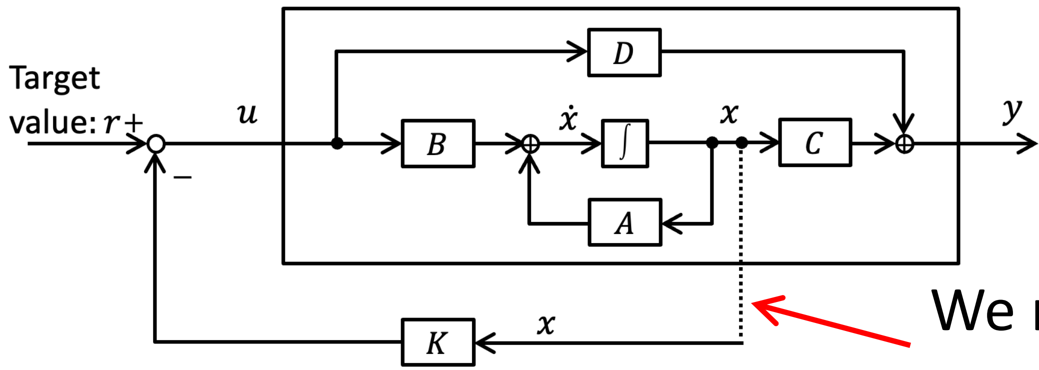
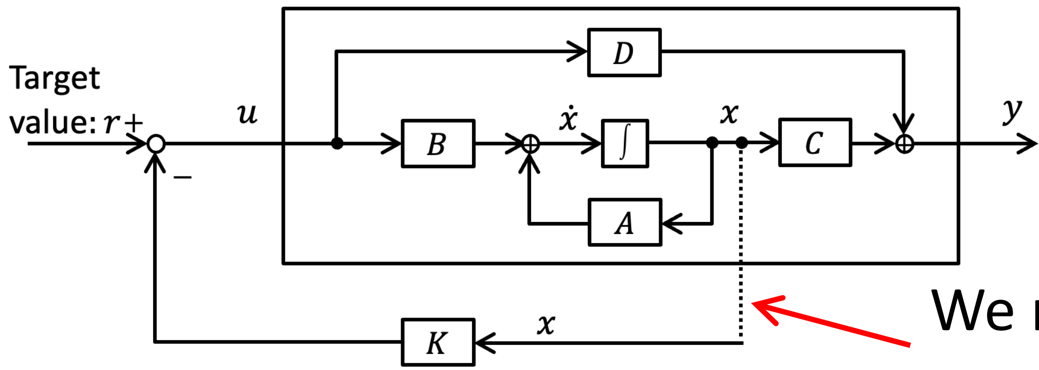
Issue with state feedback



We need all the internal states $x = [x_1, x_2, \cdots, x_n]^T$

How can we obtain all the internal information?

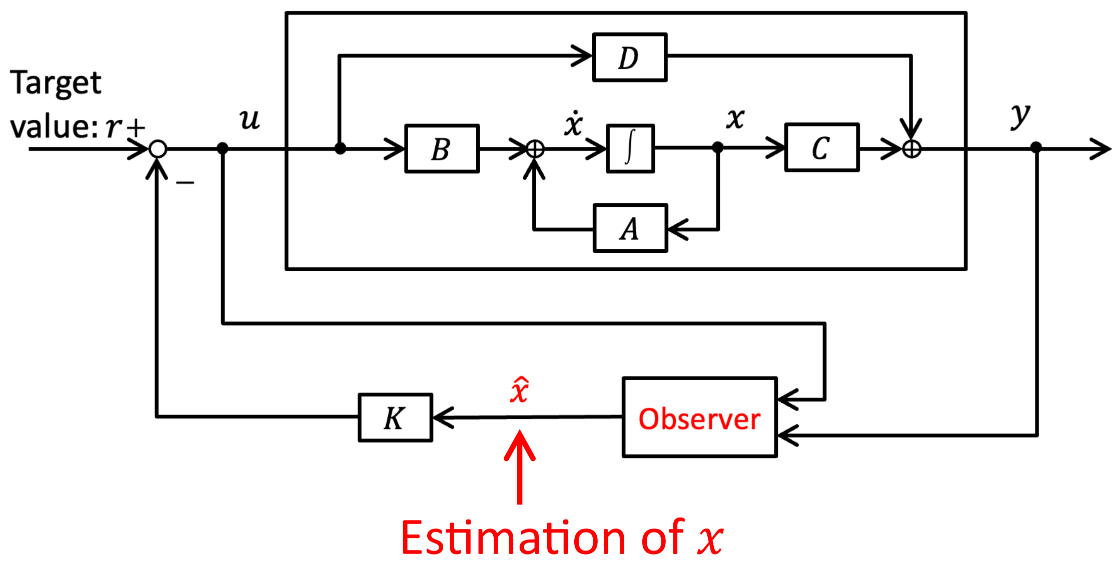
Issue with state feedback



We need all the internal states $x = [x_1, x_2, \cdots, x_n]^T$

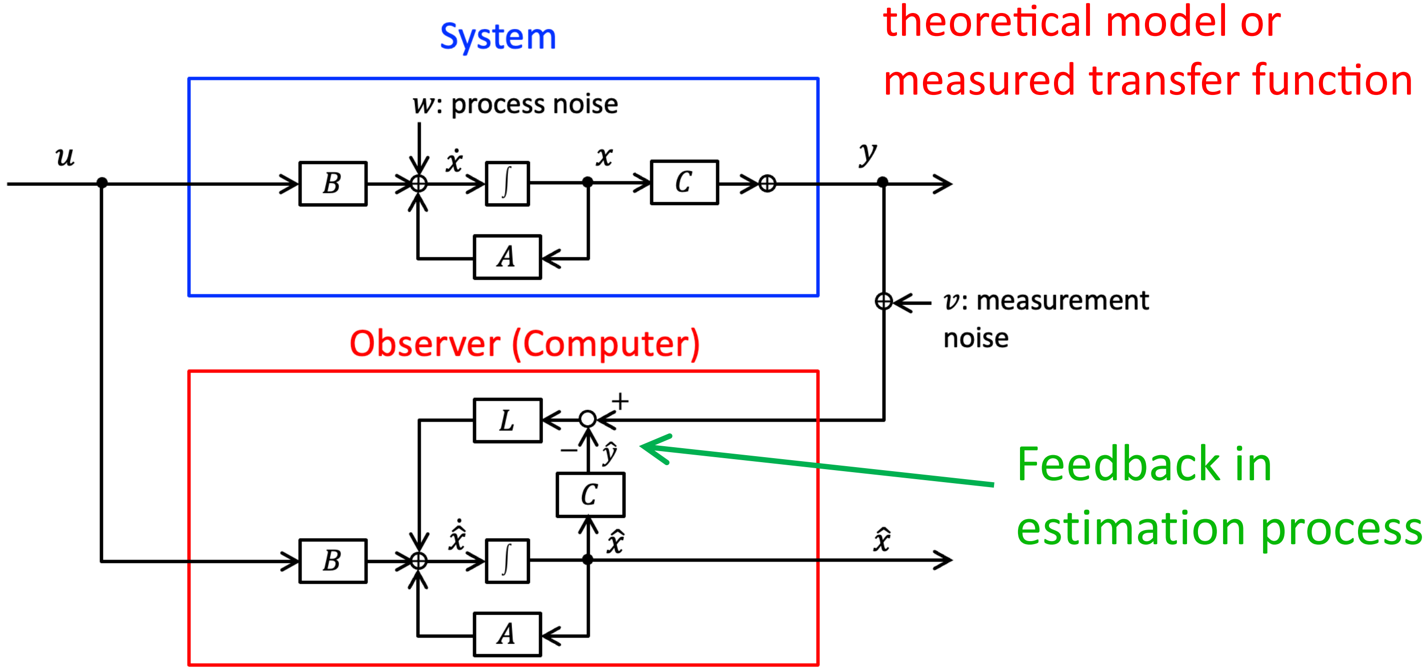
How can we obtain all the internal information?

⇒ Estimate internal states by observing inputs/outputs



Kalman filter

- Kalman filter
  - Practical observer
  - Robust to process noise w and measurement noise v
  - Use feedback of outputs estimation error $(y - \hat{y})$ for estimation of internal states x
  - Feedback gain L: Kalman gain (optimized L from Riccati eq.)
  - We need precise state-space model (A, B, C) ← Able to obtain from theoretical model or measured transfer function



State feedback w/ Kalman filter = Linear Quadratic Gaussian (LQG) controller

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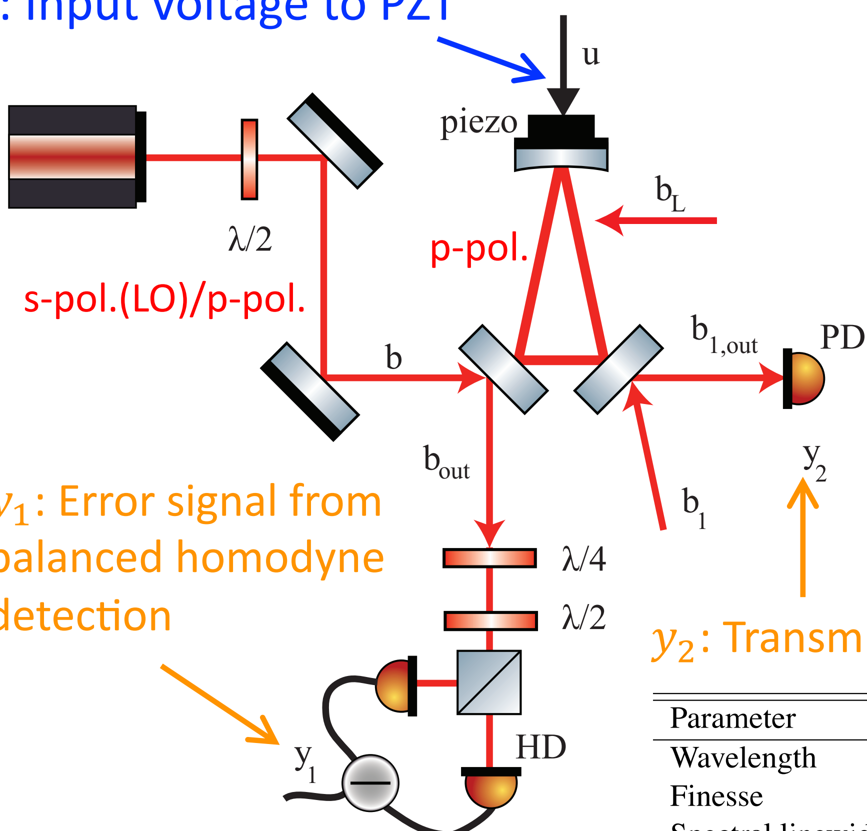
Application for three-mirror ring cavity

- For practice, application for three-mirror ring cavity is tested:
  - ➢Cavity locking with linear LQG control
  - ➢Autolocking with non-linear control

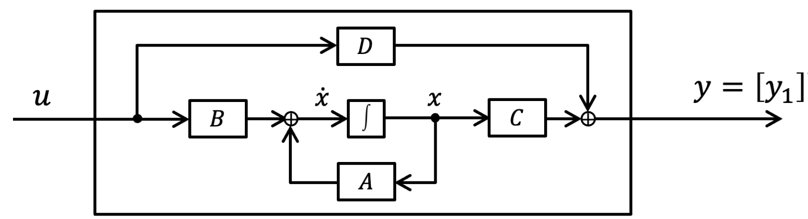
u: Input voltage to PZT



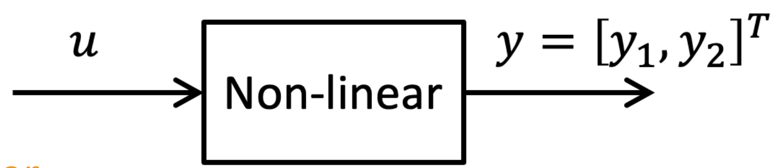
y_1: Error signal from balanced homodyne detection

y_2: Transmitted power

System of the cavity (linear around resonance)



System of the cavity (non-linear outside resonance)



Parameter	Value
Wavelength	1550 nm
Finesse	≈ 10
Spectral linewidth	≈ 65 MHz
Waist	453 μm

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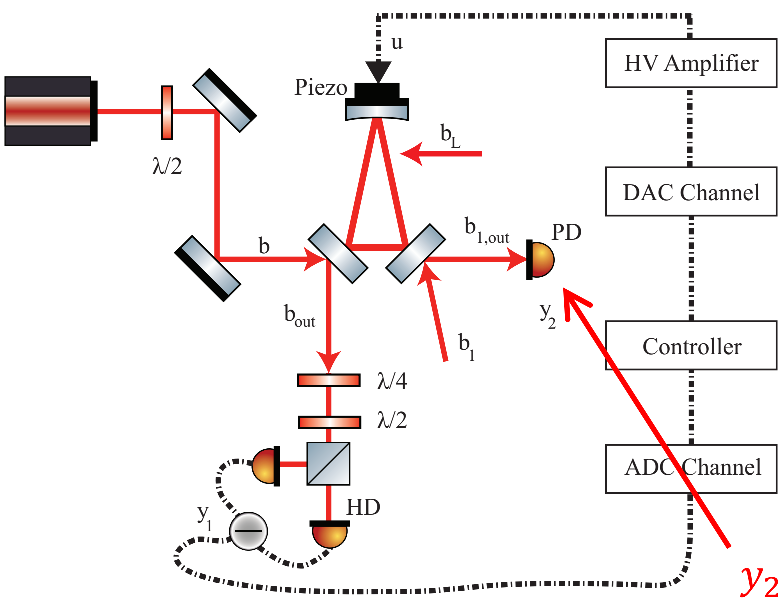
Implementation steps of locking

① Measure the transfer function of the cavity: $H(s) = Y_1(s)/U(s)$

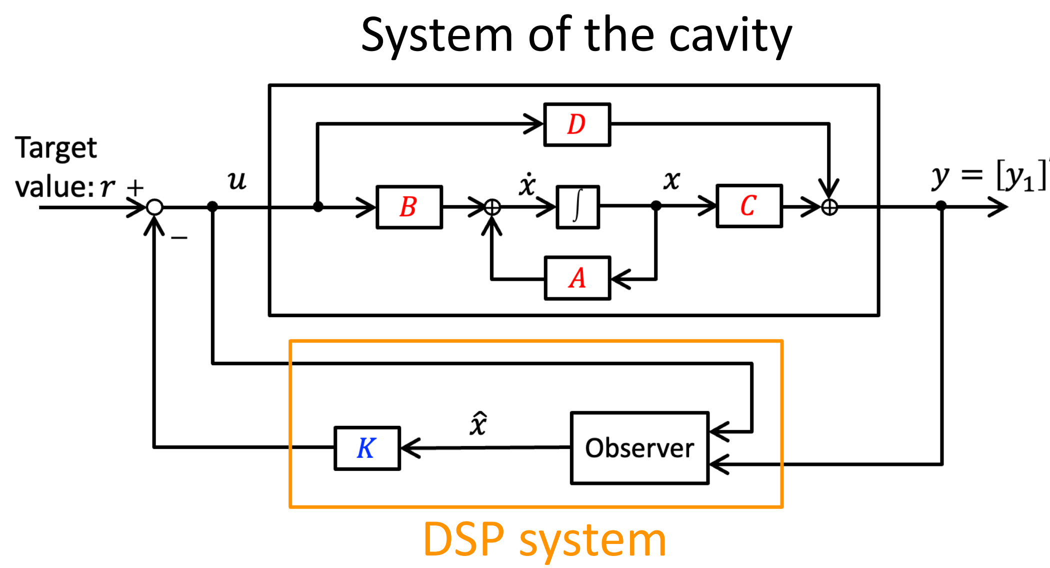
② Transform $H(s)$ to state-space representation (A, B, C, D)

③ Calculate optimized controller: K with LQG method

④ Implement (A, B, C, D) and K on a Digital signal processing (DSP) system



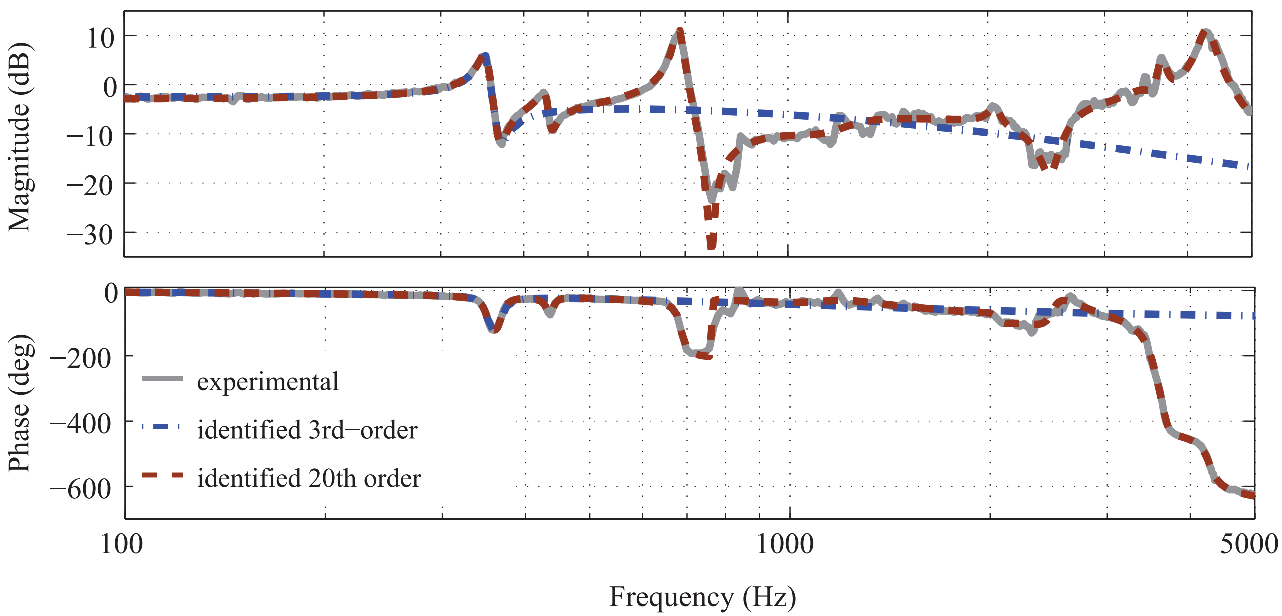
y_2 (transmitted power) is not used here



① Measurement of transfer function

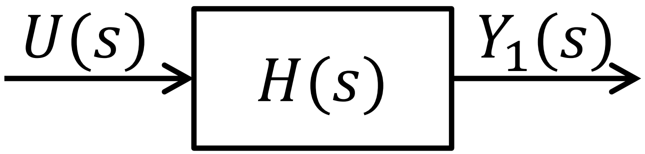
- Measured the transfer function of the cavity: $H(s) = Y_1(s)/U(s)$ with PI control
- Fitted with 3rd-order model and 20th-order model

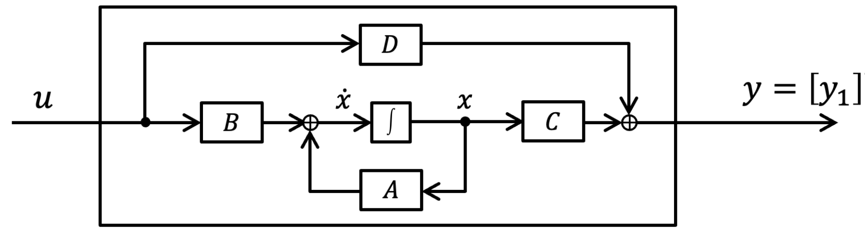
  ⇒Used 3rd-order model (up to first resonance) for easy computation

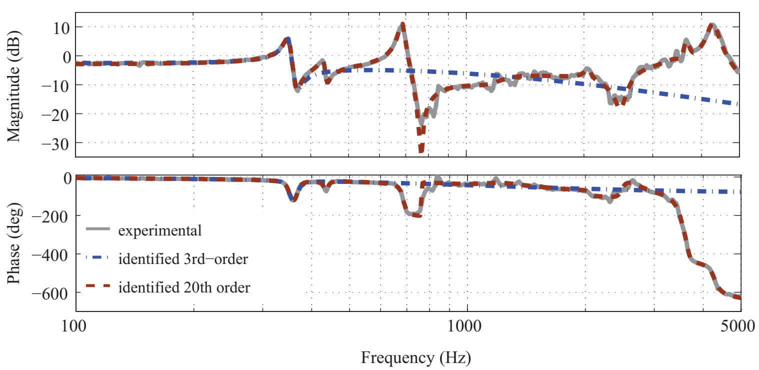


② Transform $H(s)$ to state-space model

- Transformed $H(s)$ to state-space representation (A, B, C, D) with Matlab functions (tf2ss?)







$$
\begin{cases}
A = 10^4 \cdot \begin{bmatrix} -0.0180 & -0.2865 & 0.0573 \\ 0.1693 & -0.0157 & 0.2339 \\ 0.0446 & 0.1109 & -1.1449 \end{bmatrix} \\
B = \begin{bmatrix} 2.8394 \\ 4.2852 \\ -24.9287 \end{bmatrix} \\
C = \begin{bmatrix} 24.0014 \\ 37.3086 \\ -34.4903 \end{bmatrix}^T \\
D = 0.
\end{cases}
$$

③Calculation of optimized controller: K

- To deal with unmodelled external disturbance (e.g. 1/f laser phase noise), integral control is introduced:

*State-space model in discrete time

Additional internal state: q at time instant k

$$q_{k+1} = q_k + y_k$$

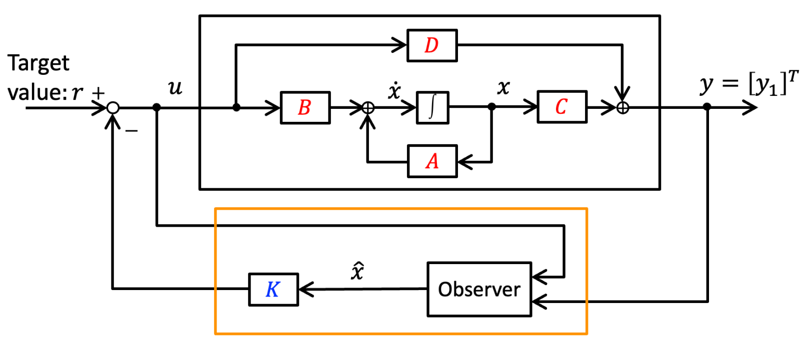
Integral of output y (error signal)

Modified state-space model

$$\begin{bmatrix} x \\ q \end{bmatrix}_{k+1} = \begin{bmatrix} A & 0 \\ C & I \end{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix}_k + \begin{bmatrix} B \\ 0 \end{bmatrix} \tilde{u}_k + \tilde{w}_k$$

$$\tilde{y}_k = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix}_k + \tilde{v}_k,$$

- Calculated optimized controller: K with LQG (cost function) method

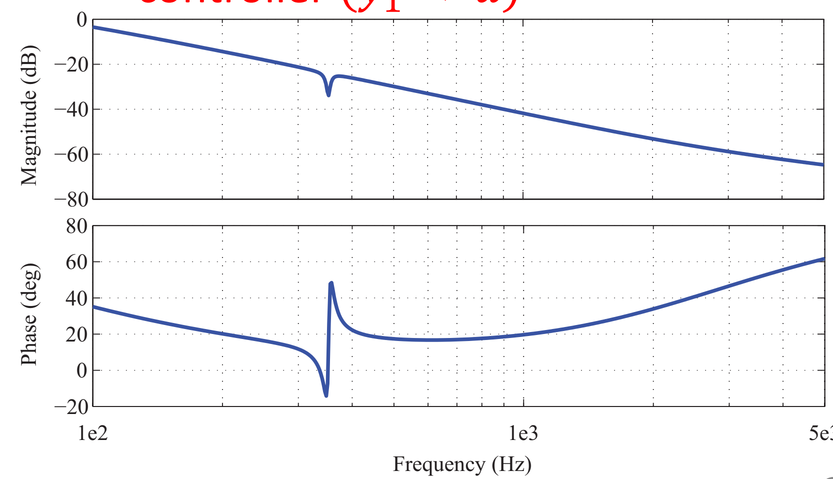


Design parameters for cost function

$$Q_L = \sigma_1^2 = 1, \qquad R_L = \begin{bmatrix} \sigma_2^2 & 0 \\ 0 & \sigma_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 10^{-7} \end{bmatrix}$$

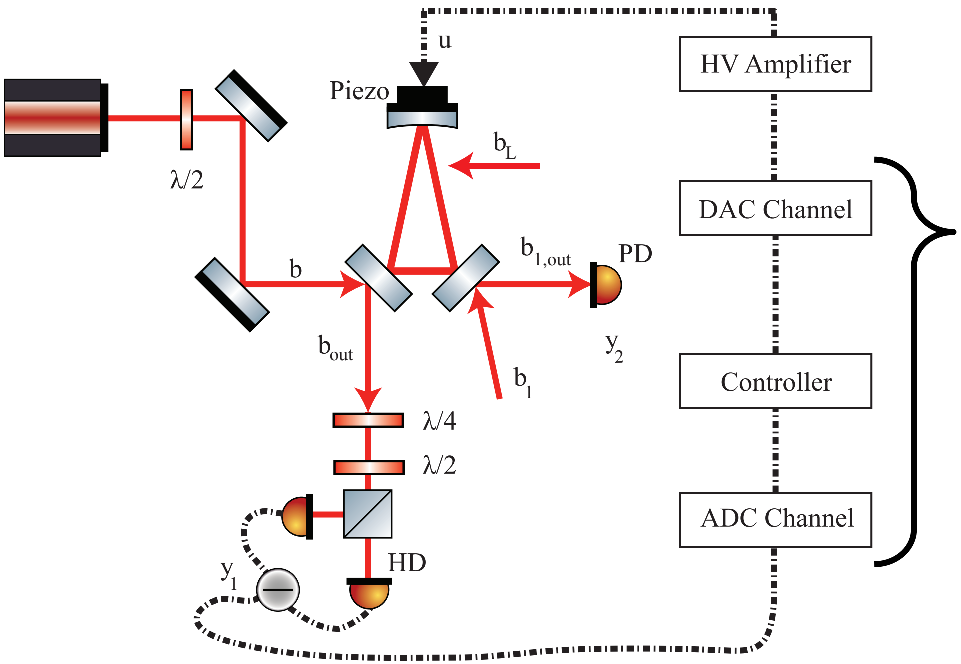
$$Q_K = q = 5 \cdot 10^5, \qquad R_K = r = 0.5.$$

Obtained transfer function of the controller ($y_1 \Rightarrow u$)



④Implementation

- Implemented the observer and the controller on DSP system

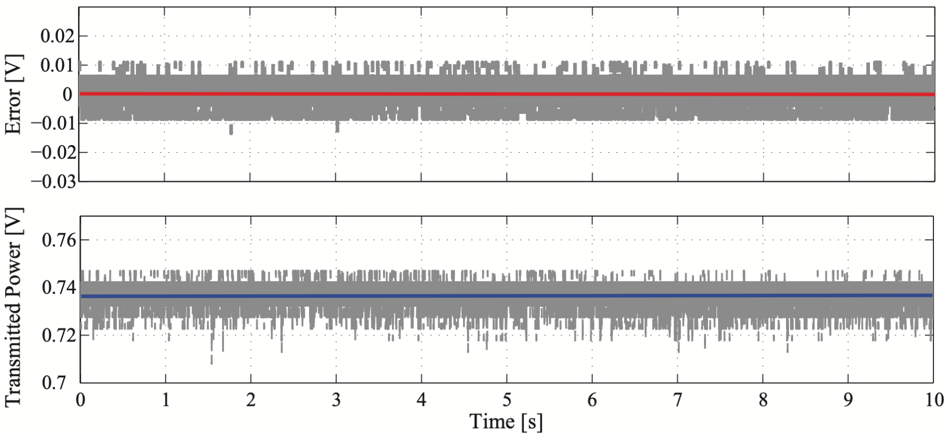


- DS1104 dSPACE DSP system
  - ➢8 DAC channels (12-bit, 300kHz)
  - ➢16 DAC channels (12-bit, 300kHz)
  - ➢Programmable with Simulink

Result

- Succeeded in locking the cavity
- UGF≃61 Hz, phase margin≃47°

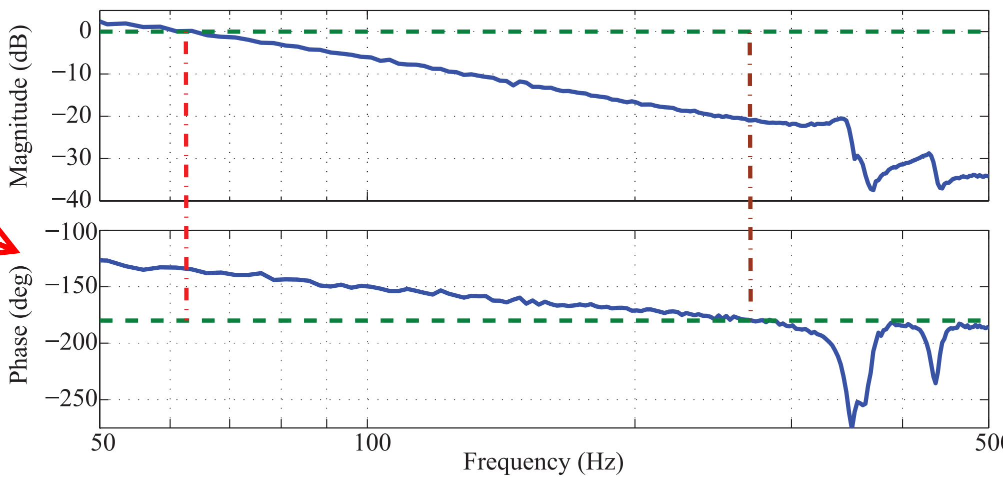
Error signal and transmitted power during lock



I'm not sure...

- Is this really optimized?
- Large phase delay ⇐ due to computation?

Open-loop transfer function



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Autolocking with non-linear control

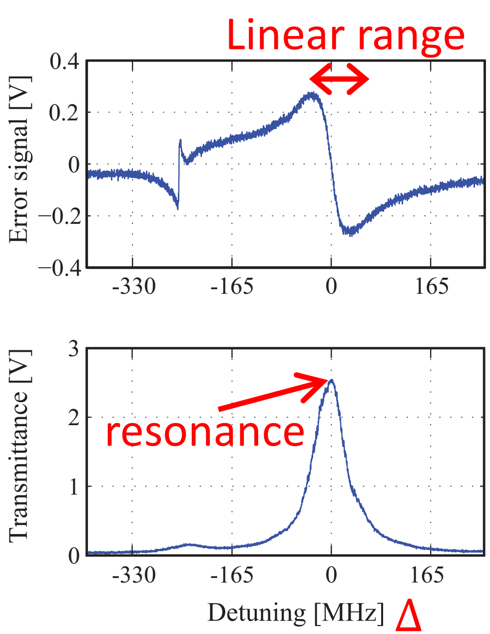
- With linear control, cavity lock can be unlocked due to large disturbance (goes out of linear range of the error signal)

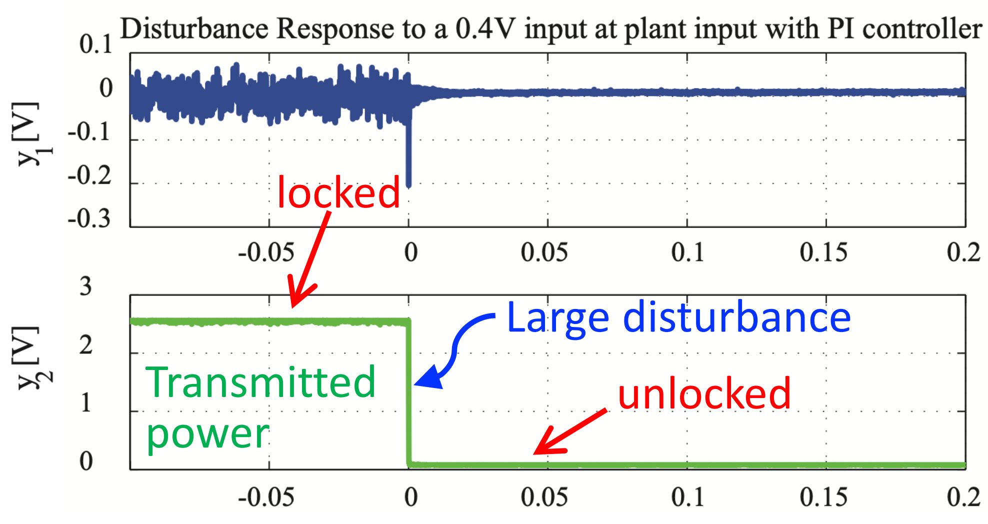
Error signal from balanced homodyne

$y_1 = f_1(\Delta)$

Transmitted power

$y_2 = f_2(\Delta)$





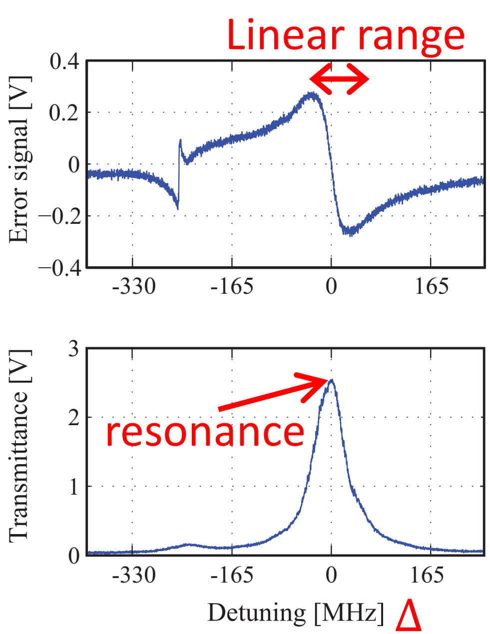
Autolocking with non-linear control

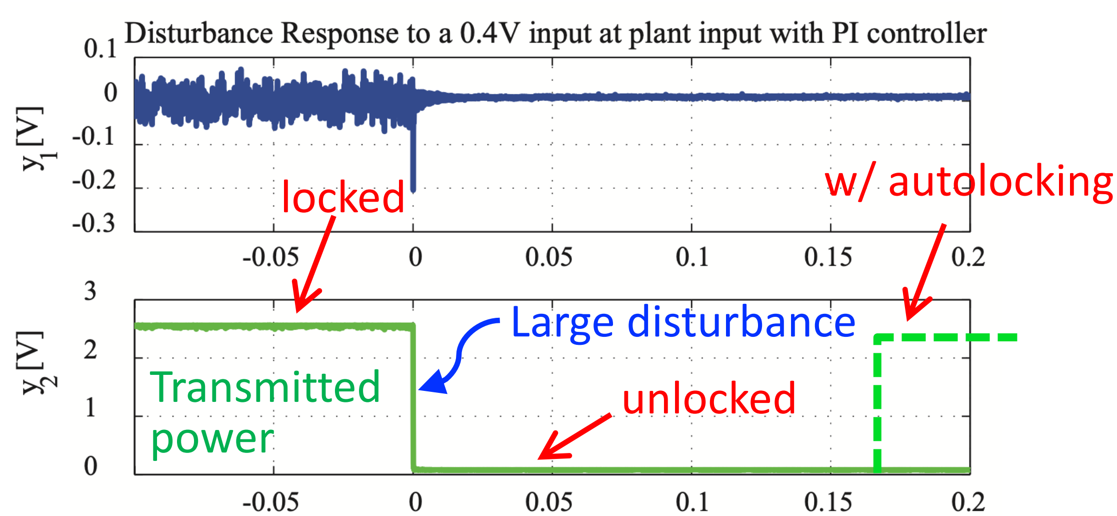
- With linear control, cavity lock can be unlocked due to large disturbance (goes out of linear range of the error signal)
- If we can obtain the detuning Δ in outside of the linear range, we can realize autolocking

➡ Use non-linear signal $y_1 = f_1(\Delta), y_2 = f_2(\Delta)$ to obtain detuning Δ

Error signal from balanced homodyne $y_1 = f_1(\Delta)$

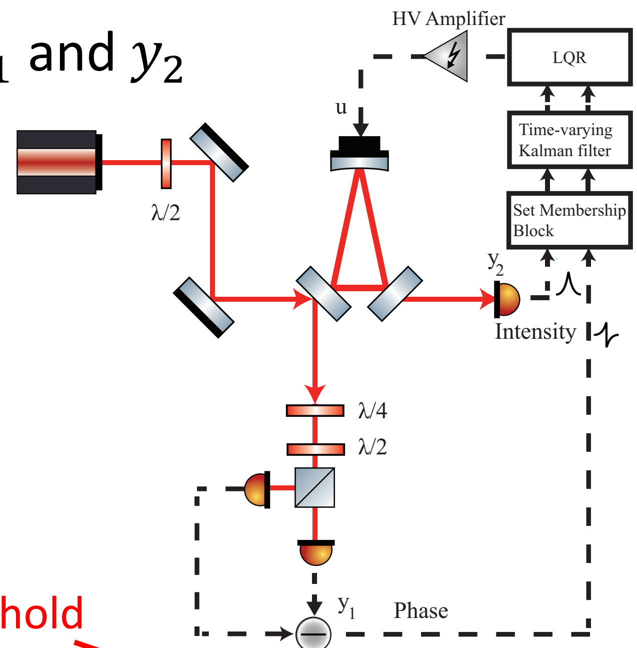
Transmitted power $y_2 = f_2(\Delta)$





Obtaining detuning Δ from non-linear signal

- We want to know detuning Δ from observed y_1 and y_2



$$y_1 = -\frac{2k_2\beta\kappa_0\Delta}{\left(\frac{\kappa}{2}\right)^2+\Delta^2}+v_1 \qquad y_2 = \frac{1}{2}\frac{\tilde{k}_2\kappa_1\kappa_0\beta^2}{\left(\frac{\kappa}{2}\right)^2+\Delta^2}+v_2$$

$$= f_1(\Delta)+v_1, \qquad = f_2(\Delta)+v_2.$$

- Calculation algorithm

① Measure y_{1k} and y_{2k} at time instant k

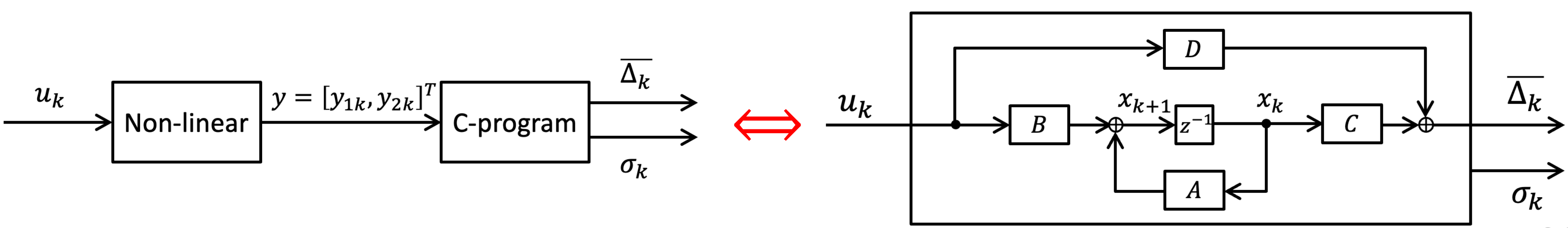
② Set of possible detuning Δ_k:

Threshold

$$S_k = \left\{\Delta_k \in \mathbb{R} \mid \left(y_{1k} - f_1(\Delta_k)\right)^2 \le \mu_1^2 \; and \; \left(y_{2k} - f_2(\Delta_k)\right)^2 \le \mu_2^2\right\}$$

③ Obtain mean: $\overline{\Delta_k}$ and standard deviation: σ_k

$u \mapsto \overline{\Delta}$ is linear



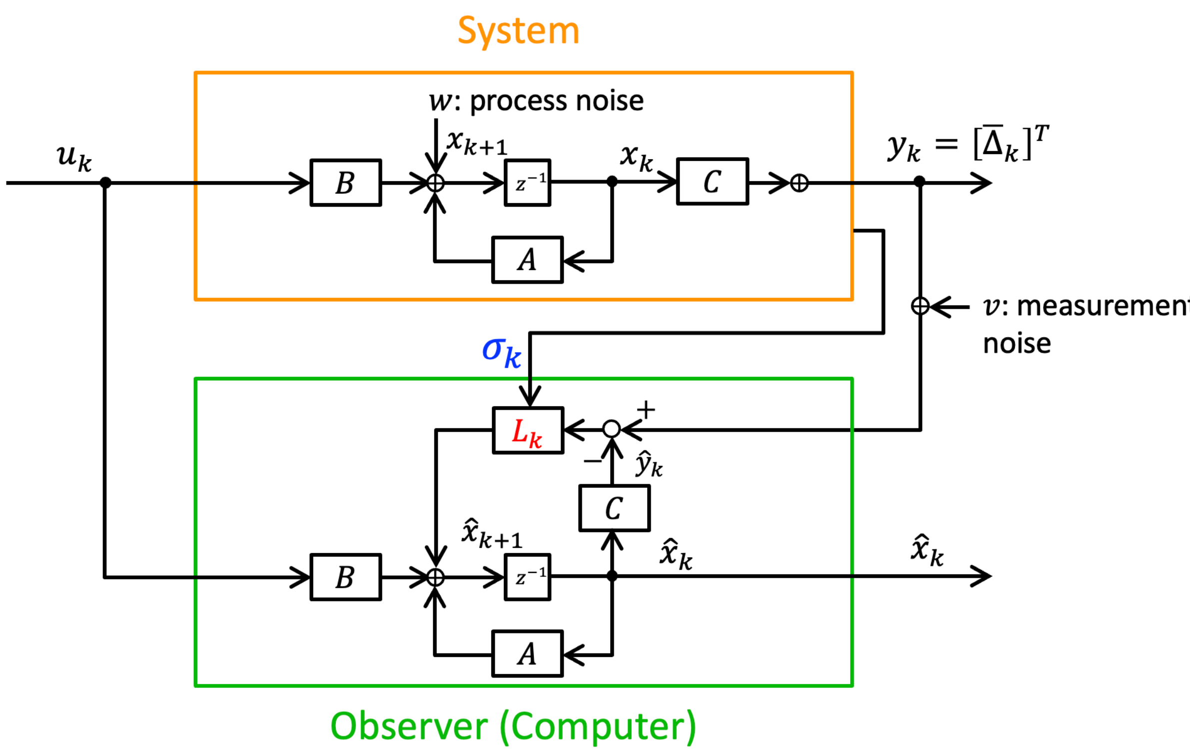
Use of standard deviation σ_k

- We can use observation error σ_k for time-varying Kalman filter

  *Kalman gain L_k = feedback gain of state (x_k) estimation process

Time varying Kalman filter
- Small $\sigma_k \Rightarrow$ Large L_k : Emphasize measured output $y_k = [\bar{\Delta}_k]^T$
- Large $\sigma_k \Rightarrow$ Small L_k : Emphasize previous prediction $\hat{x}_k$ (ignore y_k)

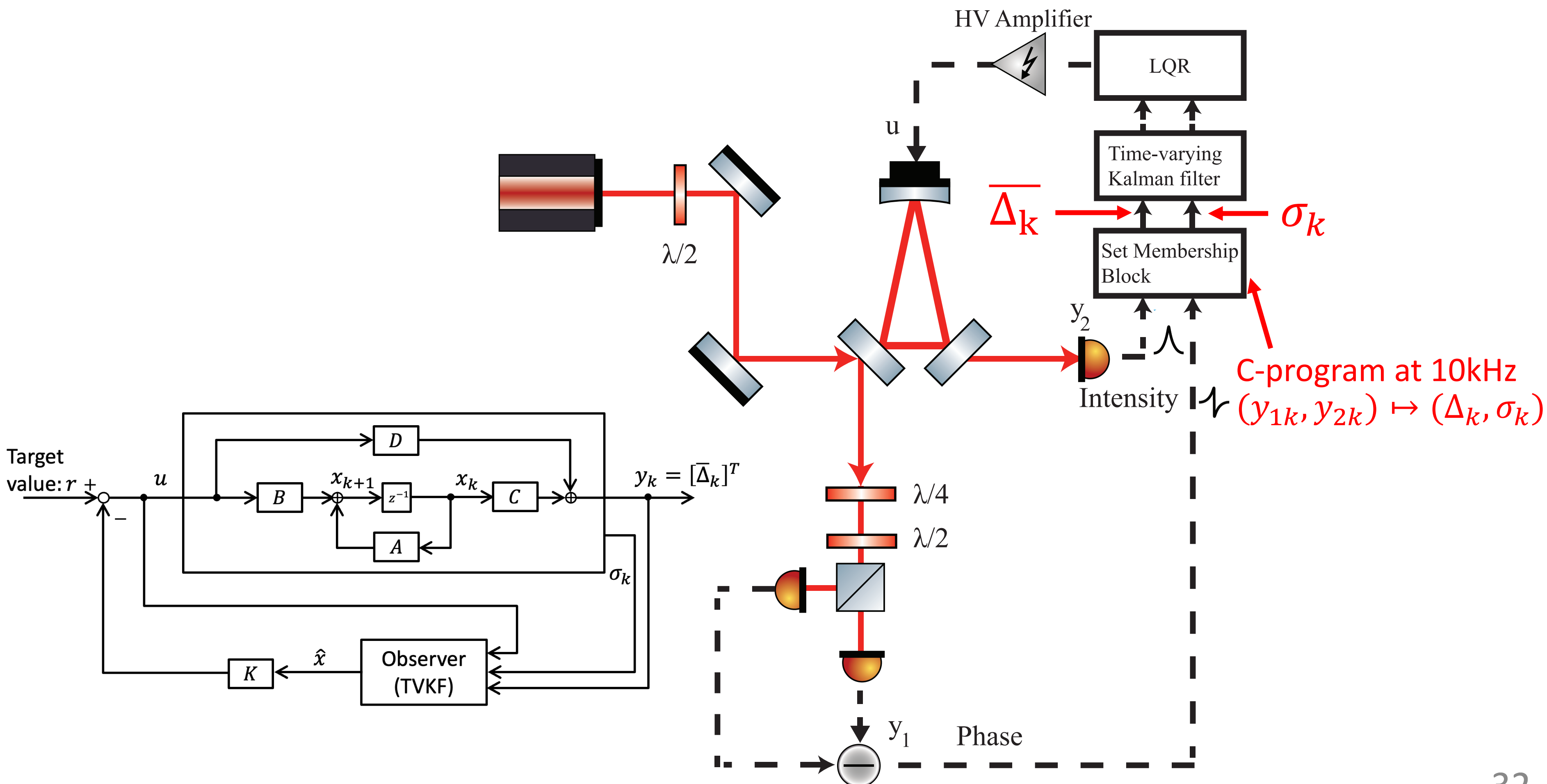
➡ Better estimation of internal state x_k



Optimal L_k can be calculated mathematically

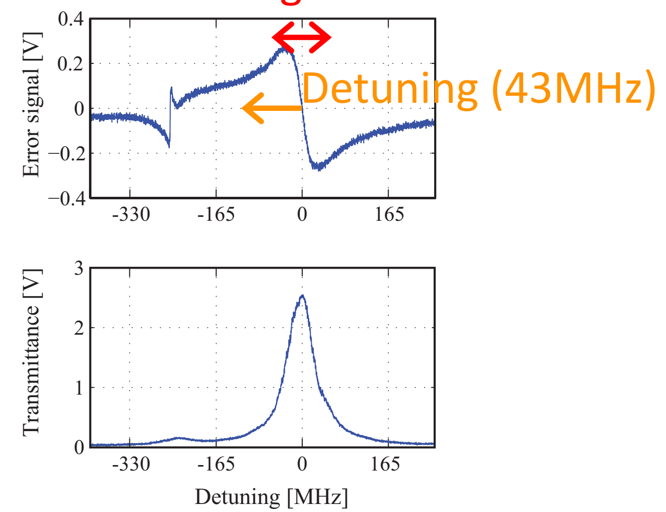
Implementation

- Implemented the observer and the controller on DSP system
- Identifying system and designing controller were done in the same way as previous cavity locking (no figures in the thesis)



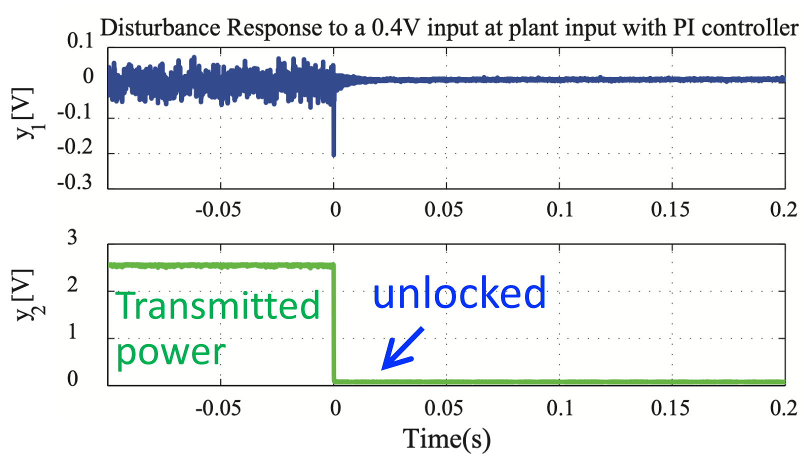
Result

- Succeeded in locking the cavity from any initial operation point (but no figures in the thesis)

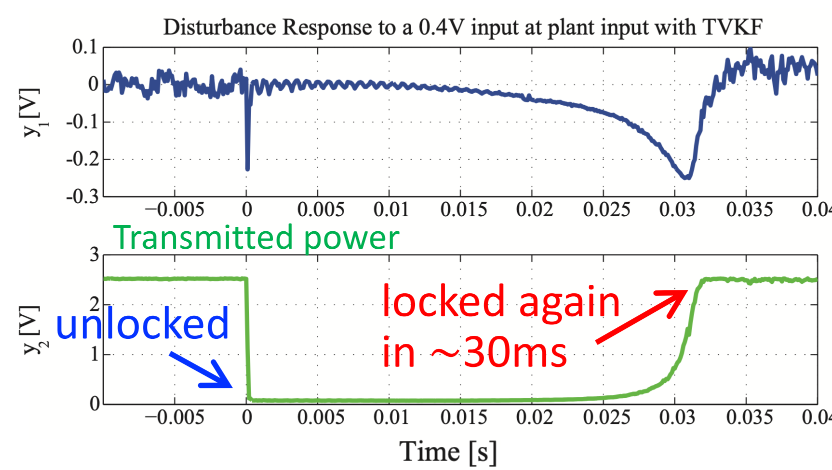


- Compared with PI control
  - ➢Injected step voltage (0.4V) to PZT (detuning ≃ 43 MHz)
  - ➢Succeeded in locking again with non-linear control

PI control



Non-linear control

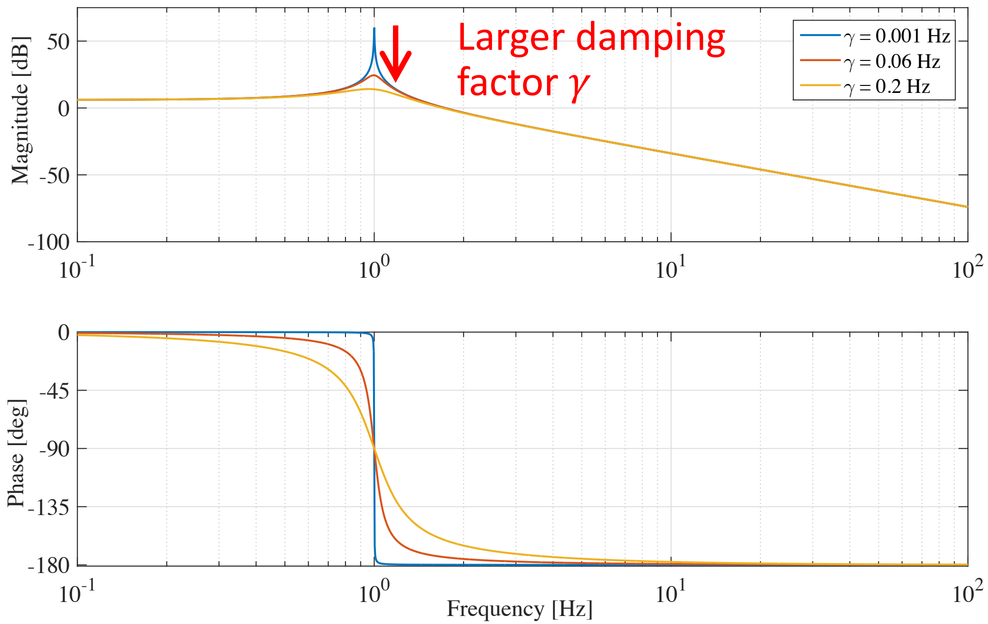


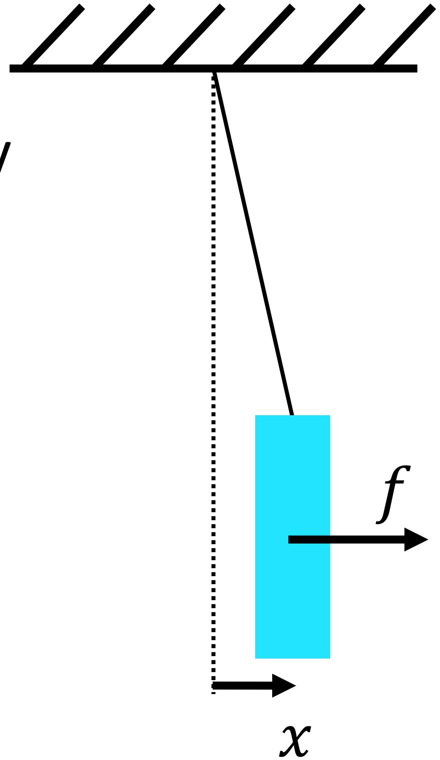
Contents

Damping of suspension

- Motion of test mass is excited on resonance

  ⇒ Need passive/active damping to extract energy





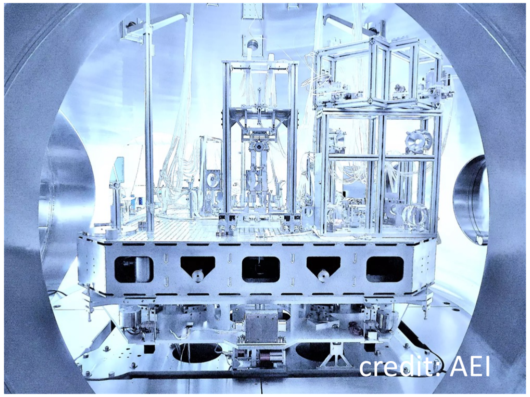
$$H(\omega) = \frac{X}{F} = \frac{1}{m(\omega_0^2 - \omega^2 + i2\gamma\omega)}$$

- Suspension system is generally a complex MIMO system (multiple pendulum, undecoupled DOFs (longitudinal, yaw, pitch))

  ⇒ Apply modern control for active damping!

Overview

- Target system
  - ➢Triple pendulum suspension used for [10 m prototype @ AEI]

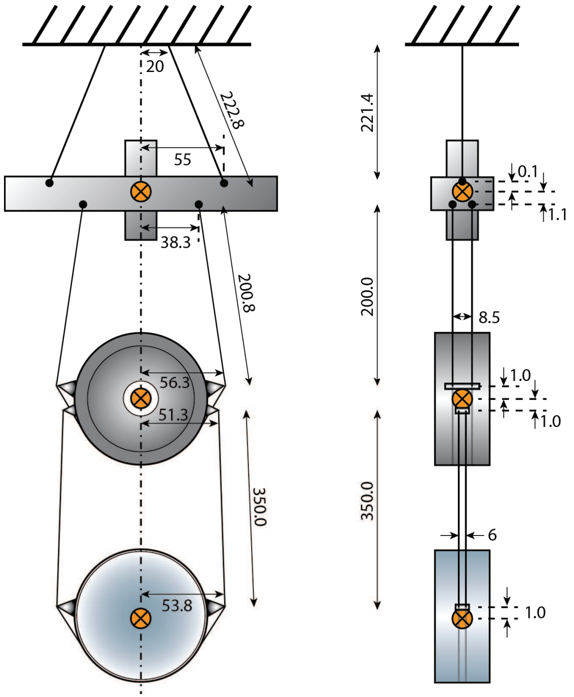


- Active damping for longitudinal, yaw and pitch

⇒ Need optimal filters for feedback control

- ➢Measured transfer functions and identified systems
- ➢Designed optimal filters with $\mathcal{H}_2$ controller synthesis

*Implementation of filters was not done in this thesis



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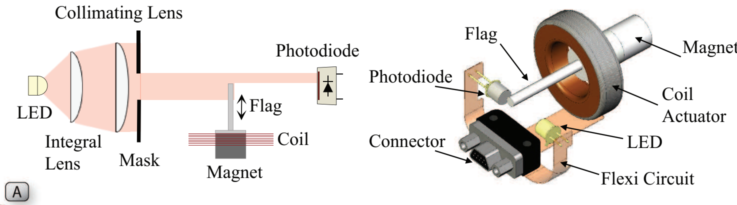
Sensors/Actuators

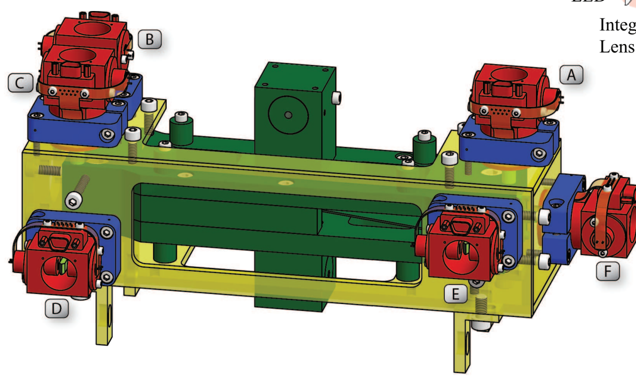
- Used 6 BOSEMs on upper mass

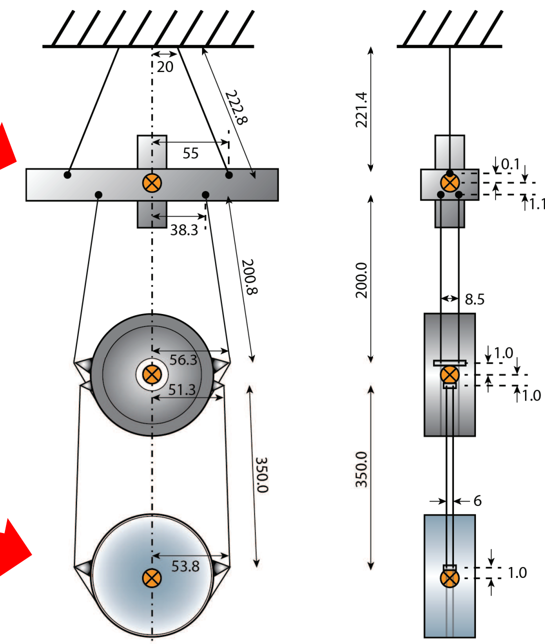
Structure of a BOSEM (Birmingham Optical Sensor and Electro-Magnetic actuators)



u_l: Longitudinal actuation (D,E)
u_y: Yaw actuation (D,-E)
u_p:Pitch actuation (B,C)

y_{Ul}: Longitudinal sensor
y_{Uy}: Yaw sensor
y_{Up}: Pitch sensor

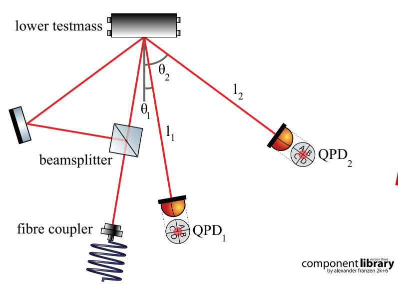




- Used 2 optical levers for lower mass

y_{Ll}: Longitudinal sensor
y_{Ly}: Yaw sensor
y_{Lp}: Pitch sensor

*Used for system identification
(not used for control signal)



- This system can be regarded as
3 inputs 6 outputs system

$$u = [u_l, u_y, u_p]^T \longrightarrow \boxed{H(s)} \longrightarrow y = [y_{Ul}, y_{Uy}, y_{Up}, y_{Ll}, y_{Ly}, y_{Lp}]^T$$

Measurement of transfer functions

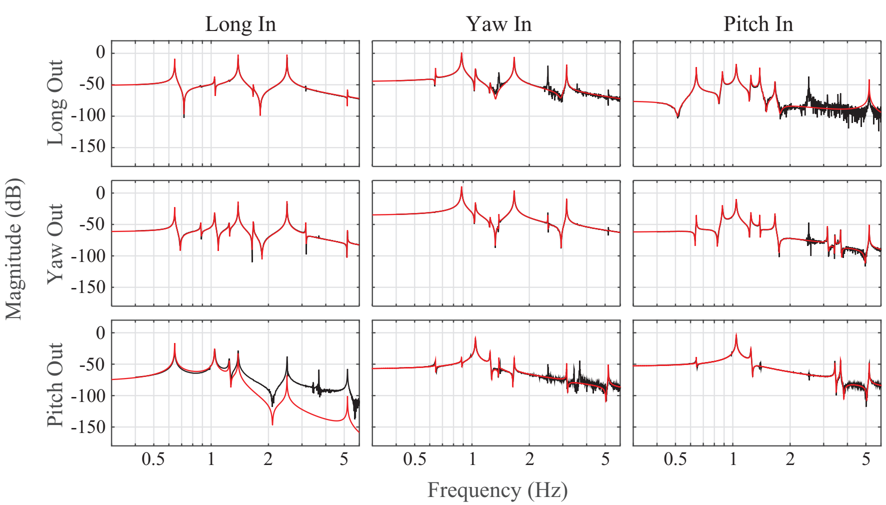
- Measured transfer function matrix using BOSEMs and optical levers
- Used LIGO CDS

$$u = [u_l, u_y, u_p]^T \rightarrow H(s) \rightarrow y = [y_{Ul}, y_{Uy}, y_{Up}, y_{Ll}, y_{Ly}, y_{Lp}]^T$$

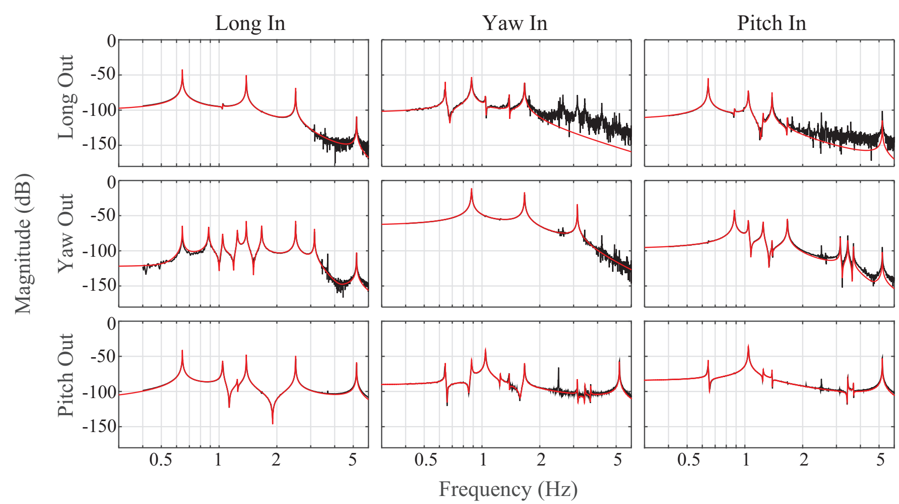
➢ Longitudinal, yaw and pitch were highly coupled (not diagonalized)

$\mathcal{H}_2$ controller synthesis does not need decoupling process

Actuators of BOSEMs ⇒ Upper mass



Actuators of BOSEMs ⇒ Lower mass

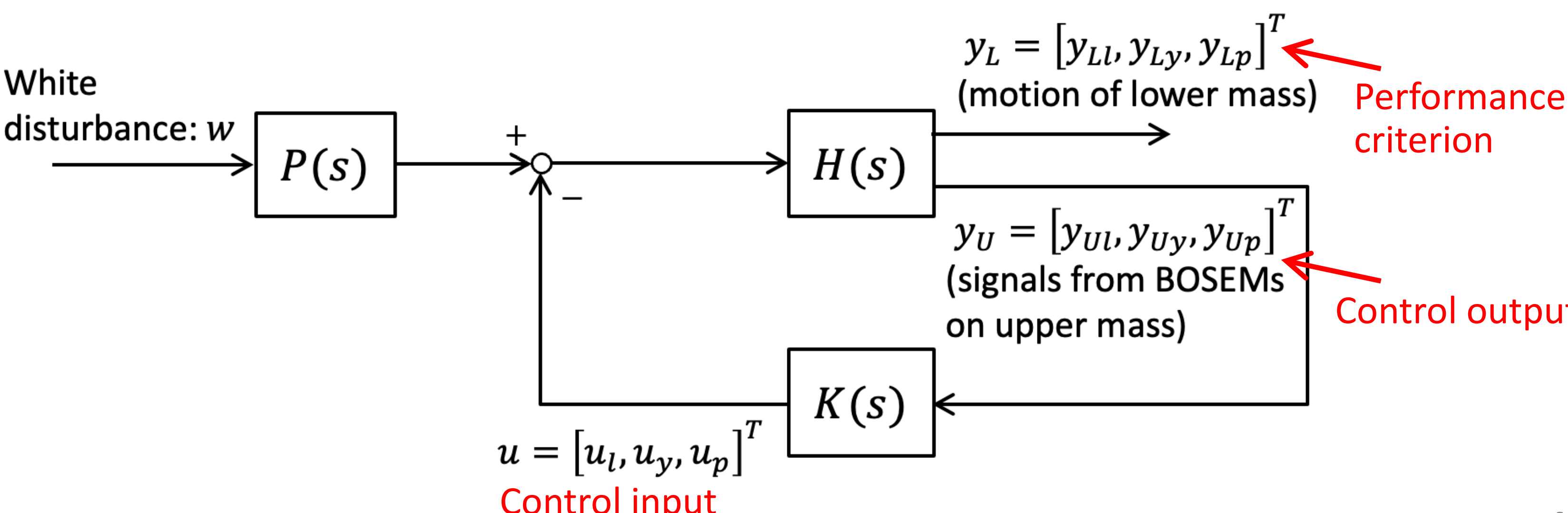


Space-state representation

- $\mathcal{H}_2$ controller synthesis minimizes performance criterion
- For $\mathcal{H}_2$ controller synthesis, transfer function matrix $H(s)$ was transformed to state-space model (A, B, C, D)
- Used Matlab *balreal* and *modred* functions

  ⇒ 60th-order system was obtained

Use this for calculating optimal feedback filters: $K(s)$



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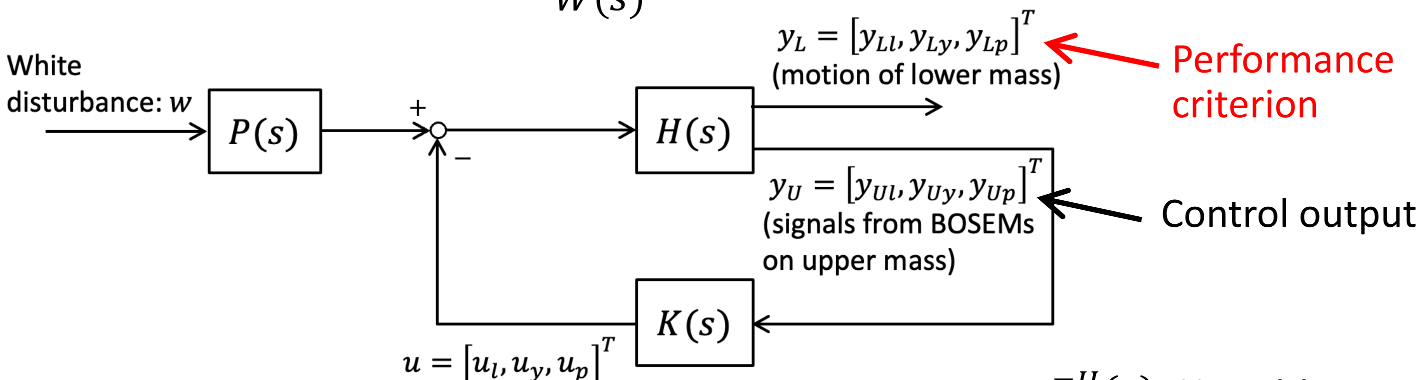
$\mathcal{H}_2$ controller synthesis

- Closed-loop transfer function from noise w to output y_L:

$$F(s) = \frac{Y_L(s)}{W(s)}$$ ← We want to minimize this



- Use $\mathcal{H}_2$ norm

$$\| F(s) \|_2 \equiv \sqrt{\int_{-\infty}^{\infty} \mathrm{Tr}\left[F^H(i\omega)F(i\omega)\right] \frac{d\omega}{2\pi}}$$

$$= \sqrt{\int_{-\infty}^{\infty} \sum_{k,l} |F_{kl}(i\omega)|^2 \frac{d\omega}{2\pi}}$$

$F^H(s)$: Hermitian transpose of $F(s)$

We can inject any modelled noise with $P(s)$

➡ $\mathcal{H}_2$ norm = RMS of all performance criterion: $\sqrt{\sum_i \langle y_{Li}^2 \rangle}$ with unit-intensity white noise

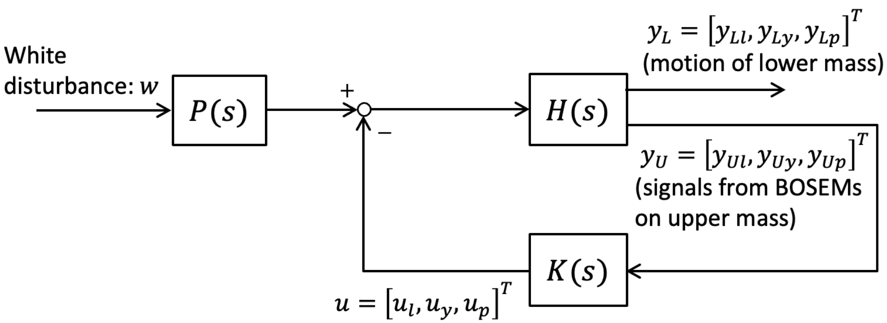
- Trade-off between control performance and control energy

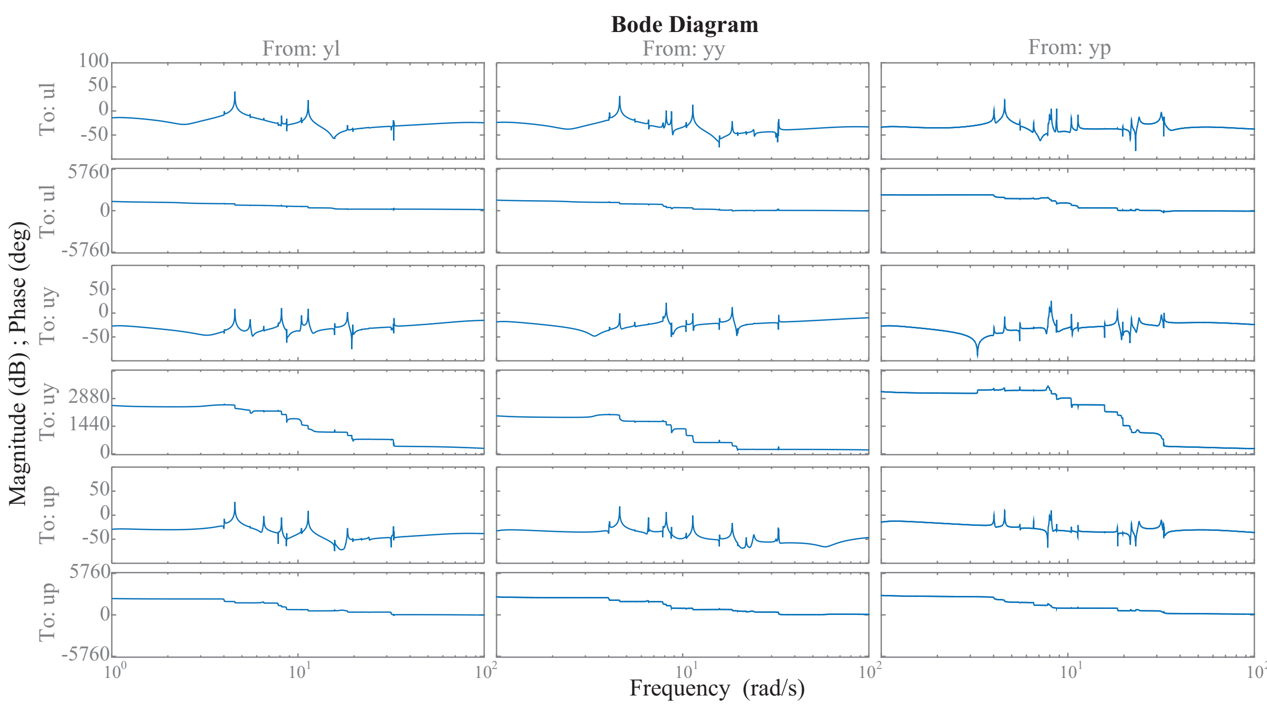
  ⇒ optimal filter K that minimize $\| F(s) \|_2$ can be obtained by solving Riccati equation

Result: Obtained controller

- Calculate optimal filters K from state-space model (A, B, C, D)
- Used Matlab *h2syn* function

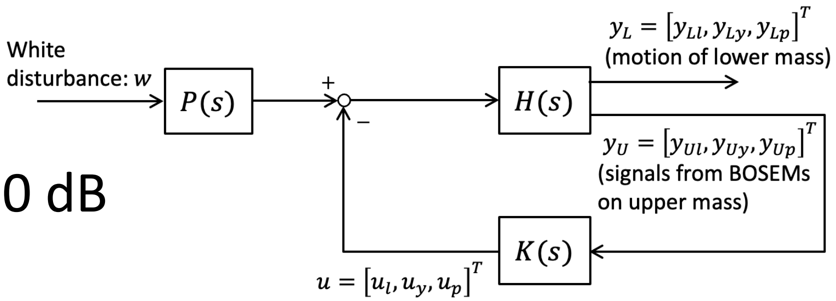
  ⇒ 75th-order filters were obtained



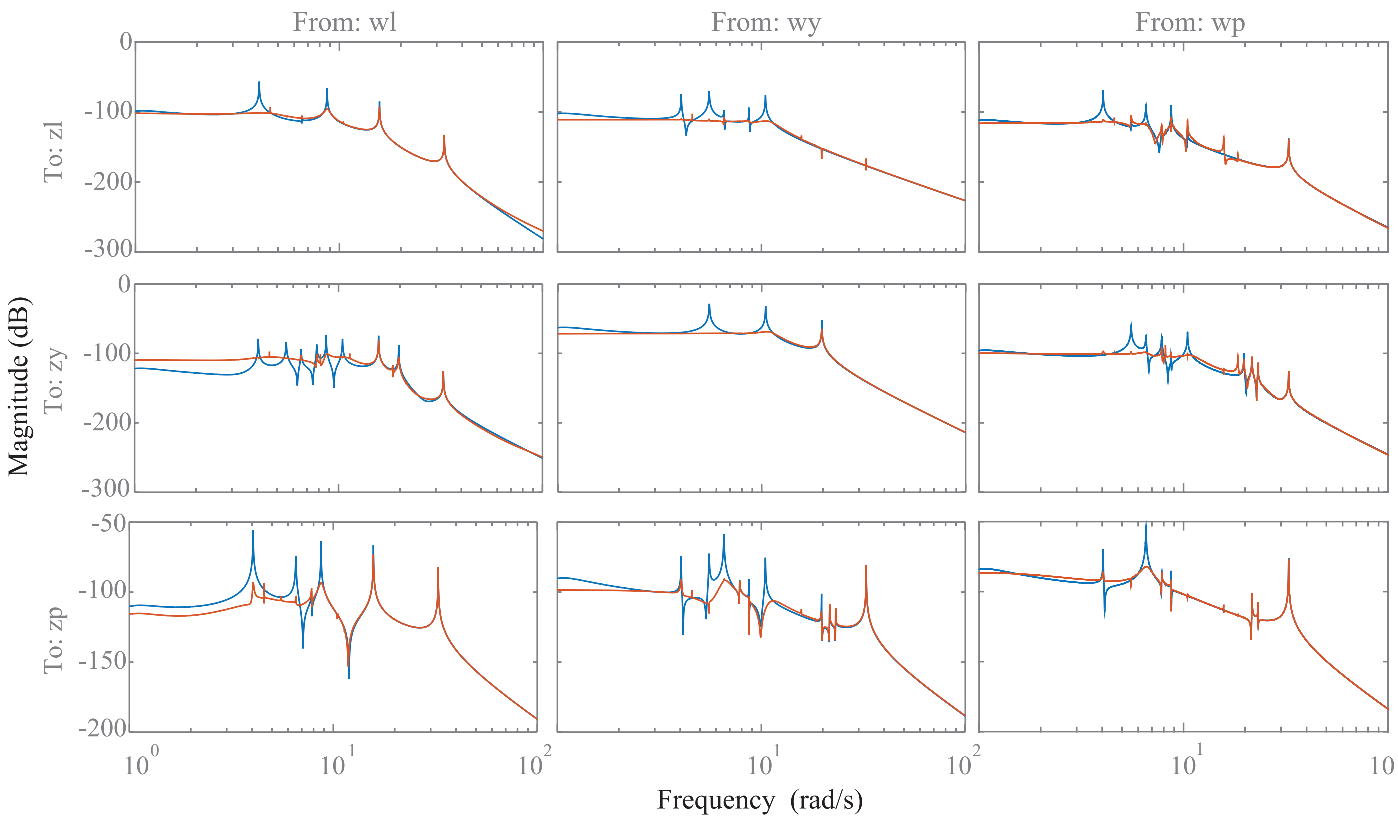


Result: Closed-loop transfer function

- Simulated closed-loop transfer functions: $F(s) = \dfrac{Y_L(s)}{W(s)}$



- Succeeded in damping modes by ~50 dB without exciting other modes



Blue: undamped case

Red: with closed-loop

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Advantages/Disadvantages of modern control

- Advantages
  - Able to deal with multiple-input multiple-output (MIMO) uncoupled system
  - Feedback control with internal state x (State feedback)
  - Able to obtain optimal filters for feedback control mathematically (with no need for professional tuning technique)

- Disadvantages
  - For a too complicated system, enormous computation is needed
  - Need precise system identification
  - Need to set design parameters of cost function manually

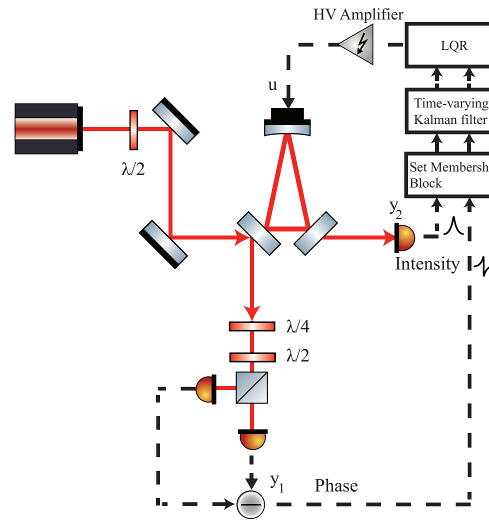
Summary

- Application of modern control for gravitational wave detectors and quantum optical experiments

■Application for a three-mirror ring cavity

➢Locked cavity with linear LQG control

➢Autolock with non-linear control



■Application for a suspension system

➢Designed optimal filters for active damping of triple pendulum suspension with $\mathcal{H}_2$ controller synthesis technique