

Phase Shift of Linear Polarizations on Mirror Coating

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2021/7/9 @Ando Lab Seminar

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- ◆ Resonant frequency difference in DANCE
- ◆ Dielectric mirror and phase shift
- ◆ How to measure the coating phase shift

Resonant frequency difference in DANCE

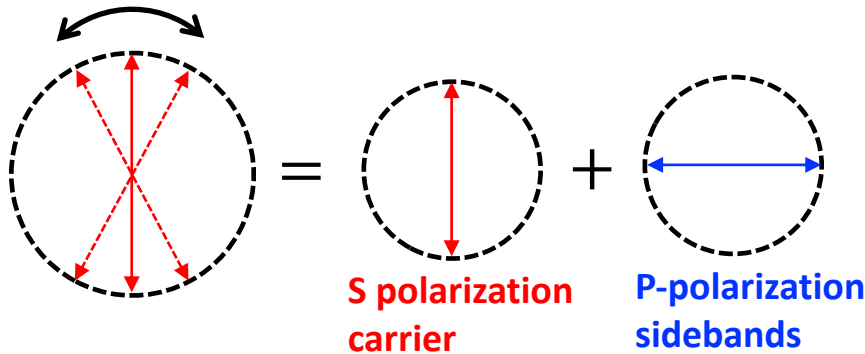
Axion-photon interaction

Axion-photon interaction causes phase velocity difference

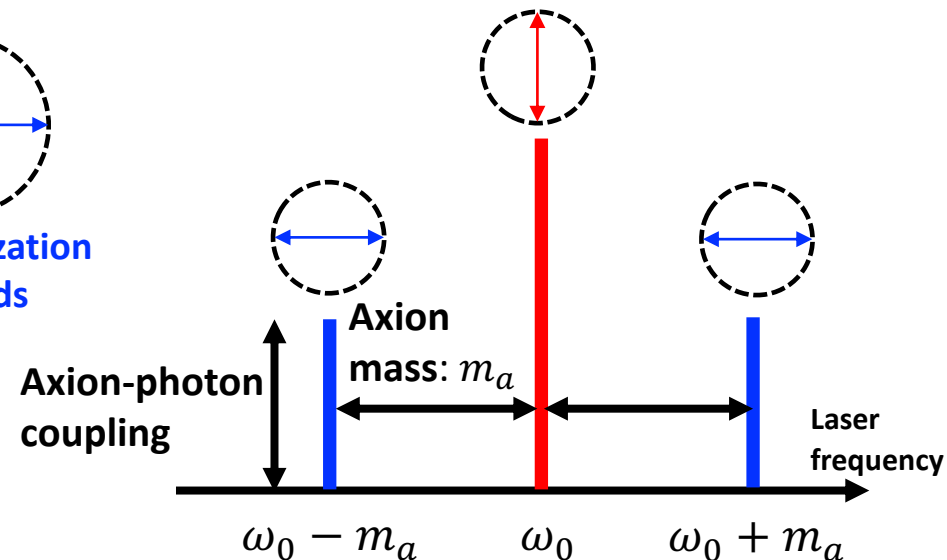
$$c_{L/R} = \sqrt{1 \pm \frac{g_{a\gamma} a_0 m_a}{k} \sin(m_a t + \delta_\tau)}$$

Left-/Right-handed polarization $\rightarrow c_{L/R}$
 Axion-photon coupling constant $\rightarrow g_{a\gamma}$
 Axion field $\rightarrow a_0$
 Axion mass $\rightarrow m_a$

Rotational oscillation of linear polarization

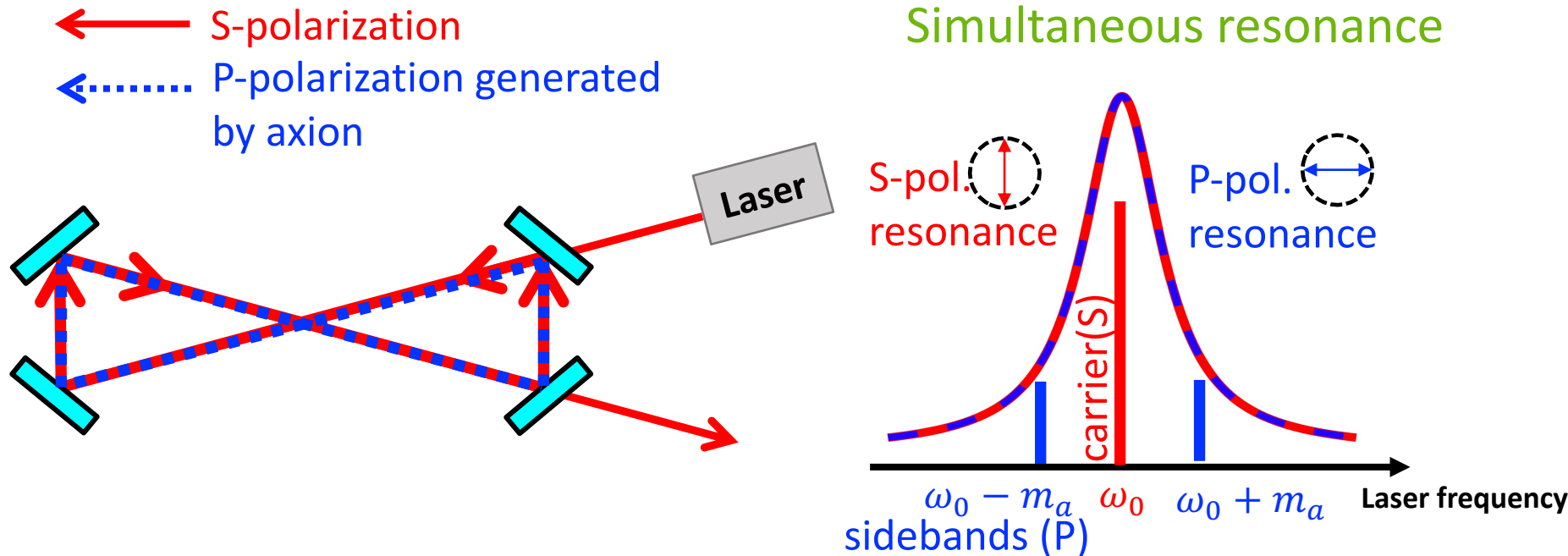


Axion signal is produced as sideband: $\omega_0 \pm m_a$ of P-pol



Enhance signal with DANCE cavity

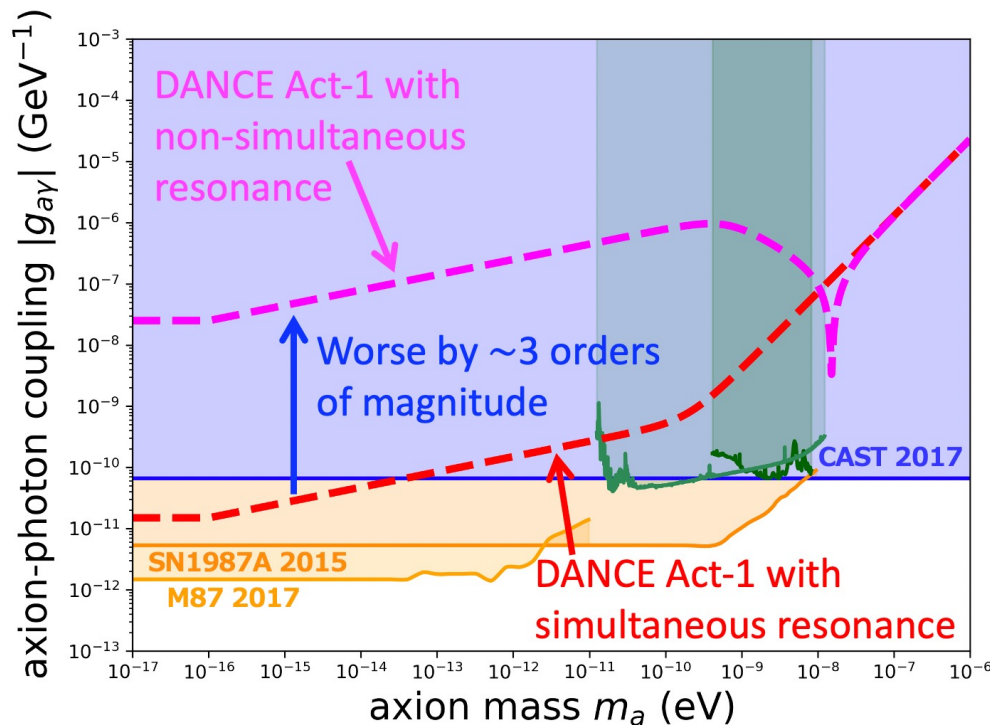
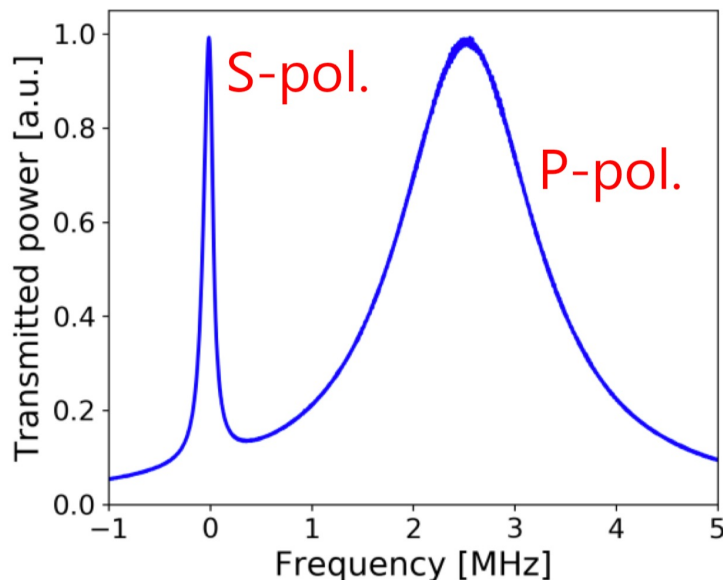
Axion signal can be enhanced with bow-tie ring cavity



Simultaneous resonance is needed to amplify both carrier and axion sidebands

Phase difference on mirror's surface

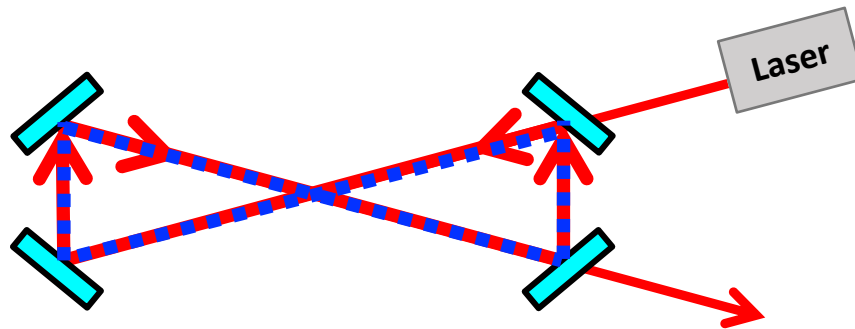
S- and P- polarizations cannot resonate simultaneously



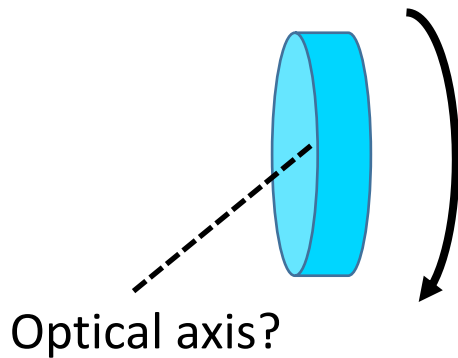
In DANCE Act-1, phase difference ≈ 3 deg

Upper limit of target phase difference = **0.06 deg** (Finesse:3000)

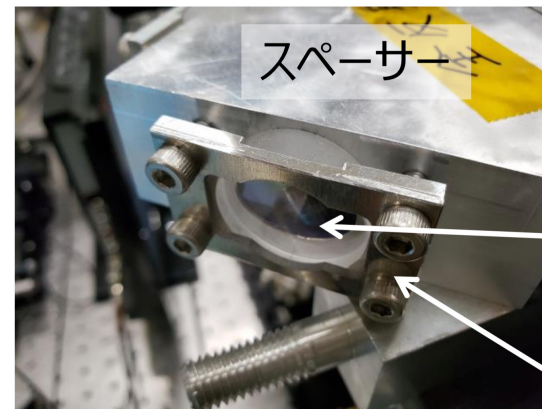
What is the cause?



Birefringence of mirror?



Photoelasticity induced by holding pressure?



Mirror

Mirror holding jig

Both did not change the resonant frequency difference...

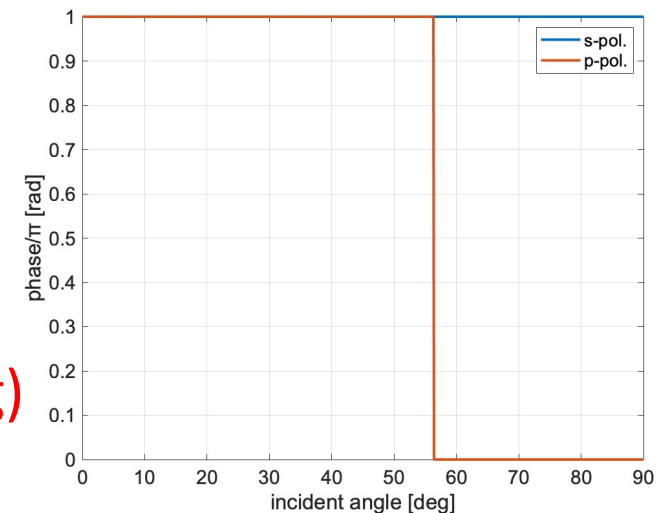
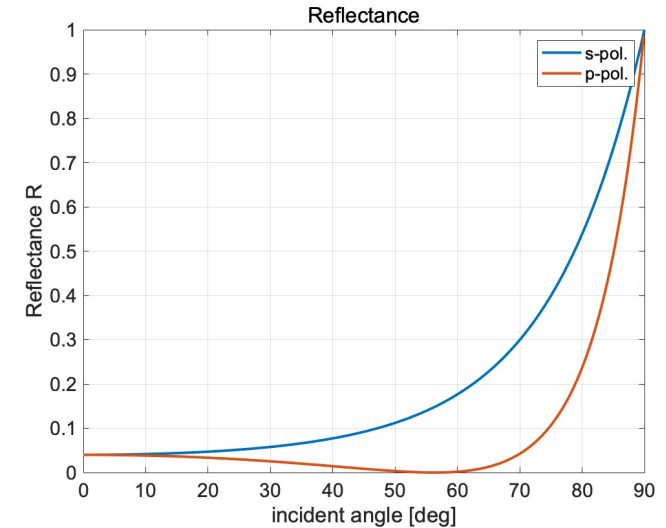
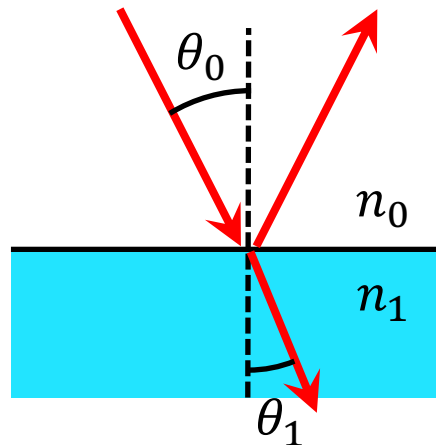
Fresnel Reflection

Paper on ADBC experiment says Fresnel reflection causes the phase difference

Fresnel equation

$$r_s(\theta_0) = \frac{n_0 \cos \theta_0 - n_1 \cos \theta_1}{n_0 \cos \theta_0 + n_1 \cos \theta_1}$$

$$r_p(\theta_0) = \frac{n_0 \cos \theta_1 - n_1 \cos \theta_0}{n_0 \cos \theta_0 + n_1 \cos \theta_1}$$

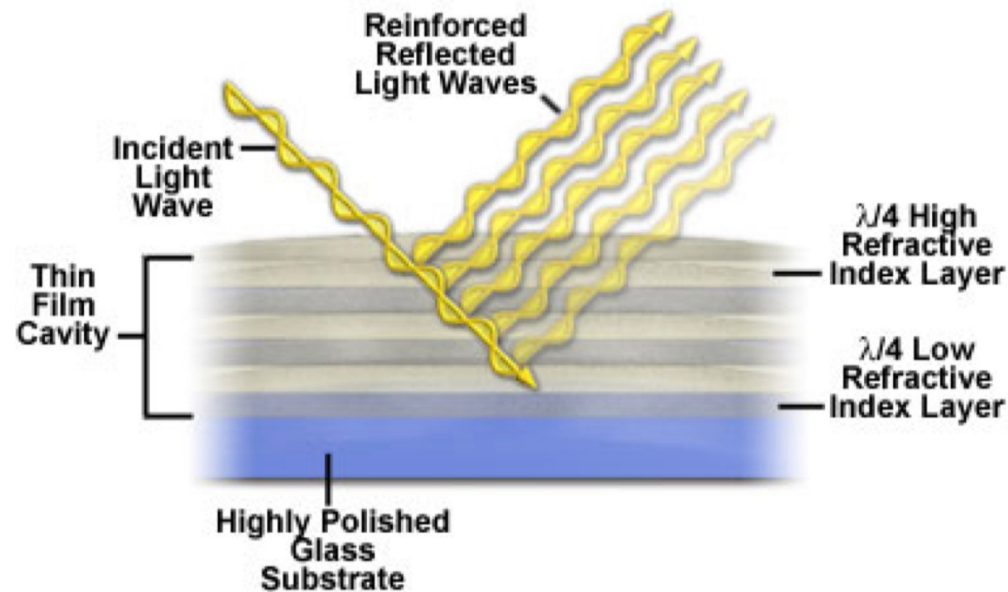


Phase shift is 0 or π (Not continuous)

Can not realize small phase shift (e.g. 3 deg)

Dielectric Mirror

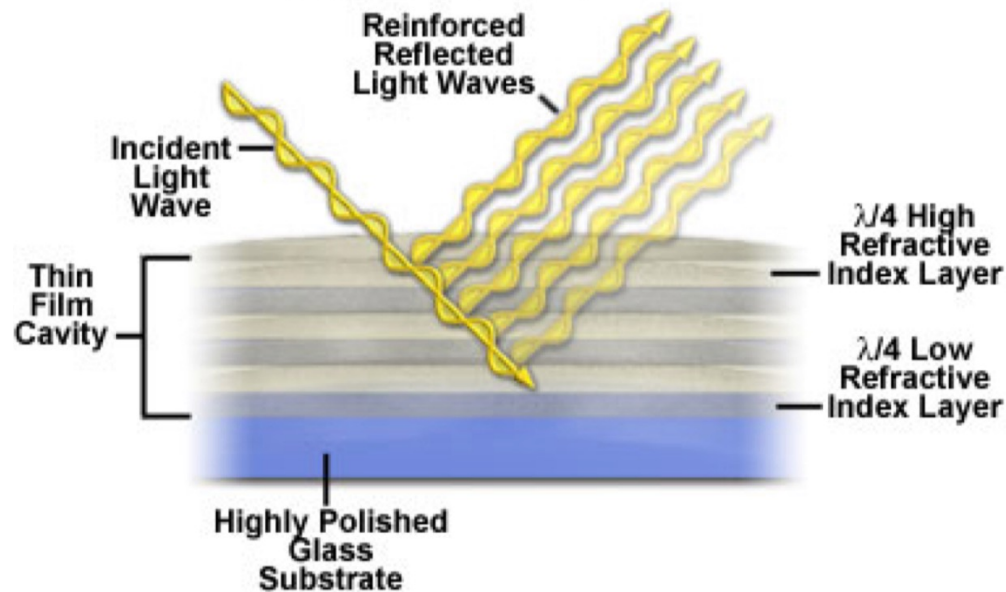
Actually, mirror is not a single boundary



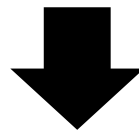
Can dielectric multilayers shift the phase continuously?

Dielectric Mirror

Actually, mirror is not a single boundary



Can dielectric multilayers shift the phase continuously?



Yes! Dielectric mirror behaves completely differently from a single boundary

Reference

◆ Tanioka-san's doctor thesis (NAOJ)

Resonant frequency difference between S- and P- pol. in a folded cavity

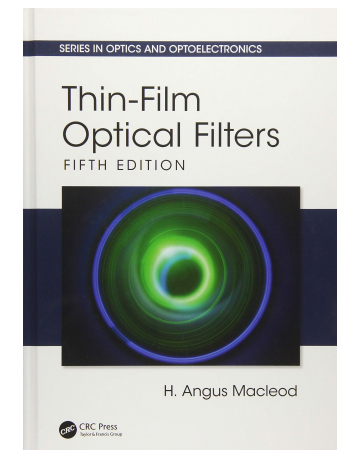
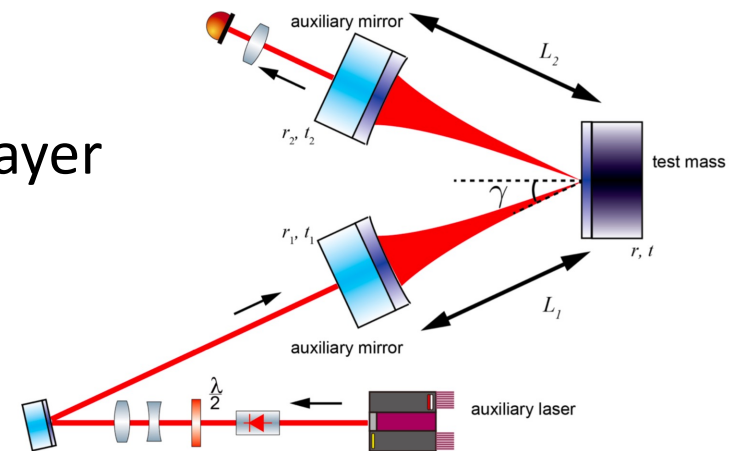
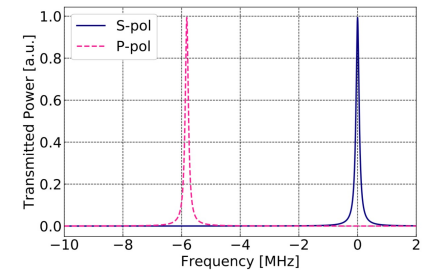


Measured the thickness of molecular layer on a cooled mirror

Used the theory of phase shift on dielectric mirror

◆ Thin Film Optical Filters

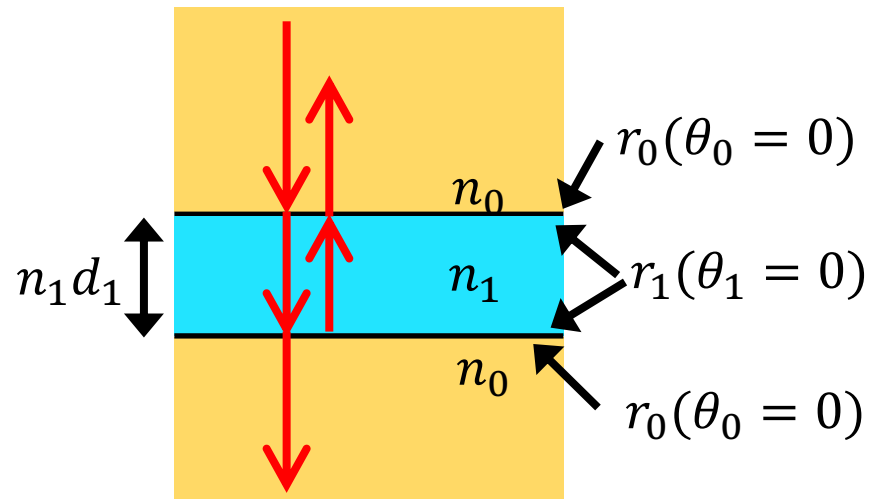
The theory of dielectric mirror is written



Dielectric mirror and phase shift

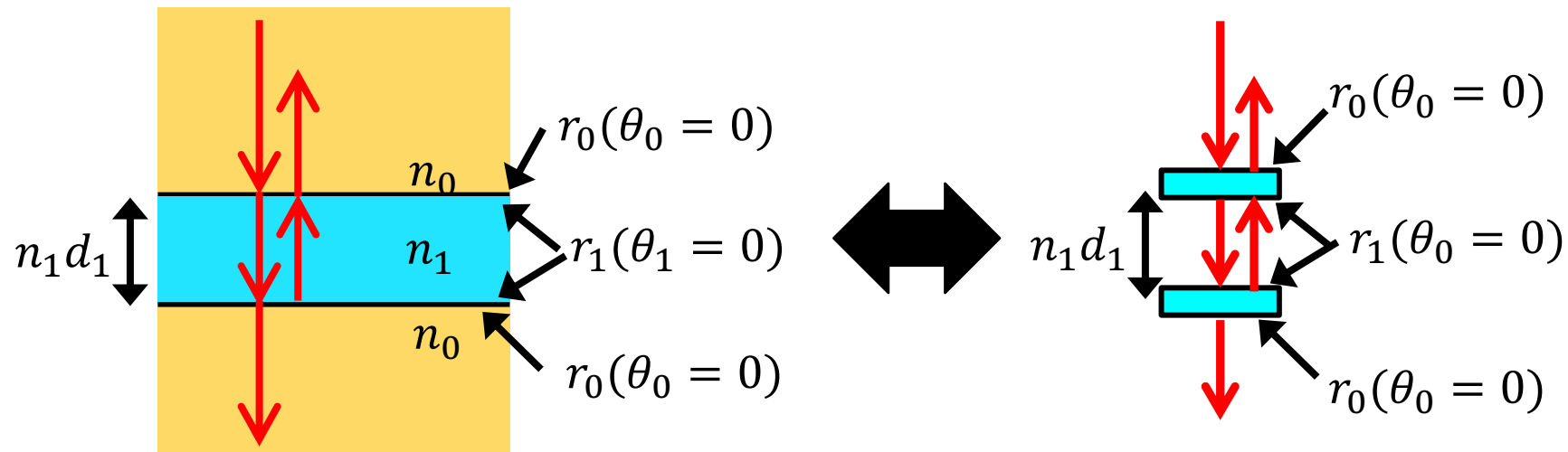
Normal reflection on a single layer

Consider a single layer (2 boundaries)



Normal reflection on a single layer

Consider a single layer (2 boundaries)



A single film is equivalent to a cavity

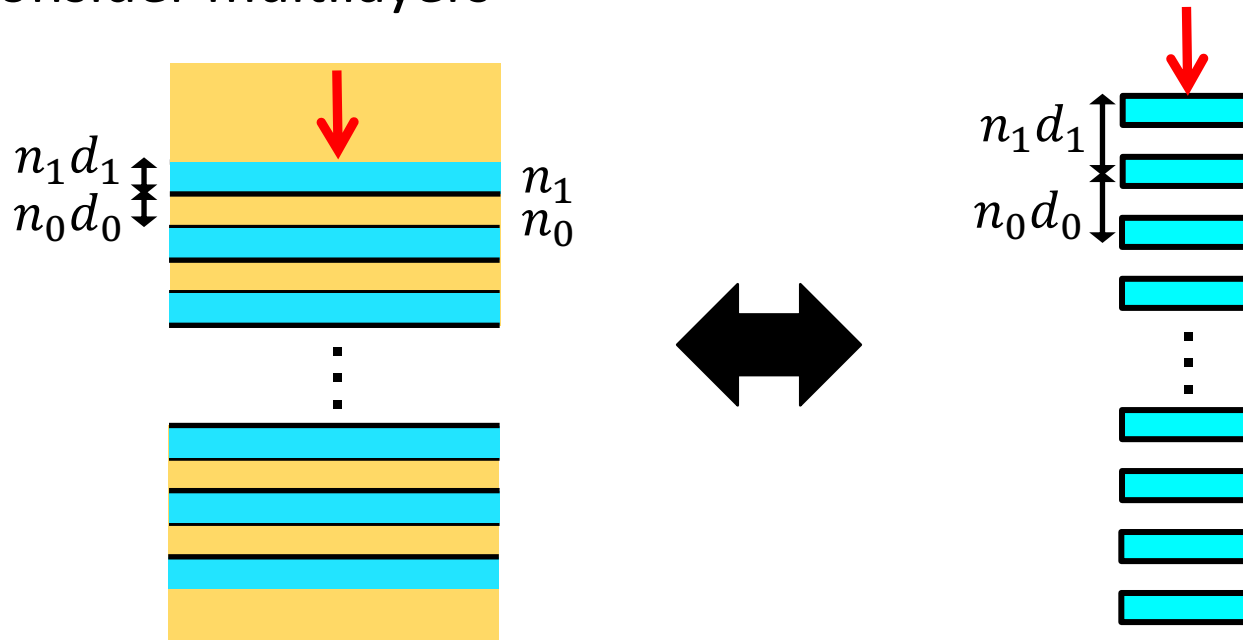
Optical thickness: $n_1 d_1 = \lambda/4$ $\Rightarrow$ Anti-resonance (**Higher reflectance**)

For example, , $n_0 = 1.44(\text{SiO}_2)$, $n_1 = 2.2 (\text{Ta}_2\text{O}_5)$

$$R_{\text{in}} = R_{\text{end}} = 0.043 \Rightarrow R_{\text{cav}} = 0.16$$

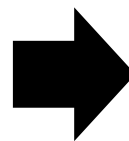
Normal reflection on multilayers

Consider multilayers



Equivalent to many coupled cavities

Optical thickness:
 $n_0 d_0 = n_1 d_1 = \lambda/4$



Anti-resonance in all cavities

Higher reflectivity than a single layer

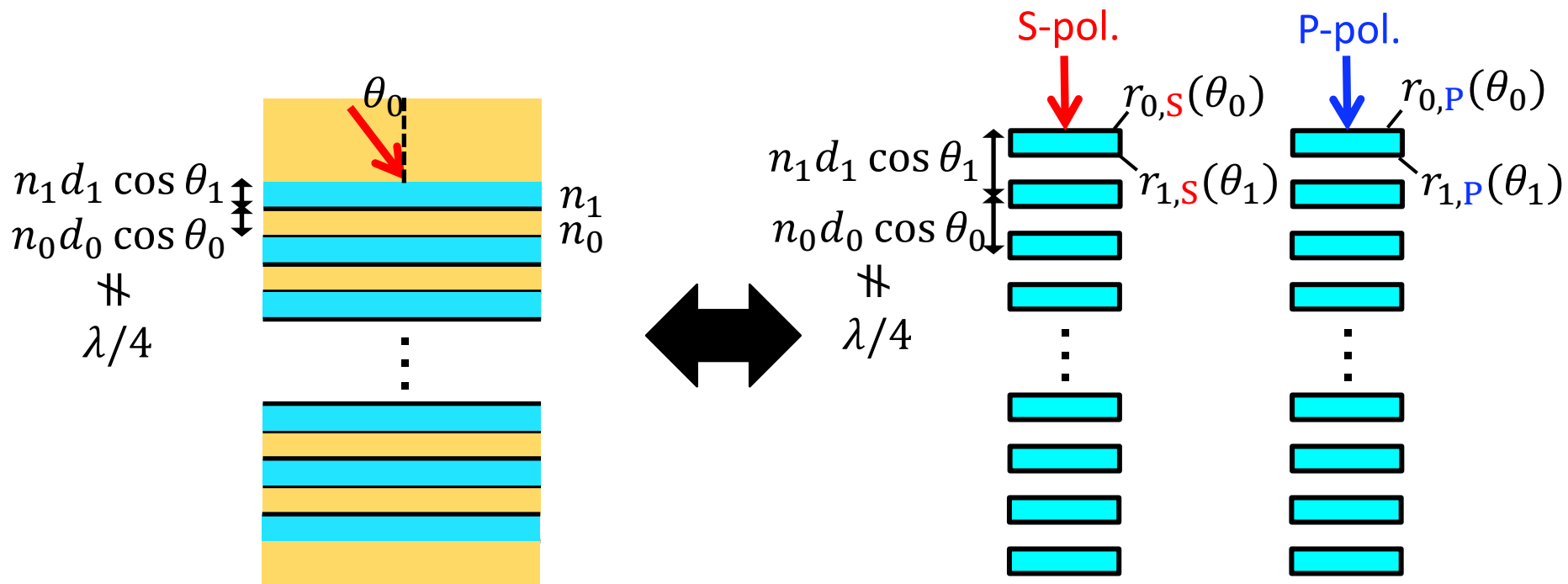
Oblique reflection on multilayers

Consider oblique incidence: $\theta_0 \neq 0$

Design: $n_0 d_0 = n_1 d_1 = \lambda/4$

Optical thickness : $n_0 d_0 \cos \theta_0, n_1 d_1 \cos \theta_1$ **Off anti-resonance**

Reflectivity of a boundary: different for S- and P-pol.



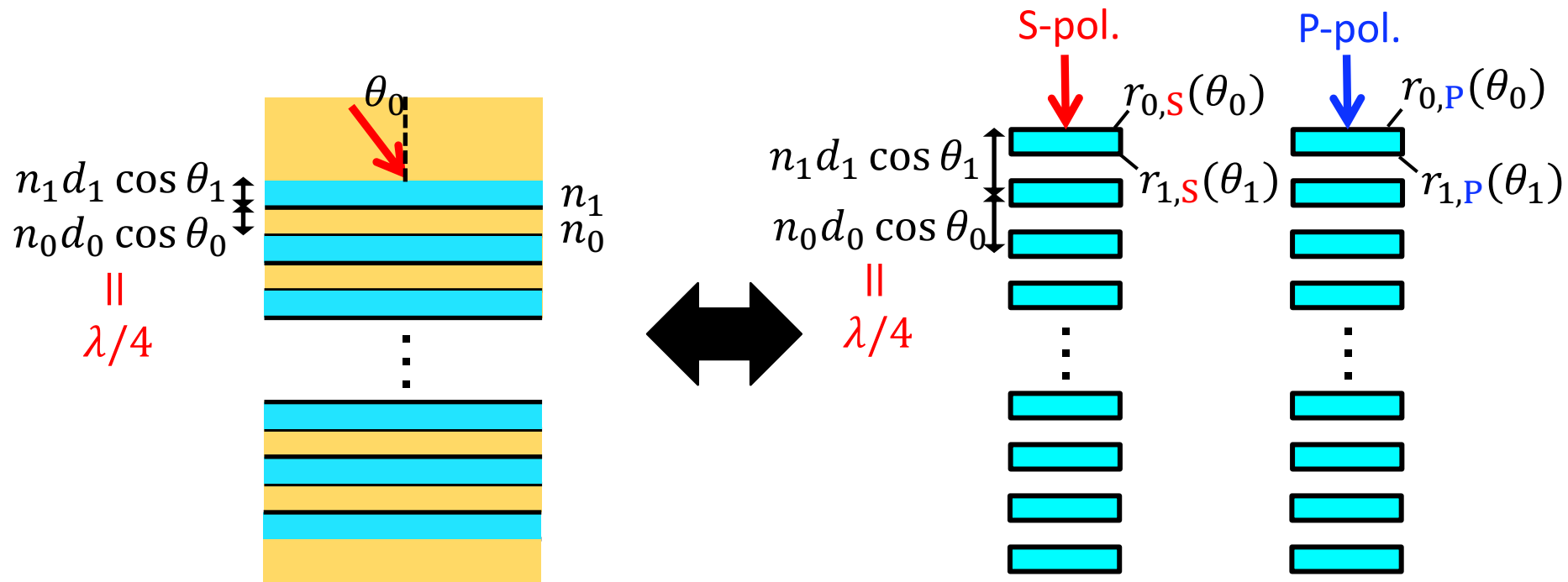
Different coupled cavities (off-antiresonance) for S- and P-pol.

➡ Different phase shift for S- and P-pol

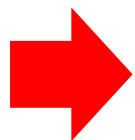
Zero phase shift mirror

Perfect anti-resonance

Set d_0 and d_1 to satisfy $n_0 d_0 \cos \theta_0 = n_1 d_1 \cos \theta_1 = \lambda/4$



Multilayers are perfectly anti-resonant for S- and P- pol.



Phase difference between S- and P-pol = 0

But difficult to realize due to the accuracy for thickness

Calculation of reflectivity

Characteristic matrix of layer k

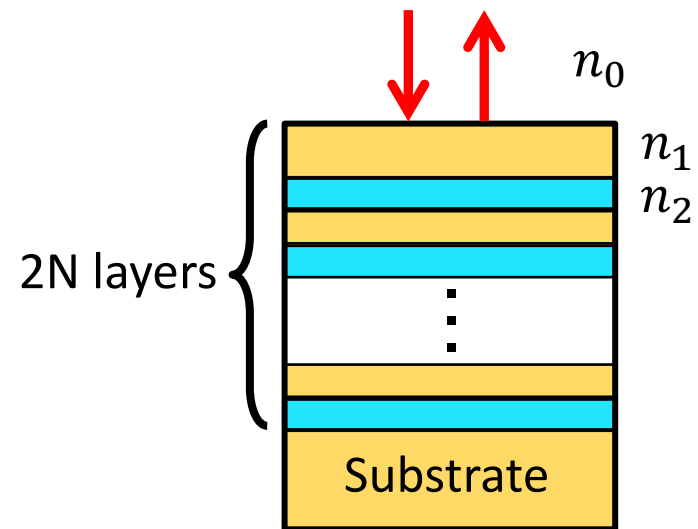
$$M_k = \begin{bmatrix} \cos \delta_k & i \eta_k \sin \delta_k / \eta_k \\ i \eta_k \sin \delta_k & \cos \delta_k \end{bmatrix}$$

$$\delta_k = \frac{2\pi}{\lambda} n_k d_k \cos \theta_k \quad \begin{aligned} \eta_{k,S} &= n_k \cos \theta_k \\ \eta_{k,P} &= n_k / \cos \theta_k \end{aligned}$$

$$\begin{bmatrix} B \\ C \end{bmatrix} = \left(\prod_{k=1}^{2N} M_k \right) \begin{bmatrix} 1 \\ \eta_{N+1} \end{bmatrix}$$

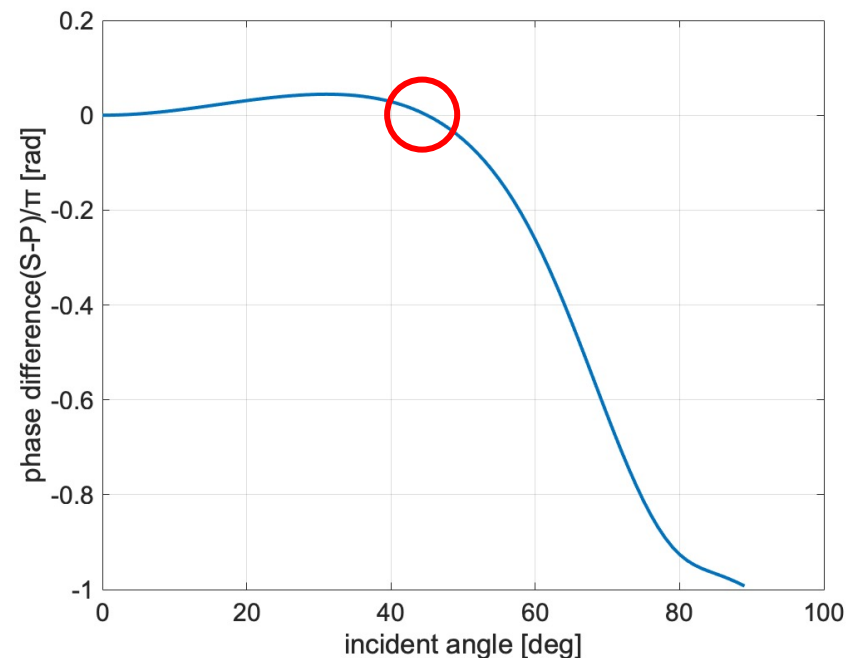
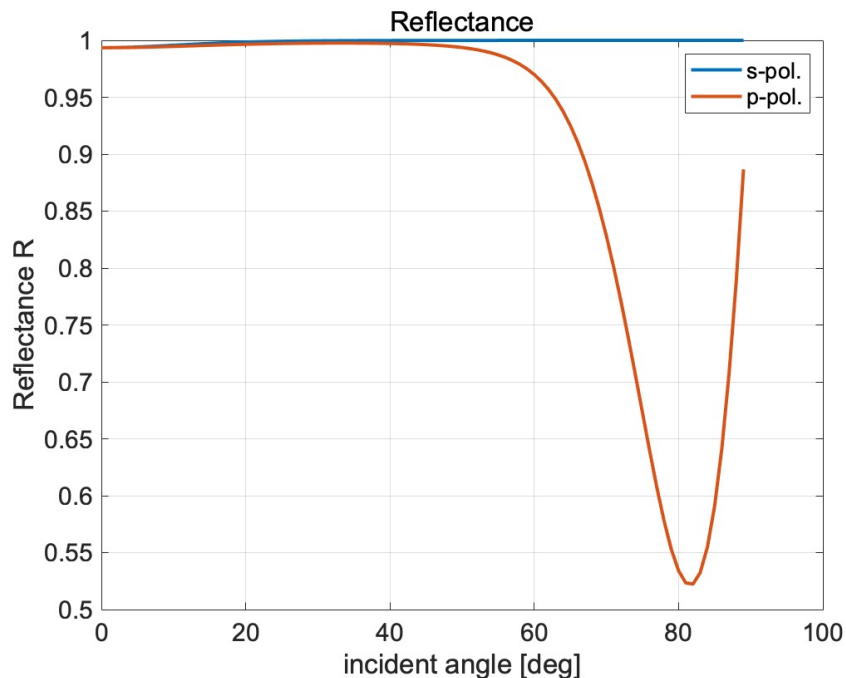
$$r(\theta_{in}) = \frac{\eta_0 - C/B}{\eta_0 + C/B}$$

Reflectivity of dielectric multilayers



I designed a high reflective mirror for 45 deg incidence ($2N=20$)

Dielectric mirror for 45 deg incidence



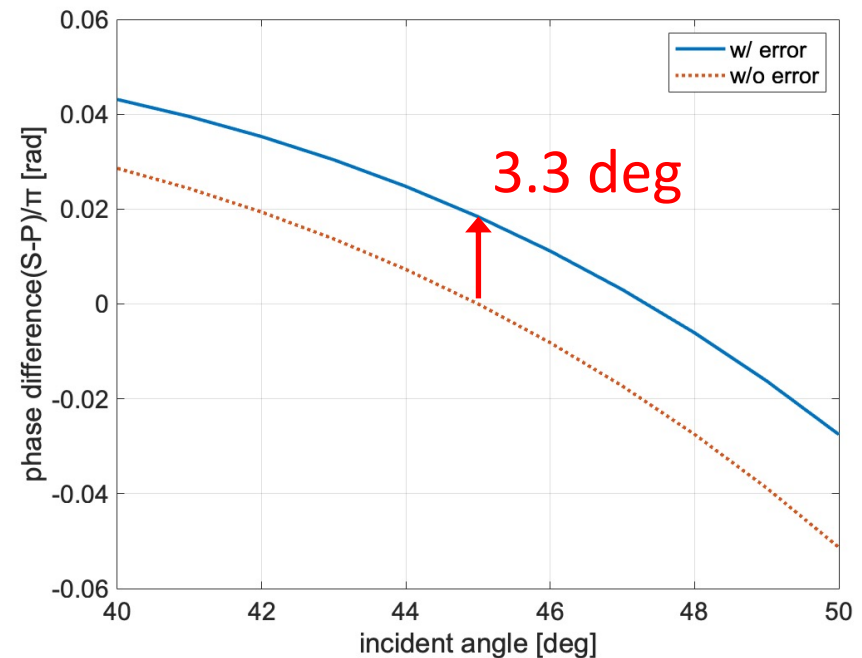
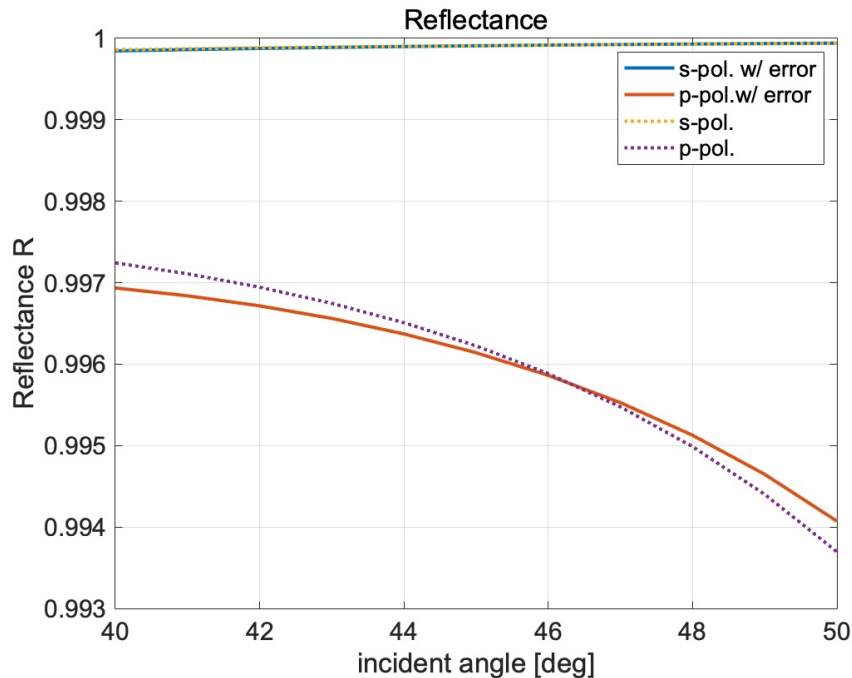
$R_S = 0.9999, R_P = 0.9962$ for 45 deg incidence

Phase difference between S and P changes continuously

Phase difference is 0 at 45 deg incidence

Error of thickness

Added error of 1%(1~2 nm) to thickness of all layers



Phase difference = 3.3 deg at 45 deg incidence

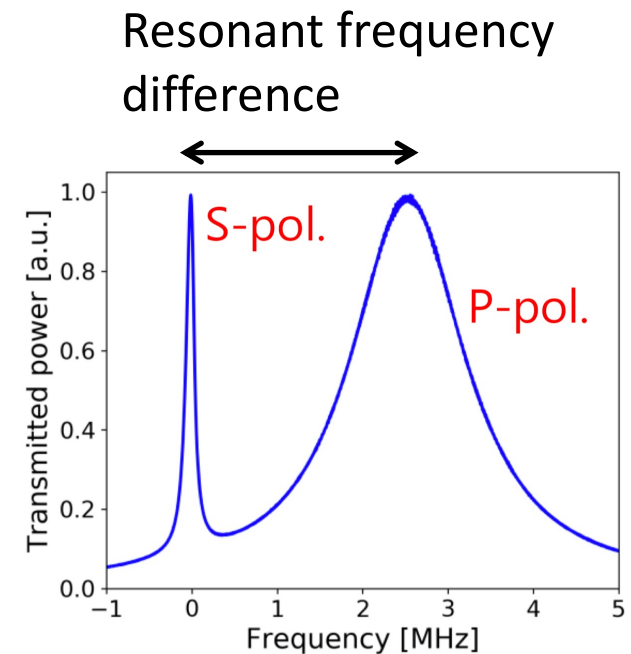
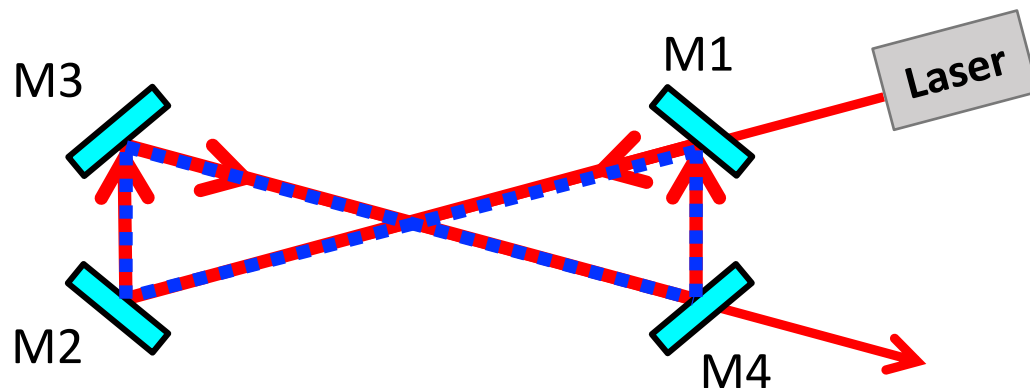
Phase difference is sensitive to error of thickness

How to measure the coating
phase shift of S- and P-pol.

Plan 0: Use DANCE cavity

From resonant frequency difference,
phase difference: $\Delta\phi = \Delta\phi_{M1} + \Delta\phi_{M2} + \Delta\phi_{M3} + \Delta\phi_{M4}$ is measured

Phase difference on a single mirror can not be measured

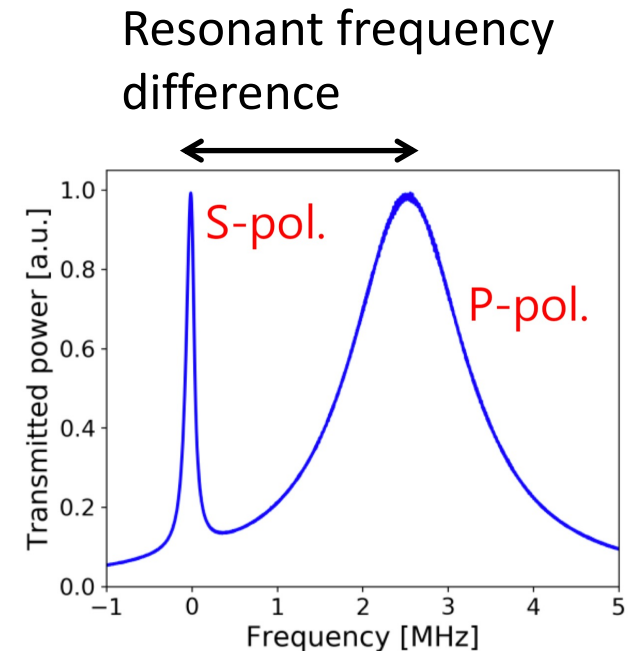
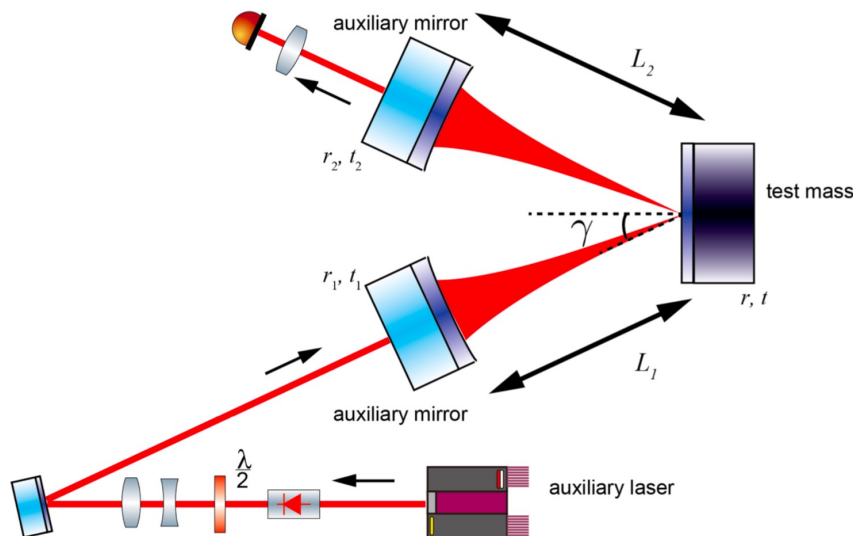


Plan 1: Use a folded cavity

Incident angle $\neq 0$ deg on only folding mirror

From resonant frequency difference,
Phase difference: $\Delta\phi = \Delta\phi_{\text{fold}}$ is measured.

Phase difference on a single mirror can be measured



Plan 2: Use modulation and demodulation

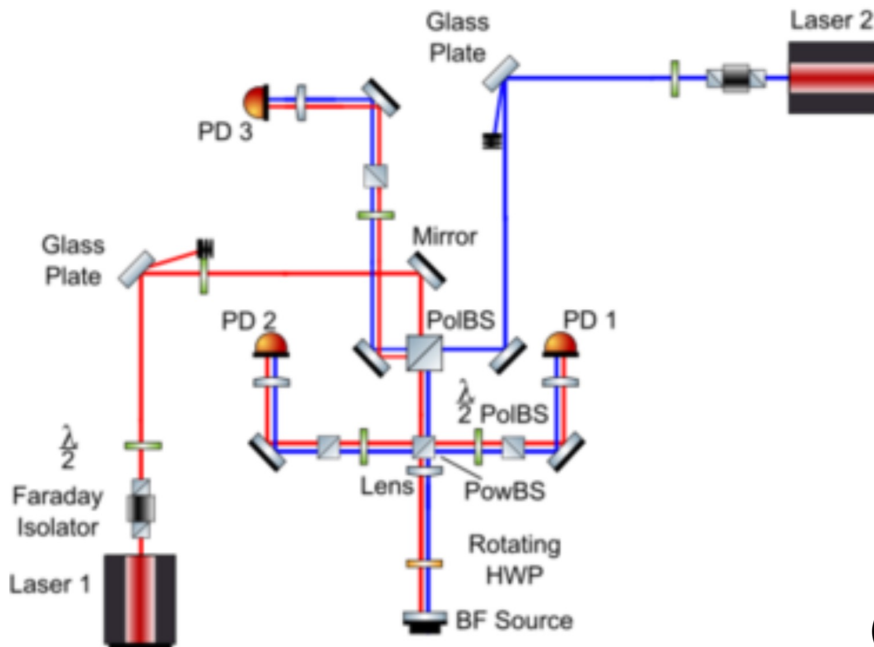
Phase shift with normal incidence was measured by ALPS group

Rotate input linear polarization at 20 Hz

Use beat note of 5 MHz of two lasers



Phase difference signal can be demodulated
accuracy: ± 0.01 deg



Characterization of Mirror Birefringence for

ALPS

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University of Florida
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August 4, 2017

(<http://www.phys.ufl.edu/REU/2017/LaBollitaHarrison.pdf>)

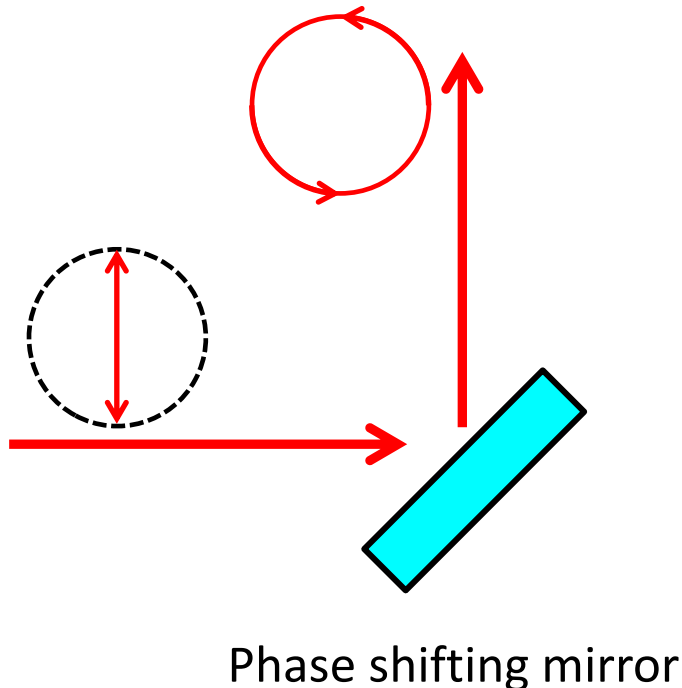
Bonus slide:

Application of phase shift on
mirror coating

Application: Creating circularly polarization

By designing the dielectric layers, phase shift of 90 deg can be realized.

Ellipticity $\varepsilon > 98\%$ was achieved



Creating circularly polarized light with a phase-shifting mirror

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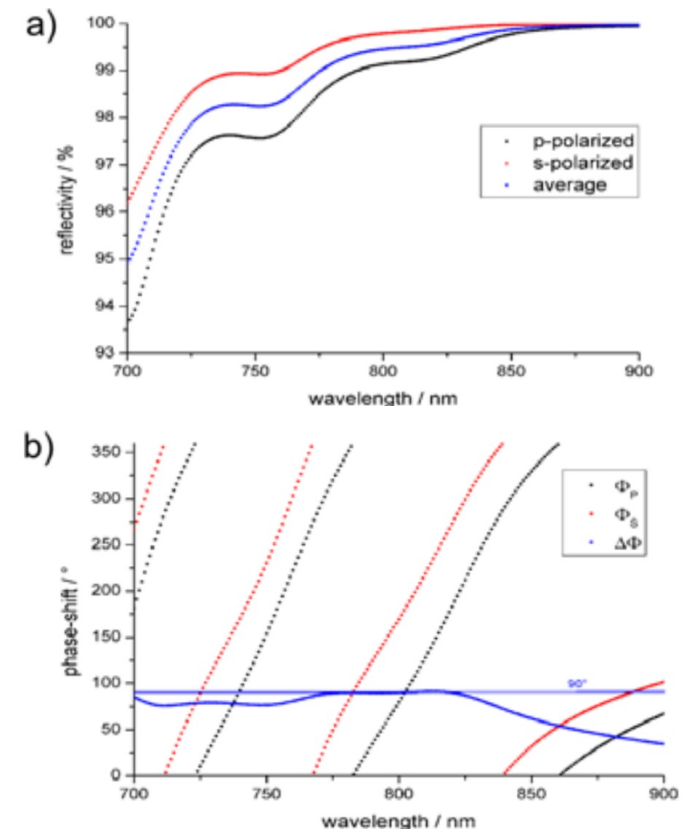
⁴EMMI Extreme Matter Institute, Planckstr. 1, 64291 Darmstadt, Germany
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Abstract: We report on the performance of a system employing a multi-layer coated mirror creating circularly polarized light in a fully reflective setup. With one specially designed mirror we are able to create laser pulses with an ellipticity of more than $\varepsilon = 98\%$ over the entire spectral bandwidth.

B. Aurand, et al.,

[Opt. Express 19, 17151 \(2011\)](#)



Summary

- ◆ Dielectric coating can be regarded as many anti-resonant coupled cavities
- ◆ Input of light from undesigned angle
⇒ phase difference between S- and P- pol.
- ◆ Difficult to make perfect zero phase shift mirror

Thank you for listening