Inflationary gravitational waves as a probe of the early Universe

中山和則 (東京大学)

with 神野隆介、諸井健夫

arXiv:1112.0084, 1208.0184, 1307.3010

DECIGO Workshop @ University of Tokyo (2013/10/27)

Contents

- インフレーション重力波
- 初期宇宙相転移
- Dark radiation
- Large B-mode インフレーション模型

インフレーション重力波

- インフレーション中、テンソル揺らぎが量子的に生成

$$ds^2 = a^2(t)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

- 無次元パワースペクトル：（スケール不変）

$$\Delta_h^2(k) = \left(\frac{H_{\rm inf}}{2\pi M_P}\right)^2$$

- $\ddot{h}_\lambda + 3H\dot{h}_\lambda + (k/a)^2 h_\lambda = 0$

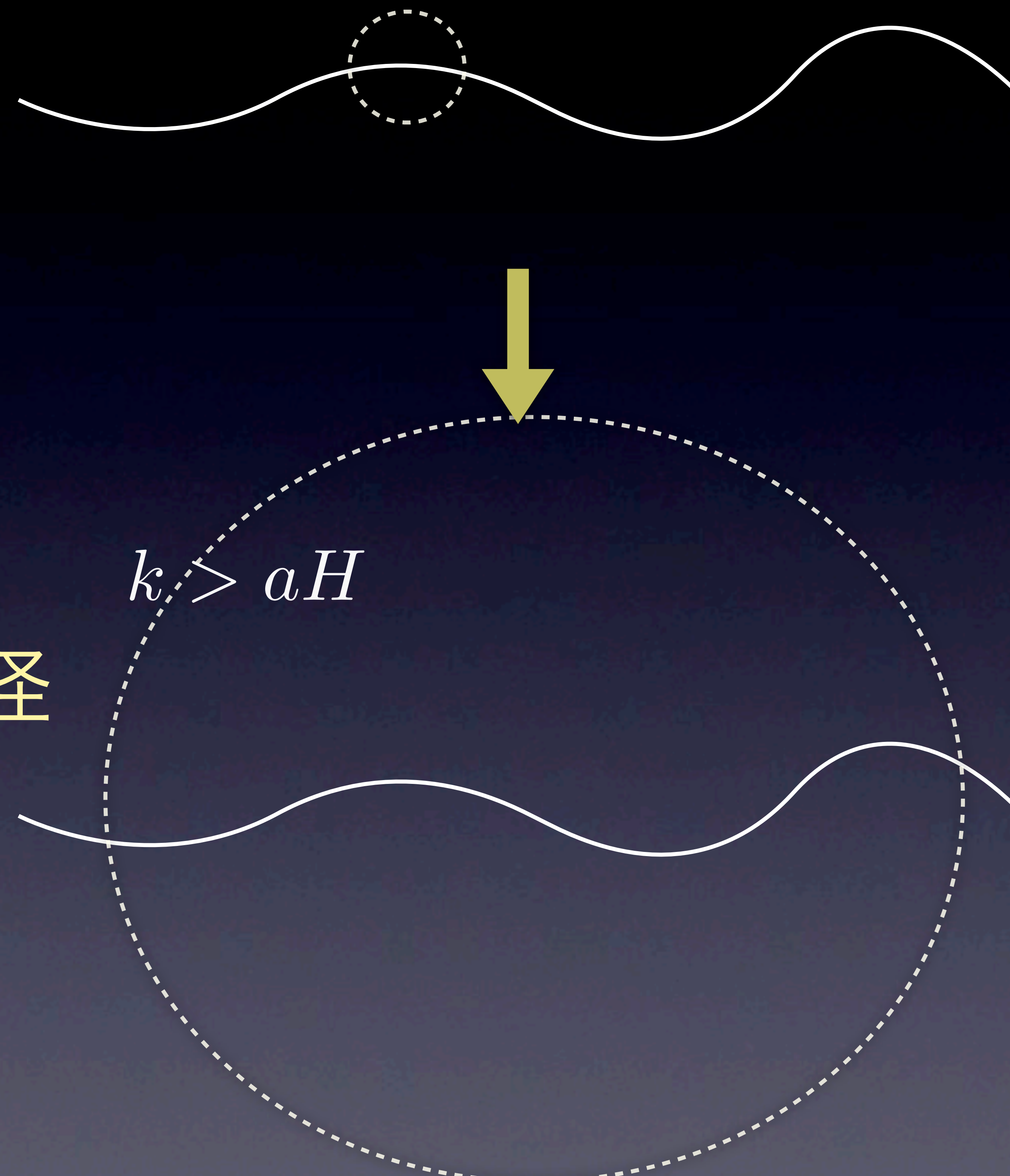
ホライズン外：$h \sim \mathrm{const}$

ホライズン内：$h \propto a(t)^{-1}$

- 波長＞ホライズン半径 $k < aH$

重力波は宇宙膨張を感じない

$$h \sim \mathrm{const}$$

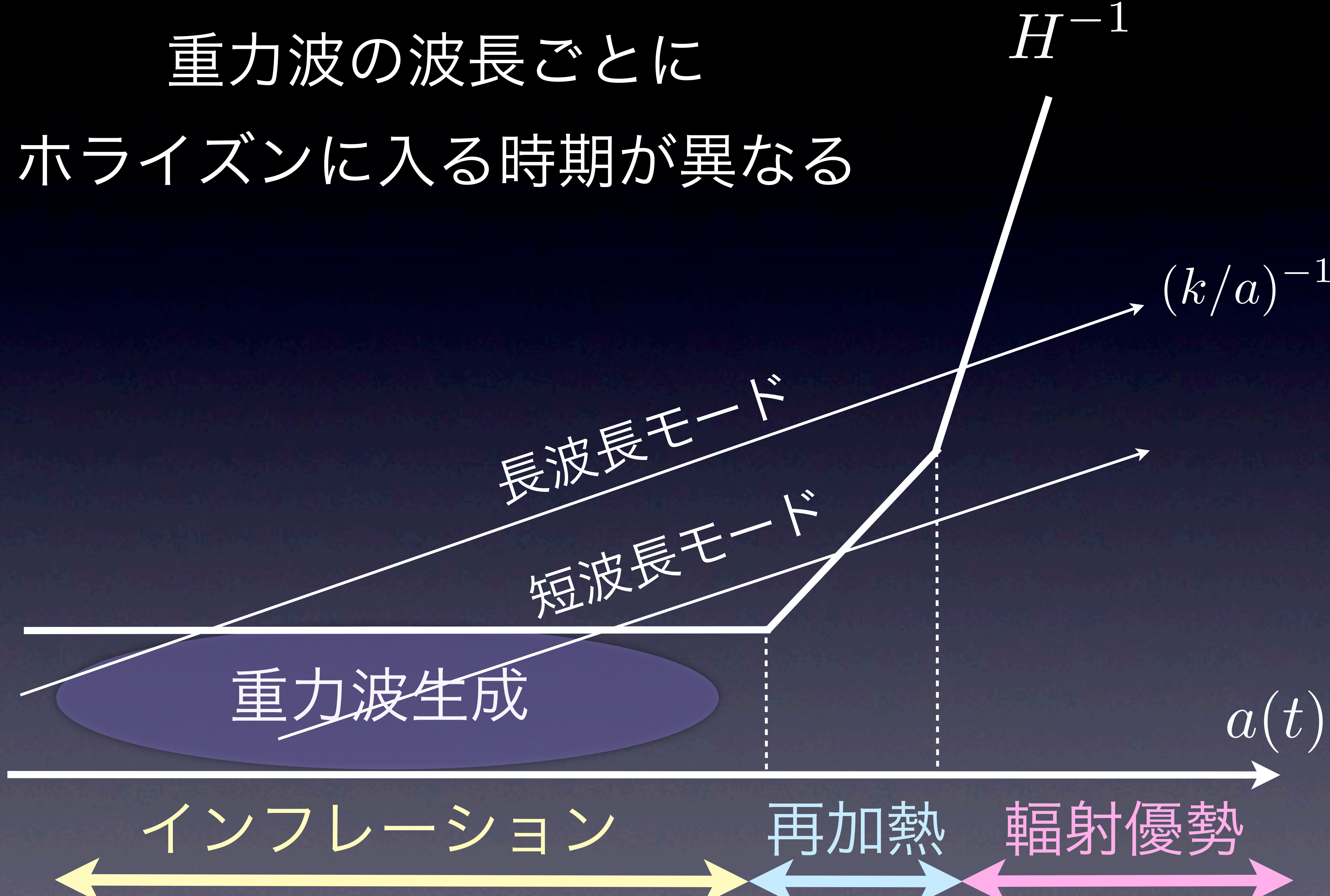


- 波長＜ホライズン半径

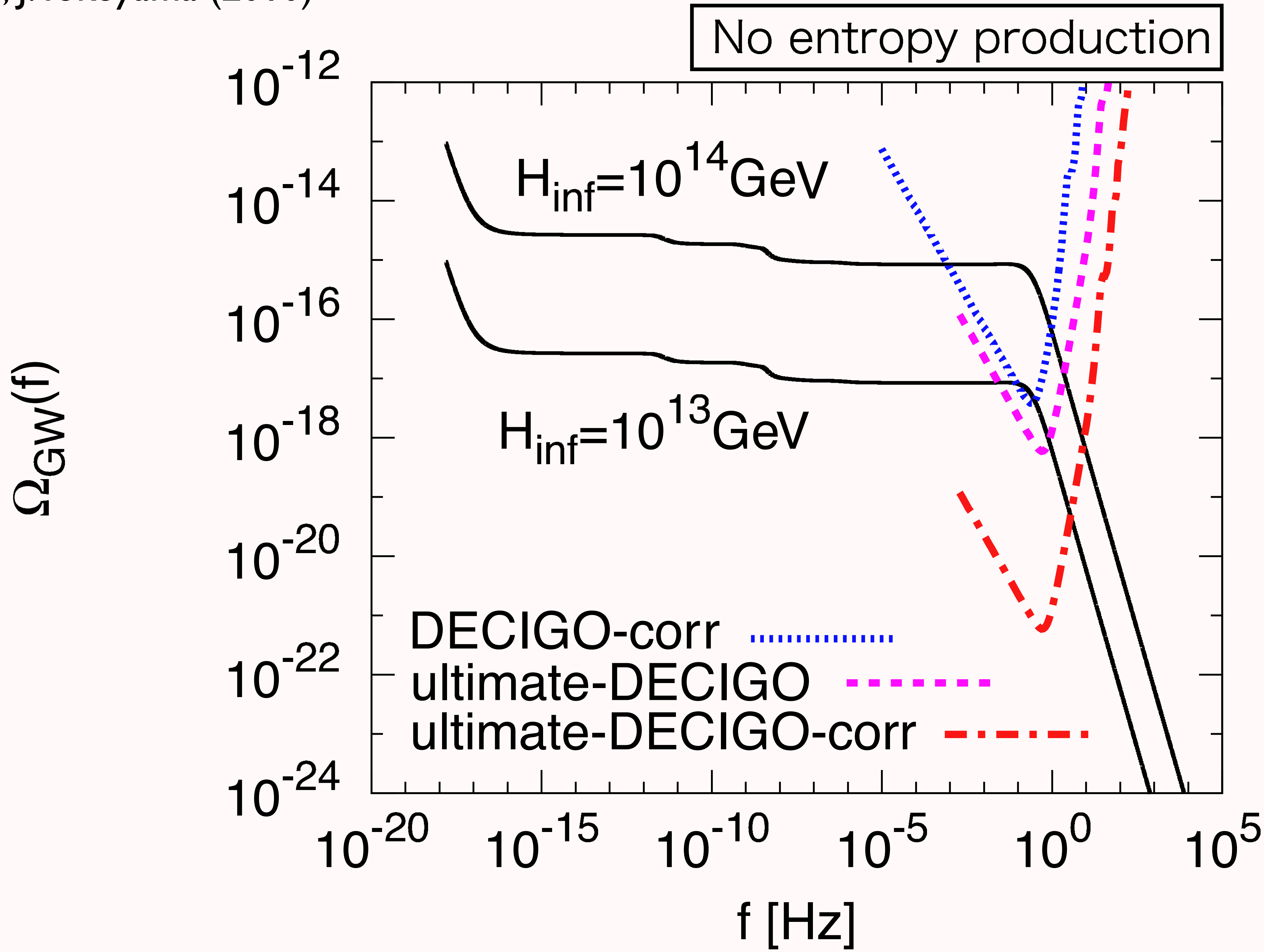
重力波は宇宙膨張を感じる

$$h \propto a(t)^{-1}$$

重力波の波長ごとに
ホライズンに入る時期が異なる



KN, J.Yokoyama (2010)



重力波スペクトルに宇宙熱史が刻まれている！

N.Seto, J.Yokoyama (2003), Boyle, Steinhardt (2005), KN, Saito, Suwa, Yokoyama (2008)
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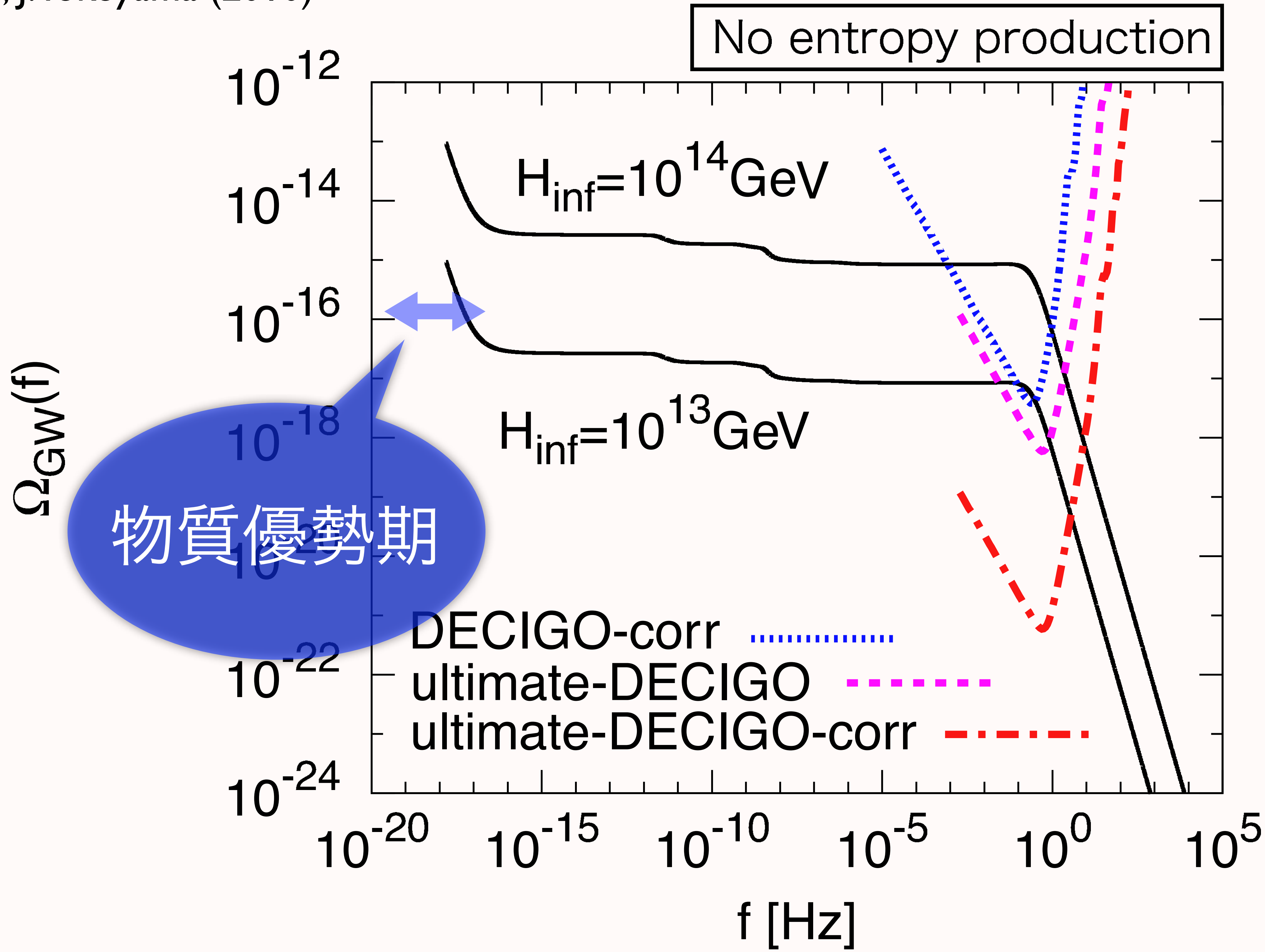

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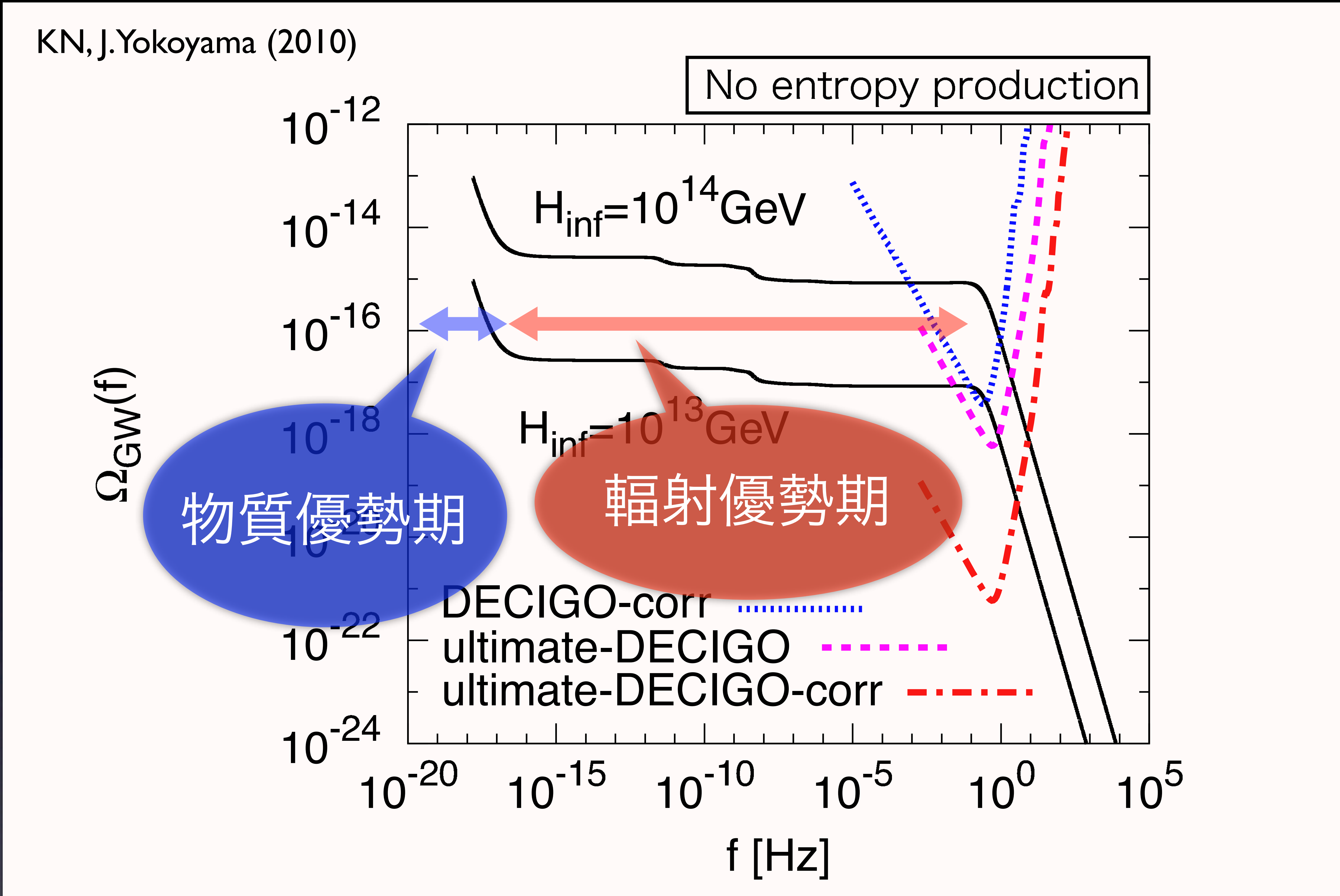

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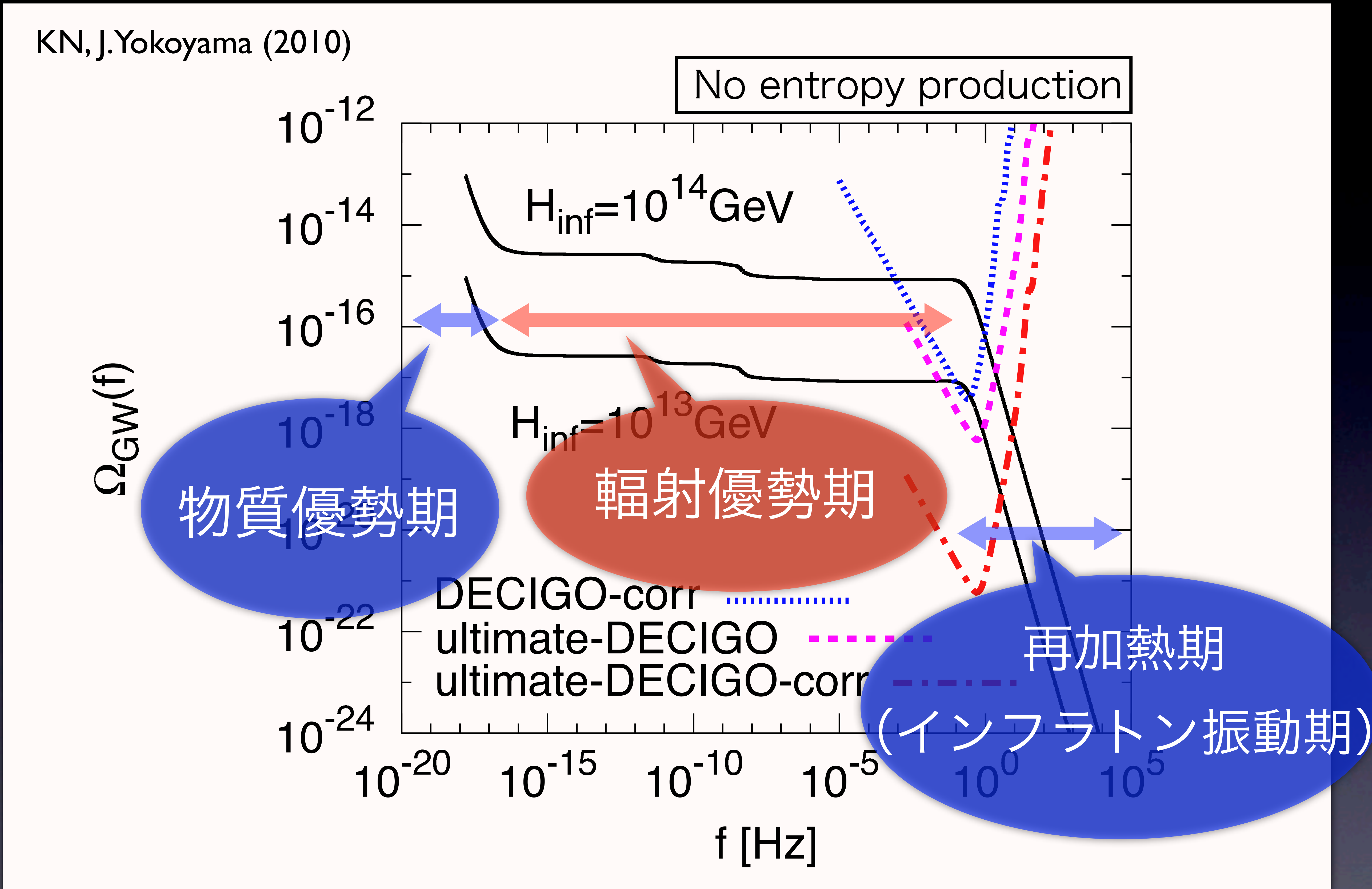

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宇宙の温度と背景重力波周波数の関係

$$f = 2.6\,\mathrm{Hz}\left(\frac{T}{10^8\,\mathrm{GeV}}\right)$$

初期宇宙相転移

R.Jinno, T.Moroi, KN, arXiv:1112.0084

宇宙における相転移

- クォーク-ハドロン相転移 (T~100MeV)
- 電弱相転移 (T~100GeV)
    - ヒッグス(-like)粒子の発見 @ LHC！
- Peccei-Quinn 相転移 (T~10^{8}-10^{12}GeV?)
- GUT (大統一理論) 相転移 (T~10^{16}GeV?)

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相転移の模型

- ある対称性の電荷を持つスカラー場 ϕ

ポテンシャル： (E.g. ヒッグス場)

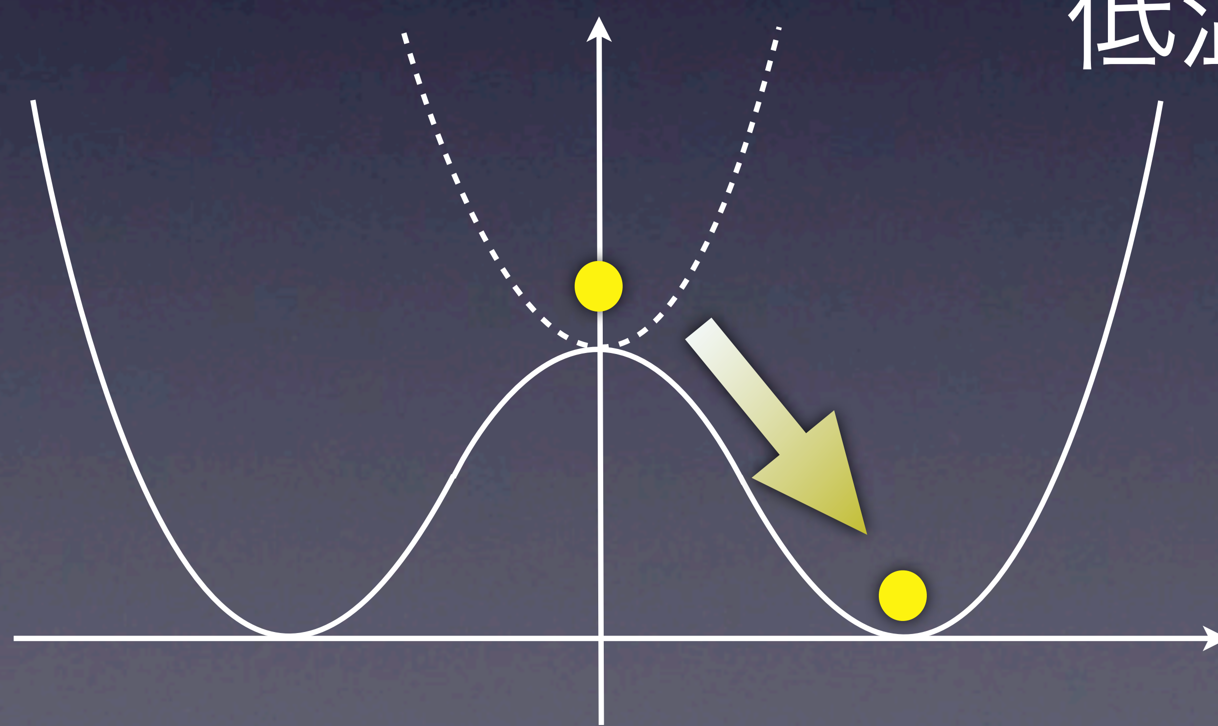
$$V(\phi) = \frac{g}{24}(\phi^2 - v_\phi^2)^2 + \frac{1}{24}hT^2\phi^2$$

高温 : $\phi = 0$

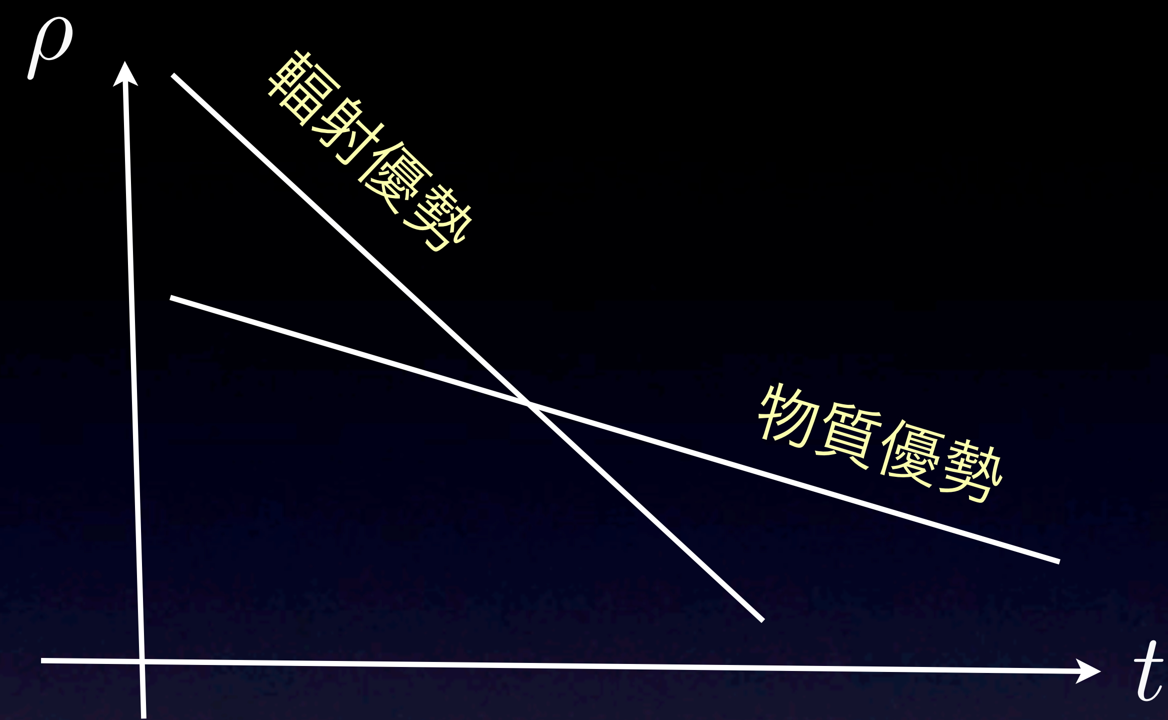
symmetric phase

低温 : $\phi = v_\phi$

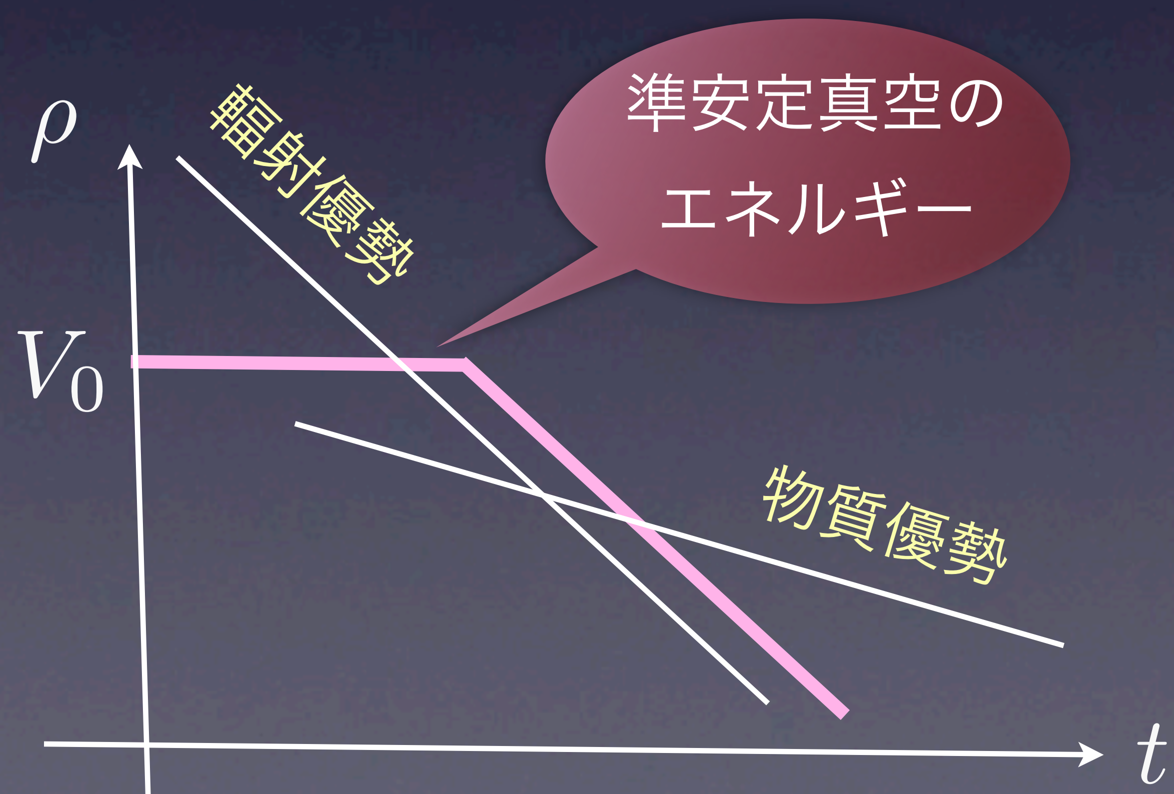
broken phase

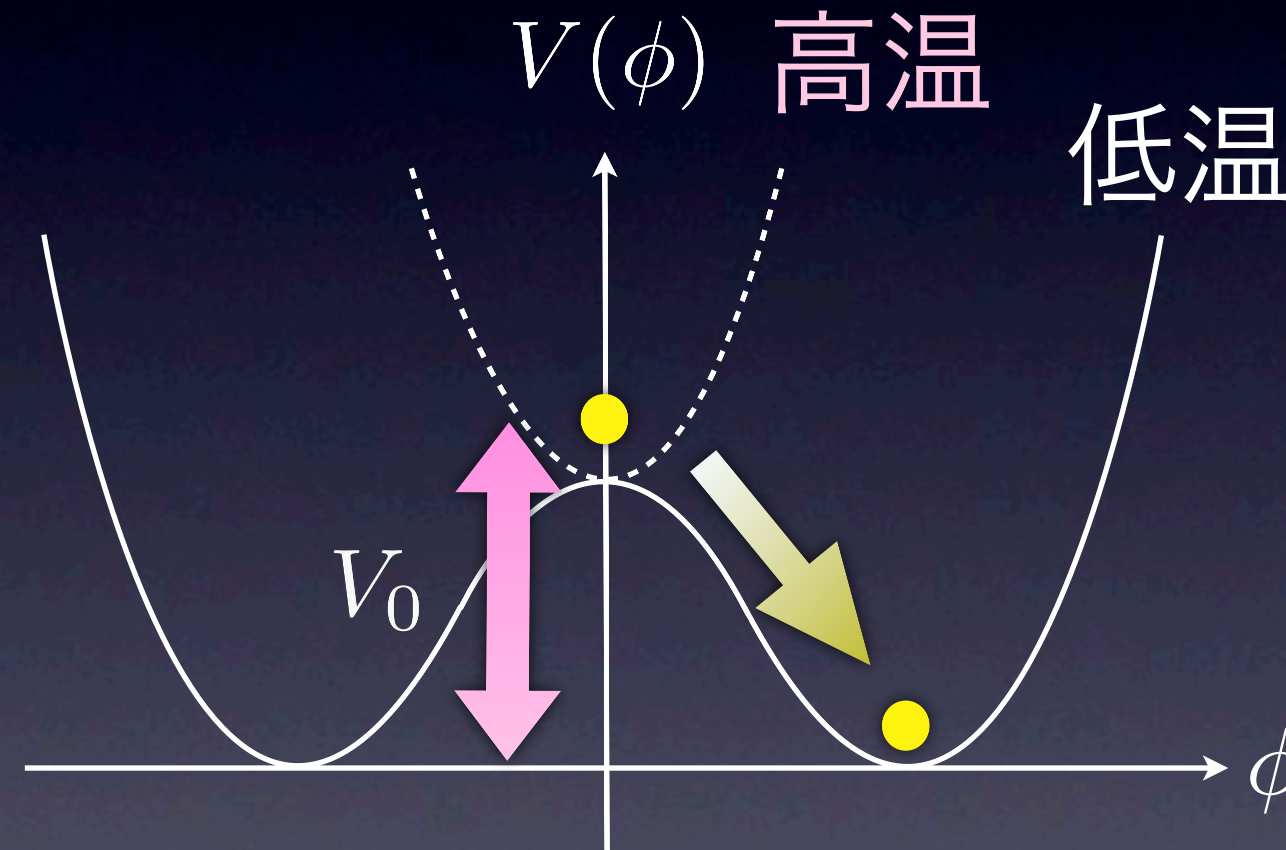


- 通常のシナリオ



- 相転移があったとき

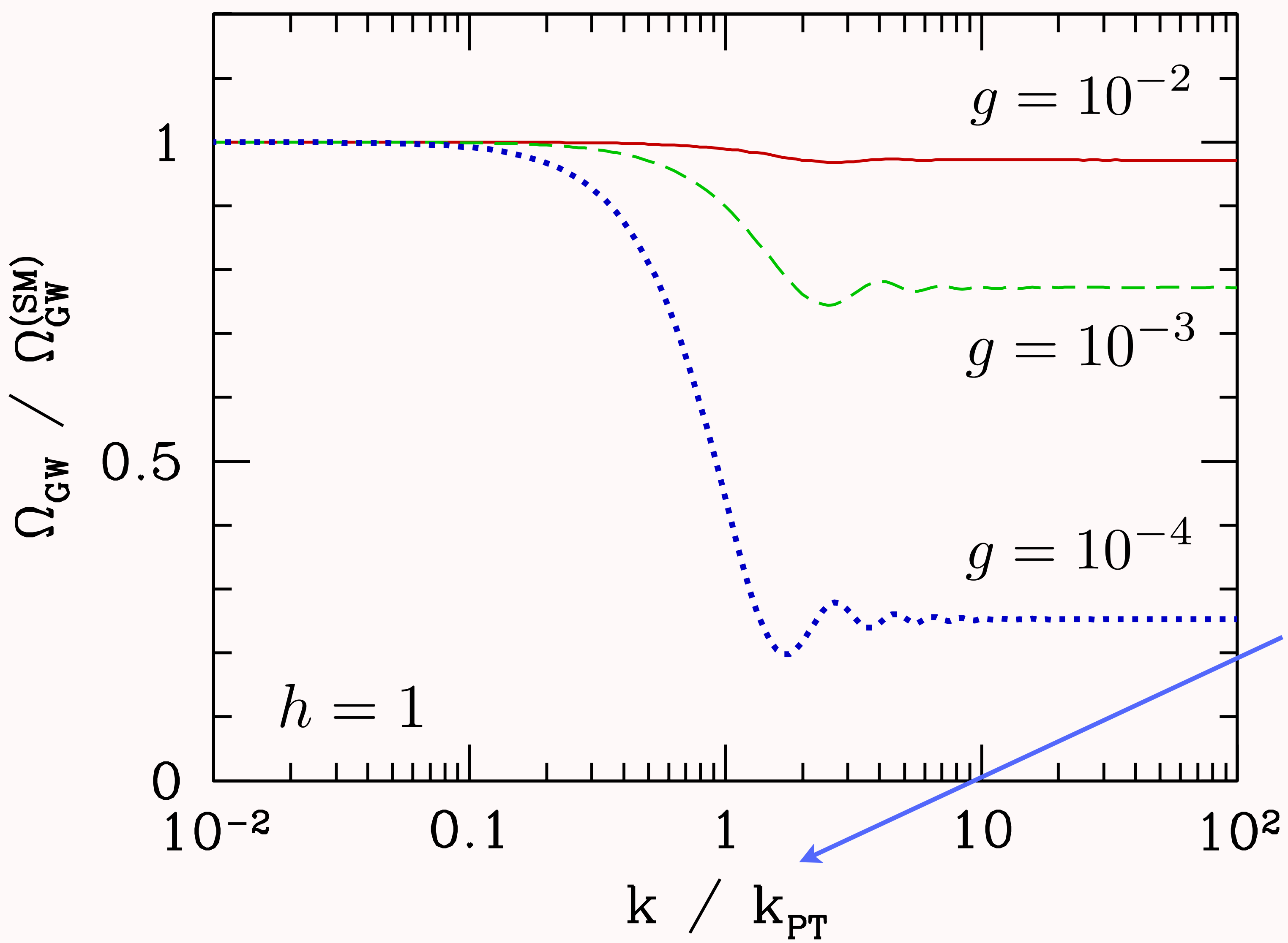




わずかな期間インフレーション
が起こる

• 相転移があったときの重力波スペクトル

$$V(\phi) = \frac{g}{24}(\phi^2 - v_\phi^2)^2 + \frac{1}{24}hT^2\phi^2$$



$$\frac{k_{\rm PT}}{2\pi} \simeq 2.7\,{\rm Hz}\left(\frac{T_{\rm PT}}{10^8\,{\rm GeV}}\right)$$

$$T_{\rm PT} \sim g^{1/4} v_\phi$$

相転移のエネルギースケールに対応

R.Jinno, T.Moroi, K.Nakayama (2012)

$k_{\rm PT} \sim 0.1 - 1\,{\rm Hz}$ ⟶ 将来DECIGOで観測可能

Dark radiation

R.Jinno, T.Moroi, KN, arXiv:1208.0184

Dark radiation

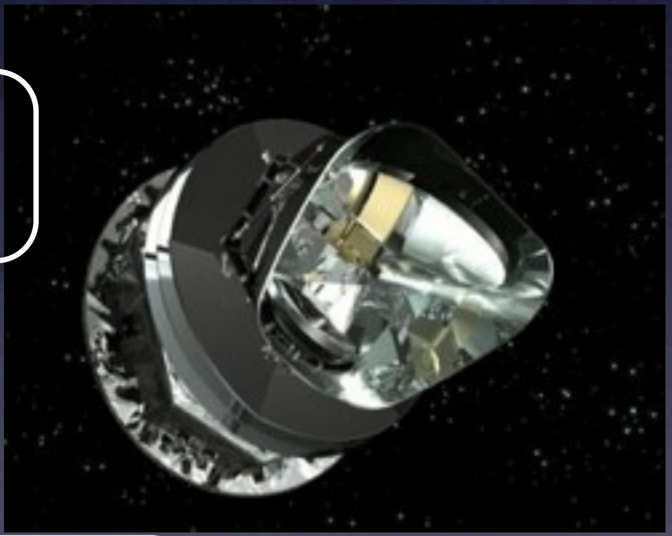
- Radiation energy density $$\rho_{\rm rad} = \left[1 + N_{\rm eff}\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma$$

$N_{\rm eff} = 3.04$ in the standard model

- CMB $$N_{\rm eff} = 3.36^{+0.68}_{-0.64} \,@\, 95\%{\rm CL}$$

Planck+WMAP pol+ high l

[Ade et al. 1303.5076]



- Helium abundance $$N_{\rm eff} = 3.51 \pm 0.35 \;@\; 68\%{\rm C.L.}$$

[Izotov, Stasinska, Guseva, 1308.2100]

Axion DR from string theory

- Kahler moduli in type IIB string : T
  Shift symmetry : $T \to T + i\alpha$
- Kahler potential : $K = K(T + T^\dagger)$

$$T - \langle T \rangle \equiv \frac{\tau + ia}{\sqrt{2K_{TT}}}$$

axion

- Moduli decay : $\tau \to 2a$

$\longrightarrow$ ***Axionic dark radiation !***

[Cicoli, Conlon, Quevedo (2012), Higaki, Takahashi (2012), Higaki, KN, Takahashi (2013)]

Dark radiation and GW

- Dark radiation affects GW spectrum in two ways

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + (k/a)^2 h_{ij} = 16\pi G \Pi_{ij}$$

Modified expansion rate

Anisotropic stress of X

cf) For standard neutinos, see

S.Weinberg (2003), Y.Watanabe, E.Komatsu (2005)

- Modified expansion rate by parent field of X

  → Modification on GW spectrum at high frequency

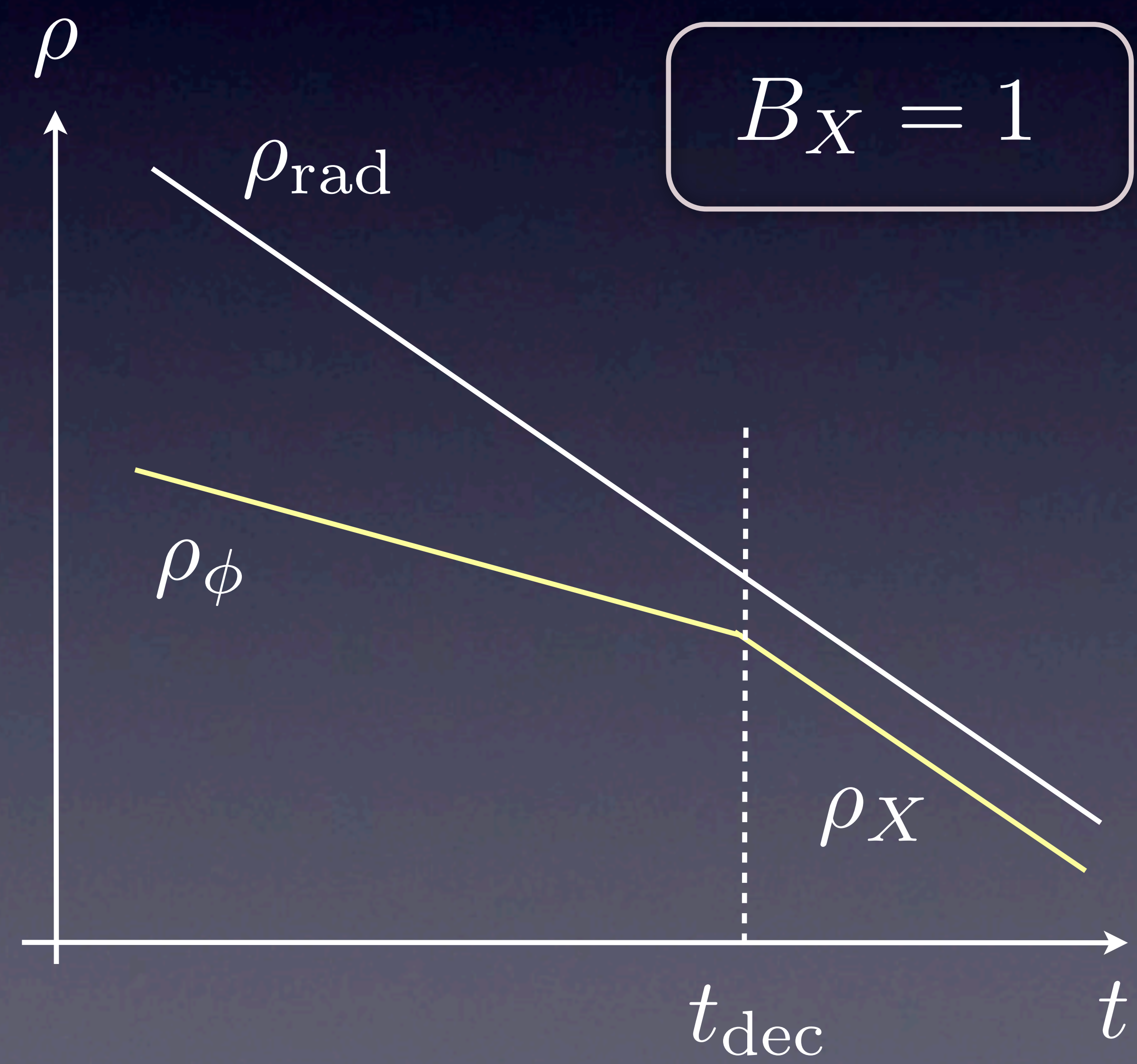
- Anisotropic stress is turned on after X production

  → Modification on GW spectrum at low frequency

A model

- A scalar field ϕ decays into X at $H \sim \Gamma_\phi$ with branching ratio B_X
- Background evolution :

$$\dot{\rho}_\phi + 3H\rho_\phi = -\Gamma_\phi \rho_\phi$$
$$\dot{\rho}_{\rm rad} + 4H\rho_{\rm rad} = \Gamma_\phi (1 - B_X)\rho_\phi$$
$$\dot{\rho}_X + 4H\rho_X = \Gamma_\phi B_X \rho_\phi$$



- ϕ nearly dominate at decay for $\Delta N_{\rm eff} \simeq 1$

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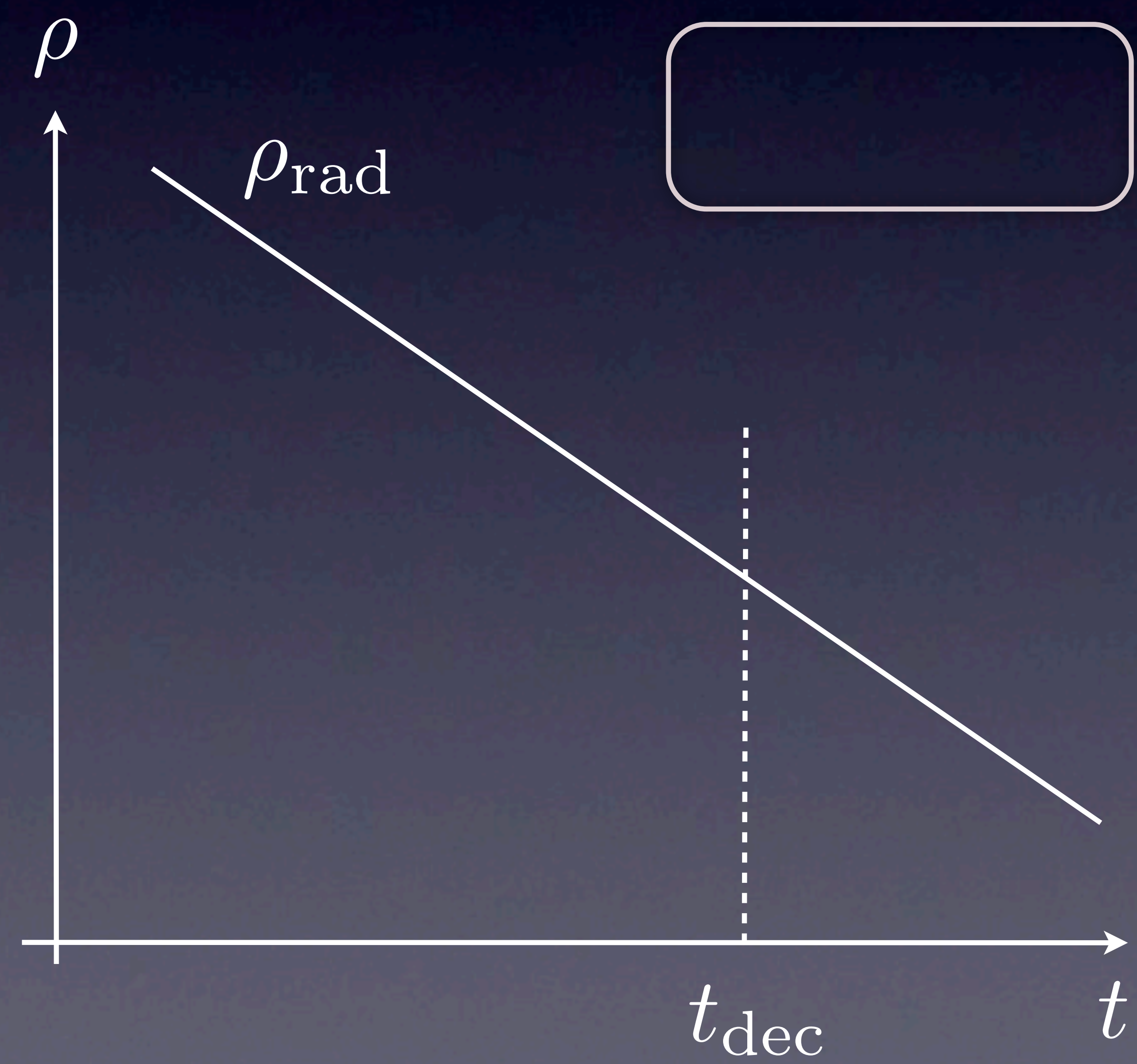

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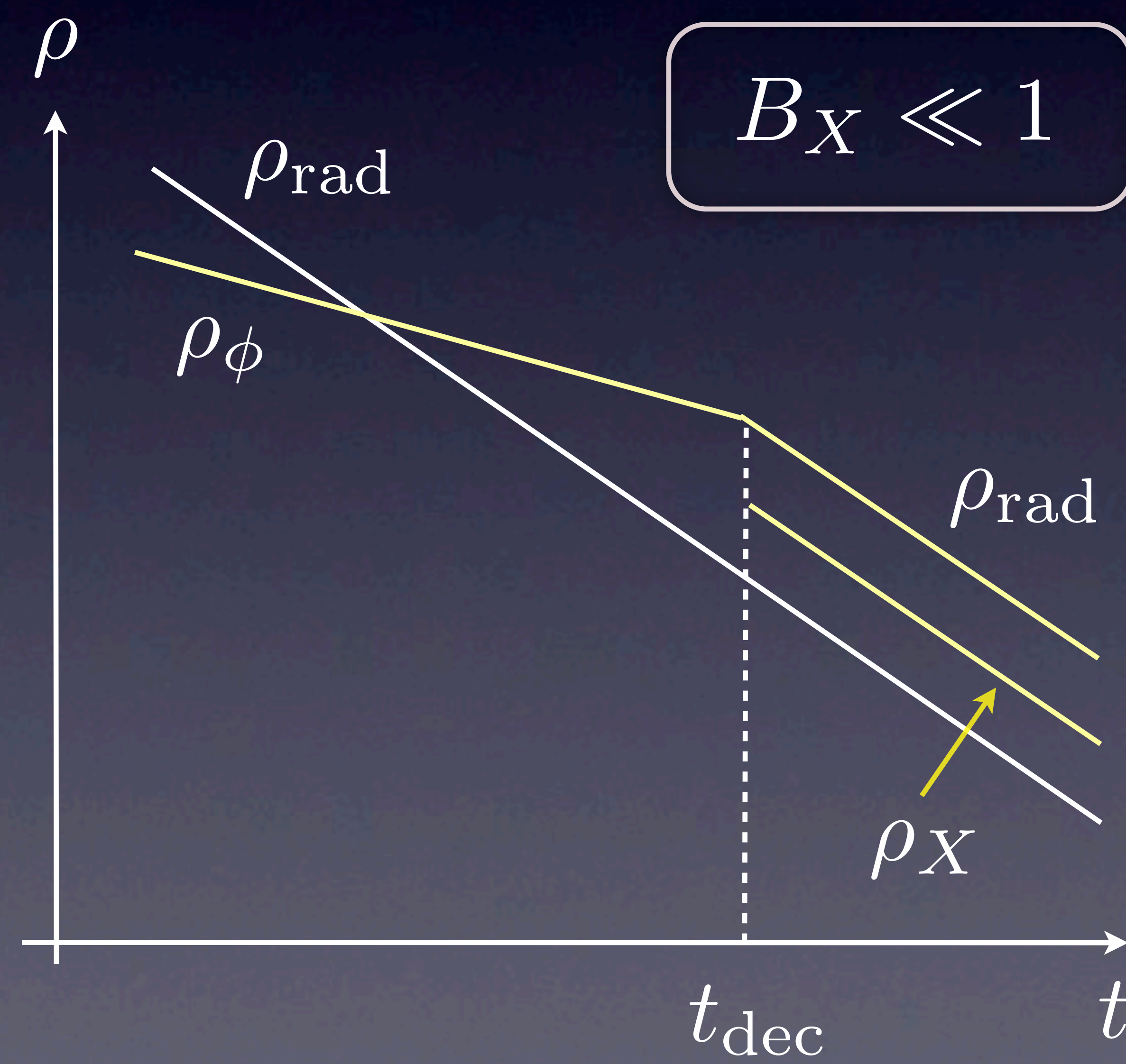
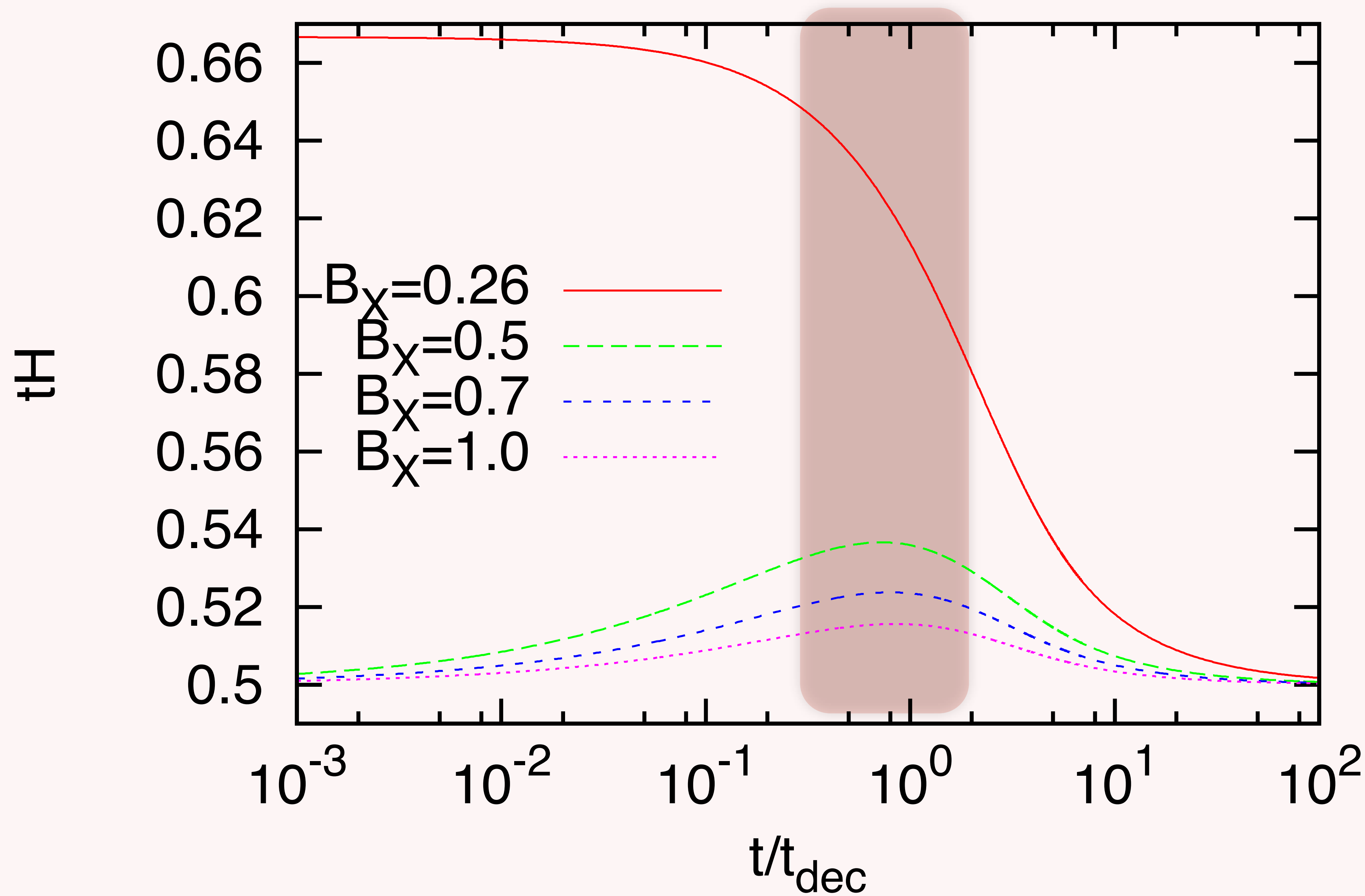


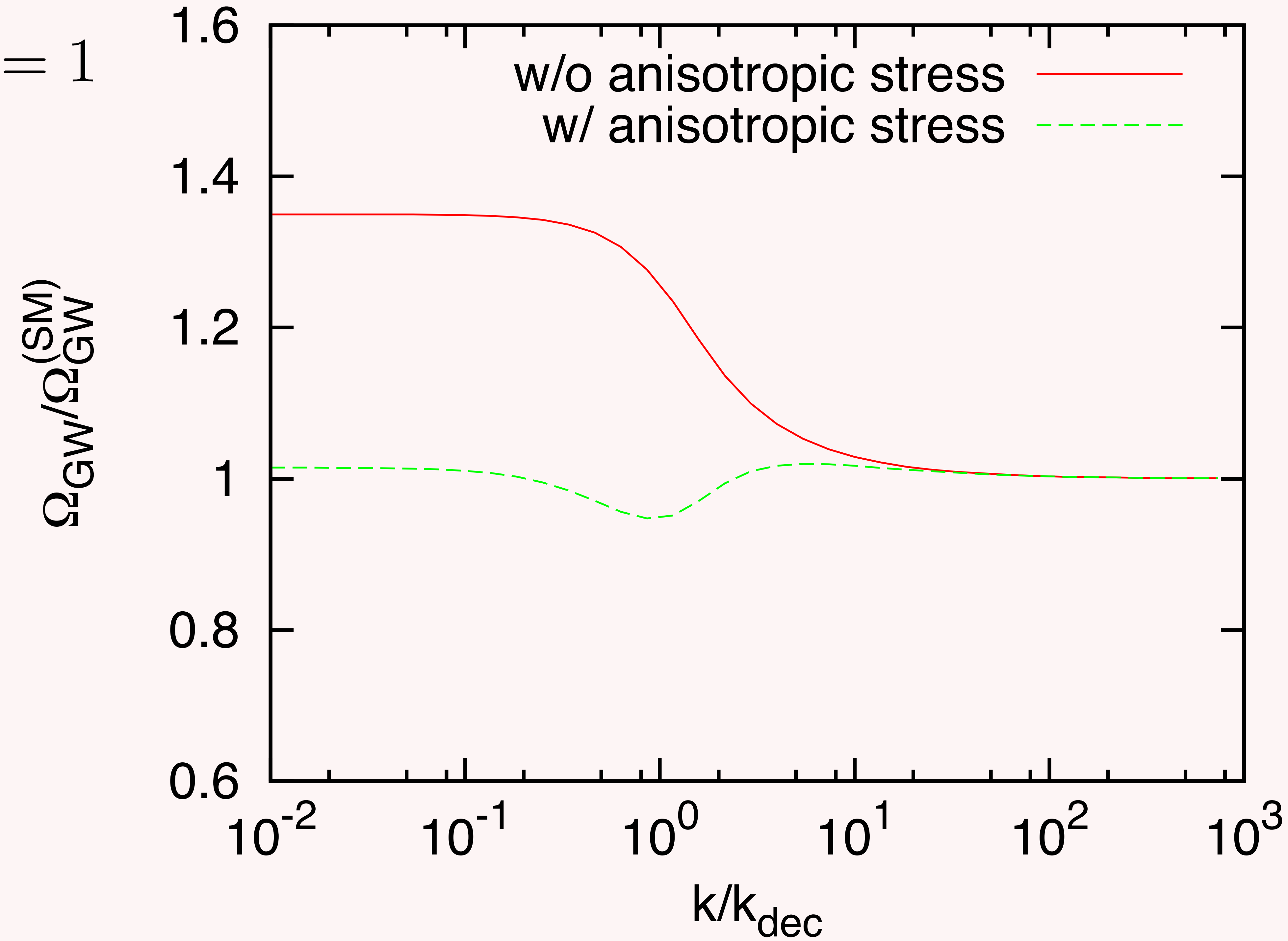
A model



Deviation from R.D., tH=0.5, around Φdecay

Numerical result

$B_X = 1$



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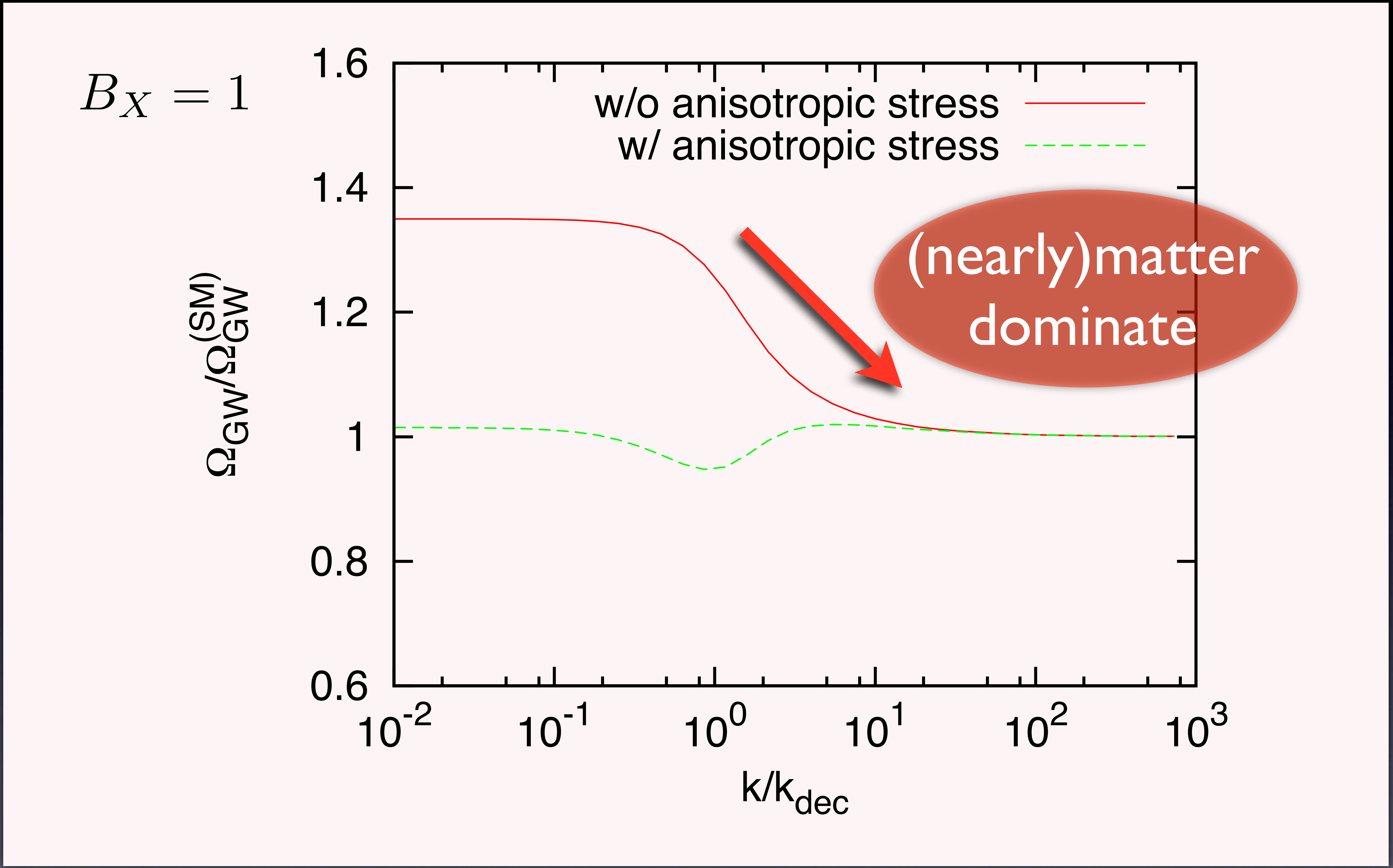

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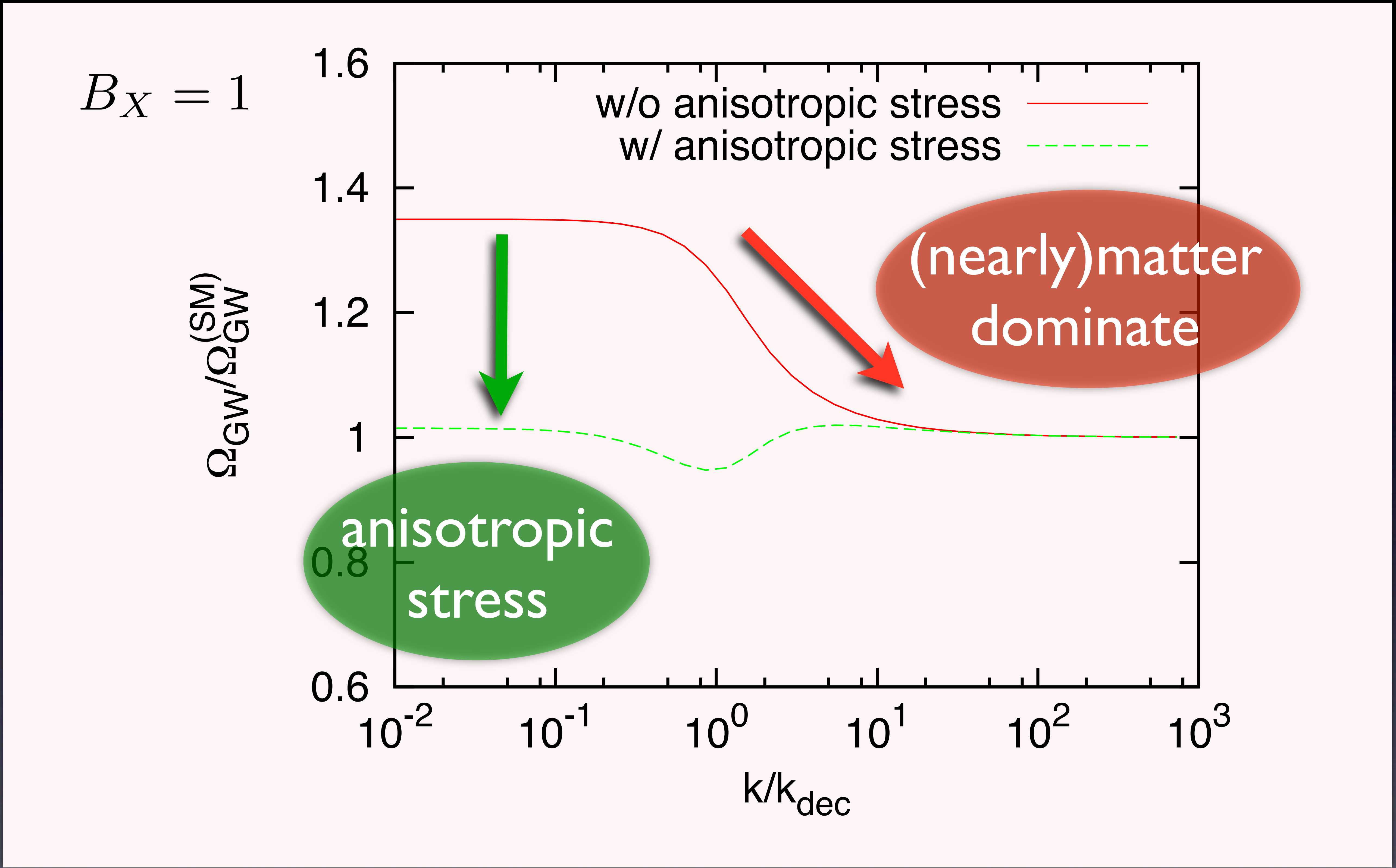

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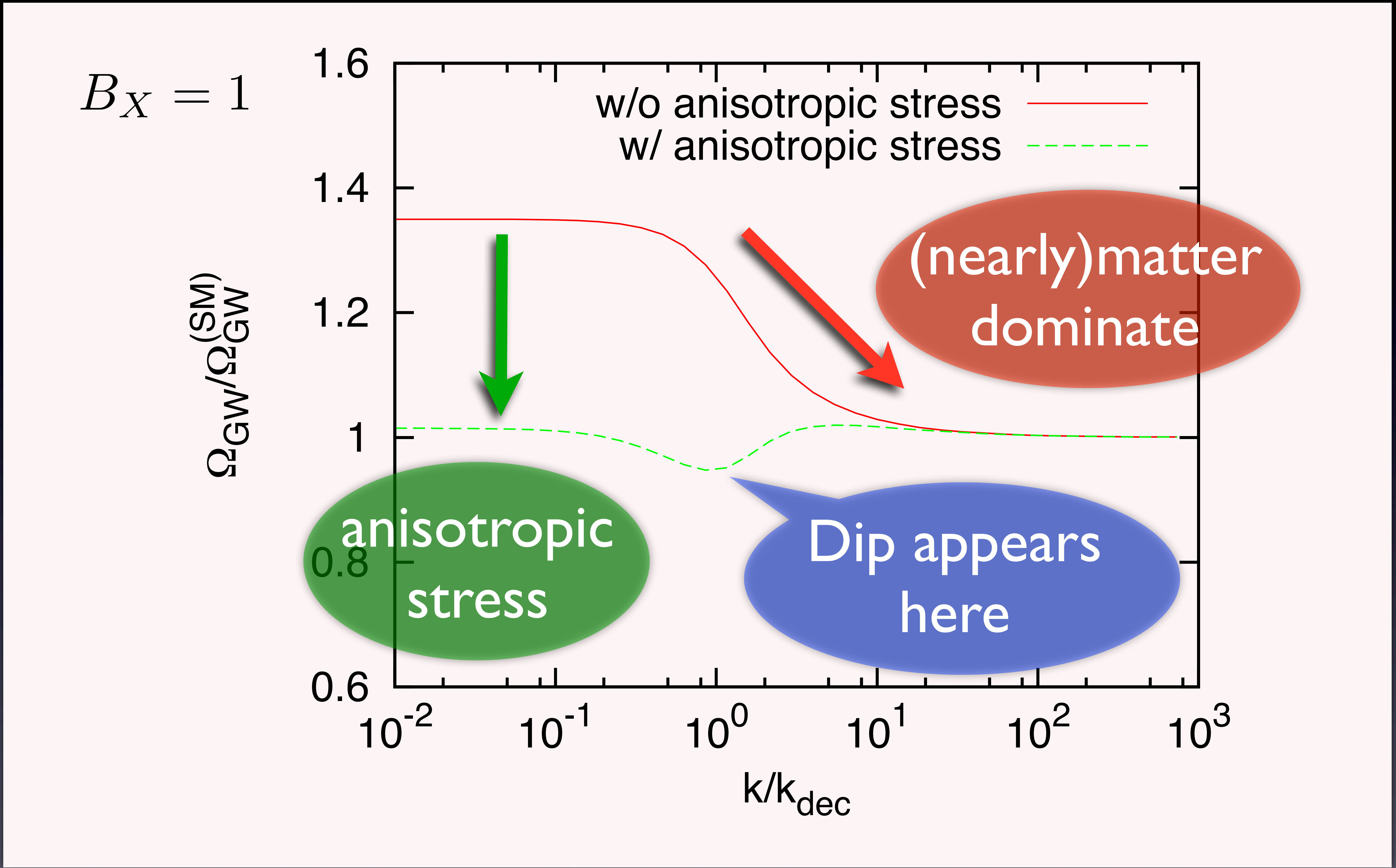

13年10月27日日曜日

Numerical result

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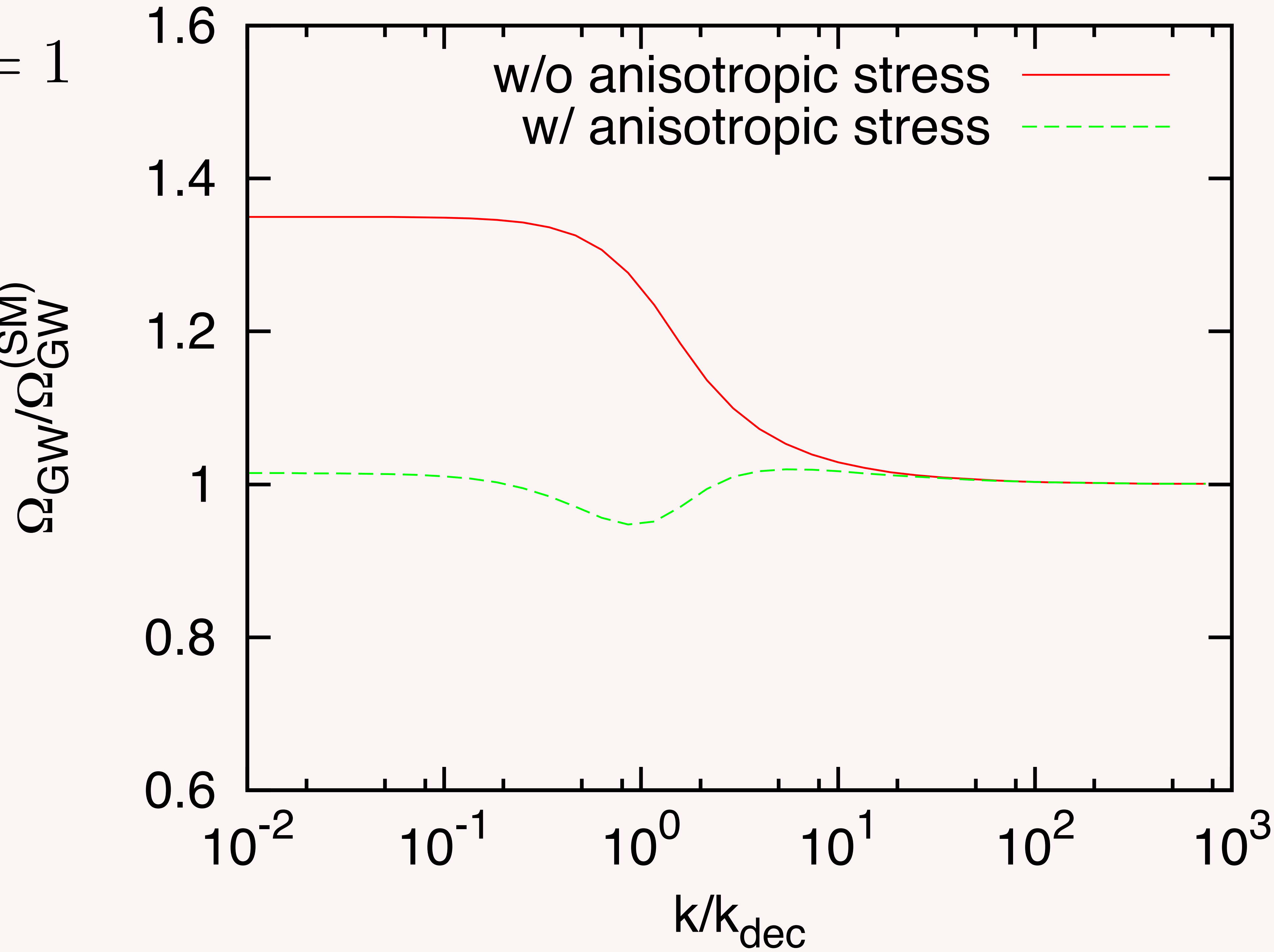

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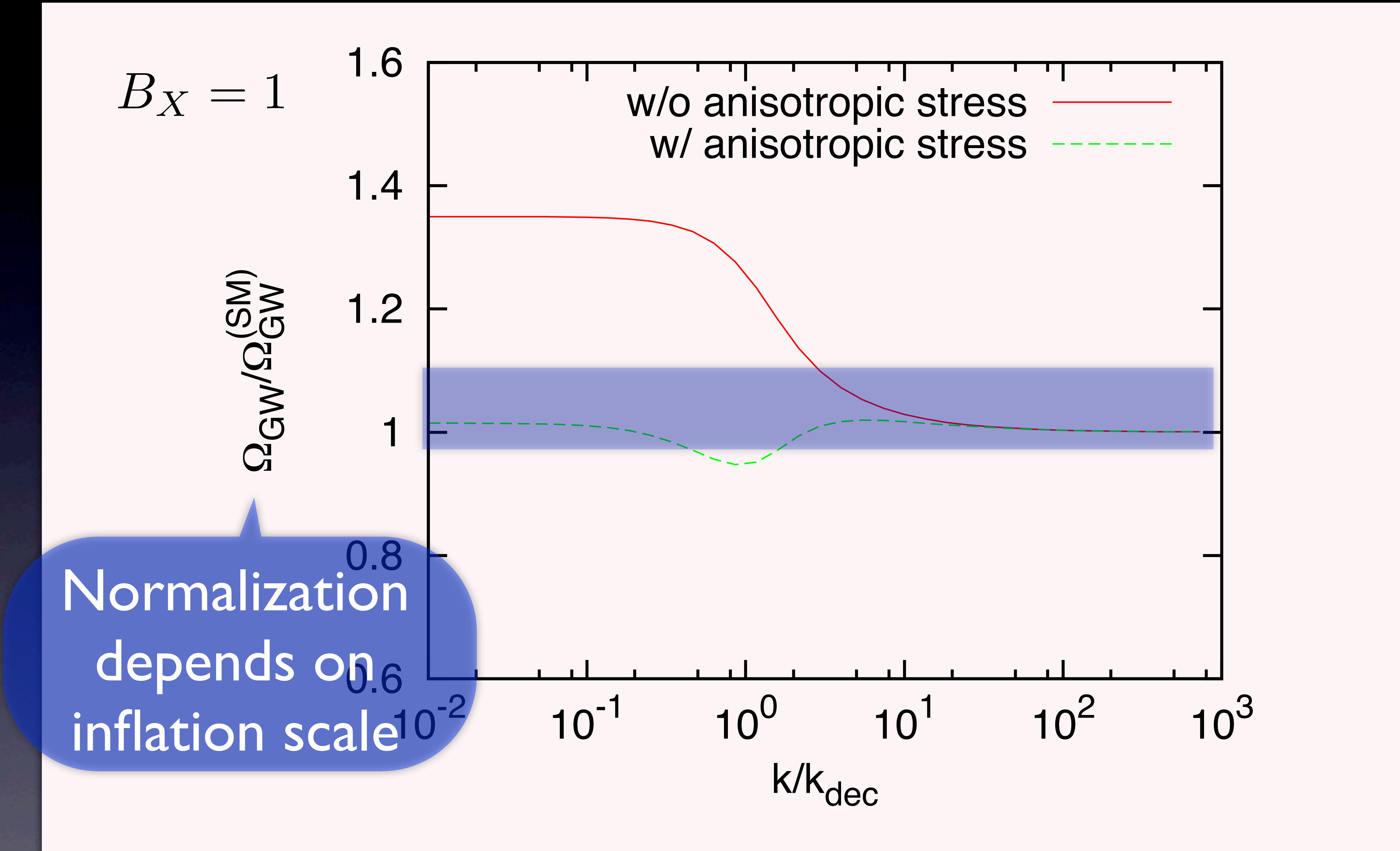

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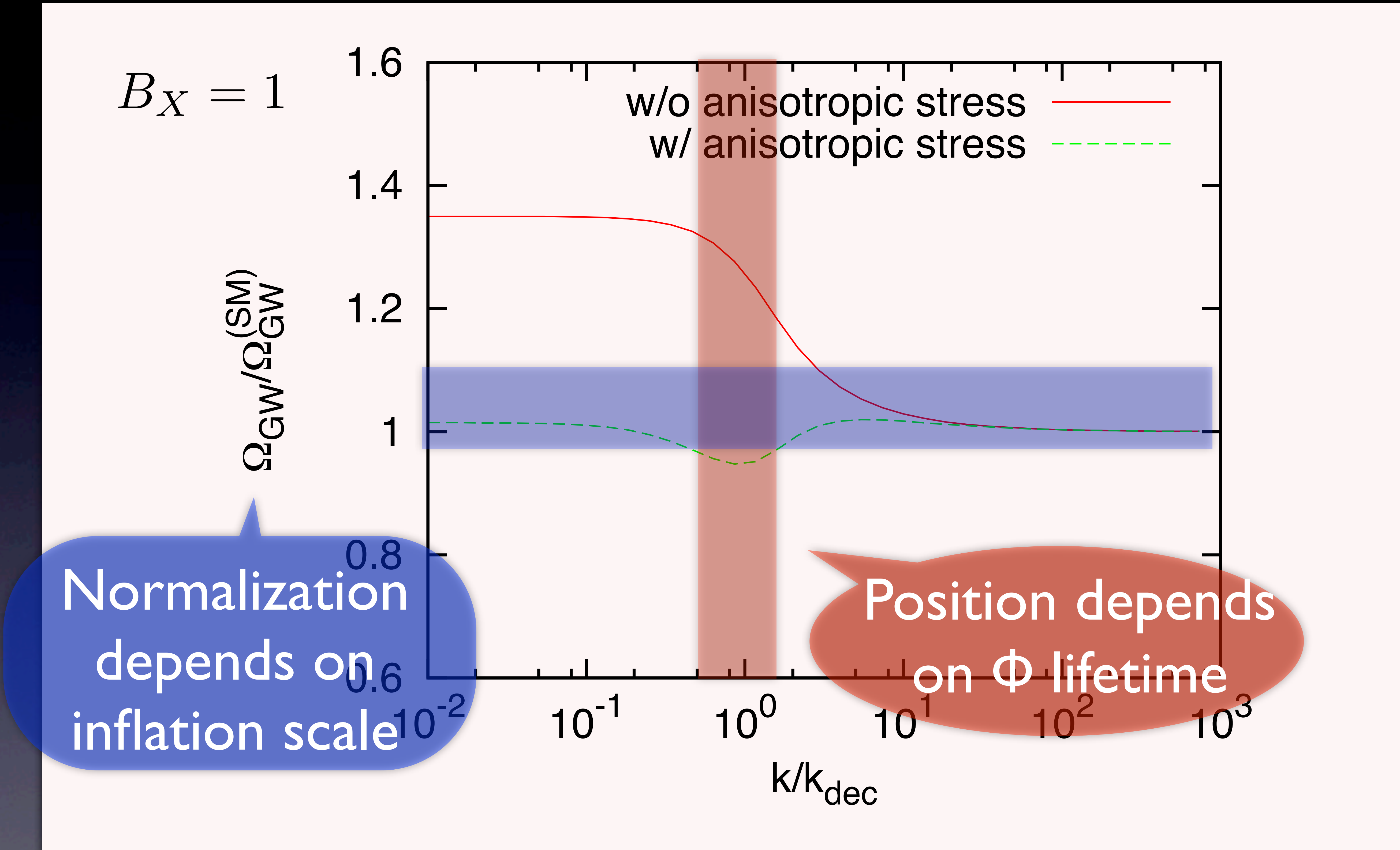

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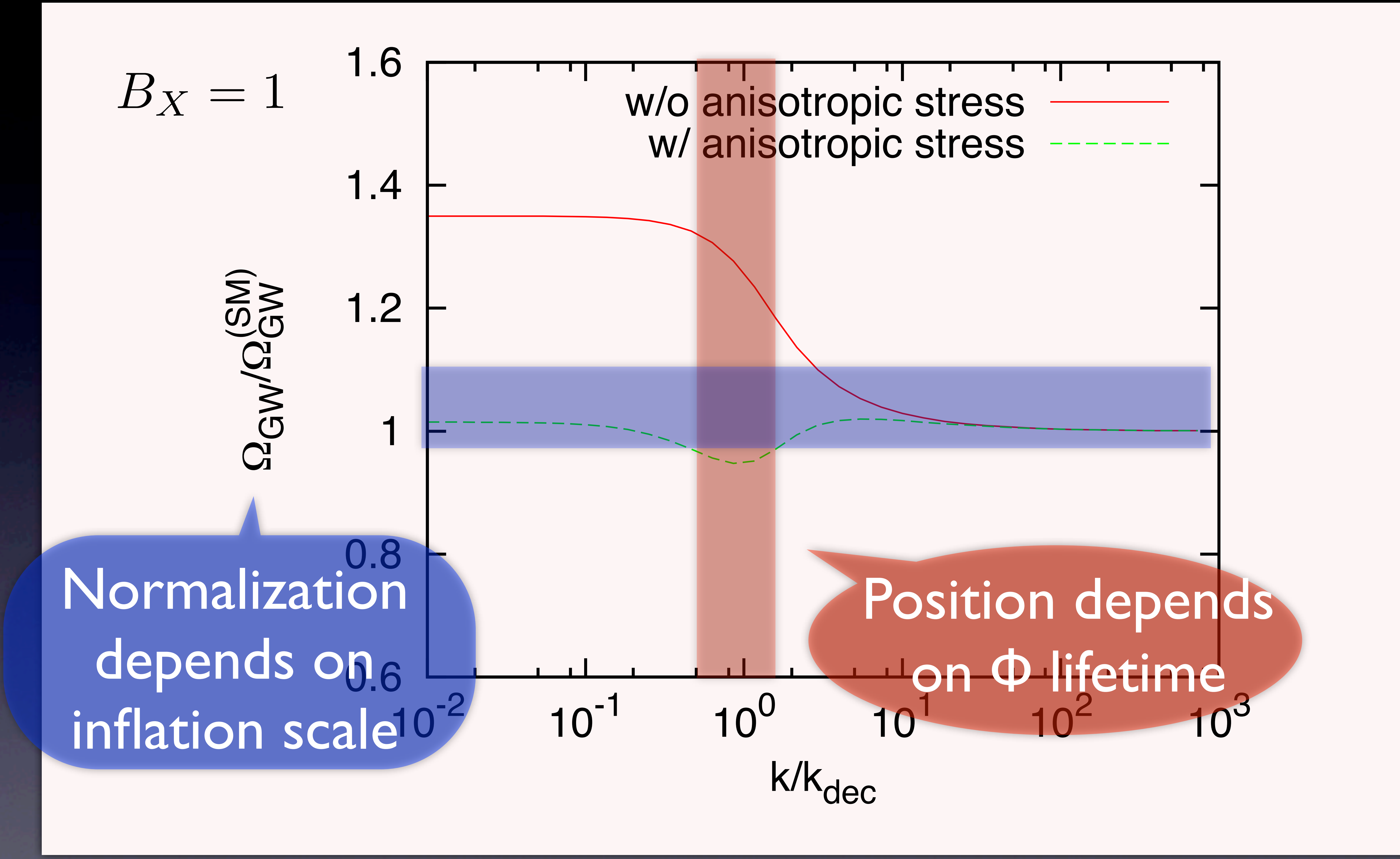
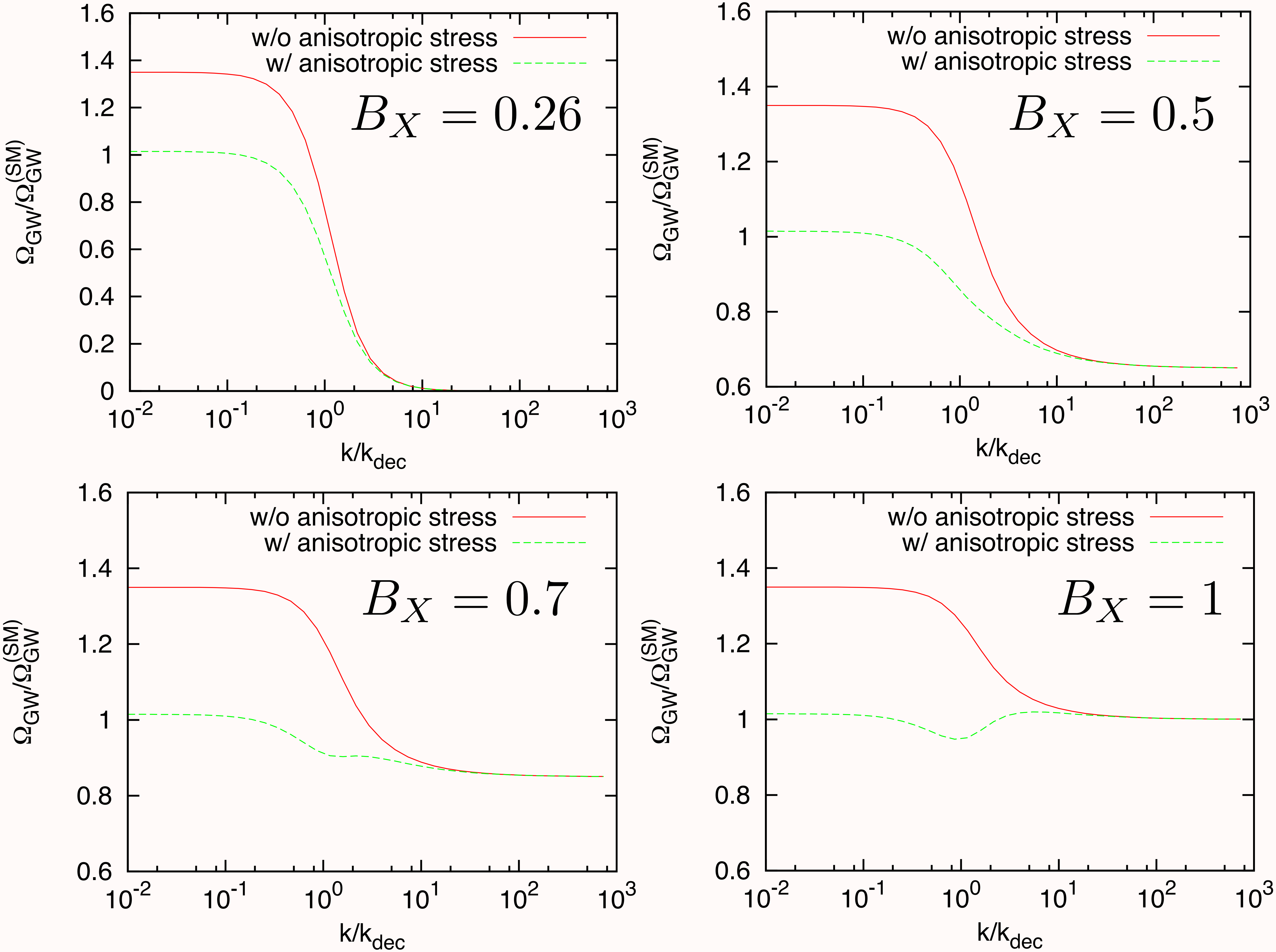


Detectable at DECIGO for $r \gtrsim 10^{-3}$ and $T_\phi \sim 10^7\,\mathrm{GeV}$



w/o anisotropic stress
w/ anisotropic stress
$B_X = 0.26$
$B_X = 0.5$
$B_X = 0.7$
$B_X = 1$
$\Omega_{GW}/\Omega_{GW}^{(SM)}$
k/k_{dec}
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

具体的模型

R.Jinno, T.Moroi, KN, arXiv:1307.3010

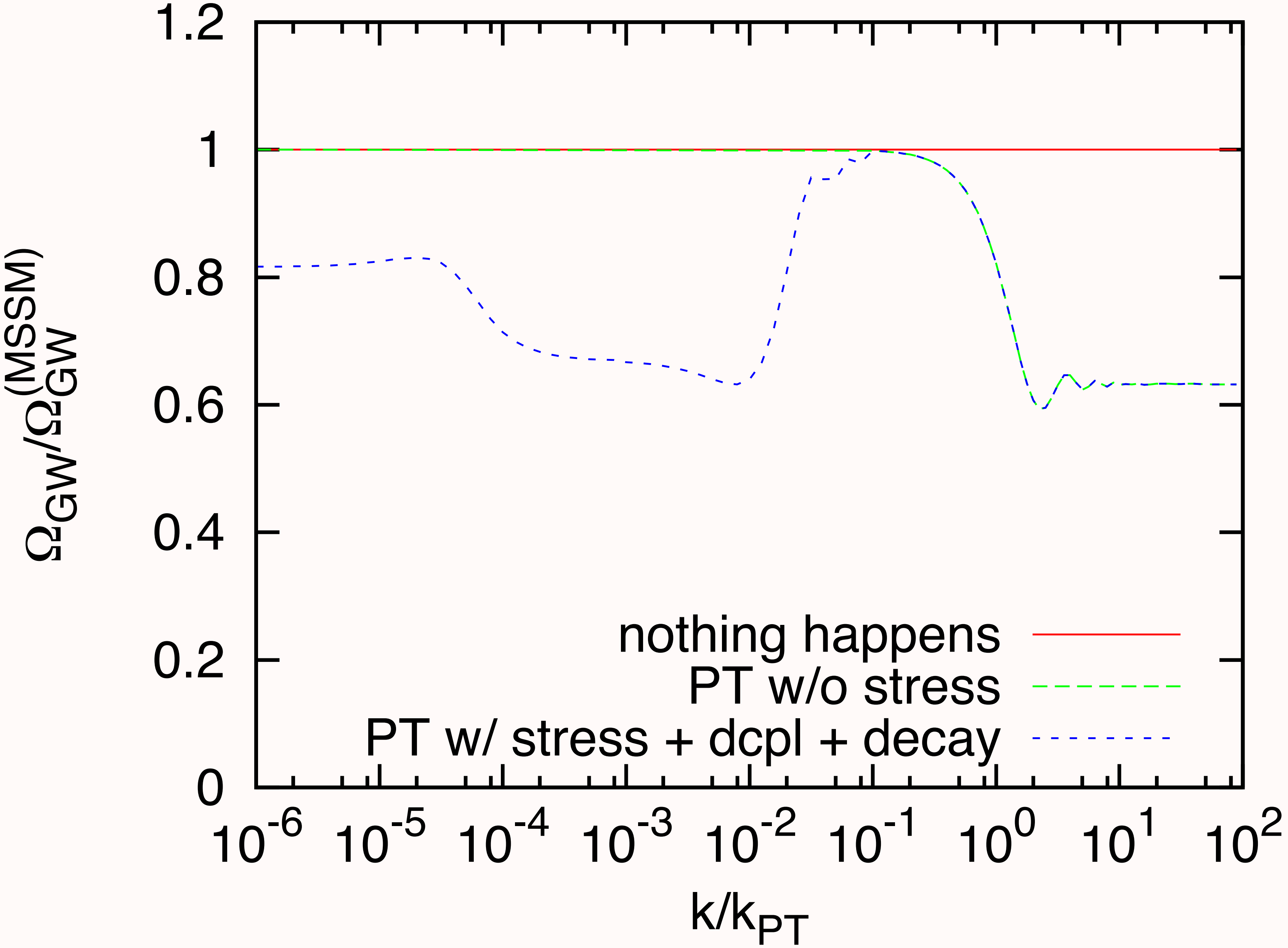
- 超対称アクシオン模型

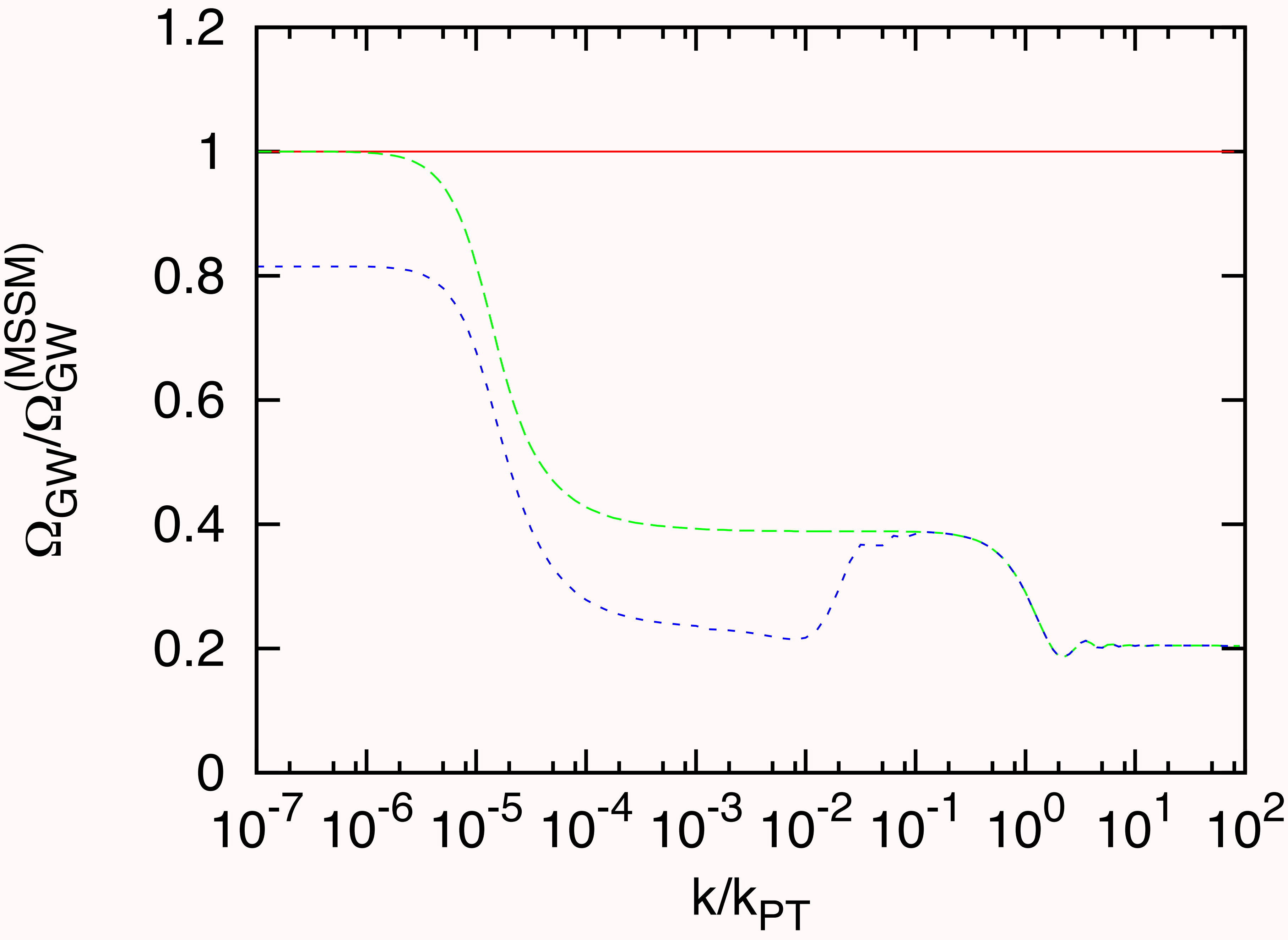
$$W = \lambda S(\Phi\bar{\Phi} - f^2) + y_1 \Phi Q \bar{Q}$$

  - 高温：$\Phi = 0$　　低温：$\Phi = f$ $\longrightarrow$ 相転移
  - 相転移後、Φ 振動期 $\longrightarrow$ 再加熱
  - Φ はアクシオンに崩壊 $\longrightarrow$ dark radiation

- 超対称マヨロン模型

$$W = \lambda S(\Phi\bar{\Phi} - f^2) + y_i \Phi N_i N_i$$

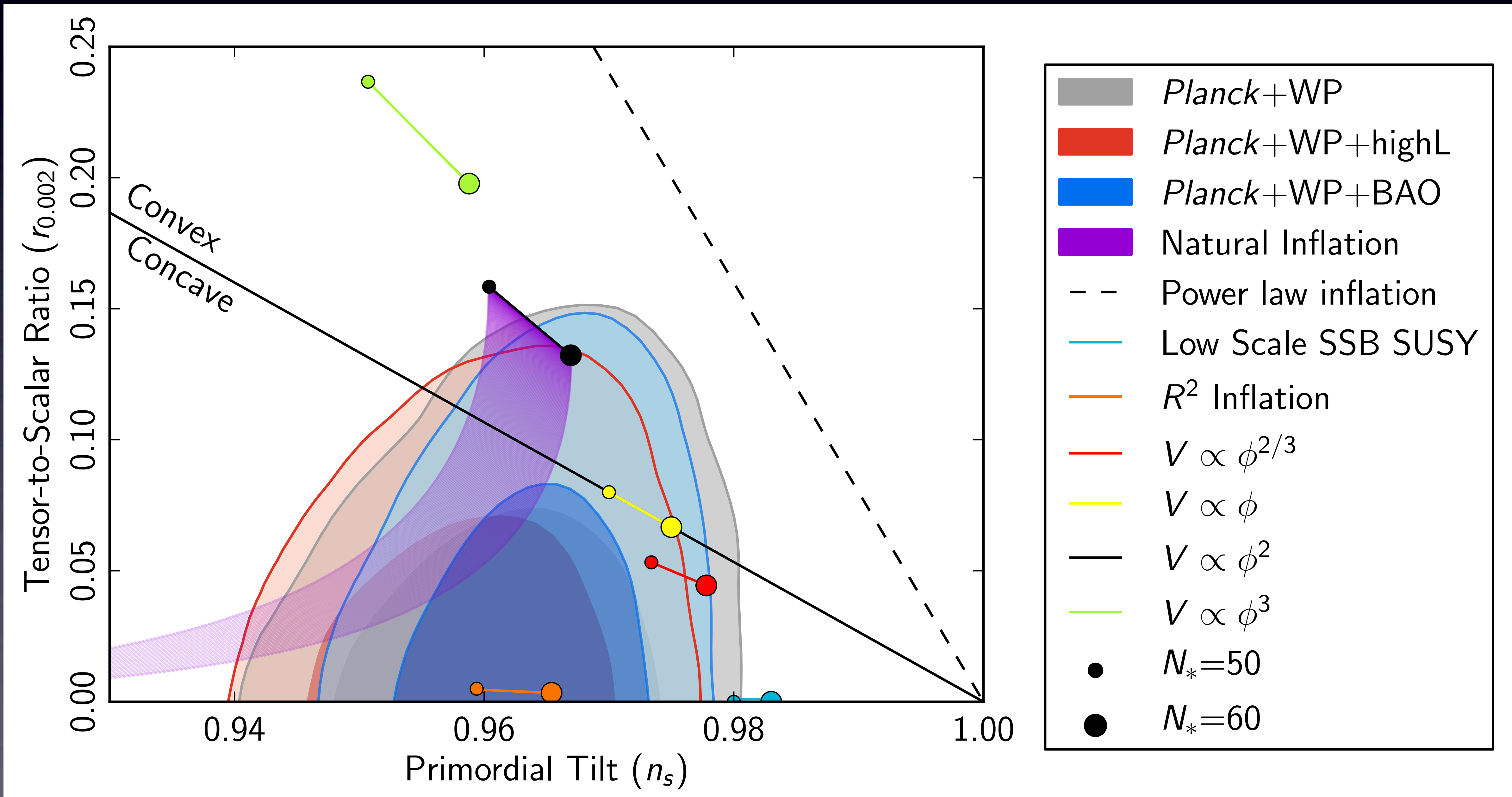

1.2
1
0.8
0.6
0.4
0.2
0
$\Omega_{GW}/\Omega_{GW}^{(MSSM)}$
nothing happens
PT w/o stress
PT w/ stress + dcpl + decay
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2}
k/k_{PT}



具体的なセットアップでは豊かな構造が現れる

Large B-mode ?

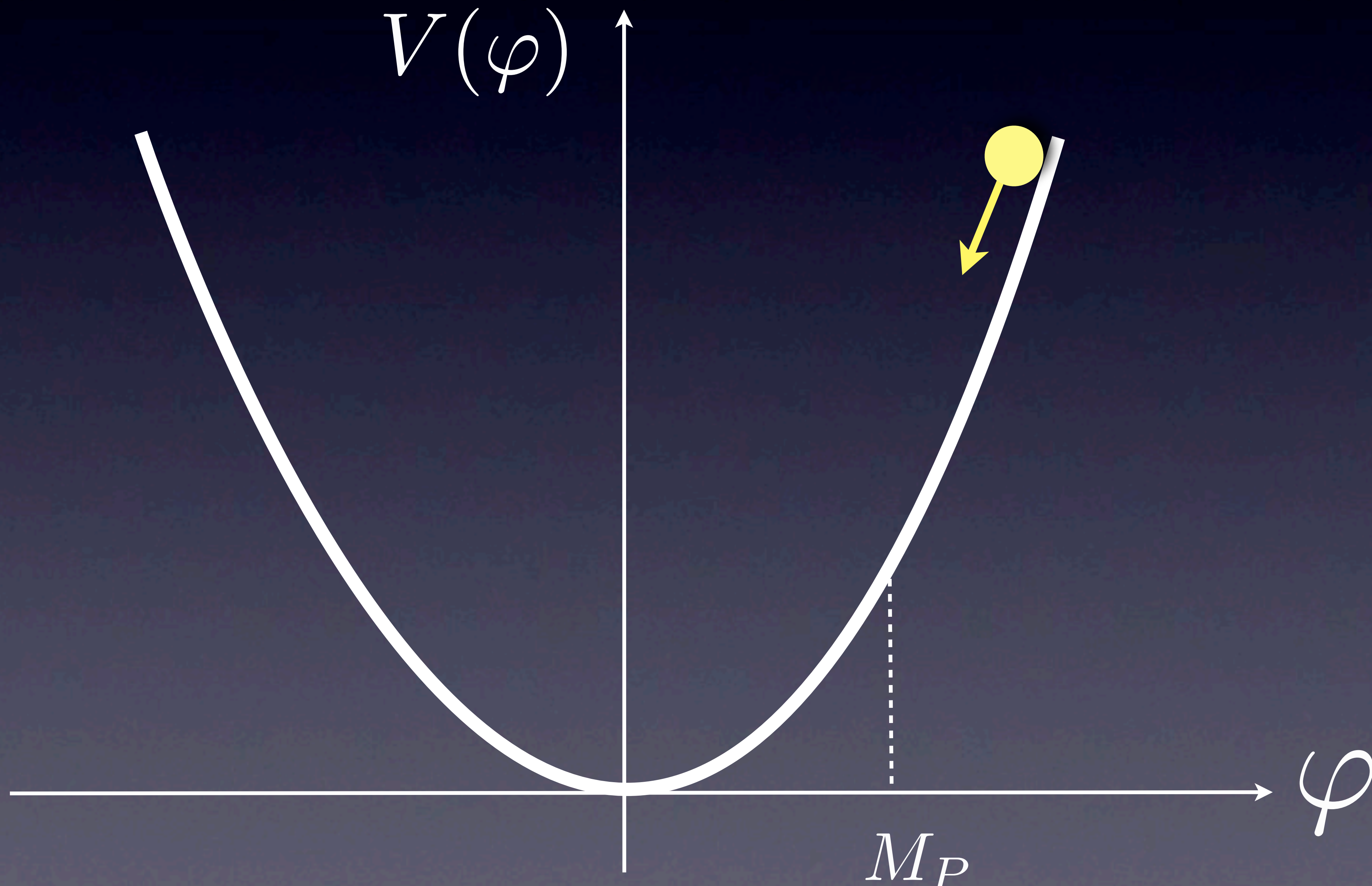
Large field model (chaotic inflation) は観測と合わない？



Planck, 1303.5082

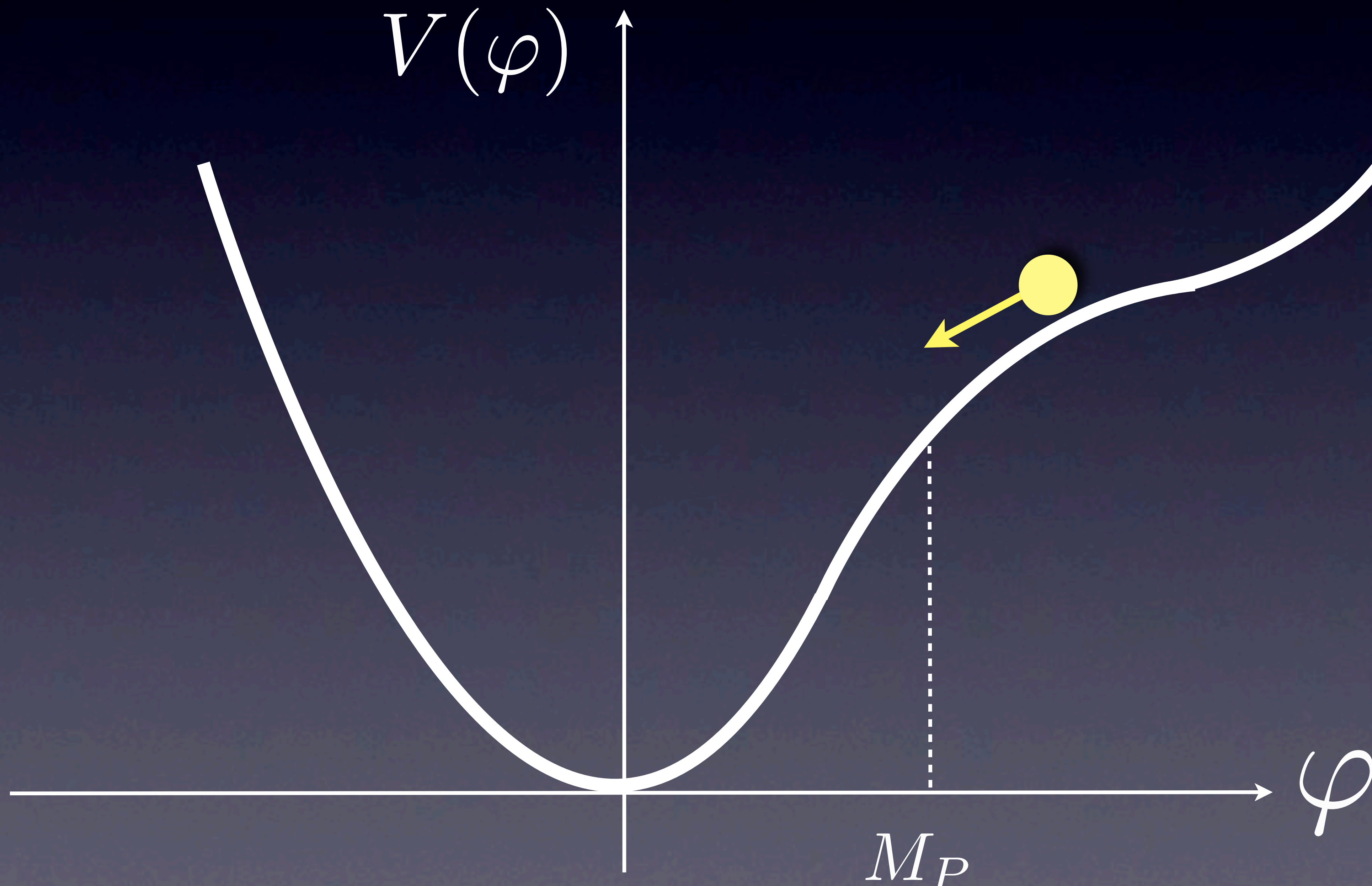
Chaotic inflation

$V(\varphi) \propto \varphi^n$



Polynomial chaotic inflation

$$V(\varphi) \propto \varphi^n(1 - c\varphi + \varphi^2)$$



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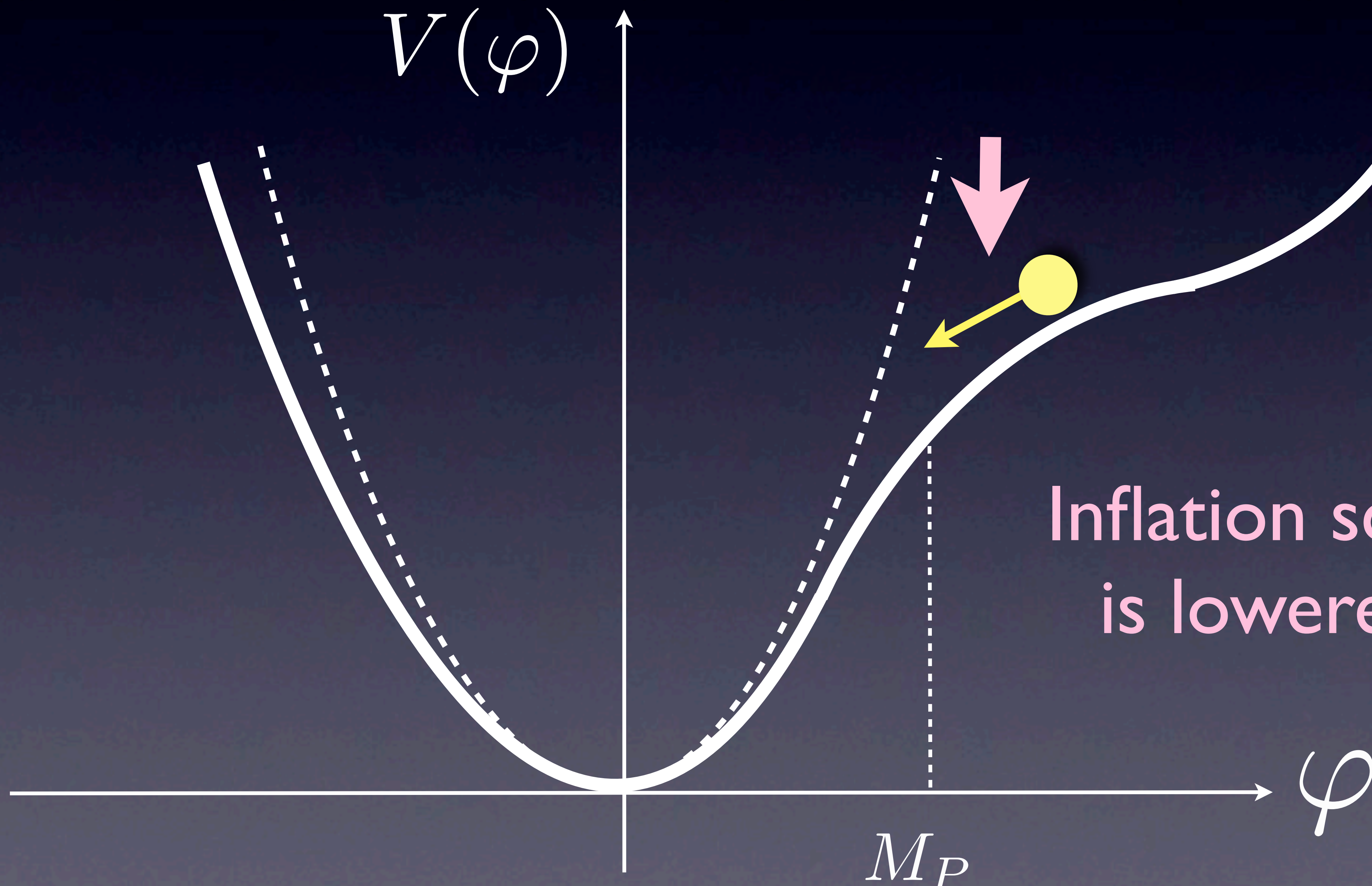
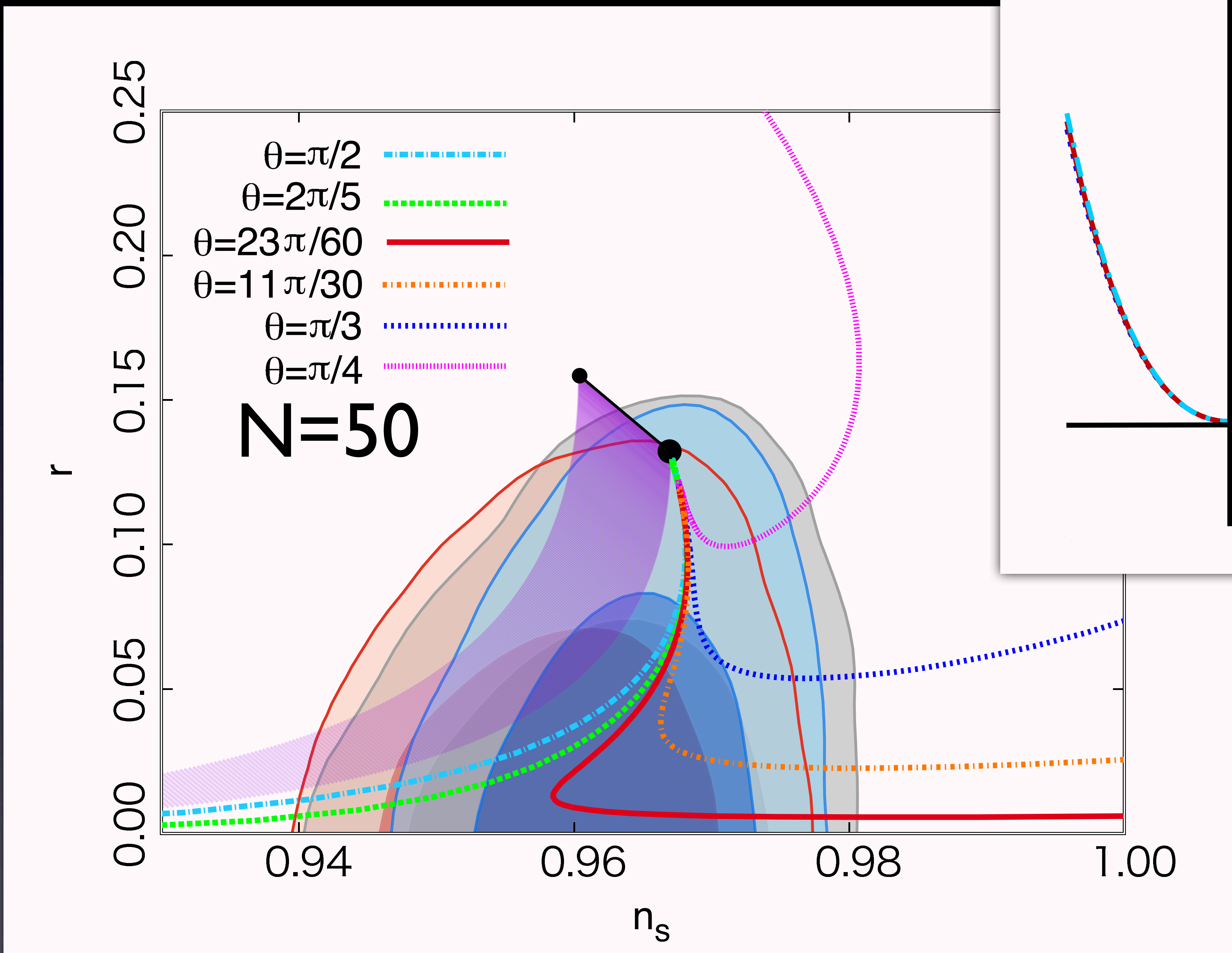
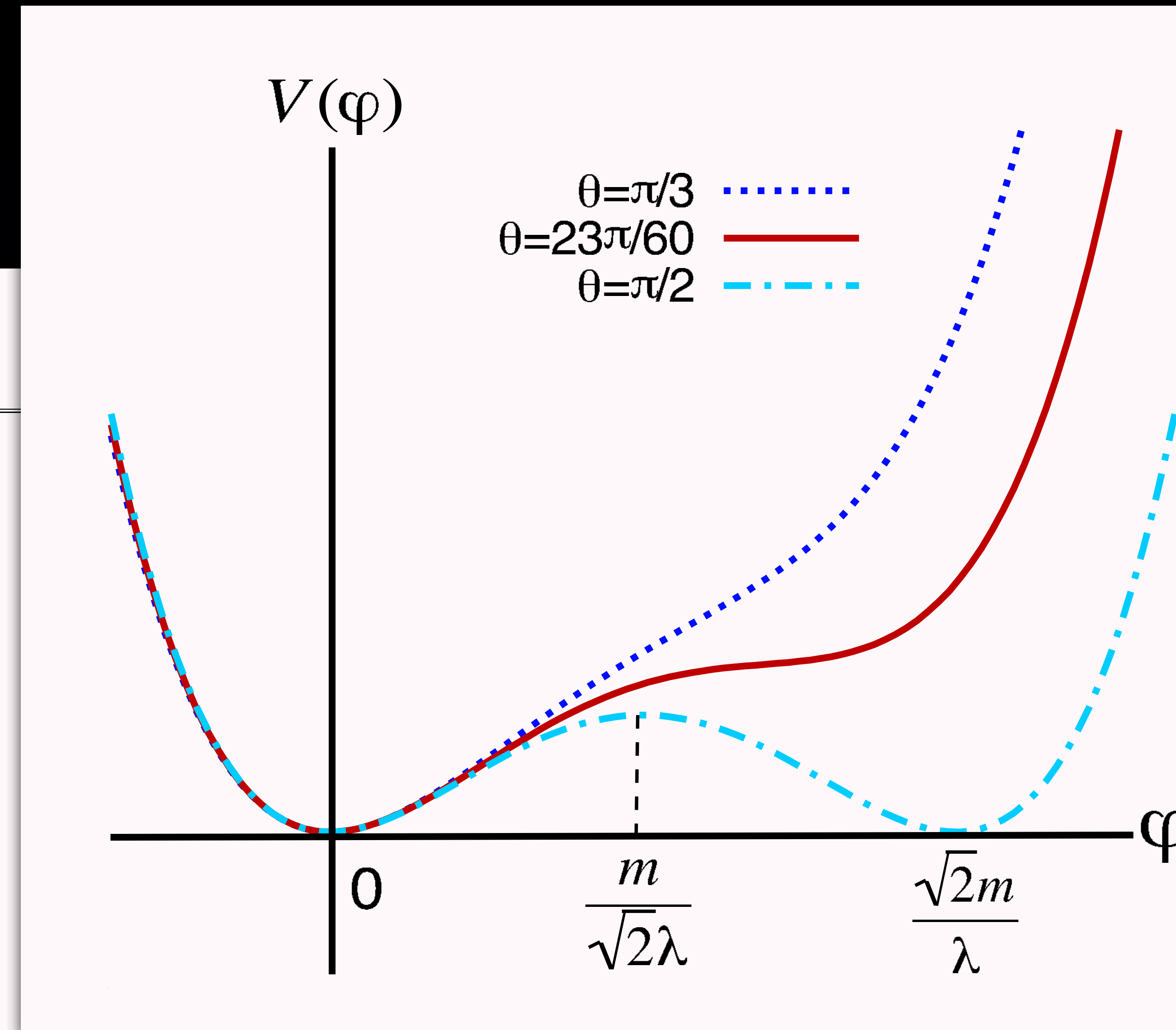


Polynomial chaotic inflation

$$V = \frac{1}{2}m^2\varphi^2\left(1 - \frac{\sqrt{2}\sin\theta\lambda\varphi}{m} + \frac{\lambda^2\varphi^2}{2m^2}\right)$$





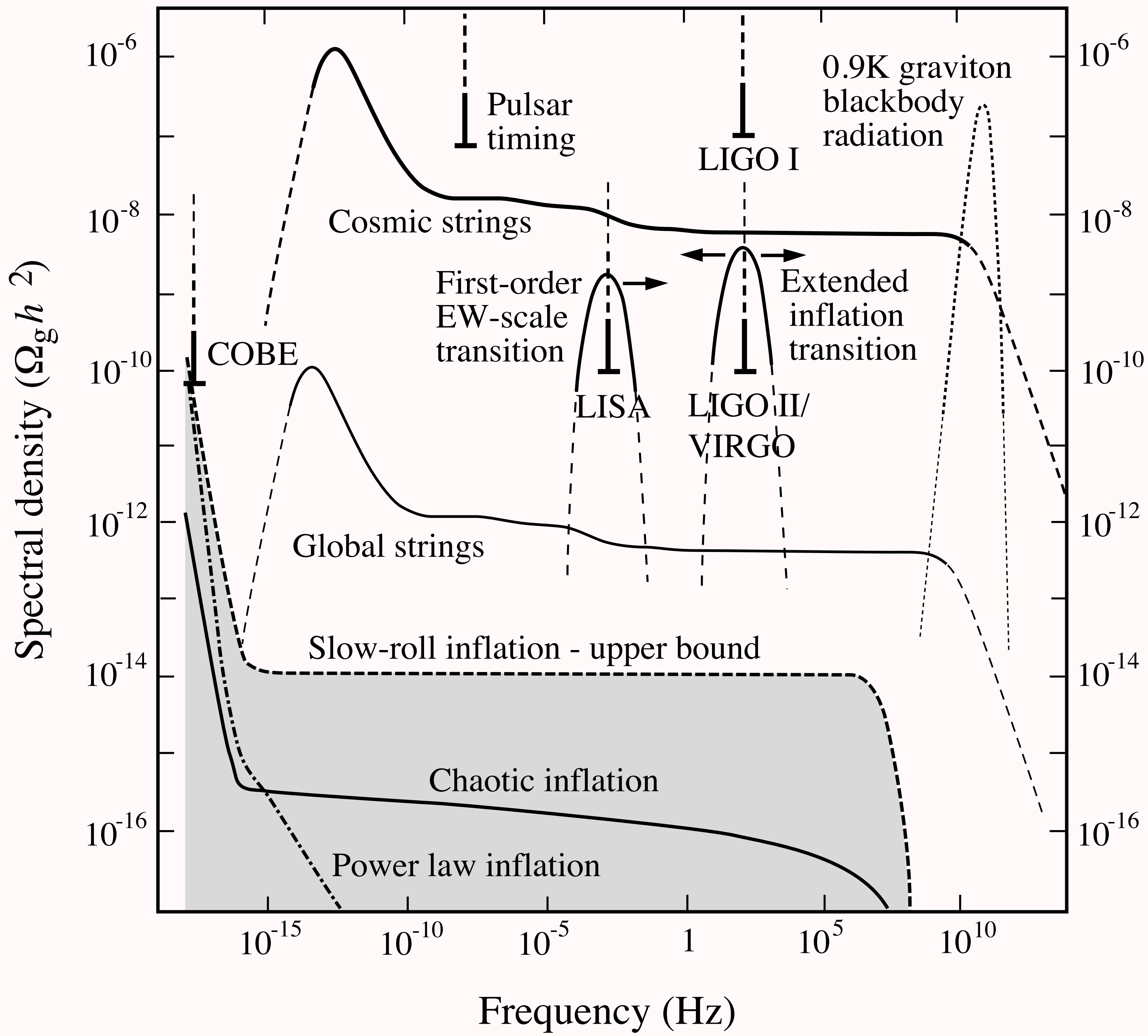
See also :
Destri, de Vega, Sanchez (2007)

KN, F.Takahashi, T.T.Yanagida, 1303.7315

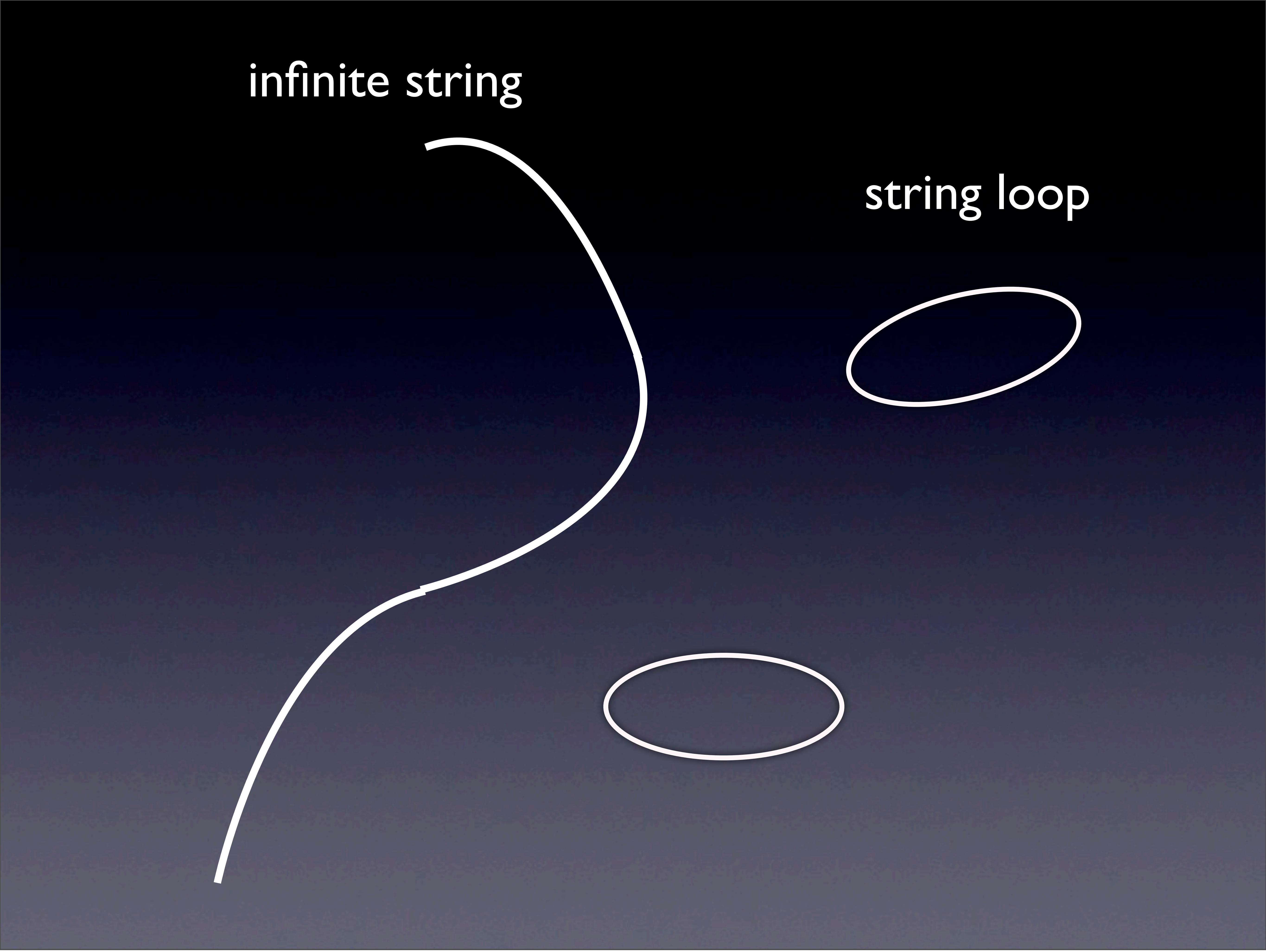
Comments

- DECIGOで検出可能な重力波を予言するインフレーション模型はある
    - Polynomial chaotic inflation
    - Higgs inflation, R^2 inflation, ...
- Hybrid inflation の場合には、cosmic string からの重力波が期待できる

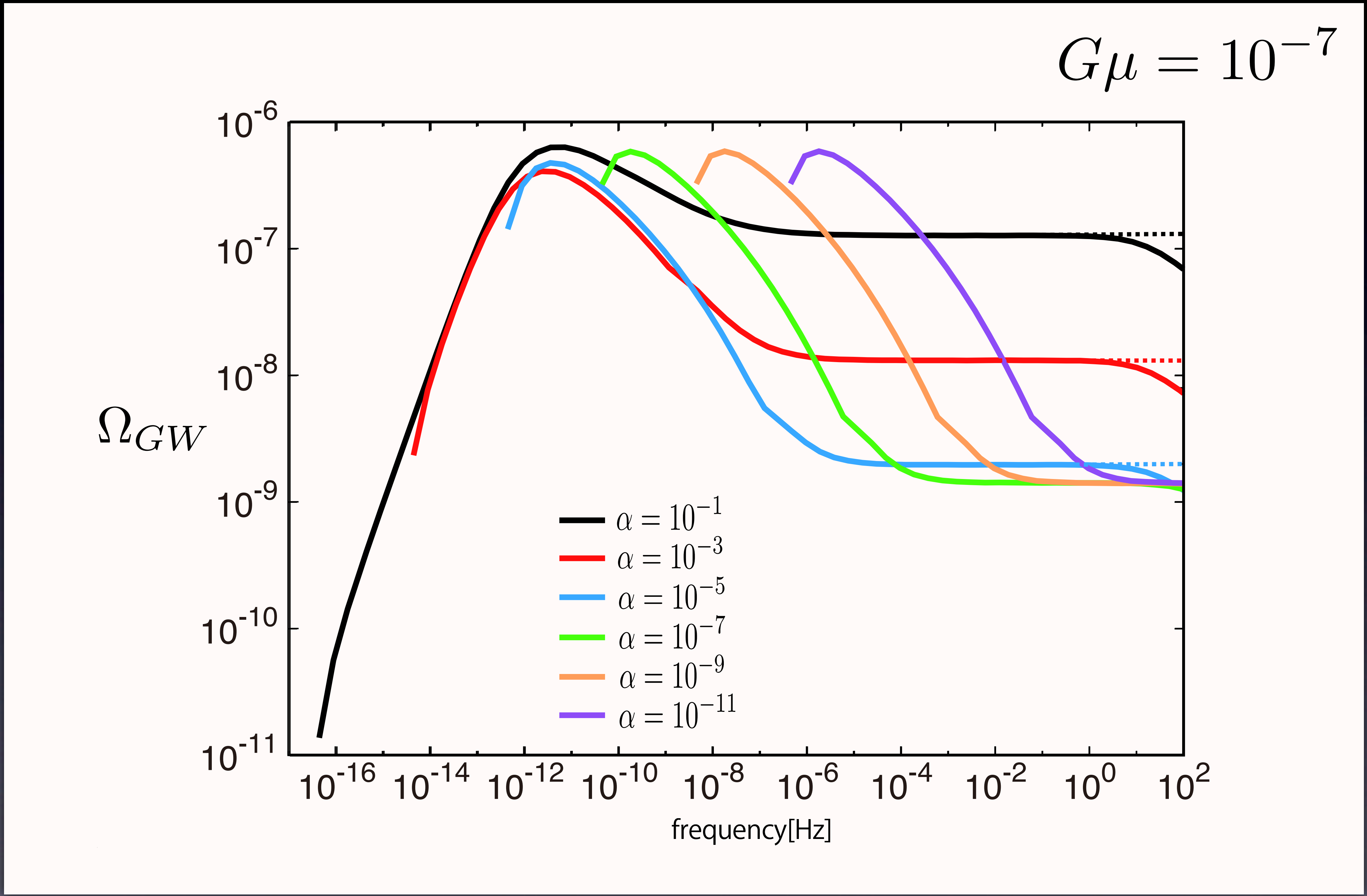
Cosmic string loop からの重力波



Battye, Caldwell, Shellard (1997)


infinite string
string loop

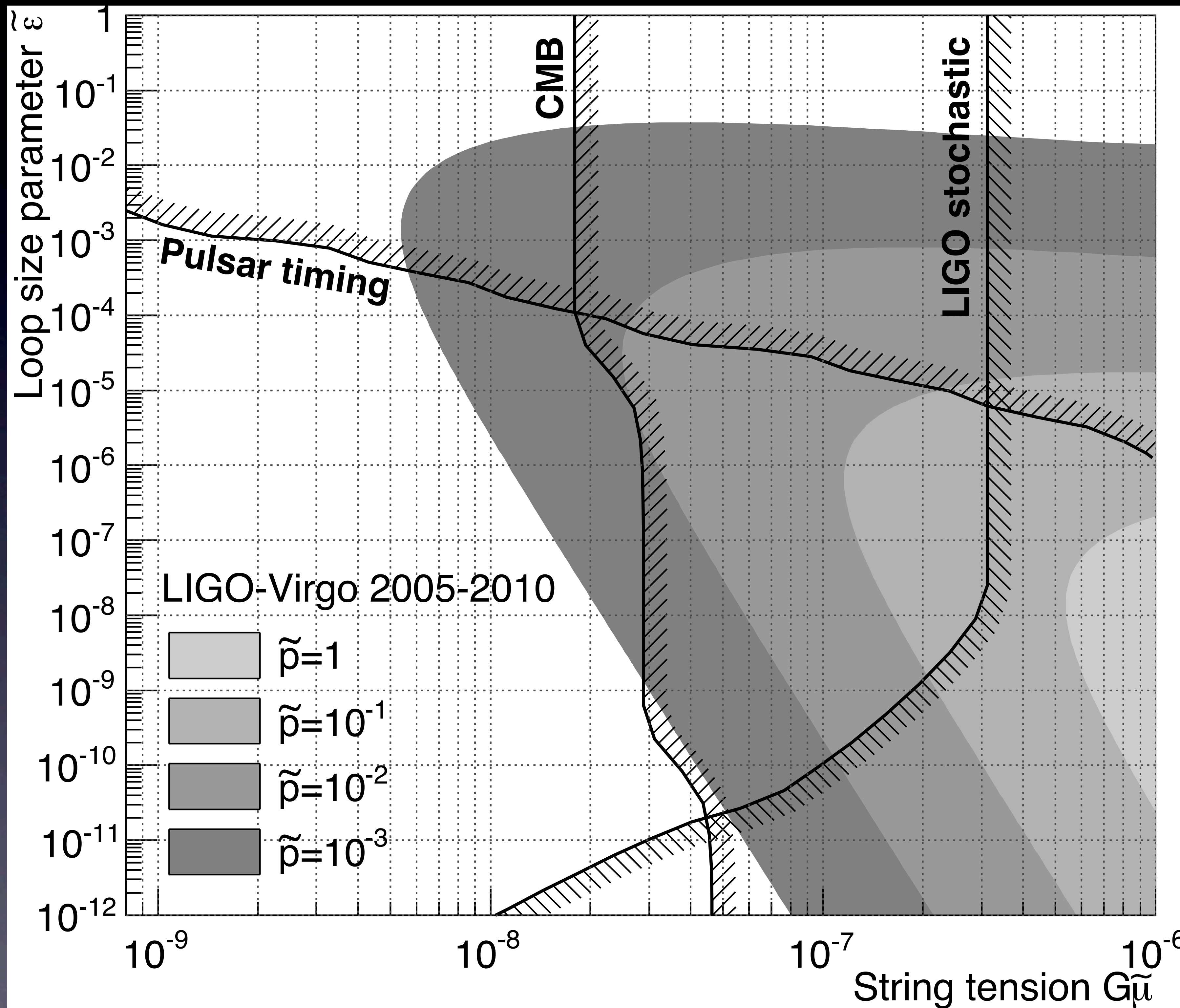
Loop size による違い



$\alpha \equiv \ell H$: loop の典型的な大きさ

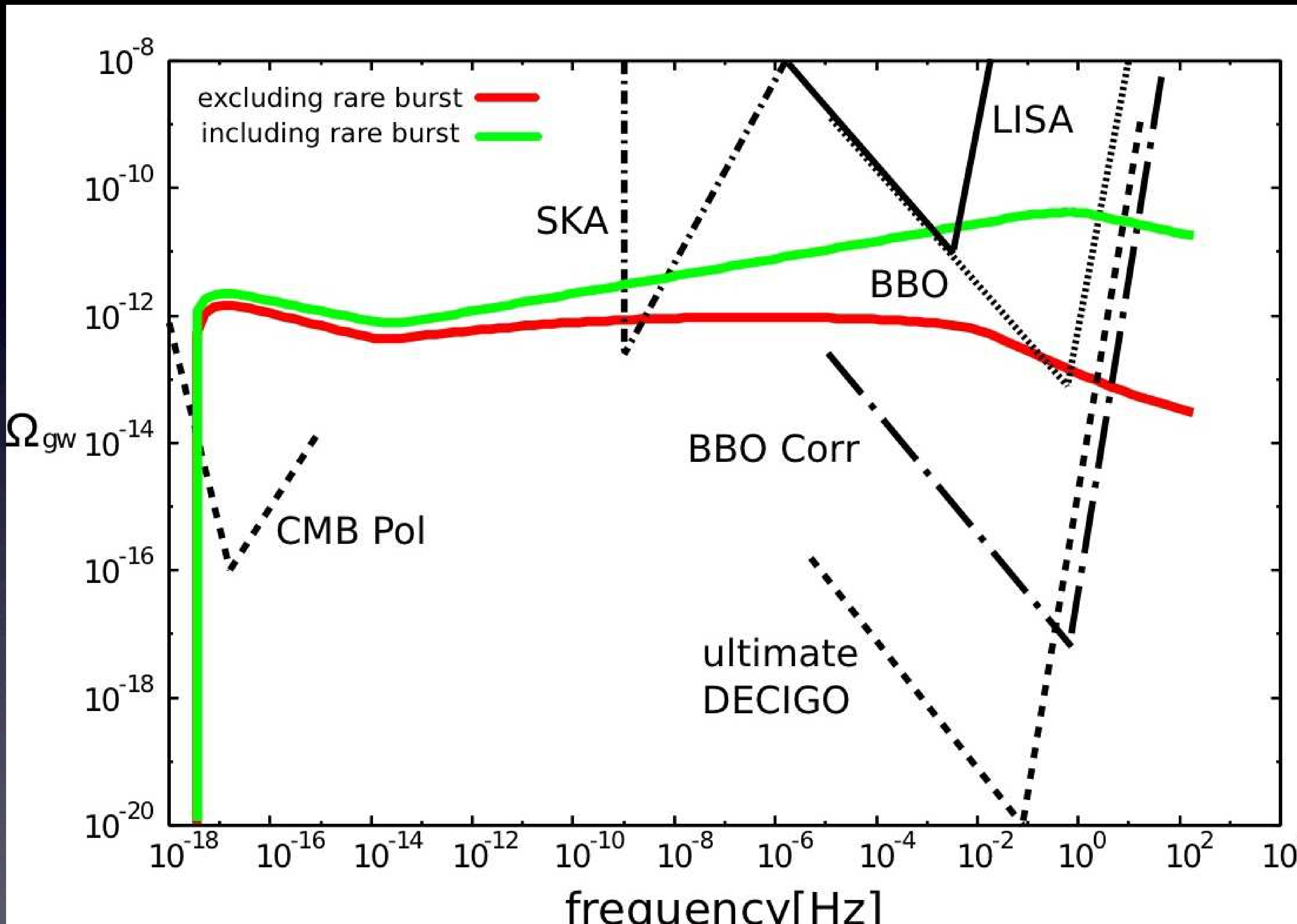
Kawasaki, Miyamoto, KN (2011)

LIGO constraint on cosmic string parameters



LIGO, 1310.2384

Infinite string からの重力波



Kawasaki, Miyamoto, KN (2010)

Summary

- インフレーション重力波観測：
  - 初期宇宙熱史（再加熱）の解明
  - 宇宙初期の相転移
  - Dark radiation の生成機構、生成時期
  - ...を通した具体的な素粒子模型の検証
- インフレーション重力波が見えなくても、Cosmic string からの重力波が見える可能性

Backup Slides

Inflationary GWs

- Inflation generates primordial GWs as quantum tensor fluctuations in de-Sitter spacetime

$$ds^2 = a^2(t)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

$$h_{ij} = \frac{1}{M_P} \sum_{\lambda=+,-} \int \frac{d^3k}{(2\pi)^{3/2}} h_k^\lambda(t) e^{i\mathbf{kx}} e_{ij}^\lambda$$

Quantization →

$$\langle h_k^\lambda h_{k'}^{\lambda'} \rangle = \frac{H_{\rm inf}^2}{2k^3} \delta^3(k - k') \delta^{\lambda\lambda'}$$

- Dimensionless power spectrum almost scale invariant

$$\Delta_h^2(k) = \left(\frac{H_{\rm inf}}{2\pi M_P} \right)^2$$

Evolution of GWs

- Eq.of.m of GW (without dark radiation)

$$\ddot{h}_\lambda + 3H\dot{h}_\lambda + (k/a)^2 h_\lambda = 0 \quad \Rightarrow \quad \begin{array}{l} h_\lambda \sim \text{const} \quad \text{for} \quad k \ll aH \\ h_\lambda \propto a(t)^{-1} \quad \text{for} \quad k \gg aH \end{array}$$

- GW energy density at horizon entry

$$\rho_{\rm GW}(k) \sim M_P^2 \Delta_h^2(k)(k/a)^2 \sim M_P^2 H_{\rm in}(k)^2 \Delta_h^2(k)$$

$$\rho_{\rm tot} \sim M_P^2 H_{\rm in}(k)^2$$

$$\Rightarrow \quad \Omega_{\rm GW}(k) = \frac{\rho_{\rm GW}(k)}{\rho_{\rm tot}} \sim \Delta_h^2(k) \sim \text{const at horizon entry}$$

$$\Rightarrow \quad \Omega_{\rm GW}^0(k) \sim \Omega_{\rm rad}^0 \Delta_h^2(k) \quad \text{at present for } k \gg k_{\rm eq}$$

GW normalization

- Standard model
  - GW spectrum at horizon entry

$$\Omega_{\mathrm{GW}}(k=aH) = \frac{\Delta_h^2(k)}{24} \qquad \Delta_h^2(k) \equiv \frac{8}{M_P^2}\left(\frac{H_{\mathrm{inf}}}{2\pi}\right)^2\left(\frac{k}{k_0}\right)^{n_t}$$

  - GW spectrum at present $(k \gg k_{\mathrm{eq}})$

$$\Omega_{\mathrm{GW}}^{(\mathrm{SM})}(k) = \gamma^{(\mathrm{SM})}\Omega_{\mathrm{rad}}^{(\mathrm{SM})} \times \Omega_{\mathrm{GW}}(k=aH),$$

Expansion history :

$$\gamma^{(\mathrm{SM})} = \left[\frac{g_*(T_{\mathrm{in}}(k))}{g_{*0}^{(\mathrm{SM})}}\right]\left[\frac{g_{*s0}^{(\mathrm{SM})}}{g_{*s}(T_{\mathrm{in}}(k))}\right]^{4/3},$$

GW normalization

- Standard model plus dark radiation
  - GW spectrum at present $(k \gg k_{\rm eq})$

$$\Omega_{\rm GW}(k) = \gamma \Omega_{\rm rad} \times \Omega_{\rm GW}(k = aH),$$

Expansion history modified by X :

$$\gamma = \frac{1 + \frac{7}{43}\left(\frac{g_{*s}(T_\phi)}{10.75}\right)^{1/3} \Delta N_{\rm eff}}{1/\gamma^{\rm (SM)} + \frac{7}{43}\left(\frac{g_{*s}(T_\phi)}{10.75}\right)^{1/3} \Delta N_{\rm eff}},$$

Radiation density :

$$\Omega_{\rm rad} = \Omega_{\rm rad}^{\rm (SM)} \times (g_{*0}/g_{*0}^{\rm (SM)}) \qquad g_{*0} = 2\left[1 + N_{\rm eff}\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]$$

Overall normalization is affected

GW normalization

- Parameterize normalization

$$\frac{\Omega_{\rm GW}(k)}{\Omega^{\rm (SM)}_{\rm GW}(k)} = C_1 \times C_2$$

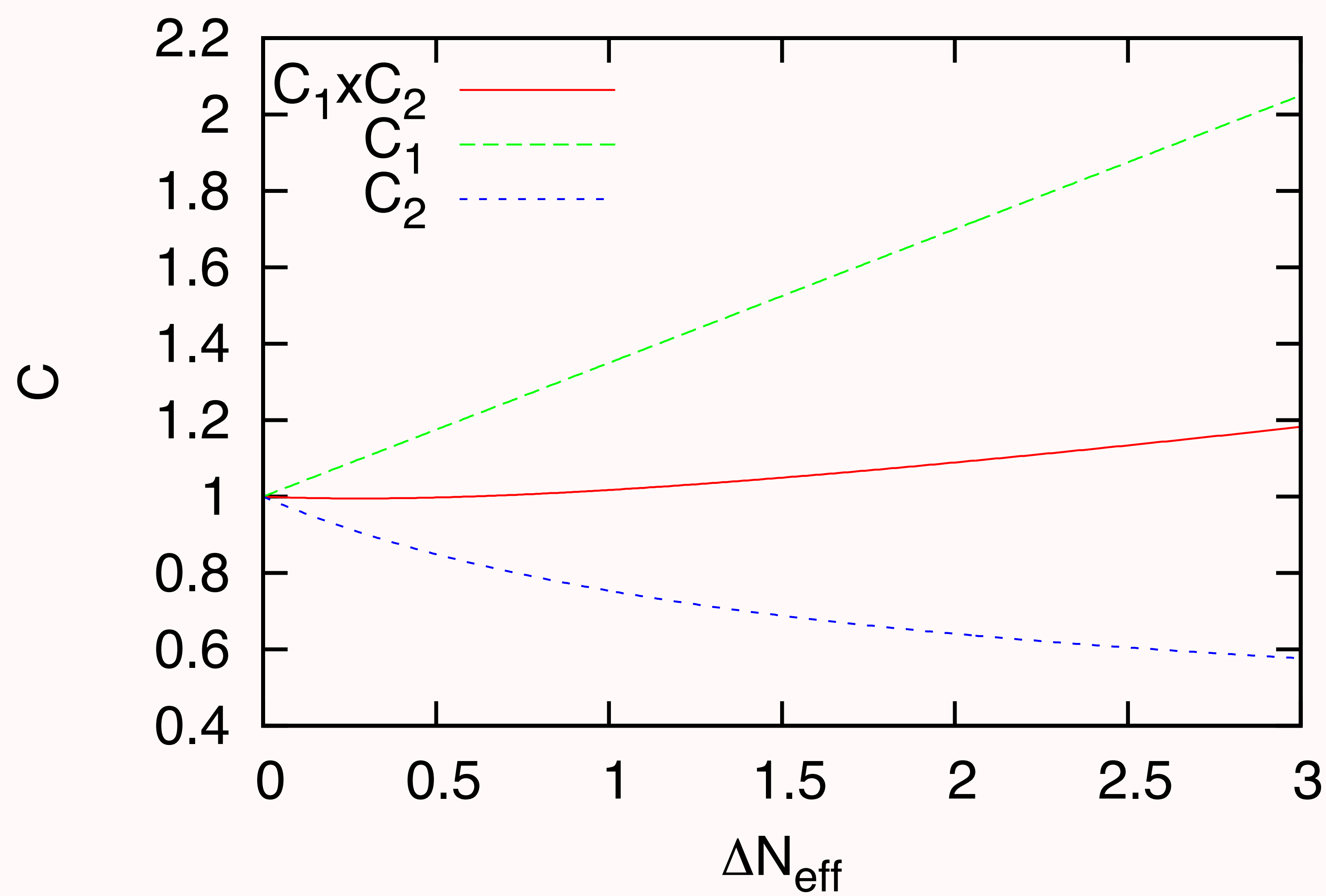
Modified BG by X :

$$C_1 \equiv \frac{\gamma}{\gamma^{\rm (SM)}} \frac{g_{*0}}{g^{\rm (SM)}_{*0}}$$

Anisotropic stress X :

C_2 analytically derived in

Dicus, Repko (2004)



C1xC2 accidentally close to unity

Anisotropic stress

- Boltzmann eq. for X

$$\frac{dF}{dt} = \frac{B_X}{4\pi(p^0)^3}\Gamma_\phi \rho_\phi \delta\left(p^0 - \frac{m_\phi}{2}\right)$$

F : distribution function of X

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{p^i}{p^0}\frac{\partial F}{\partial x^i} + \frac{1}{2}g_{ij,k}\frac{p^i p^j}{p^0}\frac{\partial F}{\partial p_k}$$

GW effect here

cf) Geodesic eq.

$$\frac{dp_i}{dt} = \frac{1}{2}g_{jk,i}\frac{p^j p^k}{p^0}.$$

- Perturbed :

$$\frac{\partial(\delta F_1 + \delta F_2)}{\partial t} + \frac{\bar{p}^i}{\bar{p}^0}\frac{\partial(\delta F_1 + \delta F_2)}{\partial x^i} + \frac{1}{2}(\delta g_{jk})_{,i}\frac{\bar{p}^j \bar{p}^k}{\bar{p}^0}\frac{\partial \bar{F}}{\partial p_i} = a\frac{\partial^2 \bar{F}}{\partial p \partial t}\delta p^0$$

$$\delta F_1(t, x^i, p_i) \equiv \bar{F}(t, (g^{ij}p_i p_j)^{1/2}/a) - \bar{F}(t, p) \qquad \delta F_2(t, x^i, p_i) \equiv F - \bar{F} - \delta F_1.$$

$$\Rightarrow \quad \frac{\partial \delta F_2}{\partial t} + \frac{\hat{p}_i}{a}\frac{\partial \delta F_2}{\partial x^i} = \frac{1}{2}\frac{\partial h_{ij}}{\partial t}\frac{\partial \bar{F}}{\partial p}p\hat{p}_i\hat{p}_j.$$

Contributes to anisotropic stress

Anisotropic stress

- EM tensor of X

$$\delta T_{ij}^{(X)} = \frac{1}{a^3}\int d^3p \left[(\delta F_1 + \delta F_2)\frac{p_i p_j}{\bar{p}^0} + \bar{F} p_i p_j \delta\left(\frac{1}{p^0}\right)\right] .= \frac{1}{a^2}\int d^3p \delta F_2 p \hat{p}_i \hat{p}_j + \frac{1}{3}a^2 h_{ij}\rho_X$$

Anisotropic stress $a^2\Pi_{ij}$

- From Boltzmann eq :

$$\delta F_2 = \int_0^\tau d\tau' \frac{1}{2}\frac{\partial h_{ij}}{\partial \tau}(\tau')\frac{\partial \bar{F}}{\partial p}(\tau') p \hat{p}_i \hat{p}_j e^{-ik\mu(\tau-\tau')}$$

- Eq.of.m of GW (with dark radiation)

$$\ddot{h}^{(\lambda)} + 3H\dot{h}^{(\lambda)} + \frac{k^2}{a^2}h^{(\lambda)} = -24H^2\frac{1}{a^4(t)\rho_{\rm tot}(t)} \times \int_0^t a^4(t')\rho_X(t')K\left(k\int_{t'}^{t}\frac{dt''}{a(t'')}\right)\dot{h}^{(\lambda)}(t',\mathbf{k})dt',$$

Anisotropic stress of X induced by GWs