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Parameter estimation with inspiral waveforms of compact binary coalescences including nontensorial gravitational waves polarizations

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Abstract

- The observation of gravitational waves(GW) from compact binary coalescences(CBC) enabled some experimental studies to probe into the nature of gravity.
- the separability of the polarization modes for the inspiral GW from the CBC systematically.

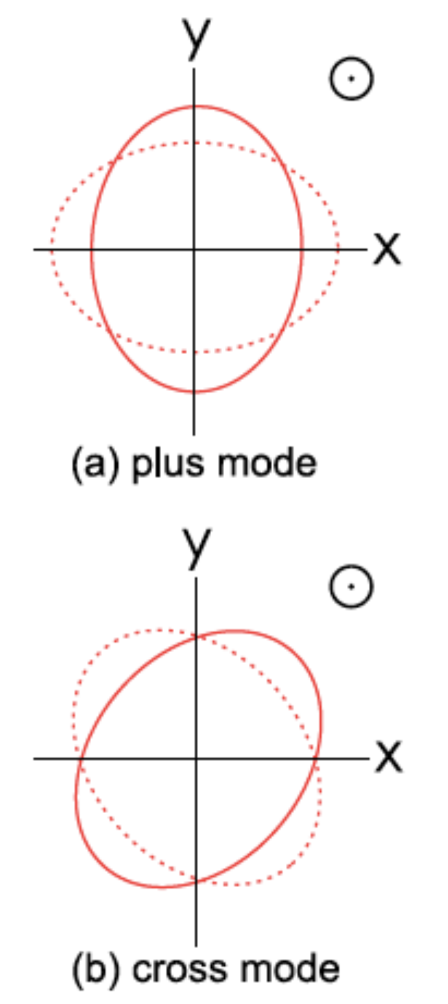
- The three polarization modes of the GW would be separable even with the global network of three detectors to some extent.
- With four detectors, even the four polarization modes would be separable.

1. Polarization of Gravitational Wave

Polarization mode of GW

The information of GW polarization modes is expected to bring more knowledge about gravity.

$$h_{ab}(t,\hat{\Omega}) = h_A(t)e^A_{ab}(\hat{\Omega})$$ $A = +,\times$



Tensor

Plus $$e^+_{ab} = \hat{e}_x \otimes \hat{e}_x - \hat{e}_y \otimes \hat{e}_y$$

Cross $$e^\times_{ab} = \hat{e}_x \otimes \hat{e}_y + \hat{e}_y \otimes \hat{e}_x$$

In General Relativity(GR),
GW has only two tensor mode(plus & cross)

A. Nishizawa et al., Physical Review D 79, 082002 (2009).

Polarization mode of GW

Generic metric theory allows 6 polarizations.

$$h_{ab}(t,\hat{\Omega}) = h_A(t) e^A_{ab}(\hat{\Omega})$$

$$A = +, \times, x, y, b, l$$

Tensor

Plus
$$e^+_{ab} = \hat{e}_x \otimes \hat{e}_x - \hat{e}_y \otimes \hat{e}_y$$

Cross
$$e^\times_{ab} = \hat{e}_x \otimes \hat{e}_y + \hat{e}_y \otimes \hat{e}_x$$

Vector

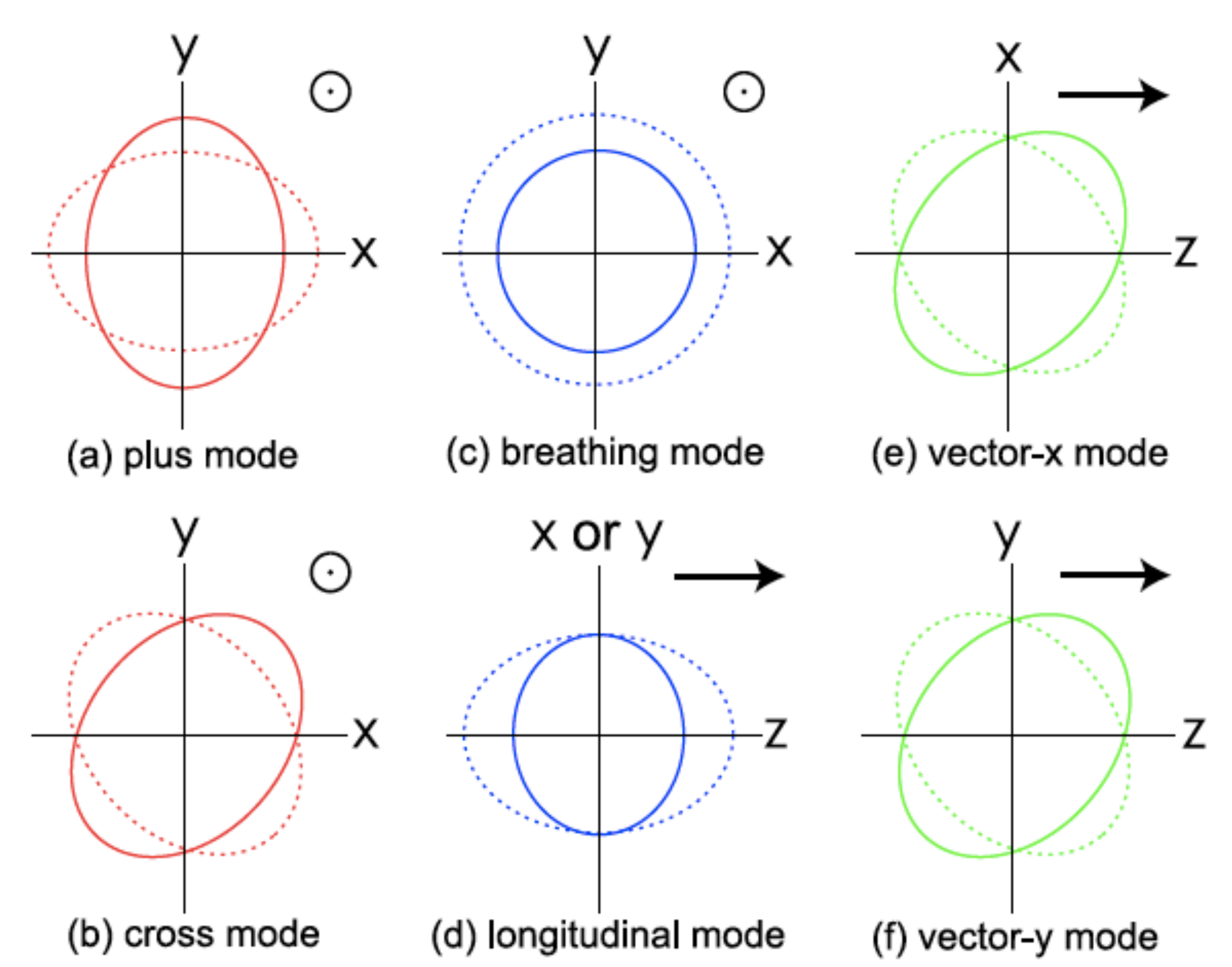
Vector x
$$e^x_{ab} = \hat{e}_x \otimes \hat{e}_z + \hat{e}_z \otimes \hat{e}_x$$

Vector y
$$e^y_{ab} = \hat{e}_y \otimes \hat{e}_z - \hat{e}_z \otimes \hat{e}_y$$

Scalar

breathing
$$e^b_{ab} = \hat{e}_x \otimes \hat{e}_x + \hat{e}_y \otimes \hat{e}_y$$

longitudinal
$$e^l_{ab} = \sqrt{2}\hat{e}_z \otimes \hat{e}_z.$$



A. Nishizawa et al., Physical Review D 79, 082002 (2009).

Test of GR and Alternative theories with GW polarization

modes allowed in a theoretical model

Theory	plus	cross	vector x	vector y	breathing	longitudinal
GR	O	O				
Kaluza-Klein theory	O	O	O	O	O	
Brans-Dicke theory	O	O			O	O
$f(R)$ theory	O	O			O	O
Bimetric theory	O	O	O	O	O	O

Test of Polarization modes of GW is a powerful tool for pursuing the nature of space-time.

Separate and Reconstruct polarization modes of GW model independently from detector signal

Polarization test with GW from CBC

In principle, (The number of polarization modes) = (The number of detectors)

→ More polarization modes can be probed with the larger number of detectors

However

Few studies have focused on the reconstruction of the polarization of GW from CBC

Because…

Waveforms of GW from CBC have the source model parameters,
which determine the frequency evolution in time and are correlated each other.

→ It is necessary to gain a better understanding of the correlations and degeneracies among the parameters in a realistic waveform of CBC including nontensorial modes

2. Parameter Estimation

Parameter estimation including nontensorial GW polarizations

the separability of the polarization modes for the inspiral GW from the CBC systematically.

Fisher Information Matrix

$$\Gamma_{ij} = 4\mathrm{Re}\int_{\mathrm{f_{min}}}^{\mathrm{f_{max}}} df \sum_I \frac{1}{S_{n,I}(f)} \frac{\partial h_I^*(f)}{\partial \lambda^i} \frac{\partial h_I(f)}{\partial \lambda^j}$$

Antenna pattern functions

Detector Signal

$$h_I(t,\hat{\Omega}) = F_I^A(\hat{\Omega}) h_A(t)$$

GW amplitude for polarization mode “A”

Inclination angle

GW amplitude depends on the inclination angle

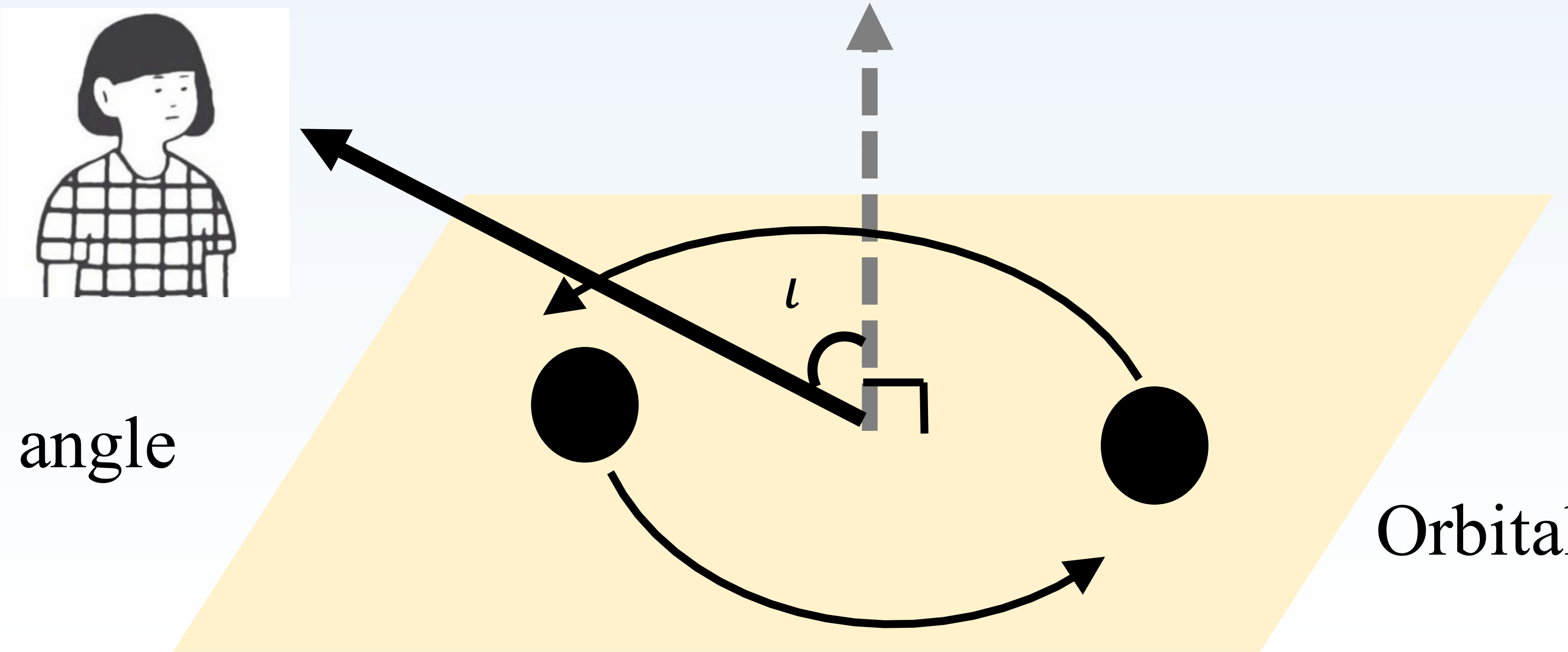
$$h_I(t,\hat{\Omega}) = F_I^A(\hat{\Omega})h_A(t)$$

geometrical factor ← inclination-angle dependence + antenna pattern

e.g. Tensor mode
$$h_I = \frac{2}{5}\mathcal{G}_{T,I}h_{\rm GR}$$

$$\mathcal{G}_{T,I} := \frac{5}{2}\{(1+\cos^2\iota)F_{+,I}(\boldsymbol{\theta_s},\boldsymbol{\theta_e}) + 2i\cos\iota F_{\times,I}(\boldsymbol{\theta_s},\boldsymbol{\theta_e})\}e^{i\phi_{D,I}(\theta_s,\phi_s,\theta_e,\phi_e)}$$

Observer



ι: inclination angle

Orbital plane

Geometrical factor for nontensorial modes

quadrupole formula

$$\mathcal{G}_{V_x,I} := \sqrt{\frac{525}{56}} \sin 2\iota F_{V_x,I}(\boldsymbol{\theta_s}, \boldsymbol{\theta_e}) e^{i\phi_{D,I}(\theta_s,\phi_s,\theta_e,\phi_e)}$$

Vector x

$$\mathcal{G}_{V_y,I} := \sqrt{\frac{15}{2}} \sin \iota F_{V_y,I}(\boldsymbol{\theta_s}, \boldsymbol{\theta_e}) e^{i\phi_{D,I}(\theta_s,\phi_s,\theta_e,\phi_e)}$$

Vector y

$$\mathcal{G}_{S_2,I} := \sqrt{\frac{225}{8}} \sin^2 \iota F_{b,I}(\boldsymbol{\theta_s}, \boldsymbol{\theta_e}) e^{i\phi_{D,I}(\theta_s,\phi_s,\theta_e,\phi_e)}.$$

scalar quadrupole

A scalar degree of freedom
e.g. Brans-Dicke theory

$$\mathcal{G}_{S_1,I} := \sqrt{\frac{45}{2}} \sin \iota F_{b,I}(\boldsymbol{\theta_s}, \boldsymbol{\theta_e}) e^{i\phi_{D,I}(\theta_s,\phi_s,\theta_e,\phi_e)}.$$

scalar dipole

Polarization Model

Model T: GR model

$$h_I = \mathcal{G}_{T,I} h_{\mathrm{GR}}$$

Model TS1: Tensor(+,×)
Scalar(dipole)

$$h_I = \{\mathcal{G}_{T,I} + \underline{A_{S_1}} \mathcal{G}_{S_1,I}\} h_{\mathrm{GR}}$$

Model TS2: Tensor (+,×)
Scalar(quadrupole)

$$h_I = \{\mathcal{G}_{T,I} + \underline{A_{S_2}} \mathcal{G}_{S_2,I}\} h_{\mathrm{GR}}$$

Model TVxS2: Tensor (+,×)
Scalar(quadrupole)
Vector-x

$$h_I = \{\mathcal{G}_{T,I} + \underline{A_{S_2}} \mathcal{G}_{S_2,I} + \underline{A_{V_x}} \mathcal{G}_{V_x,I}\} h_{\mathrm{GR}}$$

Model TVyS1: Tensor (+,×)
Scalar(dipole)
Vector-y

$$h_I = \{\mathcal{G}_{T,I} + \underline{A_{S_1}} \mathcal{G}_{S_1,I} + \underline{A_{V_y}} \mathcal{G}_{V_y,I}\} h_{\mathrm{GR}}$$

Model TV: Tensor (+,×)
Vector-x, Vector-y

$$h_I = \{\mathcal{G}_{T,I} + \underline{A_{V_x}} \mathcal{G}_{V_x,I} + \underline{A_{V_y}} \mathcal{G}_{V_y,I}\} h_{\mathrm{GR}}$$

Parameter estimation including nontensorial GW polarizations

Setup

- the inspiral waveform up to 3.0 PN in amplitude & 3.5 PN in phase $h_{\rm GR} = A_{\rm ins} e^{-i\phi_{\rm ins}}$,

$$A_{\rm ins} = \frac{1}{\sqrt{6}\pi^{2/3} d_L} \mathcal{M}^{5/6} f^{-7/6} \sum_{i=0}^{6} (\pi \mathcal{M} f)^{i/3},$$

$$\phi_{\rm ins} = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128} (\pi \mathcal{M} f)^{-5/3} \sum_{i=0}^{7} \phi_i (\pi \mathcal{M} f)^{i/3}$$

- 11 model parameters in GR + $(\log \mathcal{M}, \log \eta, t_c, \phi_c, \log d_L, \chi_s, \chi_a, \theta_s, \phi_s, \cos \iota, \psi_p)$

additional polarization parameters, $(A_{S1}, A_{Vx}$ ...) fiducial value unity

uniformly random

- Source (500)

binary black holes (BBH) with equal mass 10M⊙ at z = 0.05

binary neutron stars (BNS) with equal mass 1.4M⊙ at z = 0.01

- Detector Network

Two aLIGO detectors at Hanford- Livingston-AdV(HLV)

Two aLIGO detectors at Hanford-Livingston-AdV-KAGRA(HLVK)

all design sensitivity

Results

Model TS1

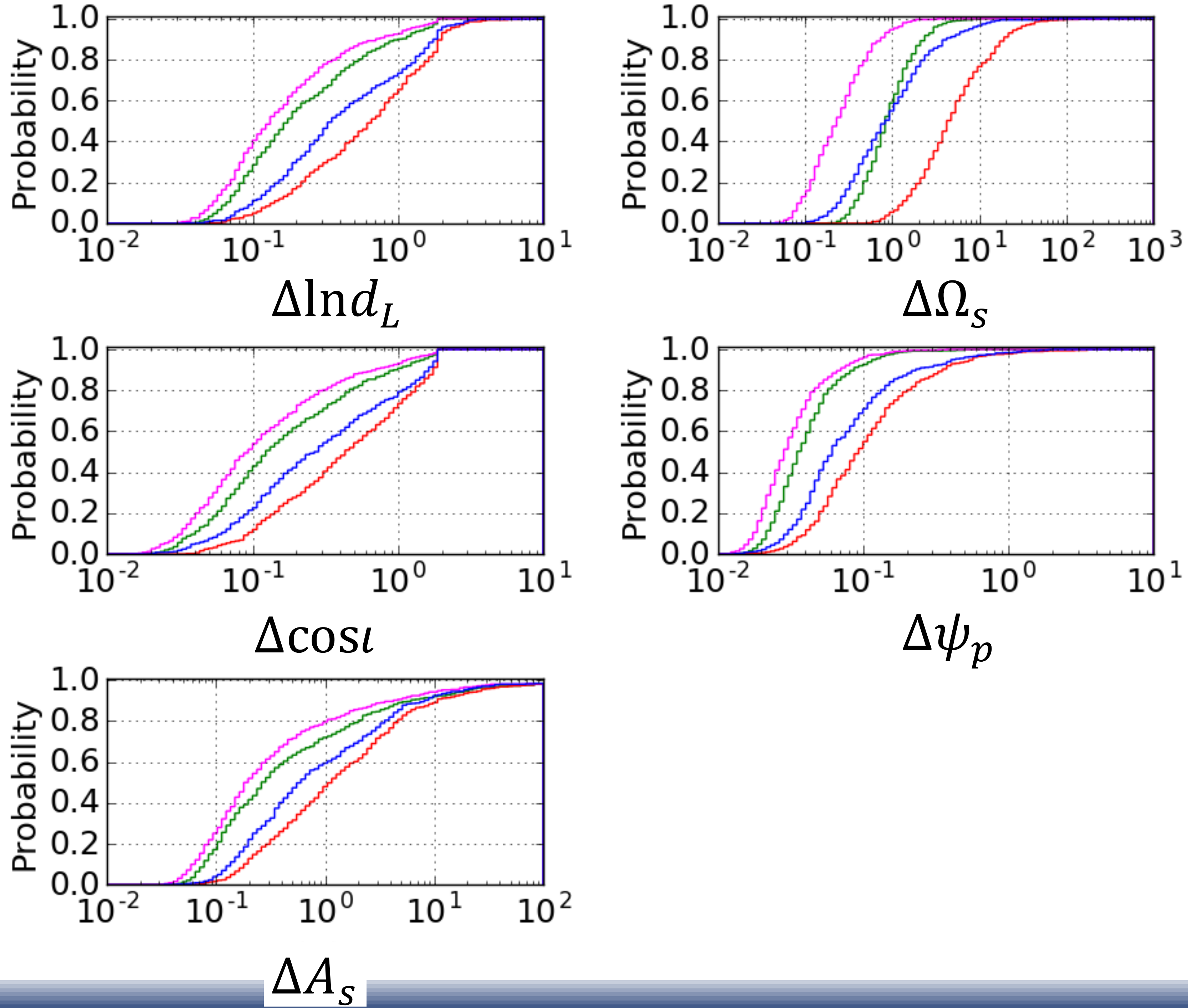
red: BBH-HLV

green: BBH-HLVK

blue: BNS-HLV

magenta: BNS-HLVK

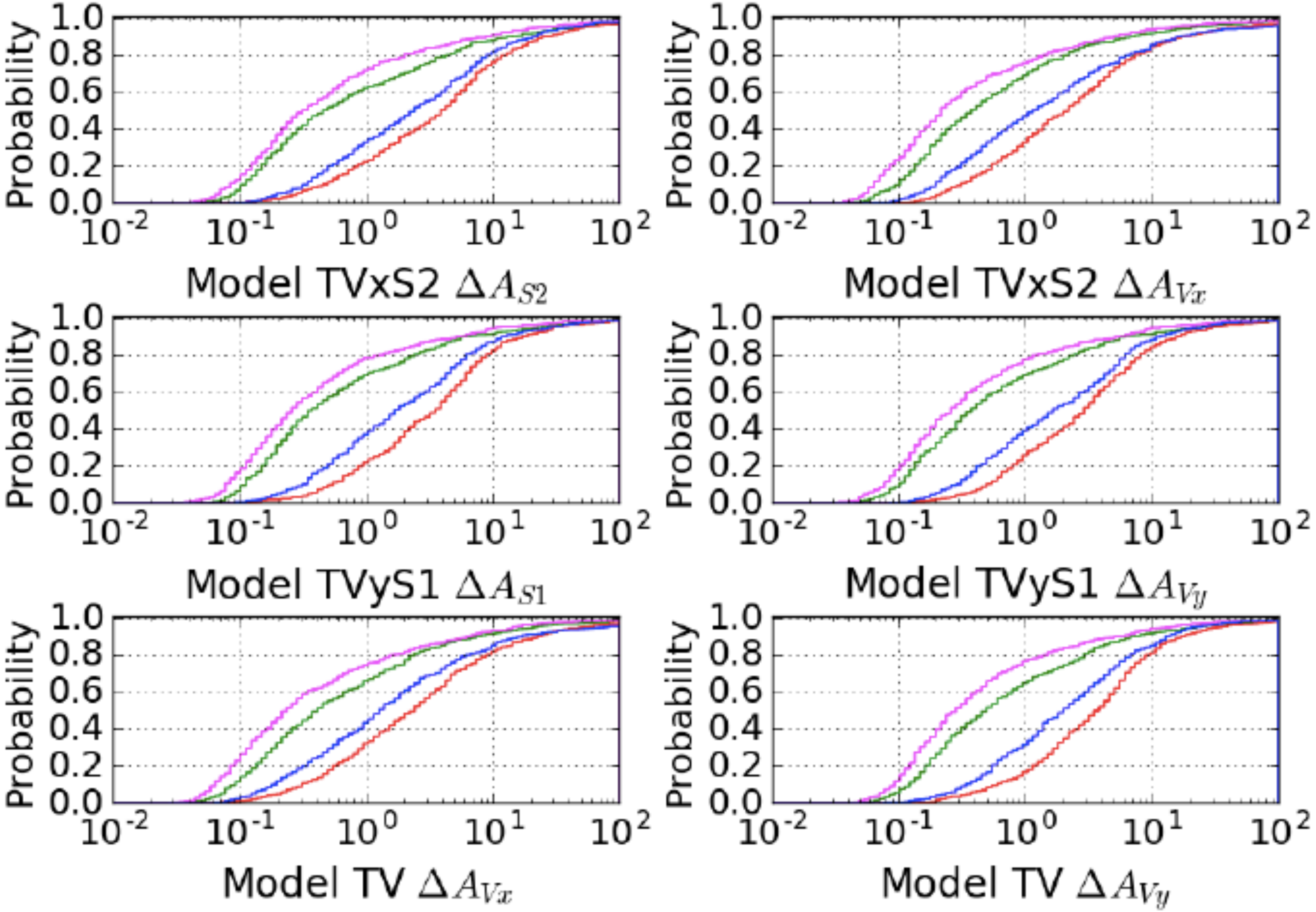
can break a degeneracy among amplitude parameters.



	BBH(HLV)	BBH(HLVK)	Improvement Factor	BNS(HLV)	BNS(HLVK)	Improvement Factor
ModelT SNR	33.3	40.2		36.4	44.3	
ModelT $\Delta \ln d_L$	0.269	0.137	1.96	0.183	0.107	1.71
ModelT $\Delta\Omega_s[\mathrm{deg}^2]$	5.91	1.77	3.34	1.39	0.517	2.69
ModelTS1 $\Delta \ln d_L$	0.678	0.179	3.79	0.359	0.134	2.68
ModelTS1 $\Delta\Omega_s[\mathrm{deg}^2]$	4.74	0.912	5.20	0.919	0.250	3.68
ModelTS1 ΔA_s	1.16	0.284	4.08	0.606	0.197	3.08
ModelTS1 $C(A_{S1}, \log d_L)$	0.998	0.989		0.996	0.984	
ModelTS1 $C(A_{S1}, \cos\iota)$	-0.553	-0.500		-0.231	-0.159	
ModelTS2 $\Delta \ln d_L$	0.676	0.182	3.71	0.358	0.134	2.67
ModelTS2 $\Delta\Omega_s[\mathrm{deg}^2]$	4.74	0.913	5.09	0.862	0.246	3.50
ModelTS2 ΔA_{S2}	1.51	0.385	3.92	0.765	0.256	2.99
ModelTS2 $C(A_{S2}, \log d_L)$	0.997	0.989		0.996	0.984	
ModelTS2 $C(A_{S2}, \cos\iota)$	-0.609	-0.564		-0.246	-0.189	

ModelTS1, TS2

- improved by factor 4 in ΔA_s(BBH), about 3 in ΔA_s(BNS)
- A_s is correlated with $\ln d_L$ and $\cos\iota$ (for all models)
- BBH estimation is worse than BNS with HLV due to the short signal of BBH.



Model TVxS2
Model TVyS1
Model TV

ModelTVxS2, TVyS1, TV more improved than in Model TS1,TS2.

	BBH(HLV)	BBH(HLVK)	Improvement Factor	BNS(HLV)	BNS(HLVK)	Improvement Factor
ModelTVxS2 $\Delta \ln d_L$	1.58	0.258	6.12	1.05	0.190	5.53
ModelTVxS2 $\Delta\Omega_s[\mathrm{deg}^2]$	6.13	0.885	6.92	0.783	0.179	4.37
ModelTVxS2 ΔA_{S2}	4.15	0.486	8.54	2.48	0.340	7.29
ModelTVxS2 ΔA_{V_x}	2.23	0.399	5.59	1.24	0.228	5.44
ModelTVyS1 $\Delta \ln d_L$	1.69	0.253	6.68	1.05	0.183	5.74
ModelTVyS1 $\Delta\Omega_s[\mathrm{deg}^2]$	6.76	0.879	7.69	0.831	0.187	4.44
ModelTVyS1 ΔA_{S1}	3.72	0.383	9.71	1.81	0.273	6.63
ModelTVyS1 ΔA_{V_y}	3.12	0.389	8.02	1.75	0.270	6.48
ModelTV $\Delta \ln d_L$	1.98	0.310	6.39	1.22	0.193	6.32
ModelTV $\Delta\Omega_s[\mathrm{deg}^2]$	5.68	0.795	7.14	0.813	0.187	4.35
ModelTV ΔA_{V_x}	2.55	0.420	6.07	1.37	0.241	5.68
ModelTV ΔA_{V_y}	3.91	0.513	7.62	2.12	0.298	7.11

Model TVxS2
Model TVyS1
Model TV

- ModelTVxS2, TVyS1, TV more improved than in Model TS1,TS2.
- the error is more than 5 times improved for both BBH and BNS

<u>even the four polarization modes would be separable with four detectors.</u>

Conclusion

・Even if the number of detectors is equal to the number of the polarization modes, it is difficult to separate the modes depending on the correlation among the amplitude parameters.

・The three polarization modes would be more separable by breaking a degeneracy of the polarization modes and even the four polarization modes would be separable **with the global four detectors network**.

・ The participation of KAGRA in the network of the GW detectors will make it possible to extract the polarization information.

ありがとうございました！