Gravitational Waves Detection via Weak Measurements

Satoru Takano

Contents

- Part1 : Quantum Measurements
  - von Neumann measurement model
  - Weak measurements and weak values

- Part2 : Review of the paper

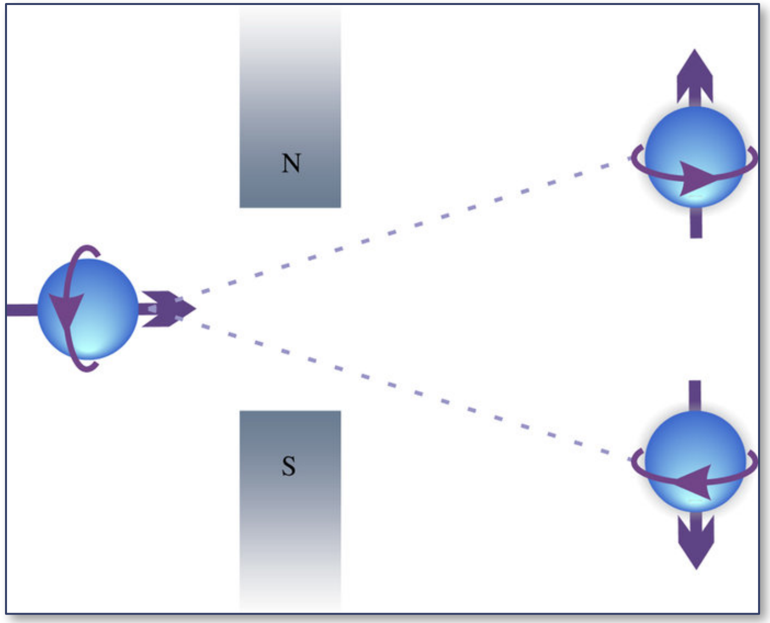
  "Gravitational Waves Detection via Weak Measurements"
  - The measurement method
  - WMA-LIGO

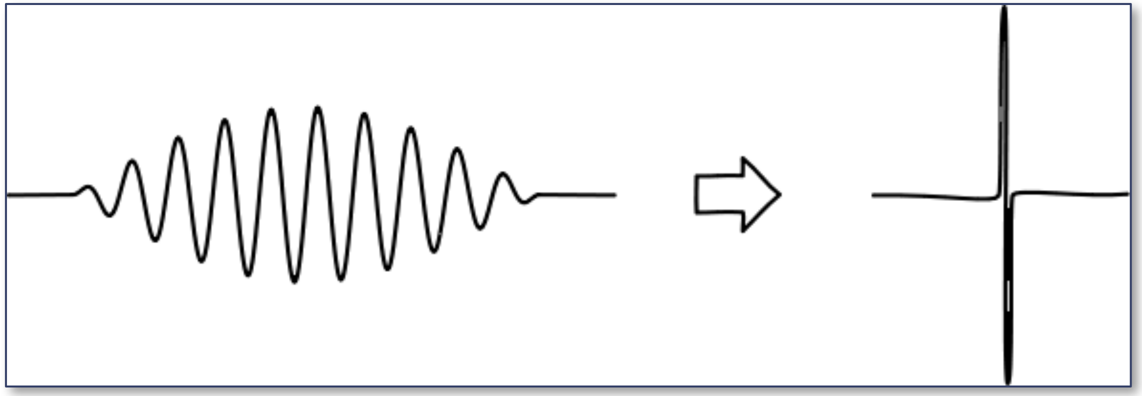
Part 1

Quantum Measurements

Quantum Measurement

- When we execute an measurement on a quantum system, ordinarily the original state is destroyed
- An example: spin measurement, wave function collapse





- We want to measure an system without destroying quantum state

➢ von Neumann’s measurement model

von Neumann measurement model

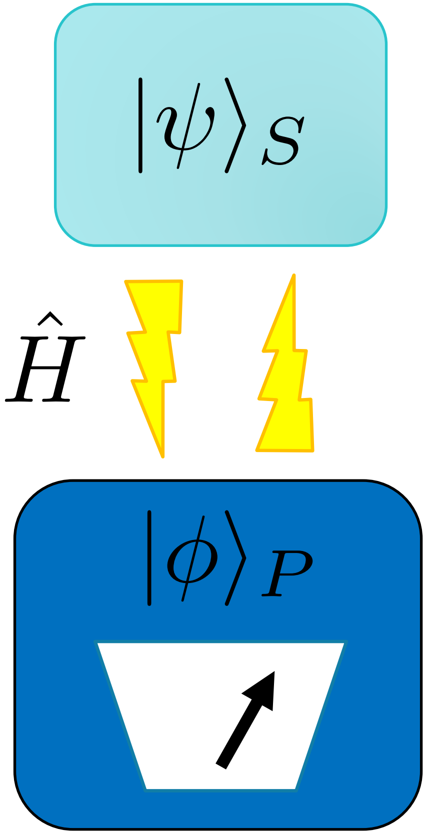
- Prepare another system called Probe or pointer : $|\psi\rangle_S, |\phi\rangle_P$
- The measured system and the probe is interacted

➢ The interaction Hamiltonian has the form like that

$$\hat{H} = \alpha \hat{A} \hat{X},$$

where $\hat{A}$ is an observable of the system,

$\hat{X}$ is a canonical variable of the probe $[\hat{X}, \hat{P}] = i$



- Whole system $|\Psi\rangle_{SP} = |\psi\rangle_S \otimes |\phi\rangle_P$
- evolves with time as: $|\Psi\rangle_{SP}(t) = e^{-i\int dt \hat{H}} |\Psi\rangle_{PS}$

Some assumptions

- For simplicity, we prepared the probe as a Gaussian in $\hat{P}$ representation:

$$_P\langle P|\phi\rangle_P = \mathcal{N} \exp\left(-\frac{P^2}{4\Delta P^2}\right)$$

- The system is supposed to be separable with eigenstates of $\hat{A}$:

$$|\psi\rangle_S = \sum c_i|a_i\rangle \,,$$

$$\hat{A}|a_i\rangle = a_i|a_i\rangle$$

- Under such a situation, we measure $\langle P\rangle$

The final state

- If the interaction time Δt is enough small,

$$\int dt\hat{H} = \alpha\Delta t\hat{A}\hat{X}$$

- Then,

$$_P\langle P|\Phi\rangle_{SP}(t) = \mathcal{N}e^{-i\int dt\hat{H}}\exp\left(-\frac{P^2}{4\Delta P^2}\right)\sum c_i|a_i\rangle$$

$$= \mathcal{N}\sum c_i \exp\left(-\frac{(P-\alpha\Delta t a_i)^2}{4\Delta P^2}\right)|a_i\rangle$$

- The result is an mixture of Gaussians

The Probability density

- The probability density of $\langle P \rangle$ is

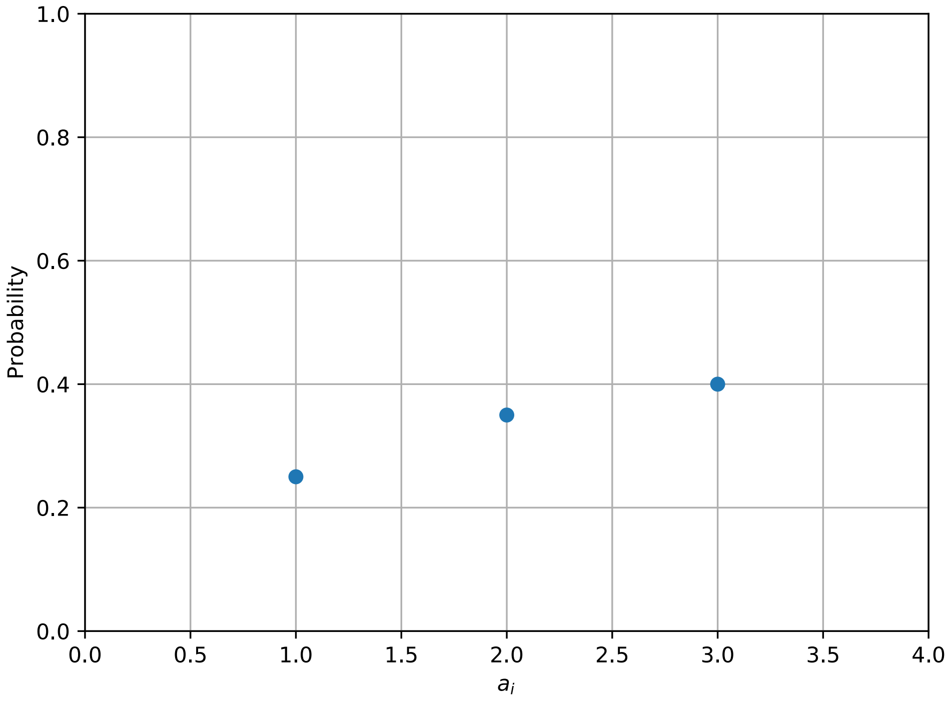
$$p(P) = |_P\langle P|\Phi\rangle_{SP}|^2 = \sum |c_i|^2 \exp\left(-\frac{(P-\alpha\Delta t a_i)^2}{2\Delta P^2}\right)$$

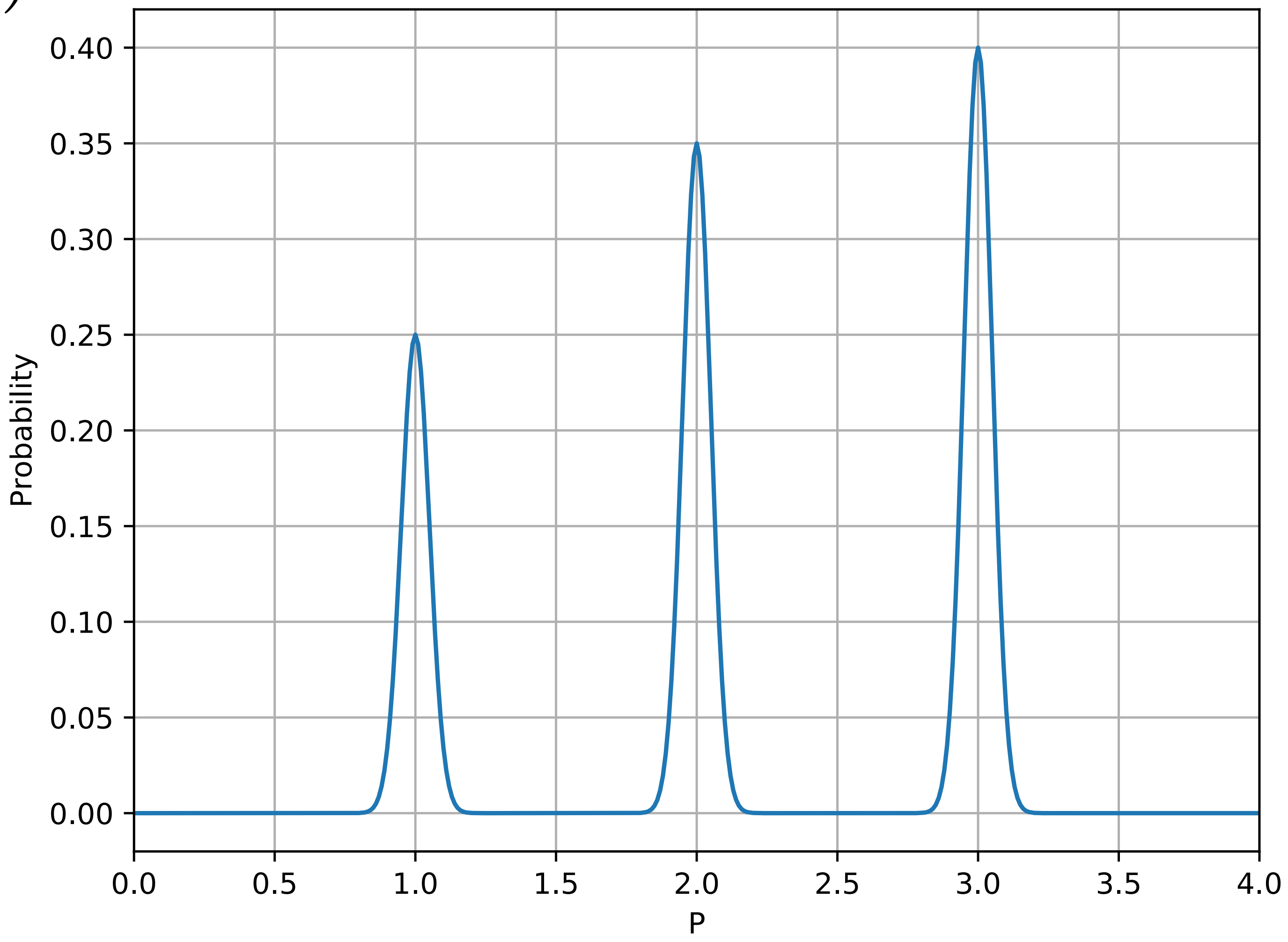
- If ΔP is much smaller than the difference of a_i
  - ➤ We will detect the value a_i with probability $|c_i|^2$
- If ΔP is much bigger than all of
  - ➤ The mean value is

$$\langle A \rangle = \sum |c_i|^2 a_i$$

An example

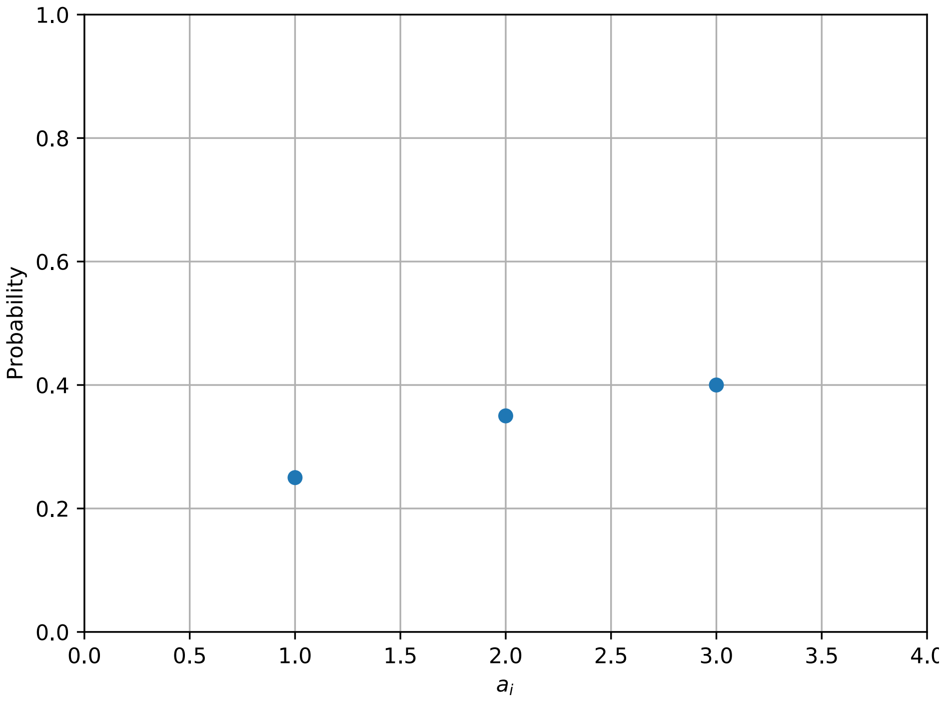
- ΔP is small $\quad (\Delta P = 0.1)$

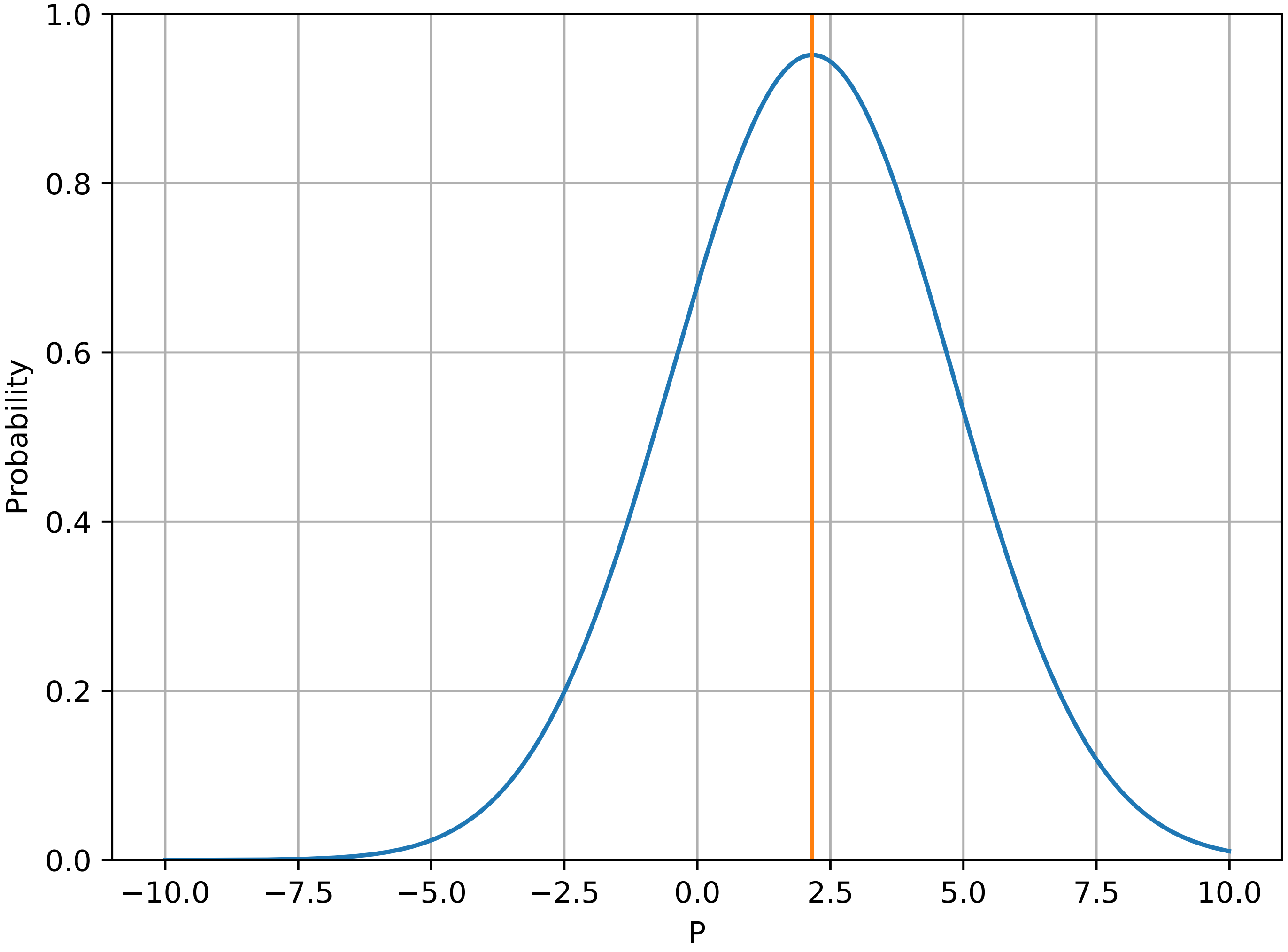




An example

- ΔP is big $(\Delta P = 5)$





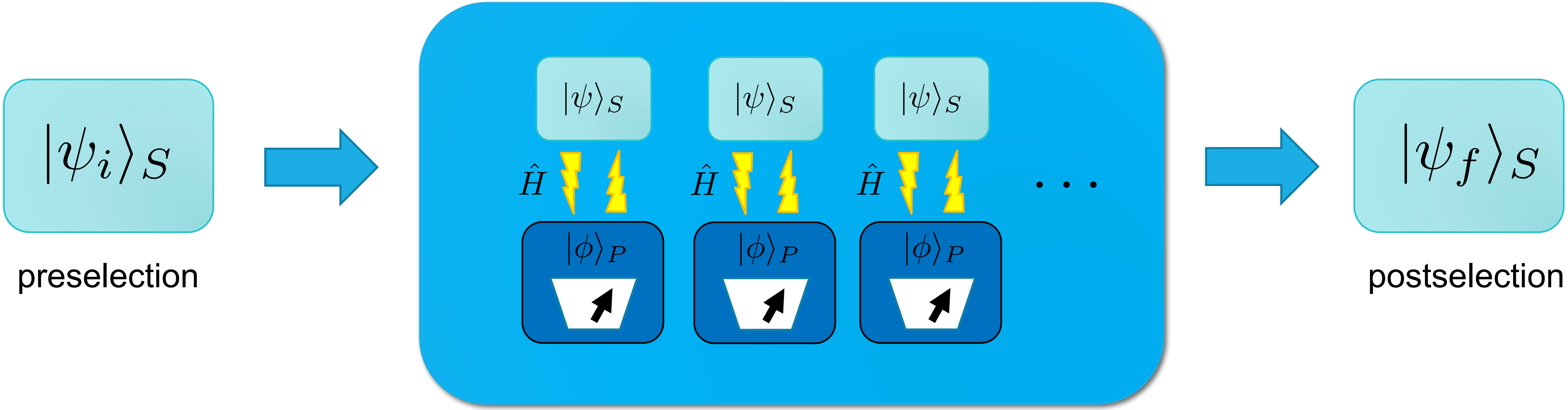
Weak measurements

- We select the initial state and the final state of the system:

$$|\psi_i\rangle_S \, , |\psi_f\rangle_S$$

these operations are called preselection, postselection

- Then, measurements are done:



Weak measurements

- The result:

$$
\begin{aligned}
{}_S\langle\psi_f|_P\langle P|\Phi\rangle_{SP} &= \mathcal{N}\langle\psi_f|e^{-i\alpha\Delta t\hat{H}}|\psi_i\rangle \exp\left(-\frac{P^2}{4\Delta P^2}\right) \\
&\simeq \mathcal{N}\langle\psi_f|\psi_i\rangle \exp\left(-i\alpha\Delta t X \frac{\langle\psi_f|\psi_i\rangle}{\langle\psi_f|\hat{A}|\psi_i\rangle}\right) \\
&\quad \times \exp\left(-\frac{P^2}{4\Delta P^2}\right)
\end{aligned}
$$

Weak value

- We define weak value as:

$$A_w = \frac{\langle\psi_f|\psi_i\rangle}{\langle\psi_f|\hat{A}|\psi_i\rangle}$$

- Then

$${}_S\langle\psi_f|_P\langle P|\Phi\rangle_{SP} = \simeq \mathcal{N}\langle\psi_f|\psi_i\rangle \exp\left(-\frac{(P-\alpha\Delta t A_w)^2}{4\Delta P^2}\right)$$

- If we measure $\langle P\rangle$, we know A_w
- The meaning of A_w is not trivial, controversing

Part 2

Review Of The Paper

Introduction

- This paper suggest a brand-new interferometer using weak measurement
- As a pointer, they use polarized photons
- The measured system is the path of the photon

  (in detail, I will explain later)
- Note: this method execute preselection and postselection, but doesn‘t use the weak value!

Notation

- Suppose we want to measure a two-level system

$$|0\rangle, |1\rangle$$

- We use photons as the pointer:

  $|H\rangle$ is the horizontal polarization state,

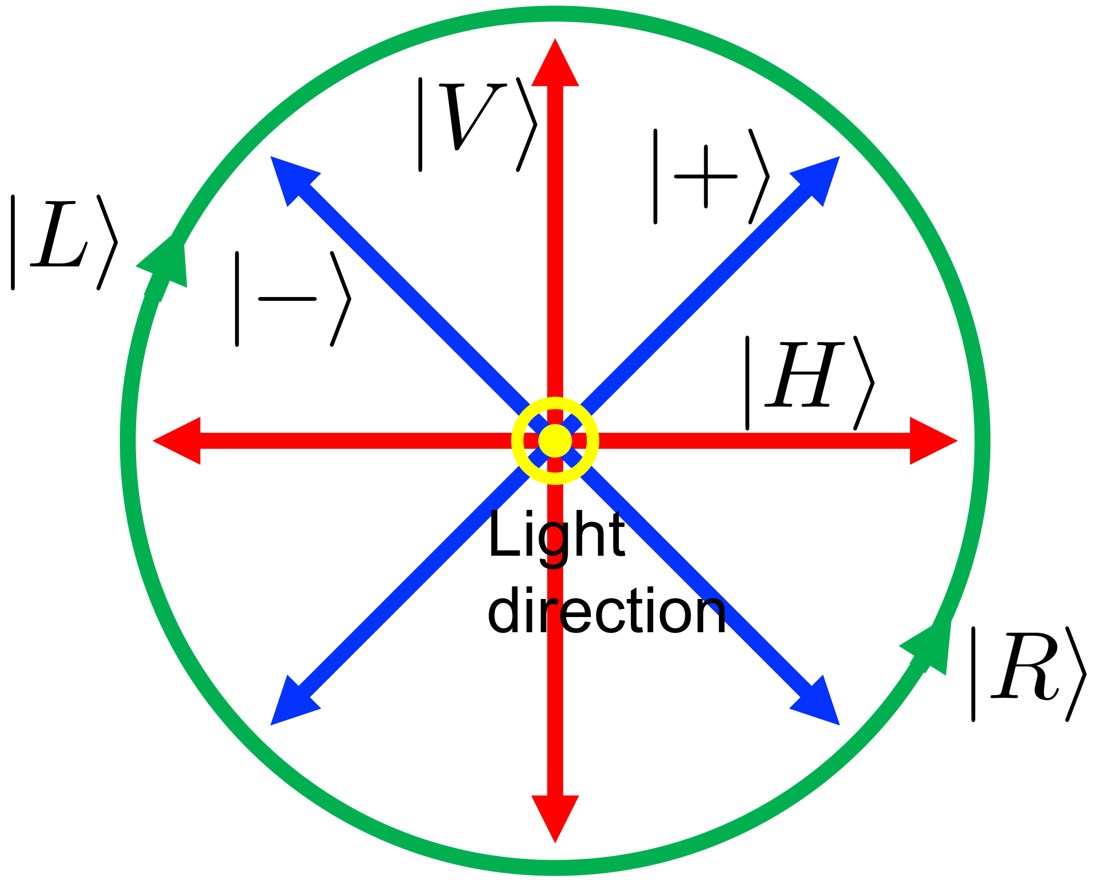
  $|V\rangle$ is the vertical polarization state

- Other representation of polarizations:

$$|+\rangle = (|H\rangle + |V\rangle)/\sqrt{2}\,, \qquad |R\rangle = (|H\rangle + i|V\rangle)/\sqrt{2}\,,$$

$$|-\rangle = (|H\rangle - |V\rangle)/\sqrt{2} \qquad |L\rangle = (|H\rangle - i|V\rangle)/\sqrt{2}$$

Polarization states



We take the basis as:

$$|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, |-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$|R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, |L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

A brand-new method

- Preparing the initial state of the pointer as

$$|\phi_i\rangle_P = |+\rangle = \frac{|H\rangle + |V\rangle}{\sqrt{2}}$$

- We take the initial and the final state of the system as before:

$$|\psi_i\rangle = \alpha|0\rangle + \beta|1\rangle\,,\ \ |\psi_f\rangle = \gamma|0\rangle + \eta|1\rangle$$

- Now, consider a new Unitary operator

$$\hat{U} = |0\rangle\langle 0| \otimes \hat{I} + |1\rangle\langle 1| \otimes (|H\rangle\langle H| + e^{i\theta}|V\rangle\langle V|)$$

- θ is the signal what we want to measure and amplify by weak measurement

Assumptions

- In this case, the final state is

$$|\Psi_f\rangle_{PS} = \hat{U}|\Psi_i\rangle_{PS}$$

$$= \alpha|0\rangle \otimes |+\rangle + \beta|1\rangle \otimes (|H\rangle + e^{i\theta}|V\rangle)/\sqrt{2}$$

$$|\phi_f\rangle_P = (\alpha\gamma + \beta\eta)|H\rangle + (\alpha\gamma + \beta\eta e^{i\theta})|V\rangle$$

- Ordinary, $\theta \ll 1$, so in the first order

$$\alpha\gamma + \beta\eta e^{i\theta} \simeq (\alpha\gamma + \beta\eta)e^{i\varphi},$$

$$\tan\varphi = \frac{\beta\eta\theta}{\beta\eta + \alpha\gamma} = \frac{\theta}{1 + \frac{\alpha\gamma}{\beta\eta}}$$

How to detect the signal

- Then, the final state of the pointer is

$$|\phi_f\rangle_P = \frac{1}{\sqrt{2}}(|H\rangle + e^{i\varphi}|V\rangle)$$

- Using this, we extract the signal φ by calculating

$$\langle\hat{\sigma}_y\rangle = {}_P\langle\phi_f|\hat{\sigma}_y|\phi_f\rangle_P = \sin\varphi$$

- The postselection probability is

$$P_{select} = |{}_S\langle\psi_f|\psi_i\rangle_S|^2 = |\alpha\gamma + \beta\eta|^2$$

Amplification factor

- Suppose $\alpha = \beta = 1/\sqrt{2}, \gamma = \cos\chi, \eta = \sin\chi$

$$\tan\varphi = \frac{\theta}{1 + \cot\chi}$$

- If we take $\chi = -(\pi/4 + \delta)$,

$$\tan\varphi \simeq \frac{\theta}{\delta}$$

so the amplification factor is

$$h = \frac{\varphi}{\theta} = \frac{\tan^{-1}(\theta/\delta)}{\theta}$$

- With recently technology, we can reach h~10^3

Trade-off of the postselection probability

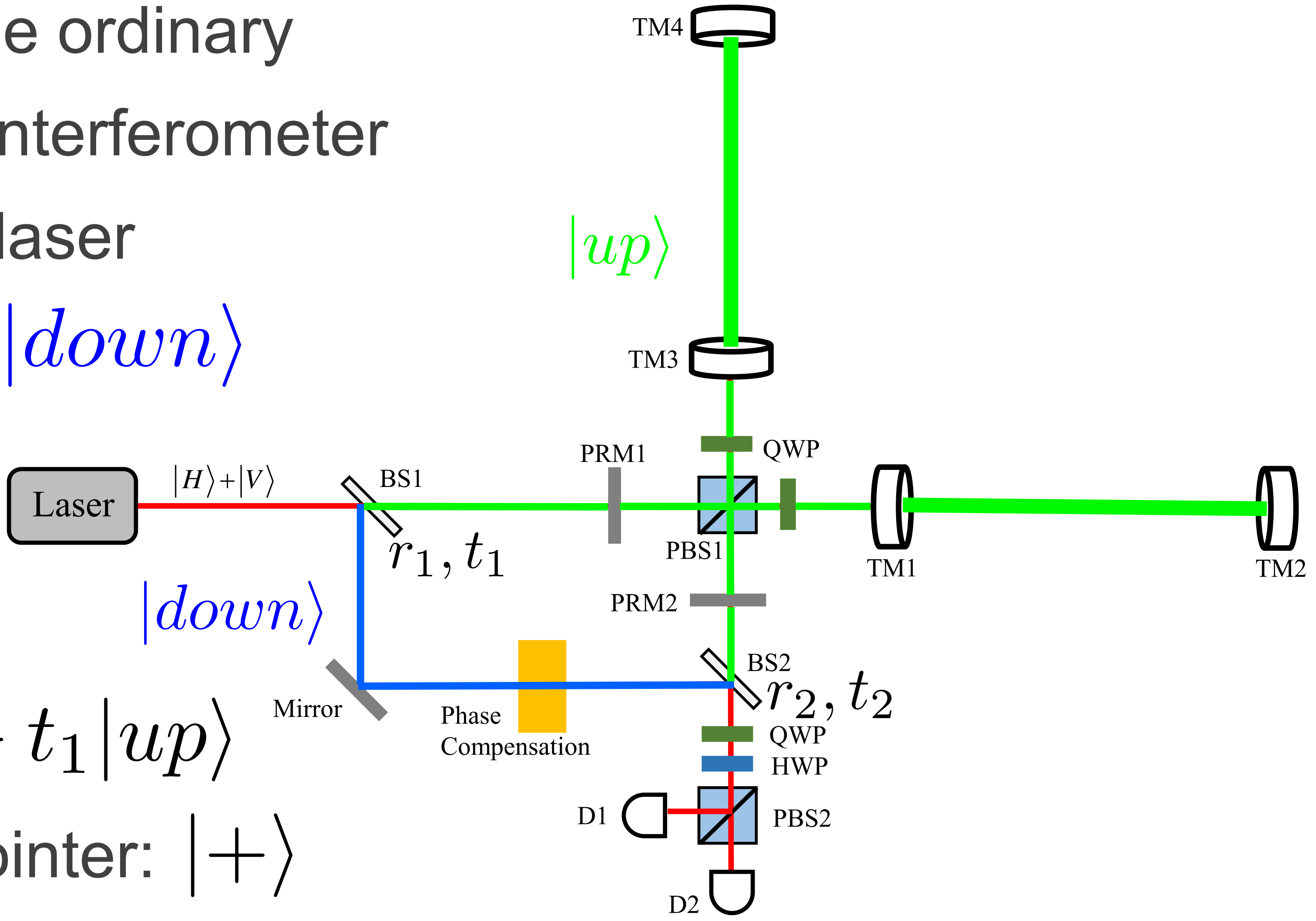
- on the other hand, the postselection probability is

$$P_{select} = \frac{1}{2}|\cos\chi + \sin\chi|^2 = \sin^2\delta$$

- There is a trade-off between
  - The amplification of the signal
  - The postselection probability

Weak Measurements Amplification-LIGO

- Basically the same as the ordinary Fabry-Perot Michelson interferometer
- Using linearly polarized laser
- The system: $|up\rangle$ and $|down\rangle$ representing two paths
- The initial state of the system:
$$|\psi_i\rangle_S = r_1|down\rangle + t_1|up\rangle$$
- The initial state of the pointer: $|+\rangle$

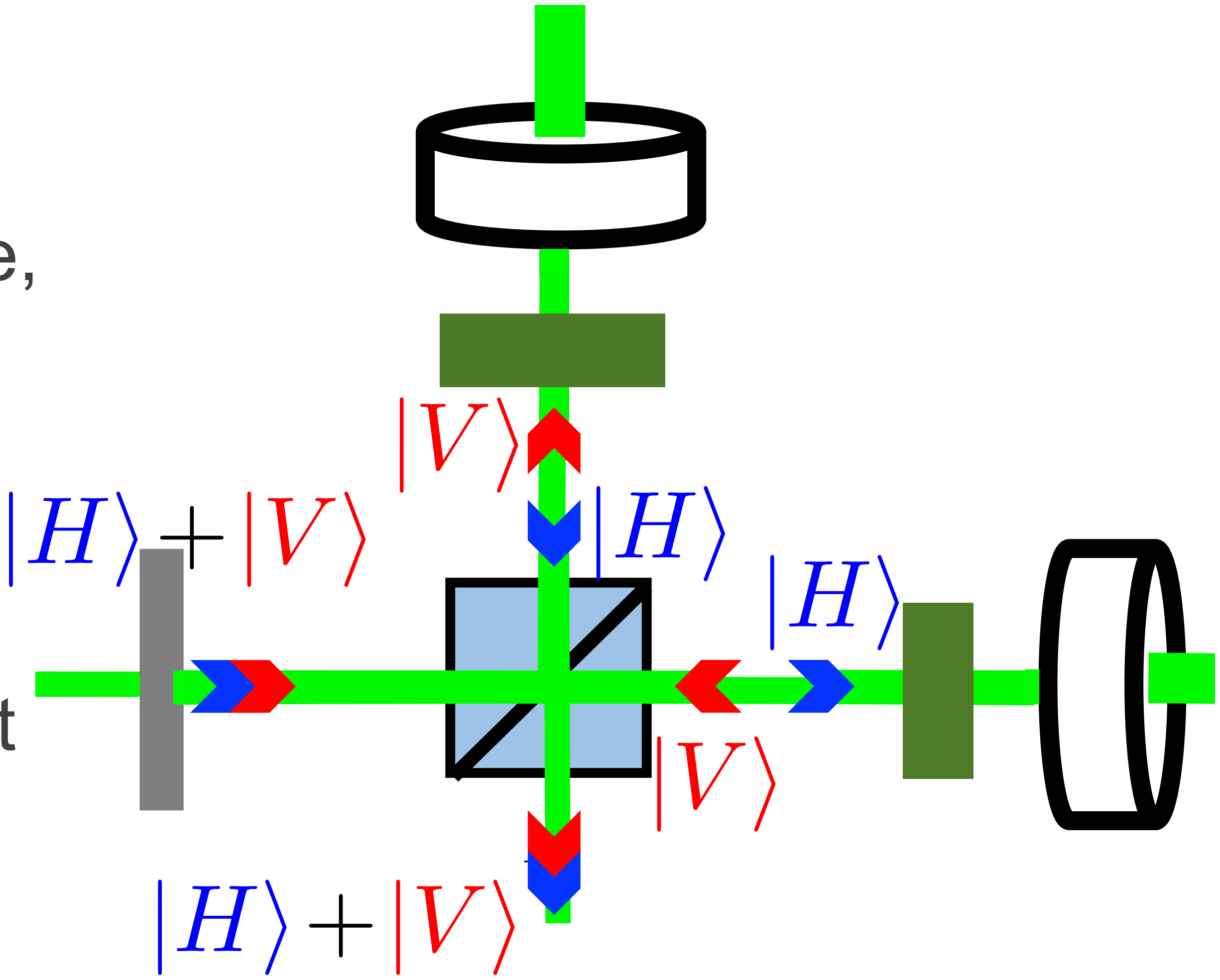


Polarization of photons in the MI

- PBS1: $|H\rangle$ is transmitted,
  $|V\rangle$ is reflected
- QWP: if photons passes it twice,

  It converts

  - $|H\rangle \rightarrow |V\rangle$
  - $|V\rangle \rightarrow |H\rangle$
- All photons go to the signal port



The preselection and the postselection

- The initial state is

$$|\Psi_i\rangle_{SP} = (r_1|down\rangle + t_1|up\rangle) \otimes |+\rangle$$

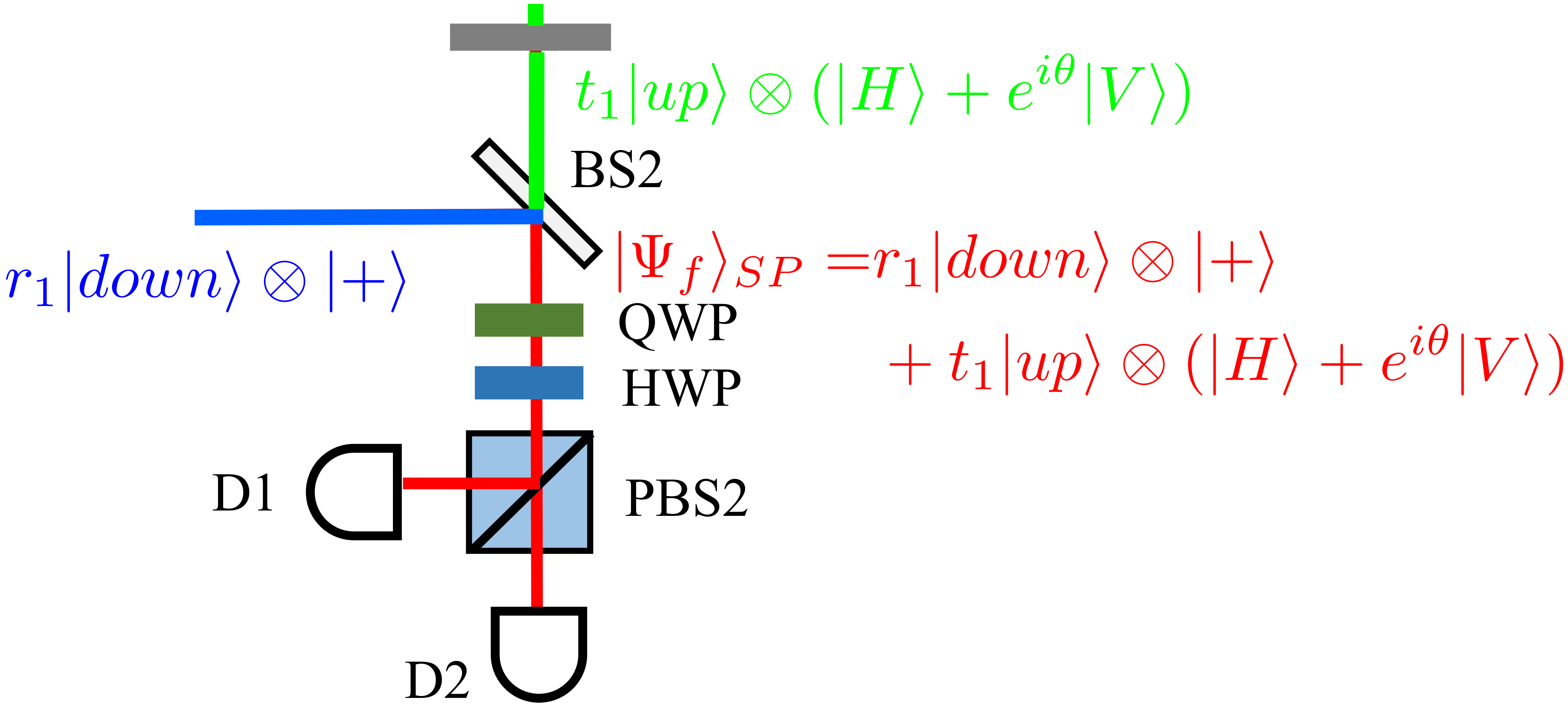
- If GW comes in, the phase between $|H\rangle$ and $|V\rangle$ is slightly change:

$$|\Psi_f\rangle_{SP} = r_1|down\rangle \otimes |+\rangle + t_1|up\rangle \otimes (|H\rangle + e^{i\theta}|V\rangle)$$

- Finally, we postselect the final states:

$$|\psi_f\rangle_S = r_2|down\rangle + t_2|up\rangle$$

Signal port



$t_1|up\rangle \otimes (|H\rangle + e^{i\theta}|V\rangle)$
BS2
$r_1|down\rangle \otimes |+\rangle$
$|\Psi_f\rangle_{SP} = r_1|down\rangle \otimes |+\rangle$
$+\, t_1|up\rangle \otimes (|H\rangle + e^{i\theta}|V\rangle)$
QWP
HWP
D1
PBS2
D2

Signal amplification

- This is the same as previous example :

$$\alpha = r_1 \, , \beta = t1$$

$$\gamma = r_2 \, , \eta = t_2$$

- Then,

$$|\phi\rangle_P = \frac{1}{\sqrt{2}}(|H\rangle + e^{i\varphi}|V\rangle)$$

$$\tan\varphi = \frac{\theta}{1 + \frac{r_1 r_2}{t_1 t_2}}$$

- If we choose r_1, r_2, t_1, t_2 so as to $r_1 r_2 + t_1 t_2 \to 0$, signal θ is much amplified

Conclusion

- Using WMA-LIGO, we can amplify the signal by the factor $h \sim 10^3$
- Shot noise and radiation pressure noises are treated as usual FPMI
- All photons go to the signal port
  - ➢ It means that the MI is at the bright fringe
  - ➢ Shot noise is worst than ordinary interferometer?

- Quantum measurements theory is difficult

End

Setting

- Suppose the interaction Hamiltonian is

$$\hat{H} = \alpha \hat{A} \otimes \hat{\sigma}_y \,,$$

where $\hat{A} = |0\rangle\langle 0| - |1\rangle\langle 1| \,,$

$$\hat{\sigma}_y = |R\rangle\langle R| - |L\rangle\langle L| = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

- The initial state is

$$|\Psi_i\rangle_{PS} = |\psi_i\rangle_P \otimes |H\rangle \,,$$

$$|\psi_i\rangle_P = \alpha|0\rangle + \beta|1\rangle$$

- The final state of the system is

$$|\psi_f\rangle_P = \gamma|0\rangle + \eta|1\rangle$$

Calculation

- If the interaction occurred between small time Δt,
  the time evolution is :

$$|\Psi_f\rangle_P = e^{-i\theta\hat{A}\otimes\hat{\sigma}_y}|\Psi_i\rangle$$

  where $\theta = \Delta t\alpha$

- Then, the final state of the pointer is

$$|\phi_f\rangle_P = \langle\psi_f|\Psi_f\rangle \simeq \langle\psi_f|\psi_i\rangle e^{-i\theta A_w\hat{\sigma}_y}|H\rangle$$

  if the weak value condition is satisfied.

- After some algebra, we get

$$|\phi_f\rangle_P = \langle\psi_f|\psi_i\rangle e^{-i\theta A_w\hat{\sigma}_y}(\cos\chi|H\rangle + \sin\chi|V\rangle)$$

$$\simeq \langle\psi_f|\psi_i\rangle(|H\rangle + \chi|V\rangle)$$

  if $\chi = \theta A_w \ll 1$

- Using $|\phi_f\rangle_P$, we can calculate A_w as follows:

$$\langle\hat{\sigma}_x\rangle = 2\theta\mathrm{Re}(A_w)\,,$$

$$\langle\hat{\sigma}_y\rangle = 2\theta\mathrm{Im}(A_w)$$