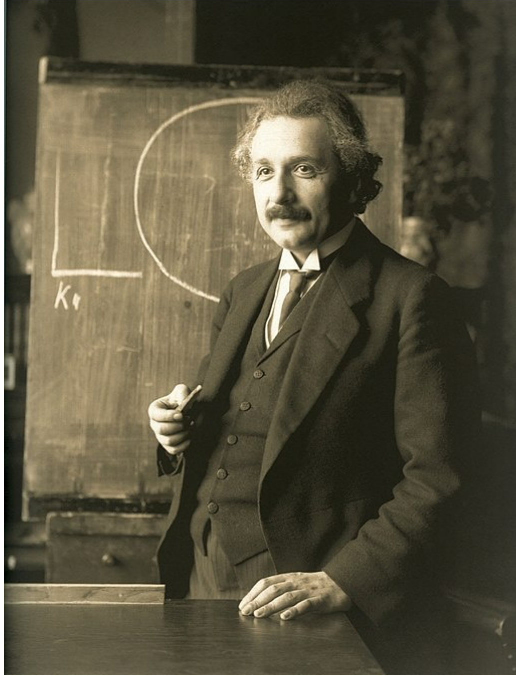
Nonequilibrium Thermodynamics and Gravitational Wave Detectors

Satoru Takano

22/11/2019 @ Ando Lab Seminar

By the way…



We so much owe to Einstein.

In 1905 (Annus Mirabilis)

- Special relativity
    - Finally it evolved to general relativity, and predict GW
- Photoelectric effect
    - Quantum mechanics. Then we get coherent light source.
- Brownian motion
    - Fluctuation dissipation theorem, stochastic thermodynamics.

All above is much related to our research!

Today’s seminar is related to third one.

Abstract

Today, I will talk some topics about nonequilibrium thermodynamics related to GW detector community.

Main topic:

- Stochastic Thermodynamics
- Fluctuation Theorem
- Information Thermodynamics

Motivation

Equilibrium state = macroscopic quantity do not change in time

How about our experiment?

- Laser: Strongly pumped
- Feedback: Controlled
- (Cryogenic: Heat flow through fibers and heatlinks)

We often consider systems far from equilibrium!

For better understanding of our experiment, we should care about physics under nonequilibrium state.

Brief History of NETD (1)

- Einstein's relation (1905):
  - Einstein, 1905
  - Relation btw diffusion of particle and viscosity

$$D = \mu k_\mathrm{B} T$$

diffusion constant

mobility, $\mu = \frac{v_d}{F}$

- Johnson noise
  - Measurement by Johnson, Theory by Nyquist (1928)
  - Thermal fluctuation of electrical voltage (or current)

$$S_V(\omega) = 4 k_\mathrm{B} T R$$

single-sided PSD
of voltage

resistance

Brief History of NETD (2)

- Fluctuation dissipation theorem (FDT)
  - Callen, Welton (1952)
  - Generalization of Johnson noise to arbitrary system

$$S_F(\omega) = 4k_\mathrm{B}TR(\omega)\frac{\hbar\omega}{2}\coth\left(\frac{\beta\hbar\omega}{2}\right)$$

$Z(\omega) = \dfrac{F(\omega)}{v(\omega)}$: generalized impedance, $R(\omega) = \mathrm{Re}[Z(\omega)]$: generalized resistance

- Levin's approach
  - Levin (1998)
  - Mathematically equivalent, calculated easily

$$S_x(\omega) = \frac{8k_\mathrm{B}T}{\omega^2}\frac{W_\mathrm{diss}}{{F_0}^2}$$

dissipation energy per period

applied force $F = F_0 \cos\omega t$

Brief History of NETD (3)

- Linear resopnse theory
  - Kubo, Nakano, … (1950s)

$$C_{\{A,B\}}(\omega) = \coth\left(\frac{\beta\hbar\omega}{2}\right) C_{[A,B]}(\omega)$$

$C_{\{A,B\}}(\omega) := \int_{-\infty}^{\infty} dt e^{i\omega t}\langle\{\hat{A}(t), \hat{B}(0)\}\rangle$: fluctuation at equilibrium

$C_{[A,B]}(\omega) := \int_{-\infty}^{\infty} dt e^{i\omega t}\langle[\hat{A}(t), \hat{B}(0)]\rangle$: resopose function

All of these relation state that fluctuation at equilibrium is related to response to external force applied

far from equilibrium ↔ equilibrium

related!

Noise out of equilibrium

Can we satisfied with FDT?

- No! There is shot noise!

$$S_I(\omega) = 2e\langle I \rangle$$

PSD of current

average current

Apparently shot noise and Johnson noise is different.

- Johnson noise depends on temperature, but shot noise doesn’t
- Johnson noise exists even average current flows, but shot noise appear only when average current exists

As a stochastic process, there noises can be described as

- Johnson noise: Gaussian
- Shot noise: Poissonian

Stochastic Thermodynamics

Conventional thermodynamics

- System is macroscopic
- Dynamics is deterministic

Stochastic thermodynamics

- System is mesoscopic (e.g. a particle driven by brownian motion)
- Dynamics is stochastic

Langevin Equation

Langevin equation

$$m\ddot{x} = -\gamma\dot{x} + F(x,\lambda) + \xi$$

$F(x,\lambda) = -\dfrac{\partial U(x,\lambda)}{\partial x}$: some controllable potential force

ξ : random force acted by the thermal bath, $\langle \xi(t)\xi(t')\rangle := D^2\delta(t-t')$

- Then, modifying the equation like this:

$$dp = \left(-\frac{\gamma}{m}x + F(x,\lambda)\right)dt + \xi dt\ , \quad dp = \frac{x}{m}dt$$

Here, $\xi dt = DdW$ is Gaussian process.

Now x and p are stochastic quantities. If you know the probability density of them, you can calculate any quantities depending x and p.

1st Law of Thermodynamics

- Next, define heat $\langle d'Q \rangle$ work $\langle d'W \rangle$as follows:

$$d'Q := (\gamma \dot{x} - \xi) \circ dx \,, \quad d'W := \frac{\partial U}{\partial \lambda} d\lambda$$

exerted force x velocity
→ energy dissipation

potential change
due to external operation
→ work done by external system

- From Langevin equation, we get:

$$dU = d'Q + d'W$$

- 1st law of thermodynamics in terms of stochastic process!

FDT Appears Naturally

- Consider average heat flow:

$$\langle d'Q \rangle = \frac{2\gamma}{m}\left(\left\langle \frac{p^2}{2m} \right\rangle - \frac{D^2}{4\gamma}\right)$$

- At equilibrium state, 1st term is equal to $k_\mathrm{B}T/2$
- And, at equilibrium there is no heat flow

$$\left\langle \frac{p^2}{2m} \right\rangle = \frac{k_\mathrm{B}T}{2} \qquad \left\langle \frac{p^2}{2m} \right\rangle - \frac{D^2}{4\gamma} = 0$$

→ $D = \sqrt{2\gamma k_B T}\,, \langle \xi(t)\xi(t') \rangle = 2\gamma k_\mathrm{B} T \delta(t - t')$

This means $S_\xi(\omega) = 4\gamma k_\mathrm{B} T$ → FDT!

Some Remarks

- Stochastic view of system can describe thermodynamics structure
  - 1st law of thermodynamics
  - 2nd law of thermodynamics (I omit its derivation)
- If the system is at equilibrium, FDT also appears naturally.
- Next, let's consider the system far from equilibrium by stochastic way
  - Fluctuation theorem

Fluctuation Theorem

A strong relation which holds even when the system is out of equilibrium.

Consider “forward process” (ordinary time evolution) and “backward process” (time-reversed and T transformation) process.

Now define the measurement probability of a quantity Ω as

- ○ Forward: $p(\Omega)$
- ○ Backward: $p^{\dagger}(-\Omega)$

Fluctuation states that

$$p^{\dagger}(-\Omega) = p(\Omega)e^{-\Omega} \qquad (k_{\mathrm{B}} = 1)$$

even the system is out of equilibrium!

FT Family

Last equation is one of FT. There are many “so-called FT”

- Crooks FT

σ: entropy generation rate

$$\frac{P^{\dagger}(-A)}{P(A)} = e^{-\sigma}$$

- Detailed FT

$$p(-\Omega) = p(\Omega)e^{-\sigma}$$

- Integral FT

zoology of theorem…

$$\langle e^{-\sigma} \rangle = 1$$

- Jarzynski equality

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$

Review of FTs: Rep. Prog. Phys. 75 126001

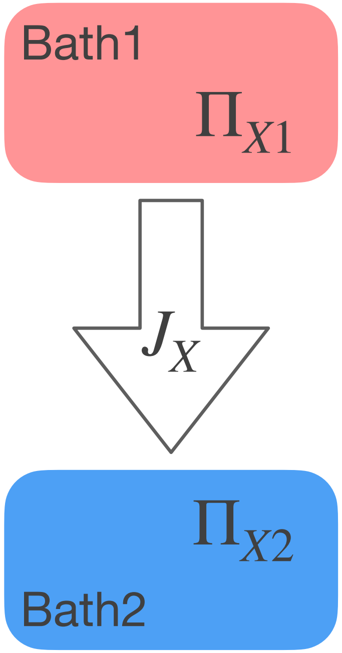
FT contains FDT (1)

- Consider two baths 1,2 and between baths current of X flows.
- Define intensive variables conjugative to X in terms of entropy and energy:

$$\Pi_X := \frac{\partial S}{\partial X}, P_X := \frac{\partial U}{\partial X} = T\Pi_X$$

- Suppose that flowing current is J_X and difference of Π_X btw 2 baths is $\Delta\Pi_X$, entropy generation rate is written as:

$$\sigma = J_X \Delta\Pi_X$$



FT contains FDT (2)

- From integral FT,

$$P(J_X) = P(-J_X)e^{\sigma} = P(-J_X)e^{J_X \Delta\Pi_X}$$

- Taking average, you'll get this:

$$\langle J_X \rangle = \langle -J_X e^{-J_X \Delta\Pi_X} \rangle$$

- Now consider the situation near equilibrium. Expanding RHS and considering up to $O(\Delta\Pi_X)$ terms,

- $$\langle {J_X}^2 \rangle \Big|_{\Delta\Pi_X=0} = 2\left.\frac{\partial\langle J_X\rangle}{\partial\Delta\Pi_X}\right|_{\Delta\Pi_X=0} = 2T\left.\frac{\partial\langle J_X\rangle}{\partial\Delta P_X}\right|_{\Delta P_X=0}$$

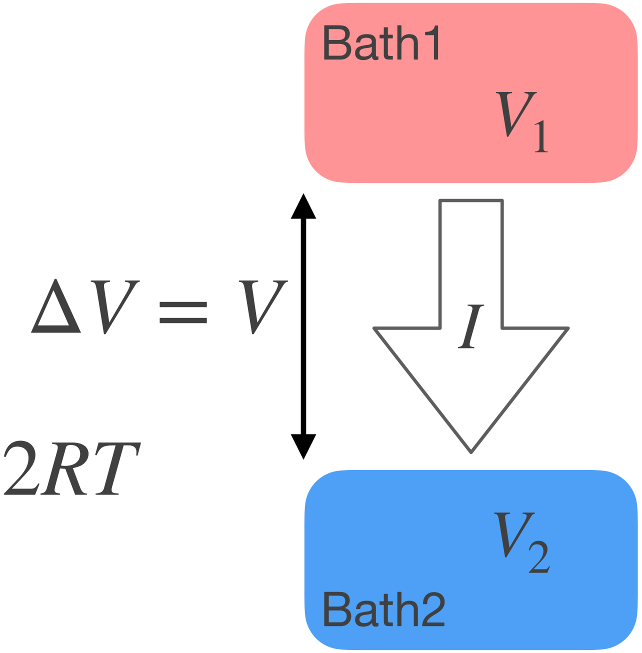
FT contains FDT: Example

Let's apply it to a circuit.

- Now, $X = q$ (charge), $\Delta P_X = V$ (voltage), $J_X = \dot{q} = I$ (current)
  - $$\frac{\partial \langle J_X \rangle}{\partial \Delta P_X} = \frac{\langle I \rangle}{V} = \frac{1}{R}$$
- Substituting this, you'll get

$$\langle I^2 \rangle = \frac{2T}{R} \quad \text{or} \quad \langle V^2 \rangle = 2RT$$

Johnson noise!



Violation of FDT under Steady Flow

Harada-Sasa (HS) equality: violation of FDT

PRL 95, 130602 (2005)

- For (over-damped) Langevin system

thermal force

$$\gamma \dot{x} = F(x) + f(x) + \xi$$

external force to measure R(t)

relation btw PSD of velocity and responce function:

$$\frac{J}{\gamma} = {v_s}^2 + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left[C(\omega) - k_{\mathrm{B}} T(R(\omega) - R(-\omega)) \right]$$

= 0 if equilibrium

≠ 0 for nonequilibrium steady state

$C(\tau) := \langle (\dot{x}(\tau) - v_s)(\dot{x}(0) - v_s) \rangle$: autocorrelation of velocity

$\langle \dot{x}(t) \rangle = v_s + \int_{-\infty}^{t} dt' R(t - t') f(t')$: response function

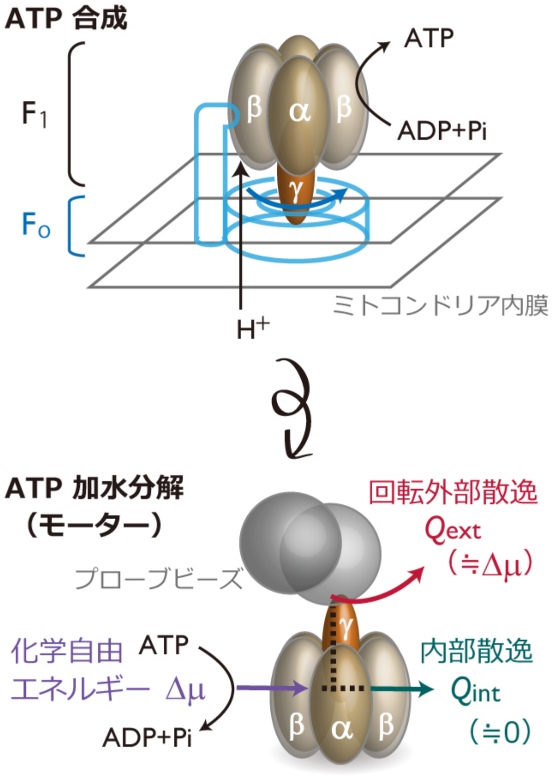
$Jdt = \langle \, (\gamma \dot{x} - \xi) \circ dx \rangle$: energy dissipation

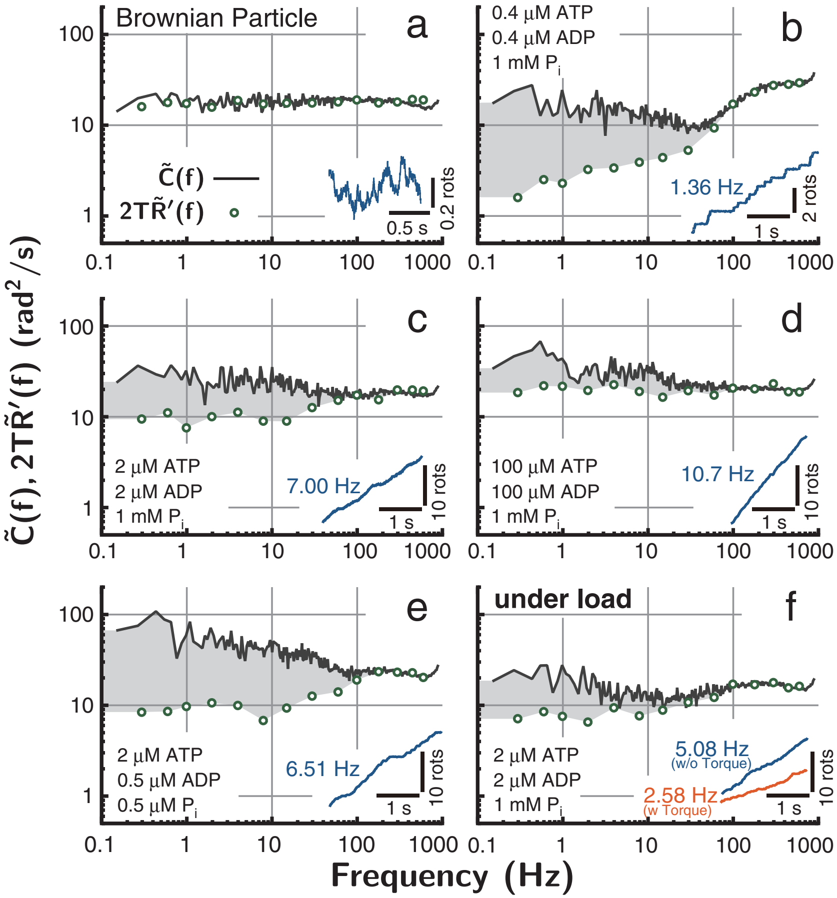
Application of HS equality

Measurement of energy loss of molecular motors

PRL 104, 198103 (2010)

- Estimation of energy dissipation at steady state (→ nonequilibrium)





line: $C(\omega)$

circle: $R(\omega)$

shaded: violation

Information Thermodynamics

Relation between work & information obtained by measurement

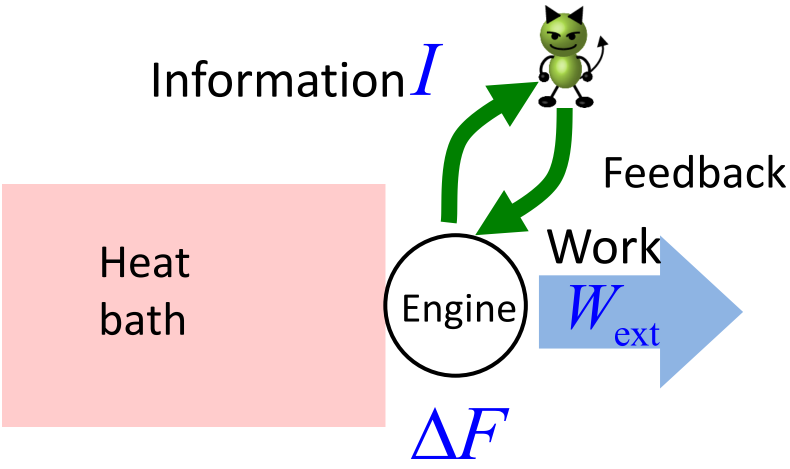
$$\Delta S_{\mathrm{system}} - \beta\langle Q\rangle \geq \Delta I$$

It says that:

Entropy change of the system must be grater than entropy change of the bath + information obtained by the measurement

Equivalently,

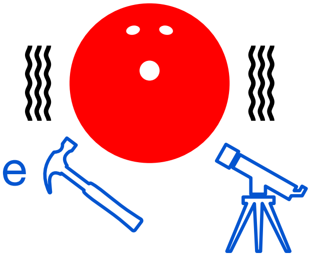
$$W_{\mathrm{ext}} + \Delta F \leq k_{\mathrm{B}}\Delta I$$



Cooling Limit by Feedback

What can we extract from information thermodynamics?

- ▶ Cooling limit by feedback control



feedback force

measurement

- For a Langevin system like

$$m\ddot{x} = -\gamma\dot{x} + F_{\mathrm{fb}}(x,\lambda) + \xi$$

- Effective temperature is bounded by mutual information:

$$\frac{T - T_{\mathrm{eff}}}{T} \leq \left\langle \sum_{i}^{t_i \in [0, \frac{m}{\gamma}]} I_i \right\rangle$$

$$\frac{k_{\mathrm{B}} T_{\mathrm{eff}}}{2} := \left\langle \frac{m\dot{x}^2}{2} \right\rangle$$

mutual information
obtained by measurement

- How much Information you got during system’s timescale puts limit for feedback cooling

PRE 84, 021123 (2011)

Application to our research

Harada-Sasa equality

- Energy loss measurement → Q value ?

Feedback cooling limit

- Better way to damp oscillation?

Other concern

- How about quantum case? → under discussion

My ambition:

- Unified description of quantum noise due to photons and thermal noise of oscillators
- Analytical calculation of nonequilibrium thermal noise of cooled pendulum (like Komori-san’s paper)

References

Stochastic thermodynamics

- K. Sekimoto, Stochastic energetics, Vol. 799. Springer (2010)
- 関本謙，ゆらぎのエネルギー論，岩波書店 (2004)

Fluctuation Theorem

- U. Seifert, Rep. Prog. Phys. 75 126001
- Documents by N. Shiraishi
- Documents by S. Ito

Information thermodynamics

- T. Sagawa and M. Ueda, arXiv:1111.5769
- Documents by T. Sagawa