Improved wave front sensor using a Gouy phase compensation cavity

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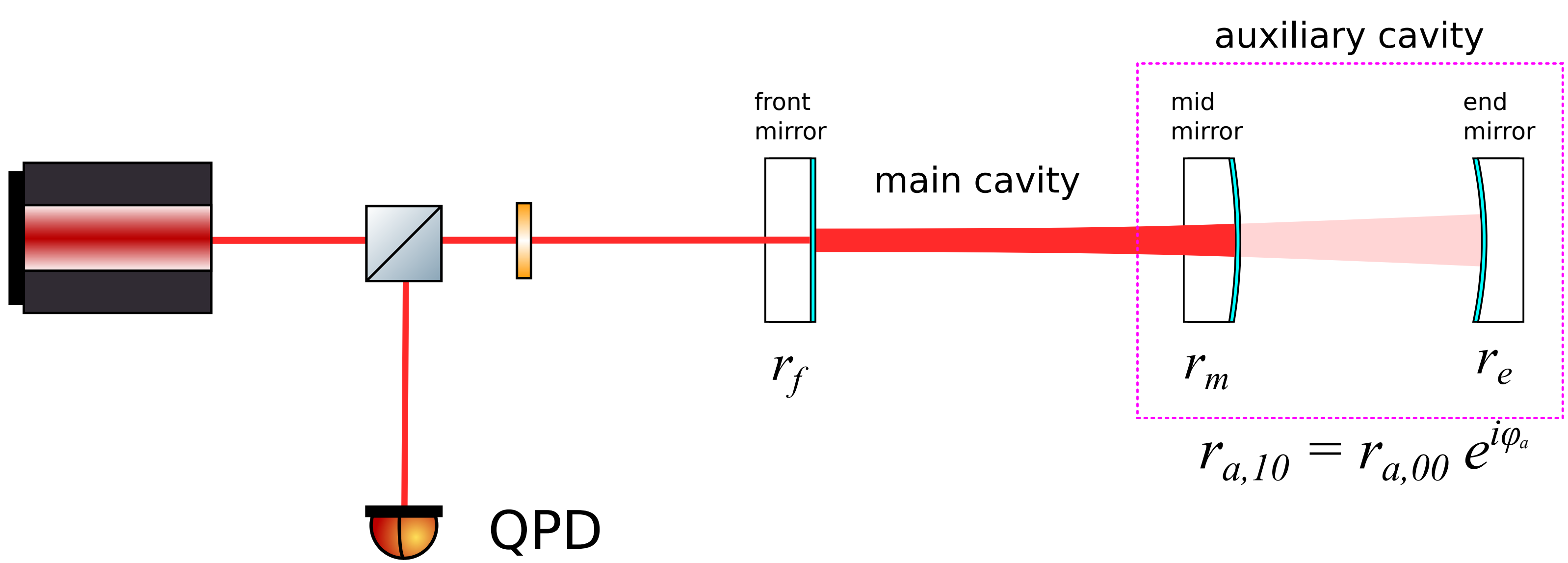
Ando lab. seminar on 22 Dec. 2017

Abstract

- We are developing an improved angular sensor
- sensitivity of wave front sensor(WFS) can be improved by using auxiliary cavity for HG_{10} mode resonance
- it gives us many merits and a few demerits compared with conventional angular sensors



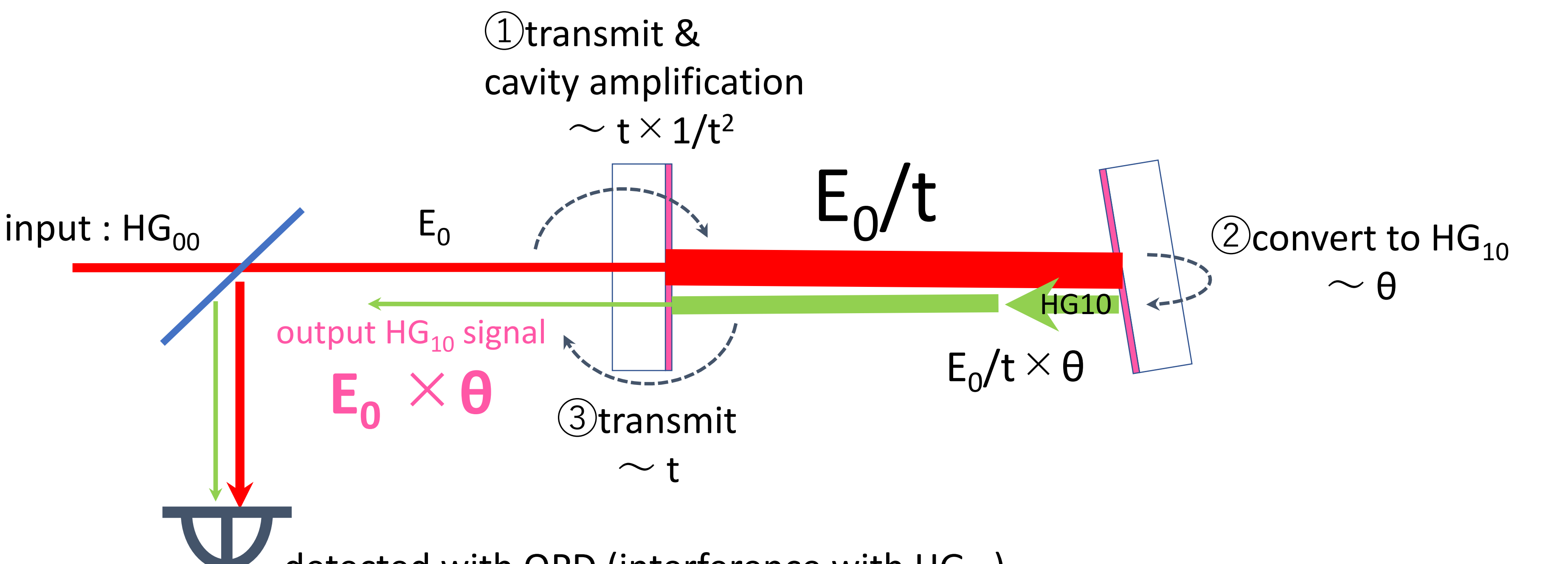
Contents

- principle
- merits & demerits
- technical issues
- design for TOBA
- summary

principle

principle of wave front sensing

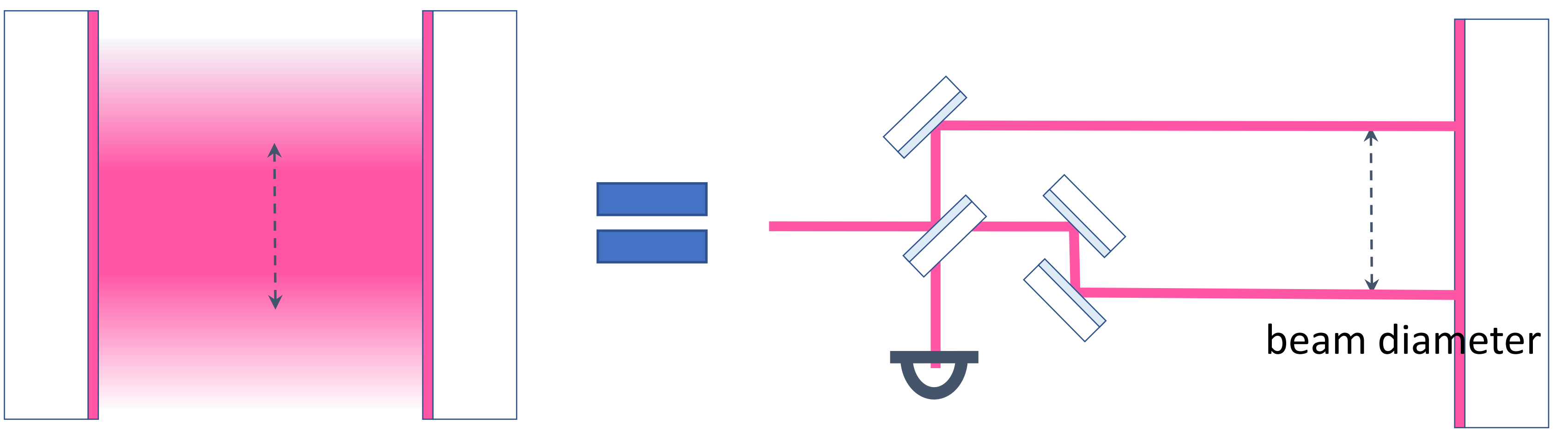
- tilted mirror converts HG_{00} to HG_{10}
- HG_{10} mode from the cavity is detected as angular signal
- no cavity amplification on HG_{10} mode signal (same as optical lever)



detected with QPD (interference with HG_{00})
(or single PD with jitter modulation (beat with HG_{10} sideband))

similar to MI

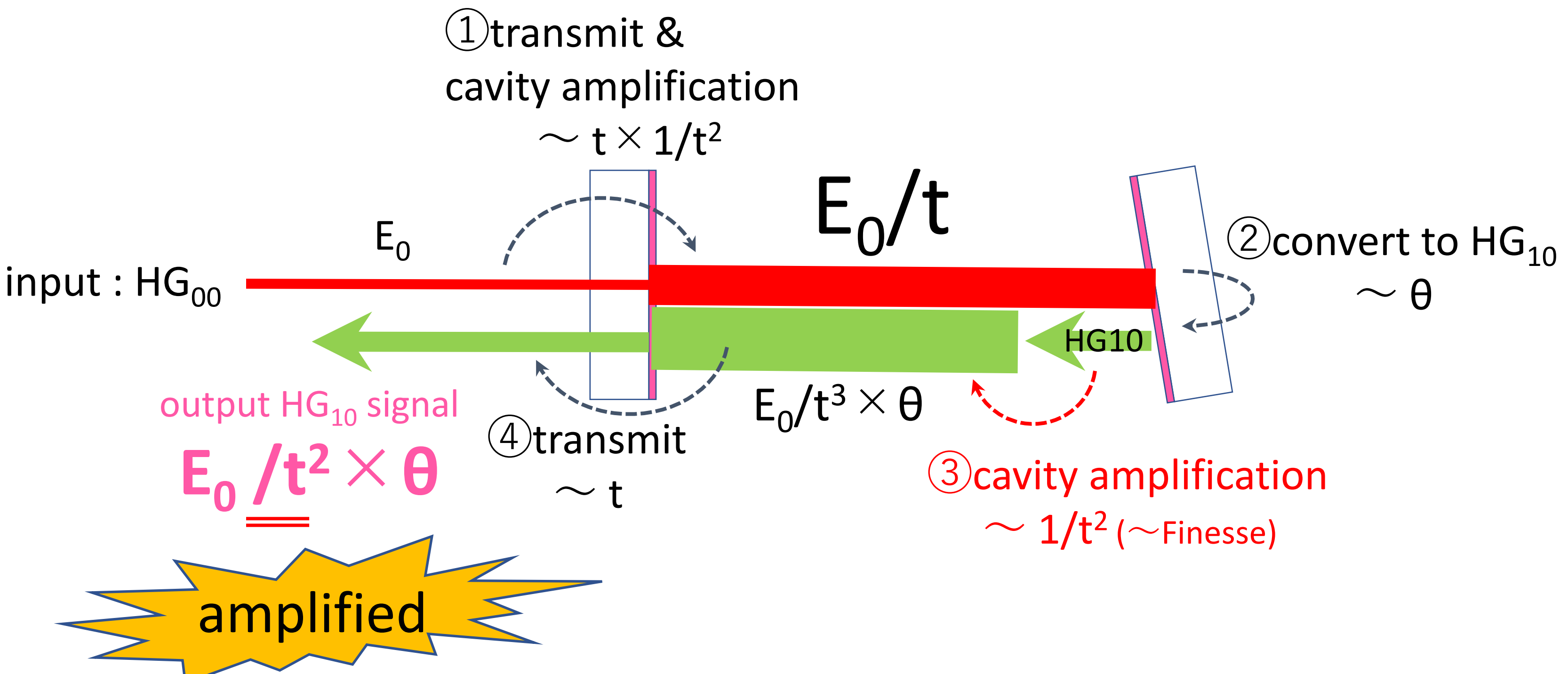
- sensitivity of conventional WFS is in the same order as Michelson interferometer whose arm separation is beam diameter
- because 10 mode signal is not amplified by the cavity



$\Rightarrow$ low sensitivity

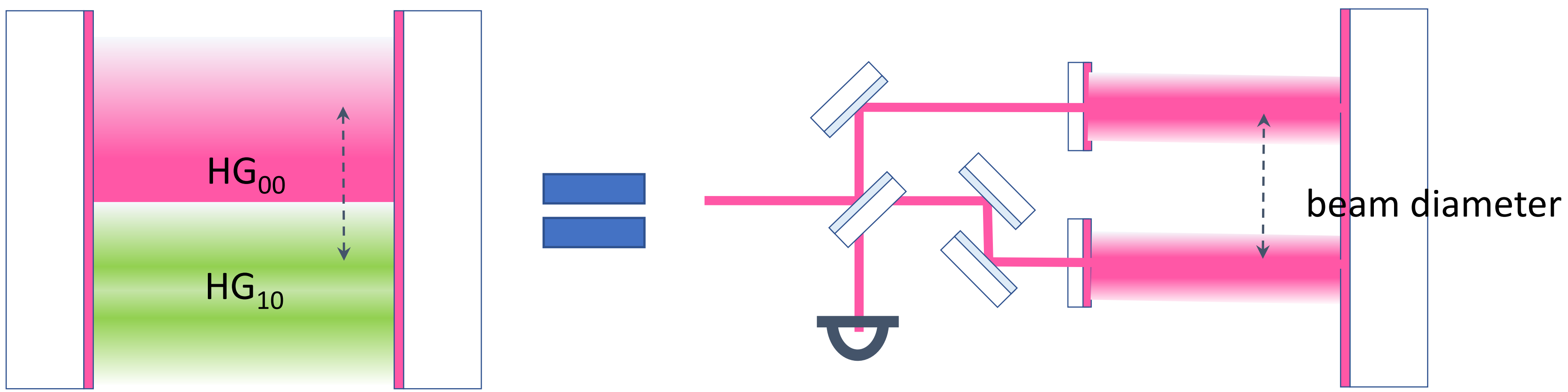
improvement idea

- If HG_{10} mode can resonate inside the cavity, generated signal is amplified



similar to FPMI

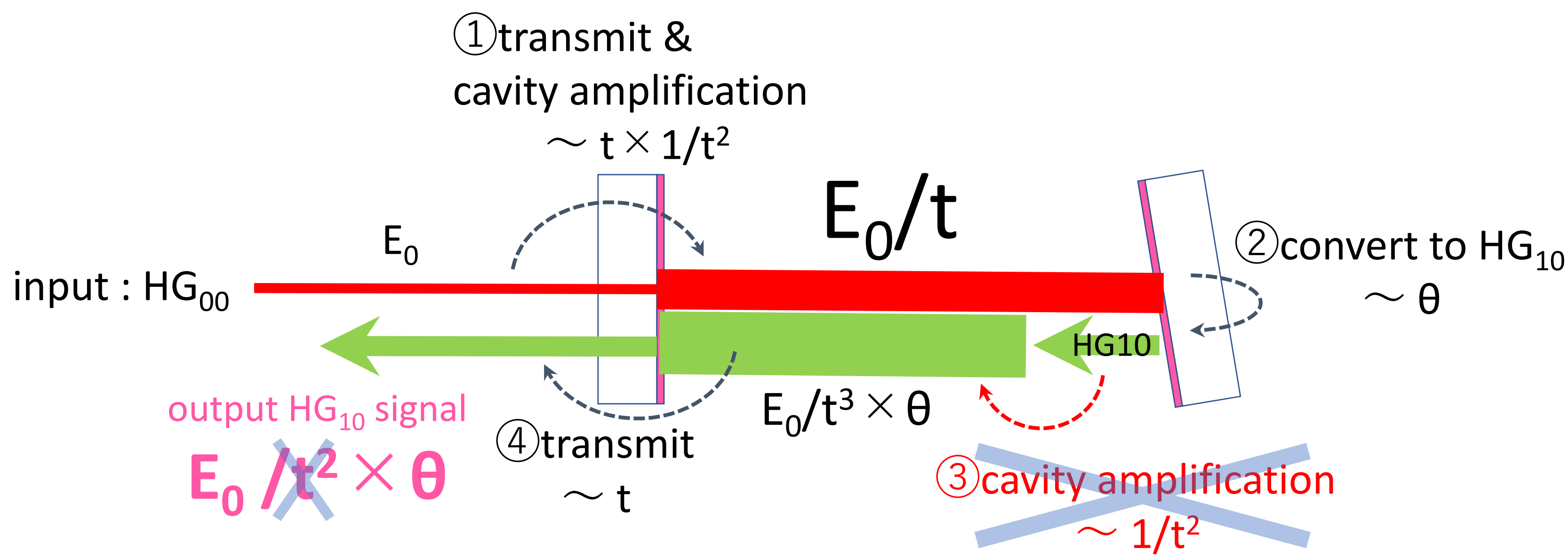
- sensitivity of improved sensor is in the same order as Fabry-Perot Michelson interferometer whose arm separation is beam diameter



$\Rightarrow$ high sensitivity

normal cavity cannot realize this

- HG_{10} mode gets different phase from HG_{00} mode by Φ_{Gouy} therefore they cannot resonate at the same time in a normal cavity

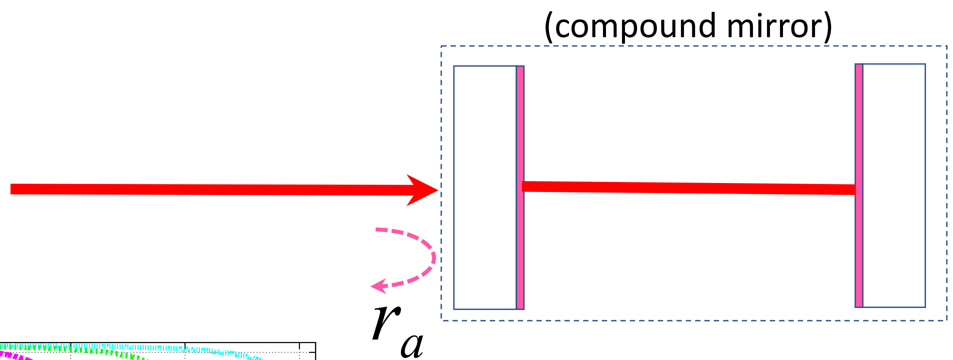


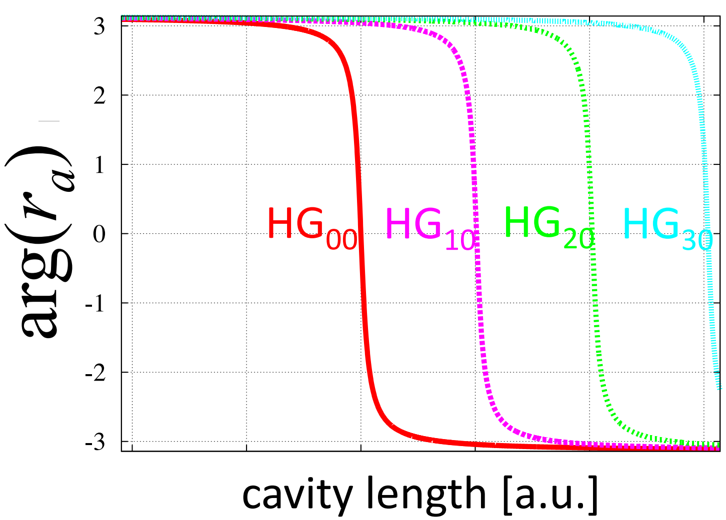
(round-trip phase)

$$\varphi_{rt,10} = \varphi_{rt,00} + \Phi_{Gouy}$$

idea : auxiliary cavity for Gouy phase compensation

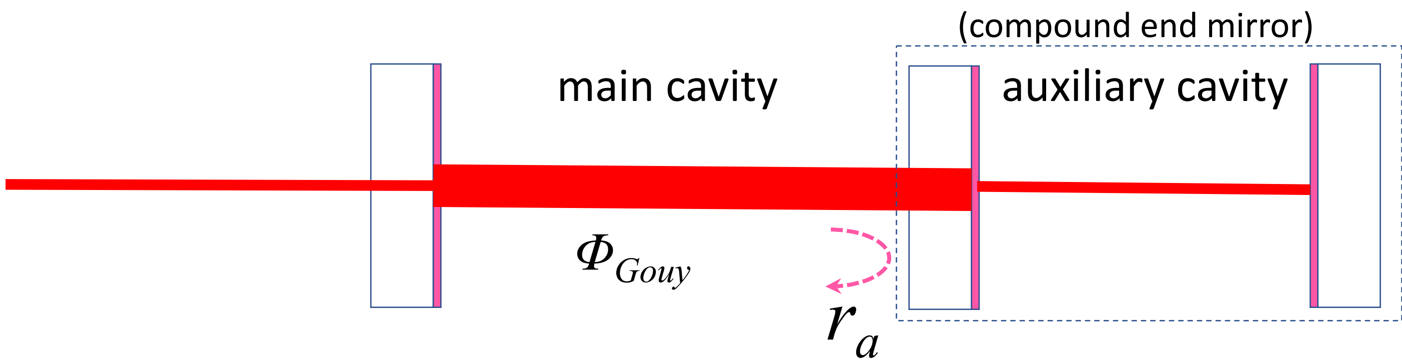
- cavity has different reflection phase for each modes





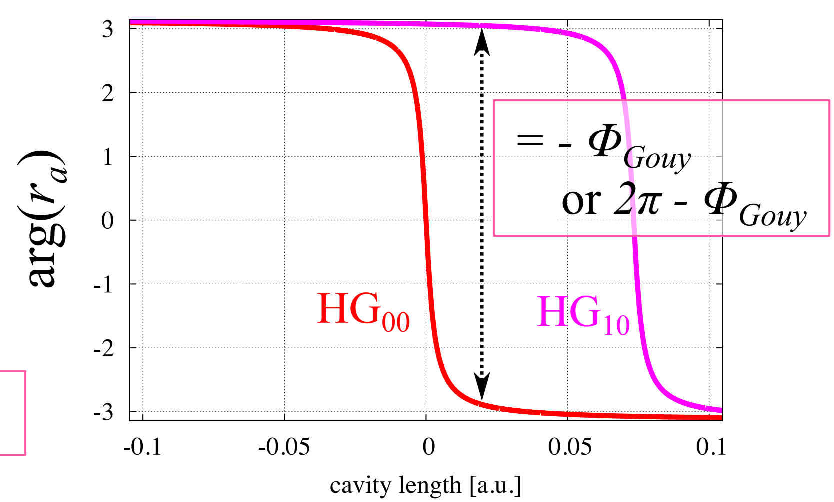
idea : auxiliary cavity for Gouy phase compensation

- reflection phase of the auxiliary cavity can compensate Φ_{Gouy} in the main cavity



round-trip phase in the main cavity

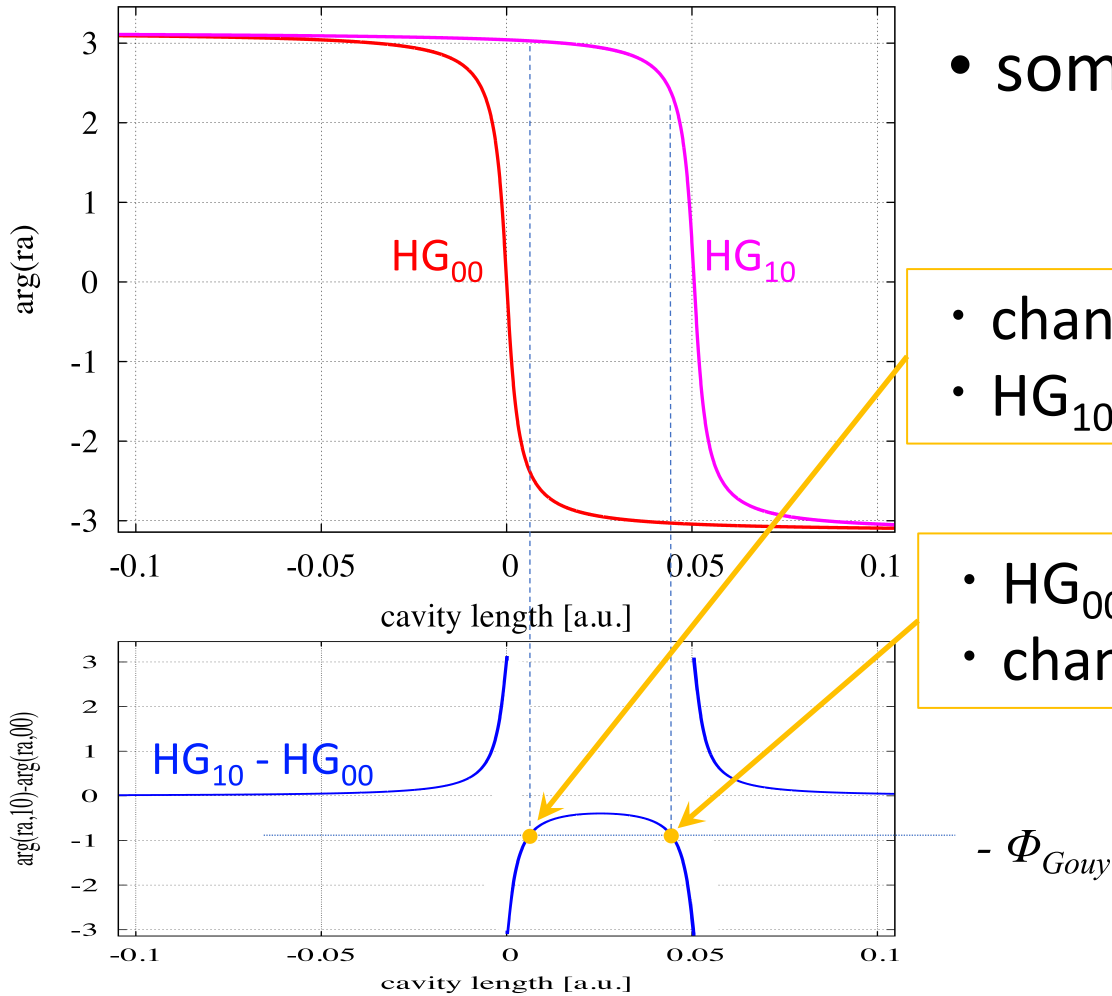
$$\varphi_{rt,10} = \varphi_{rt,00} + \Phi_{Gouy} + \arg(r_{a,10}) - \arg(r_{a,00}) = \varphi_{rt,00} + 2n\pi$$



⇒ resonate HG_{10} and HG_{00} at the same time

lock points of auxiliary cavity

- detuned from resonance
- some solutions

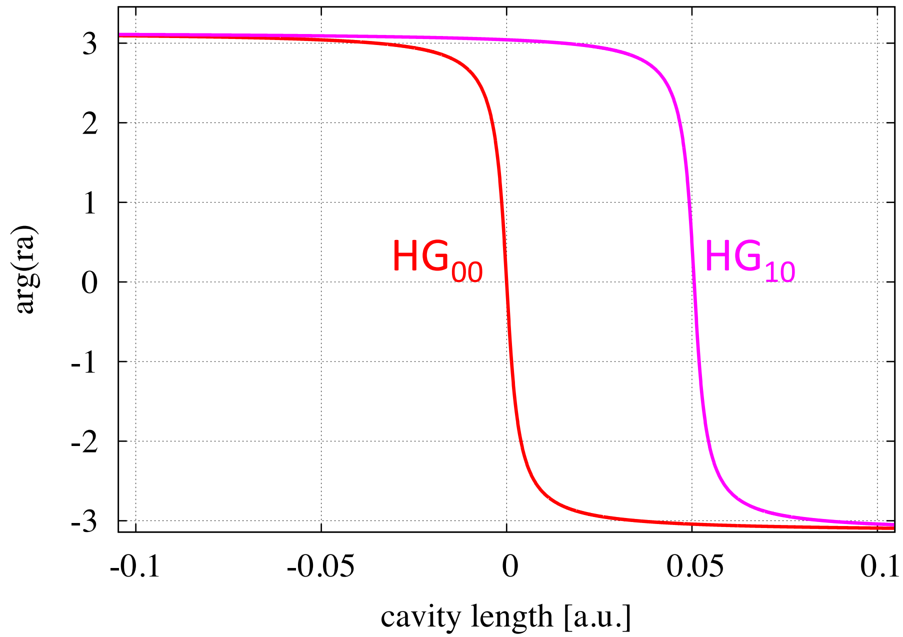


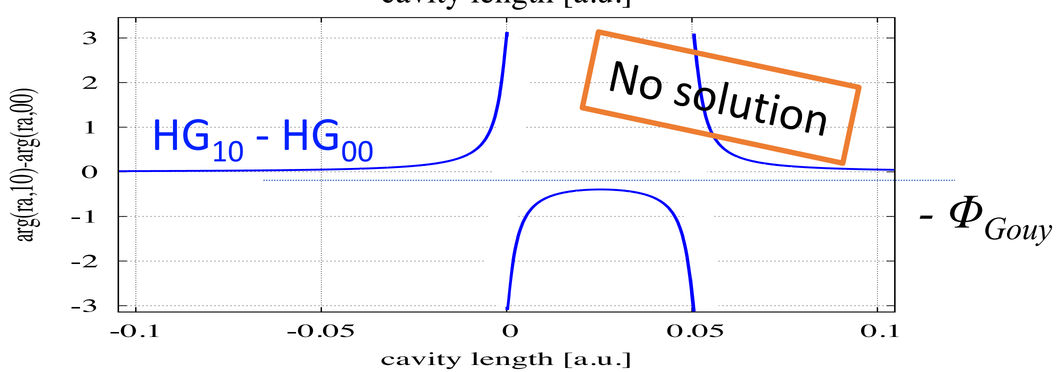
- changing HG_{00} phase
- HG_{10} is anti-resonant

or

- HG_{00} is anti-resonant
- changing HG_{10} phase

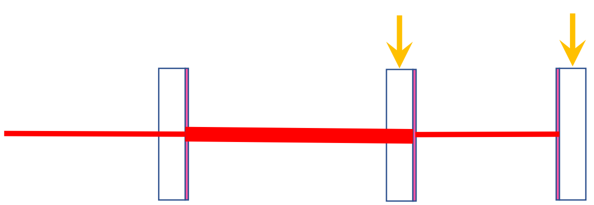
solutions don't always exist





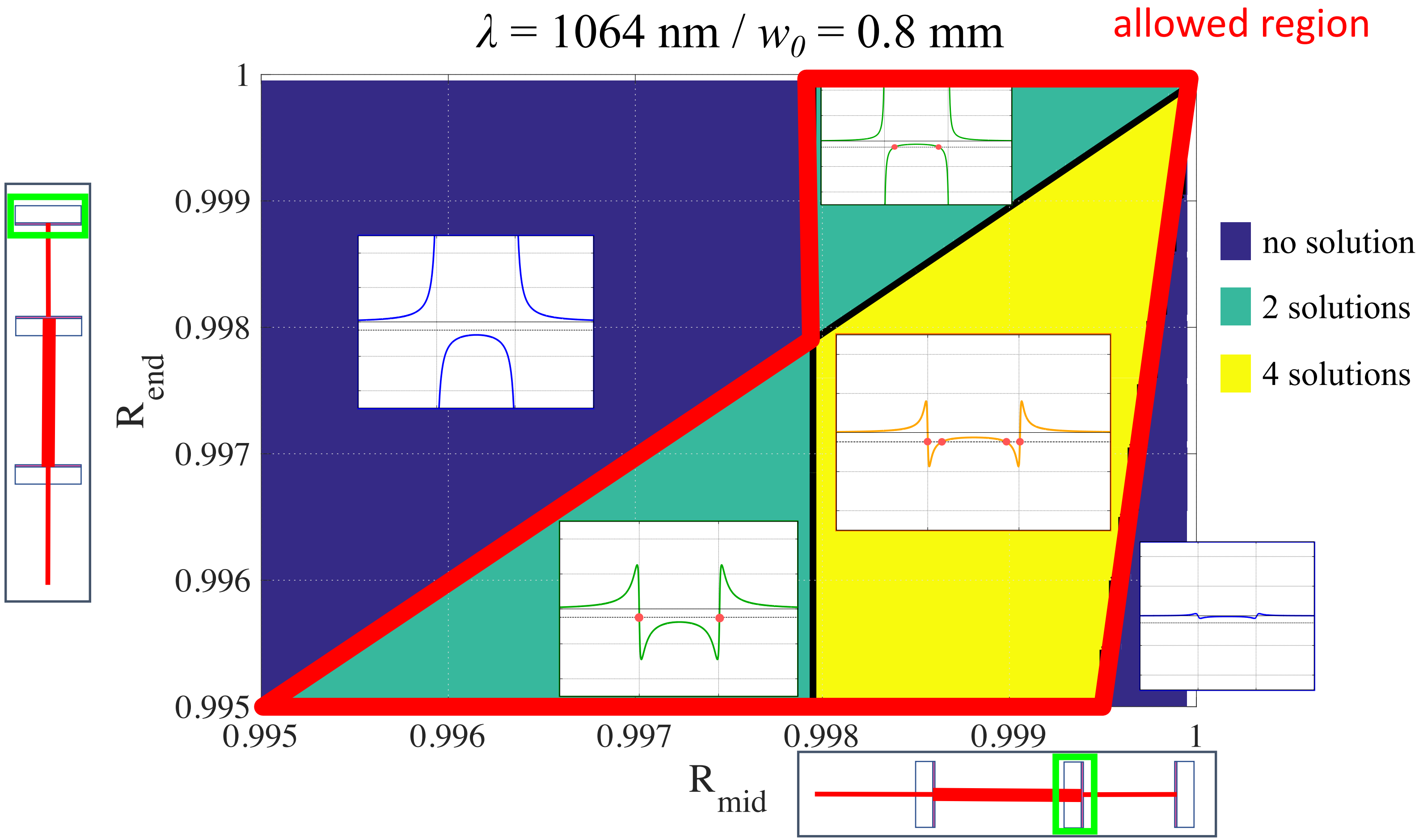
- require...
  - enough large Φ_{Gouy}
    - long cavity
    - small beam

    (these are limited by chamber size and thermal noise requirement)
- proper reflectivity of auxiliary cavity mirrors



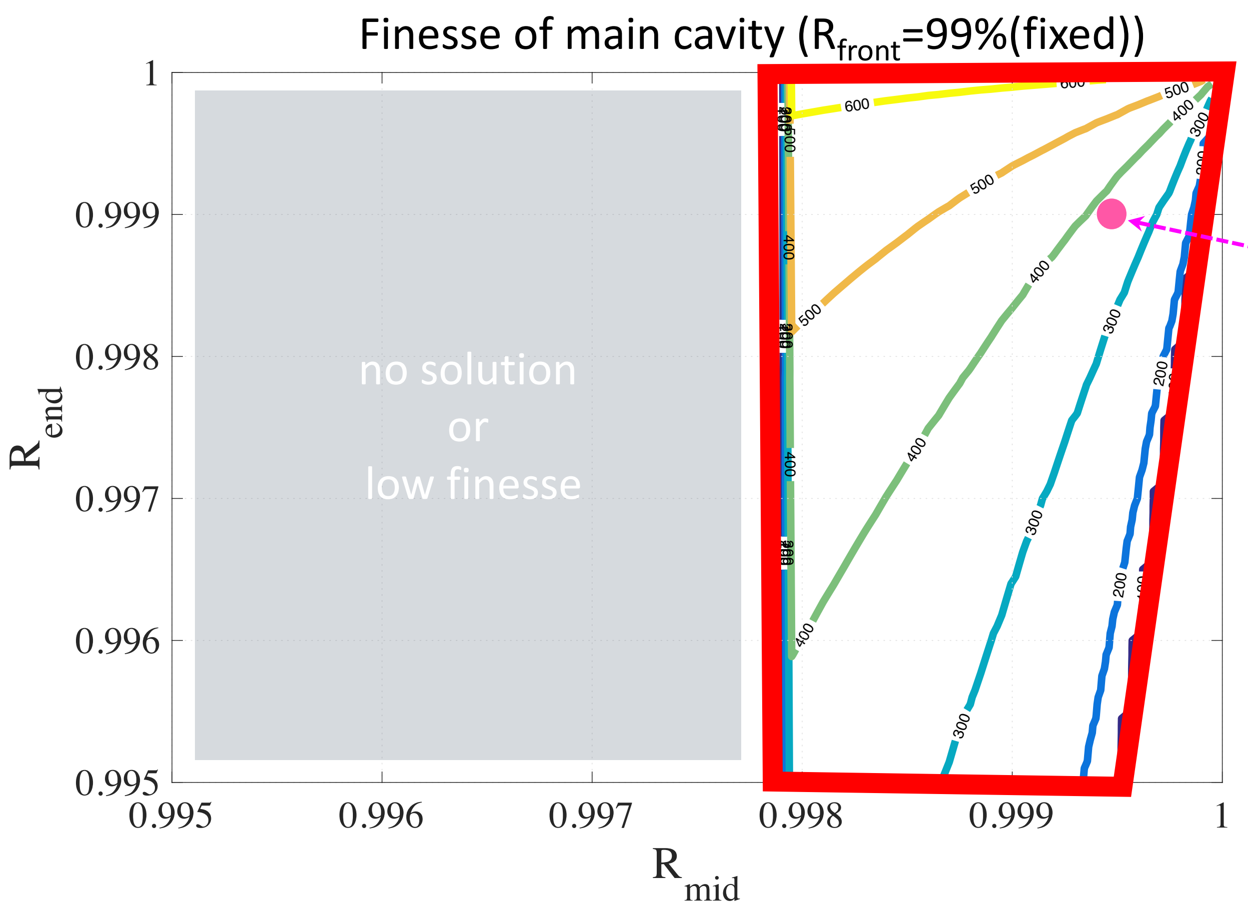
allowed reflectivity

- cavity length (10cm) & beam size (0.8mm) are fixed



preferred reflectivity

- enough finesse of the main cavity is required for angular signal amplification

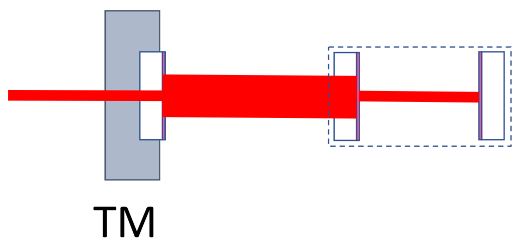


(example)
99.95% / 99.9%
→ F = 360
(these parameters are chosen in following calculations)

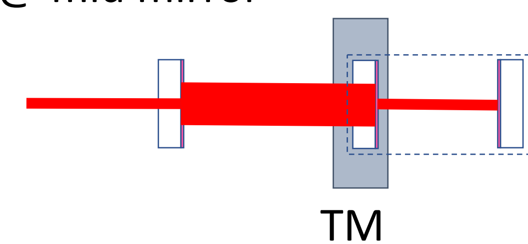
possible configurations

- test mass can be put anywhere

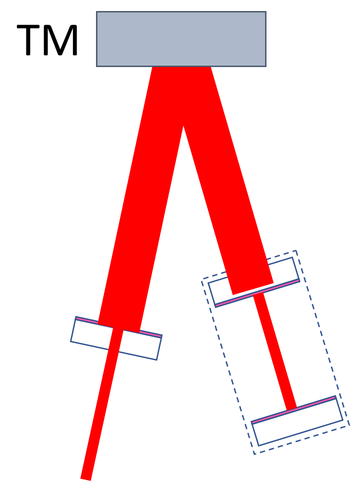
@ front mirror



@ mid mirror



inserted in cavity



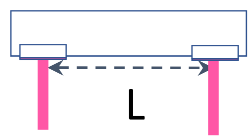
merits & demerits

low shot noise

- ⇔ high angular response (almost same as MI)

Michelson (P_{in}:input power, L:bar length)

$$\sim \; P_{in}\,\frac{L}{\lambda}\;\;[\mathrm{W/rad}]$$



new WFS (w:beam radius, F:finesse)

$$\sim \; P_{in}\,\frac{w}{\lambda}\,F\;\;[\mathrm{W/rad}]$$

new WFS with 1mm beam & finesse 300

⇔ MI with 30cm bar

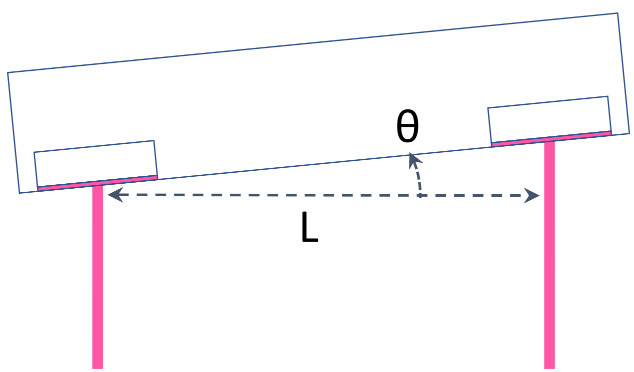
- same as MI shot noise level

$\sim$ 10^{-15} rad/rtHz

large linear range

than Michelson interferometers

- roughly larger by (bar length) / (beam diamerer)



$\theta < \lambda/L \sim 1\text{urad}$

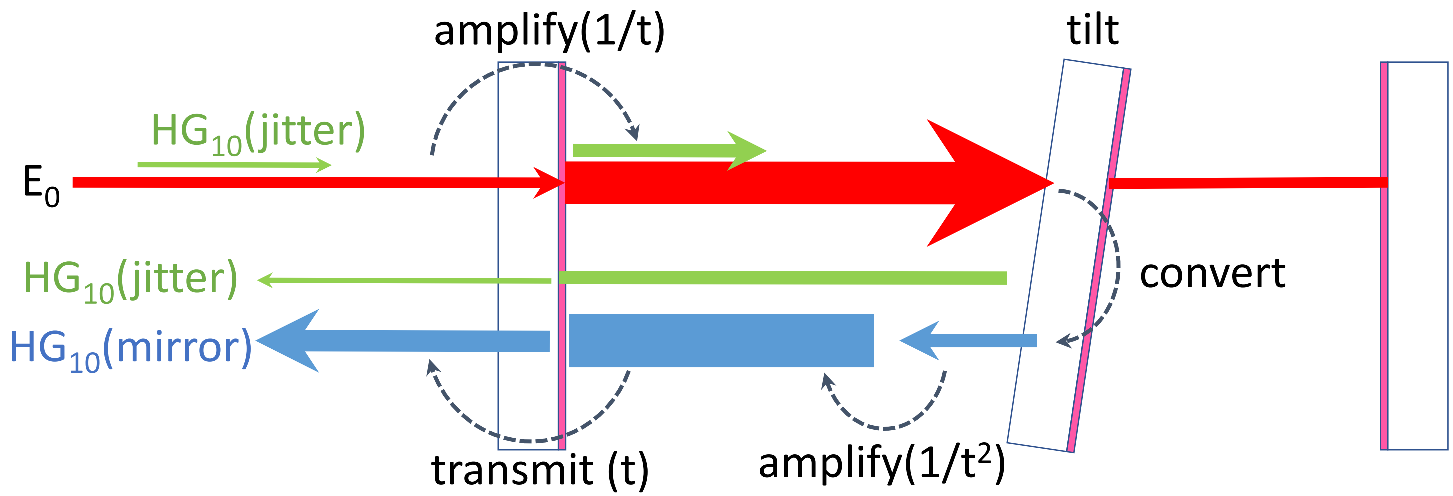
$\theta < \lambda/w \sim 0.1\text{mrad}$

- less(no) angular control is required ⇔ less(no) actuator noise
- (practically, range of PD should be considered)

low beam jitter noise

than conventional WFSs

- suppressed by finesse



beam jitter θ_{jitter} : generate HG_{10} <u>outside the cavity</u>

$$\Rightarrow E_{10,jitter} = E_0 \times \theta_{jitter} \times \underset{\text{(amplify)}}{1/t} \times \underset{\text{(transmit)}}{t}$$

cavity mirror tilt θ_{mir} : generate HG_{10} <u>inside the cavity</u>

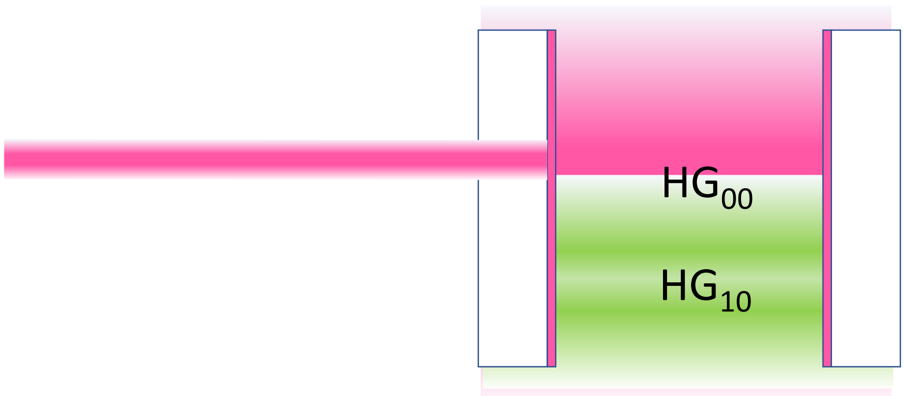
$$\Rightarrow E_{10,mirror} = E_0 \times \underset{\text{(amplify)}}{1/t} \times \theta_{mir} \times \underset{\text{(amplify)}}{1/t^2} \times \underset{\text{(transmit)}}{t}$$

$$E_{10,jitter} \sim t^2 \times E_{10,mirror}$$

$$(\sim E_{10,mirror} / F)$$

no(low) frequency noise

- no frequency noise in wave front sensor



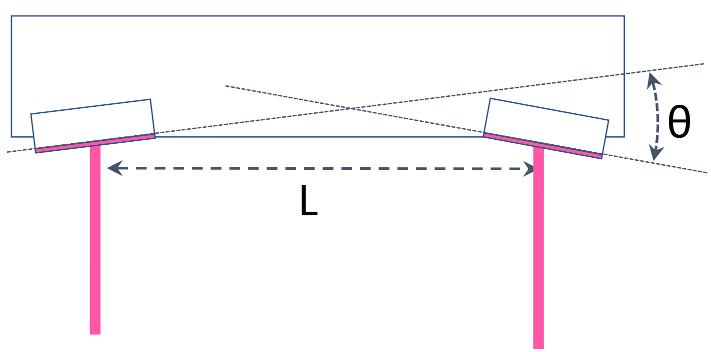
- (spatial nonuniformity of frequency fluctuation can be noise ...?)

small trans-coupling(?)

than Michelson interferometers

- making flatness is easier in smaller scale
- especially this is good point for large bars (L>50cm) which are difficult to make flatness between both ends

Michelson interferometer



~ bar size (~m)

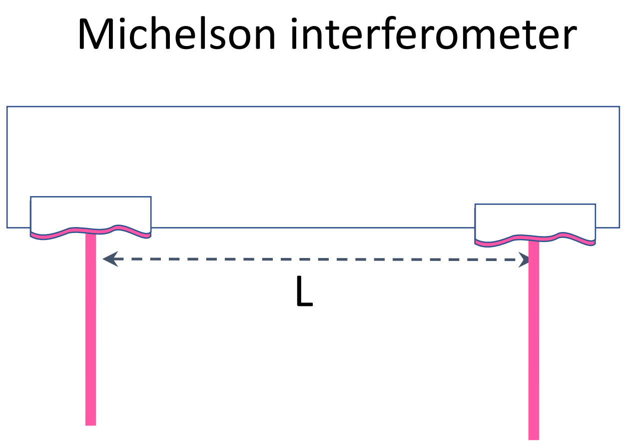
WFS

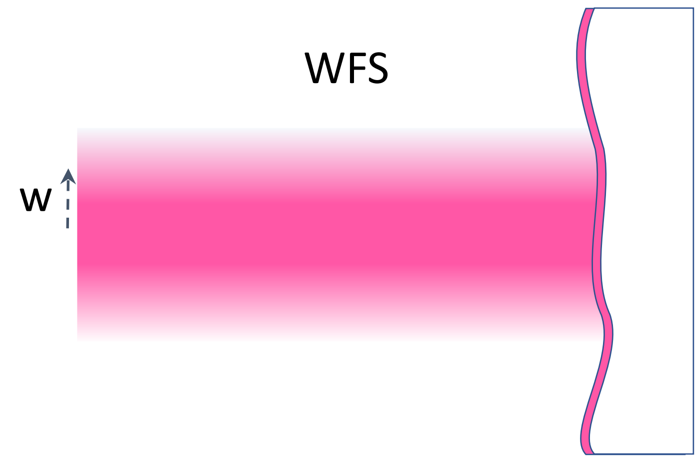
~ beam size (~mm)

higher thermal brownian

than Michelson interferometers

- higher by (bar length) / (beam diameter)





noise ~ (fluctuation) / L

noise ~ (fluctuation) / w

summary of merits & demerits

- new sensor is the best !!

	conventional WFS	Michelson interferometer	new WFS
shot noise	☹	☺	☺
linear range	☺	☹	☺
beam jitter	☹	😐?	☺
frequency noise	☺	☹	☺
trans-coupling	☺	☹	☺
thermal noise	☹	☺	☹

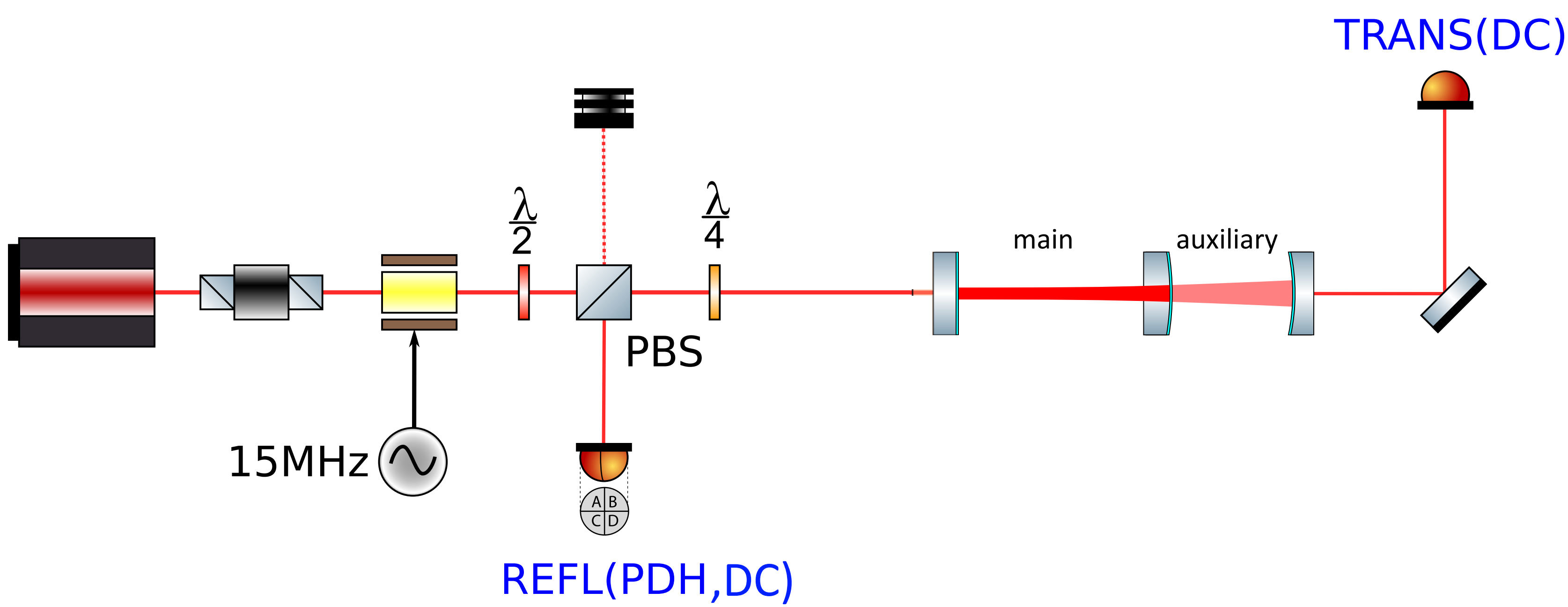
← not dominant in TOBA

technical issues

①length signal extraction

- main cavity : Refl PDH signal
- auxiliary cavity : DC signal (Trans – (const) × Refl)

to set dc offset 0

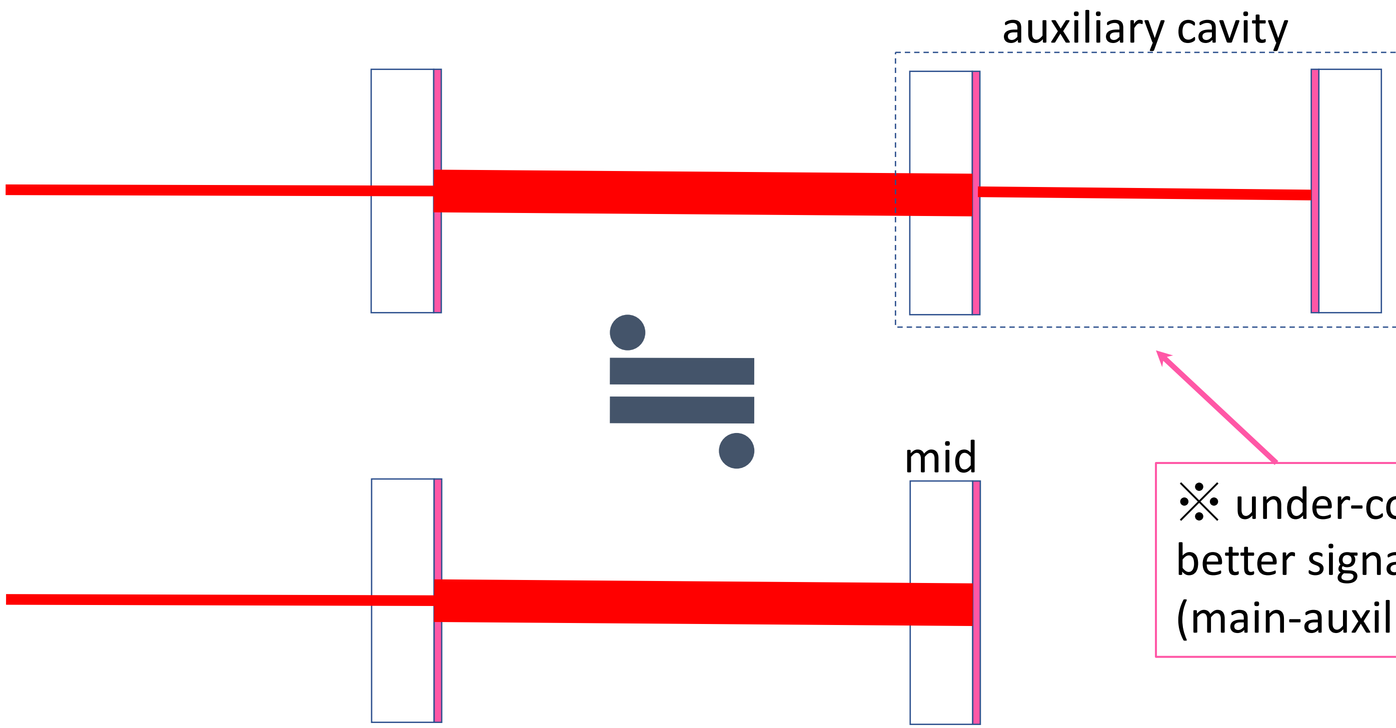


main cavity length signal

- auxiliary cavity is detuned from HG_{00} resonance

  → auxiliary cavity works as well as single mid mirror for HG_{00}

  ⇒ refl PDH siganal mainly contains main cavity length signal

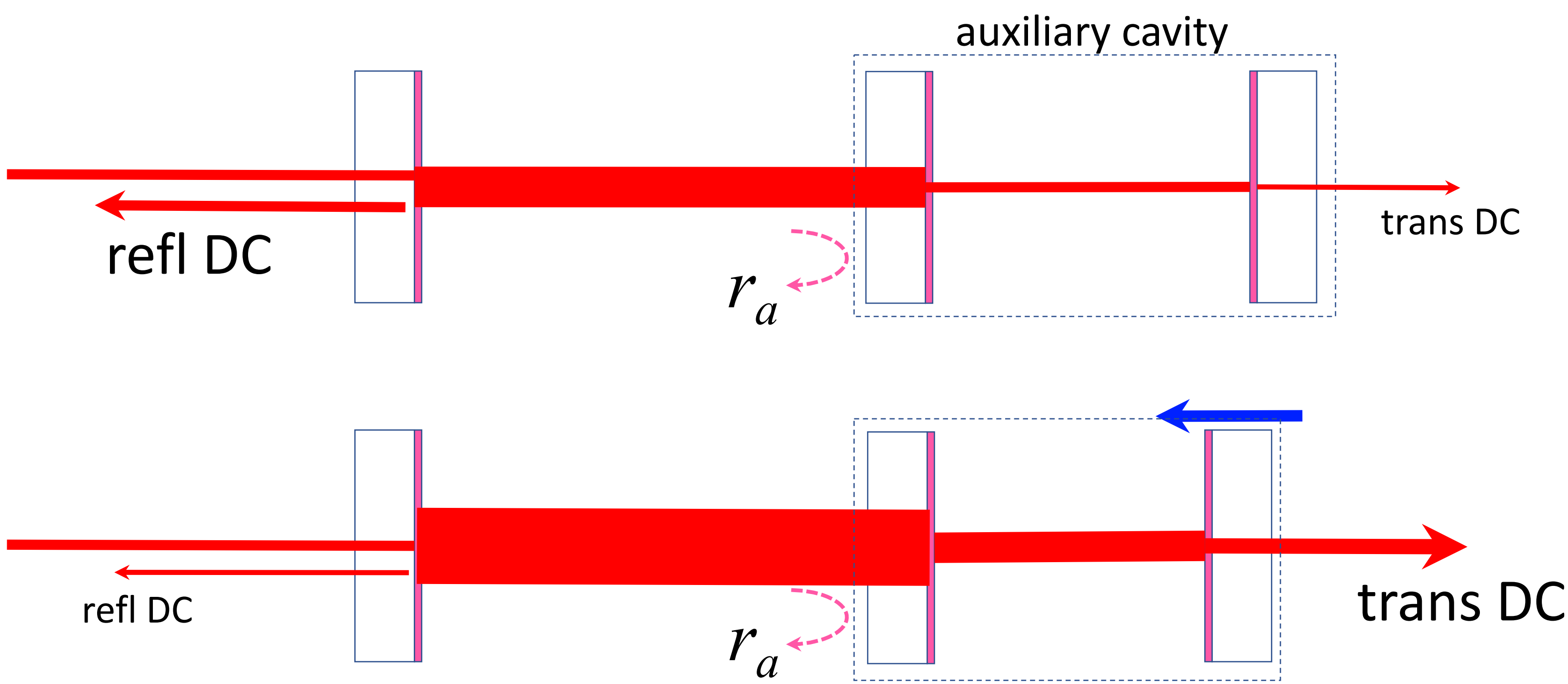


※ under-coupled cavity gives better signal separation (main-auxiliary)

auxiliary cavity length signal

- reflectivity of auxiliary cavity $\mathrm{abs}(r_a)$ depends on auxiliary cavity length

  $\Rightarrow$ trans/refl DC power changes



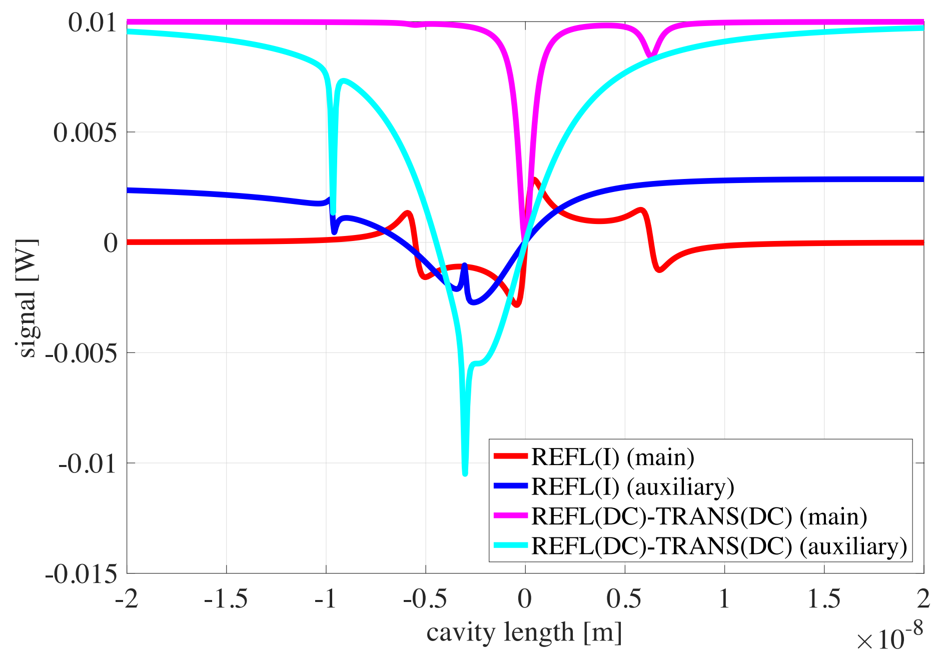
signal separation

- not bad

(R_{front} = 99% ,

R_{mid} = 99.95% ,

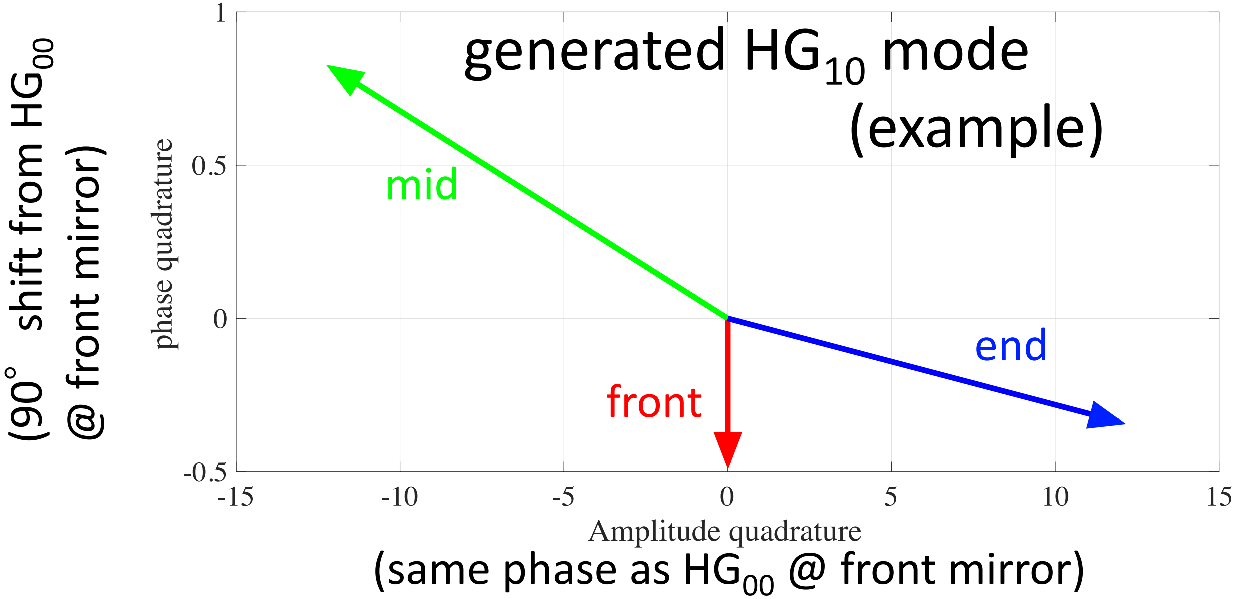
R_{end} = 99.9%)



sensing matrix	main cavity	auxiliary cavity
PDH signal (refl)	1	0.09
DC signal (refl-trans)	0	1

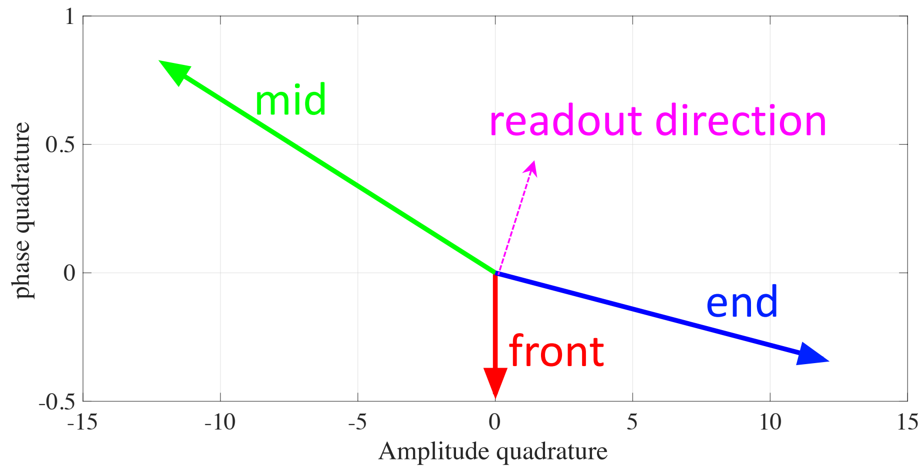
②coupling from other mirrors

- every mirror can generate HG_{10} mode



coupling cannot be eliminated

- not enough degrees of freedom to separate signals
  - 2 readout quadrature vs 3 mirrors
- (by choosing readout direction, coupling from only one mirror can be removed)



(semi-)monolithic mirrors may be necessary

⇒ how much is the angular fluctuation of mirrors?

③length-angle coupling

- cavity length control (mid, end mirror)

  actuator asymmetry

- shake mirror angle

  signal coupling

- appear in signal

(very rough estimation)

$\sim 10^{-9}$ m/rtHz @0.1Hz

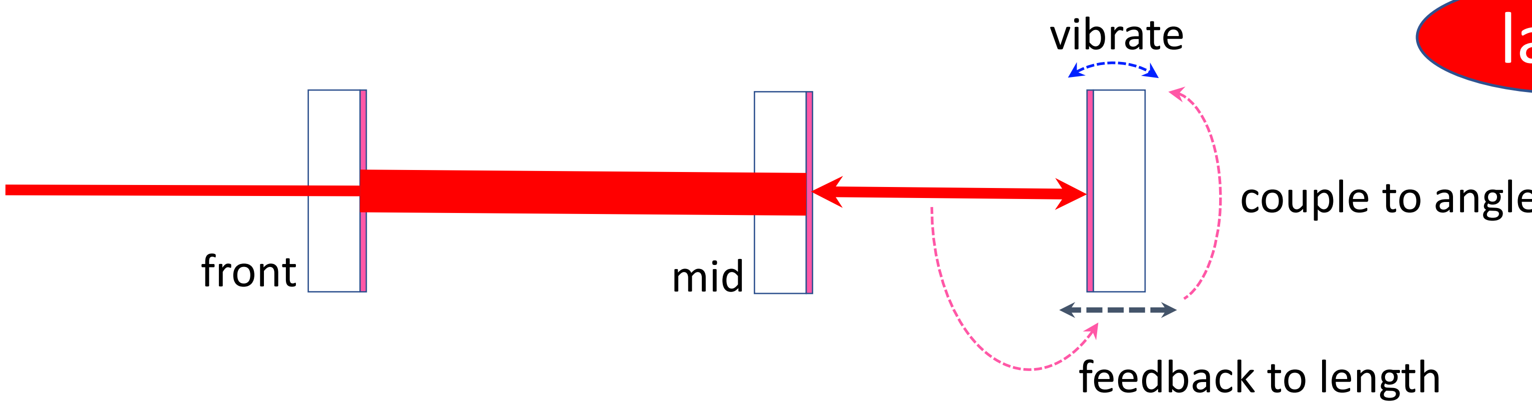
0.1 rad/m

$\sim 10^{-10}$ rad/rtHz @0.1Hz

read in minimum coupling quadrature

$\sim 10^{-12}$ rad/rtHz @0.1Hz

large



(not evaluated in detail yet)

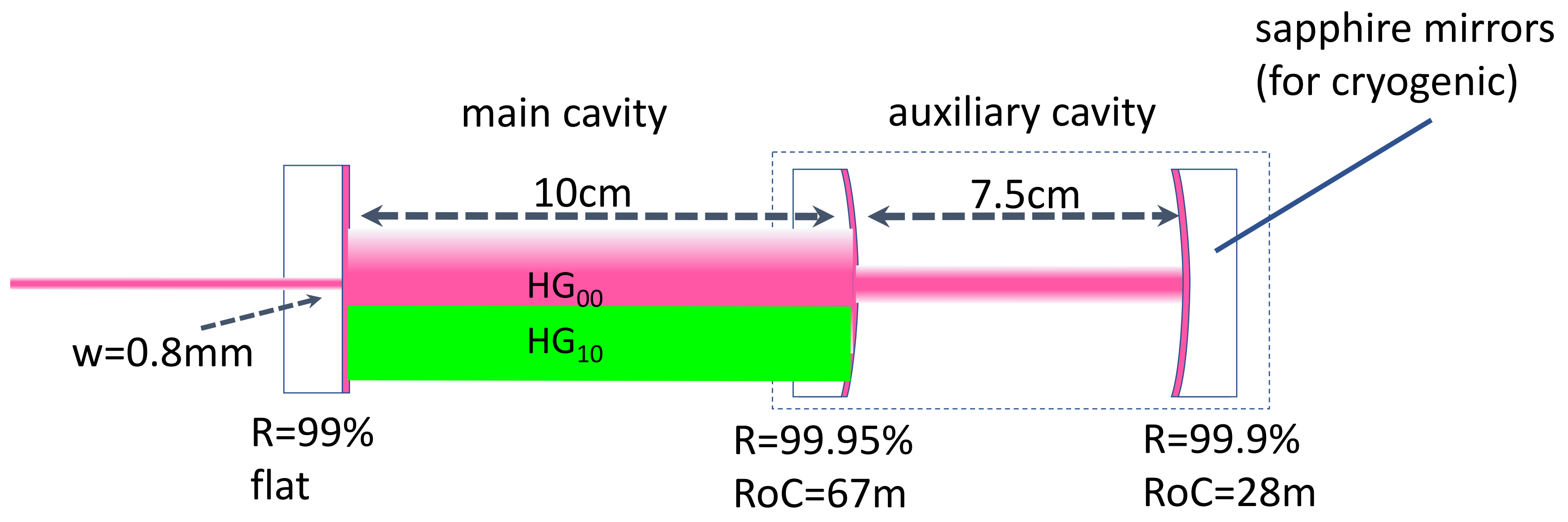
④other technical issues

- Do nonexplicit coupling routes exist? from...
  - frequency fluctuation
  - polarization fluctuation
  - intensity fluctuation
  - and so on...

design for TOBA

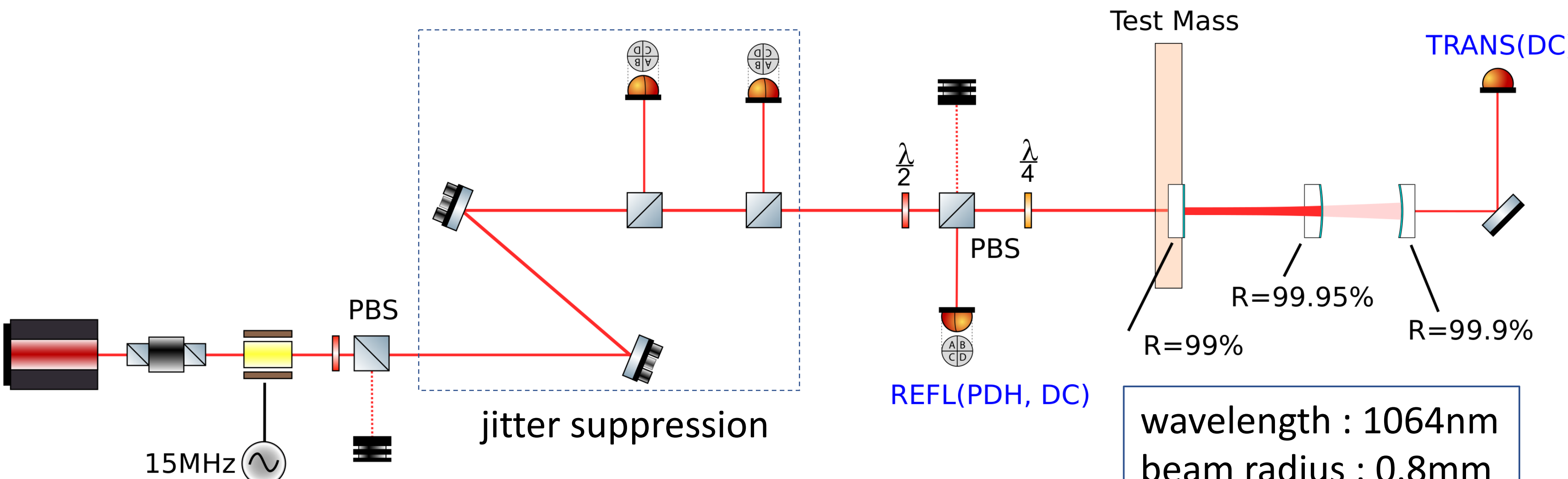
cavity design

- cavity length : 10cm(main) / 7.5cm(auxiliary)
  - limited by chamber size
- beam radius : 0.8mm @front mirror (waist)
  - from thermal noise requirement
- Finesse of main cavity : 360



total configuration (plan)

- put the front mirror on the test mass
- mid & end mirrors are (semi-)monolithic
- beam jitter is suppressed by control with two QPDs



wavelength : 1064nm
beam radius : 0.8mm
cavity length : 10cm
finesse : 360

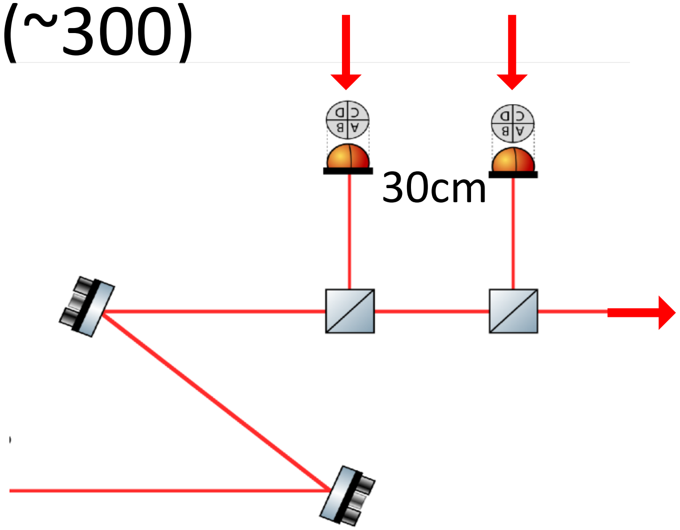
jitter suppression

- QPD noise : ~ 10^{-7} V/rtHz @0.1Hz
- position sensitivity of QPD (w=0.4mm) : ~ 30 V/mm

  → position sensing noise : 3×10^{-12} m/rtHz
- measure beam center at two points (separated by 30cm)

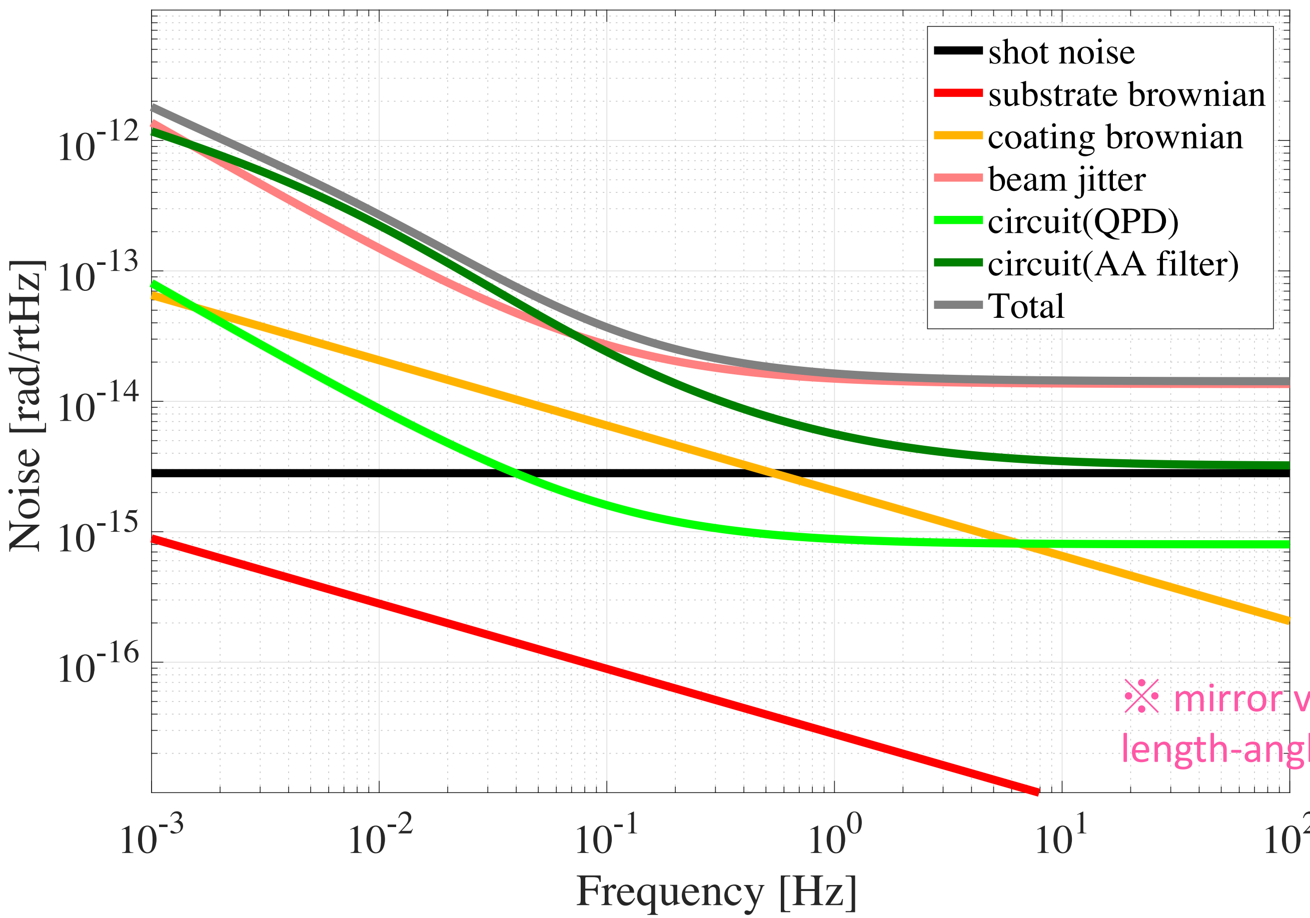
  → angular jitter : 10^{-11} rad/rtHz
- jitter noise is suppressed by finesse(~300)

  → 3×10^{-14} rad/rtHz



sensor noise level

- 4×10^{-14} rad/rtHz @ 0.1 Hz ⇐ target : 5×10^{-14} rad/rtHz

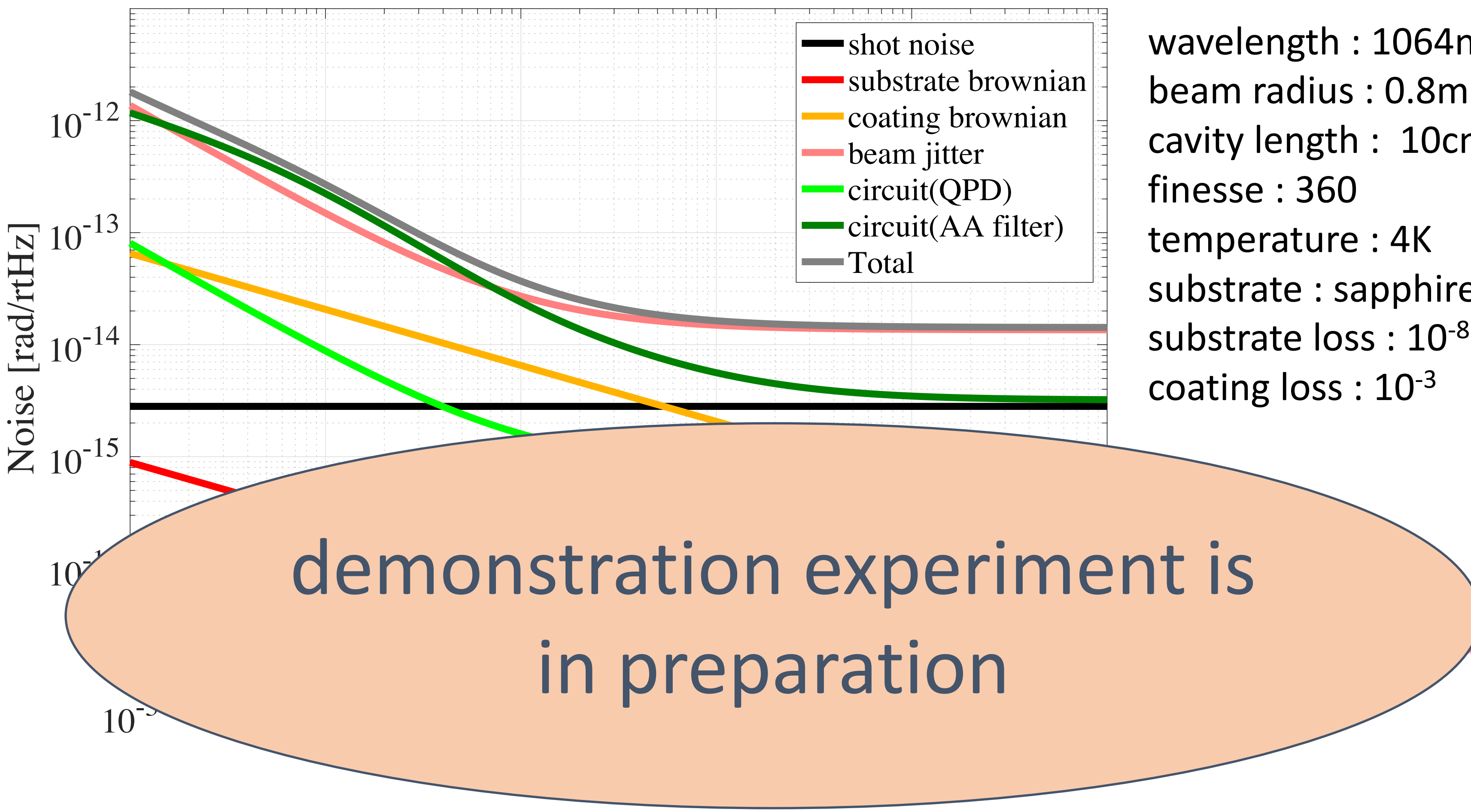


wavelength : 1064nm
beam radius : 0.8mm
cavity length : 10cm
finesse : 360
temperature : 4K
substrate : sapphire
substrate loss : 10^{-8}
coating loss : 10^{-3}

※ mirror vibration & length-angle coupling are not included

sensor noise level

- 4×10^{-14} rad/rtHz @ 0.1 Hz ⇐ target : 5×10^{-14} rad/rtHz



wavelength : 1064nm
beam radius : 0.8mm
cavity length : 10cm
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temperature : 4K
substrate : sapphire
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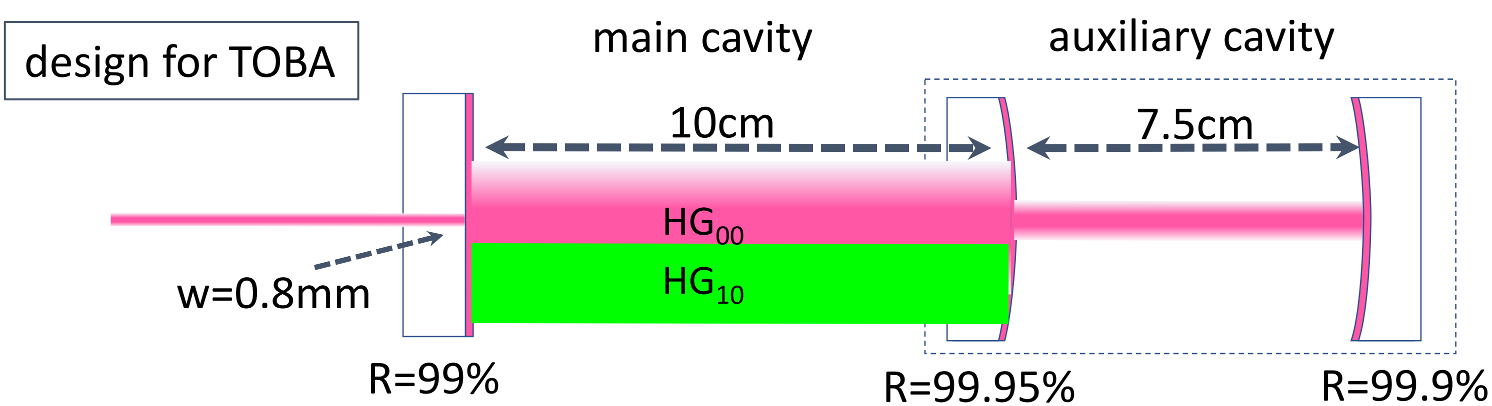
coating loss : 10^{-3}

summary

summary

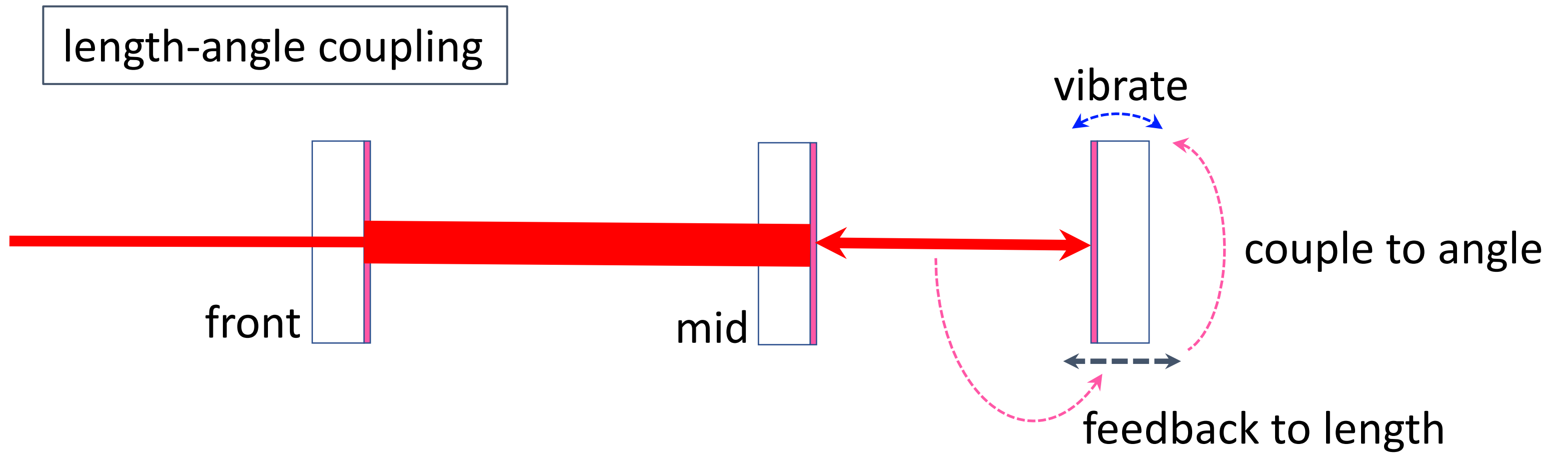
- sensitivity of wave front sensor can be improved by using auxiliary cavity for HG_{10} mode resonance
- improved sensor has low shot noise, jitter noise, frequency noise, and large linear range, but high thermal brownian noise
- optical design for TOBA is fixed : (noise) 4×10^{-14} rad/rtHz

(demonstration experiment is in preparation)



remaining issues

- measure or estimate angular fluctuation of (semi-) monolithic optics
- calculate length-angle coupling in detail and think how to reduce it



end