BEC-phononic Gravitational Wave Detector

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Paper

- "Phonon creation by gravitational waves" (C. Sabin et. al., 2014)

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Phonon creation by gravitational waves

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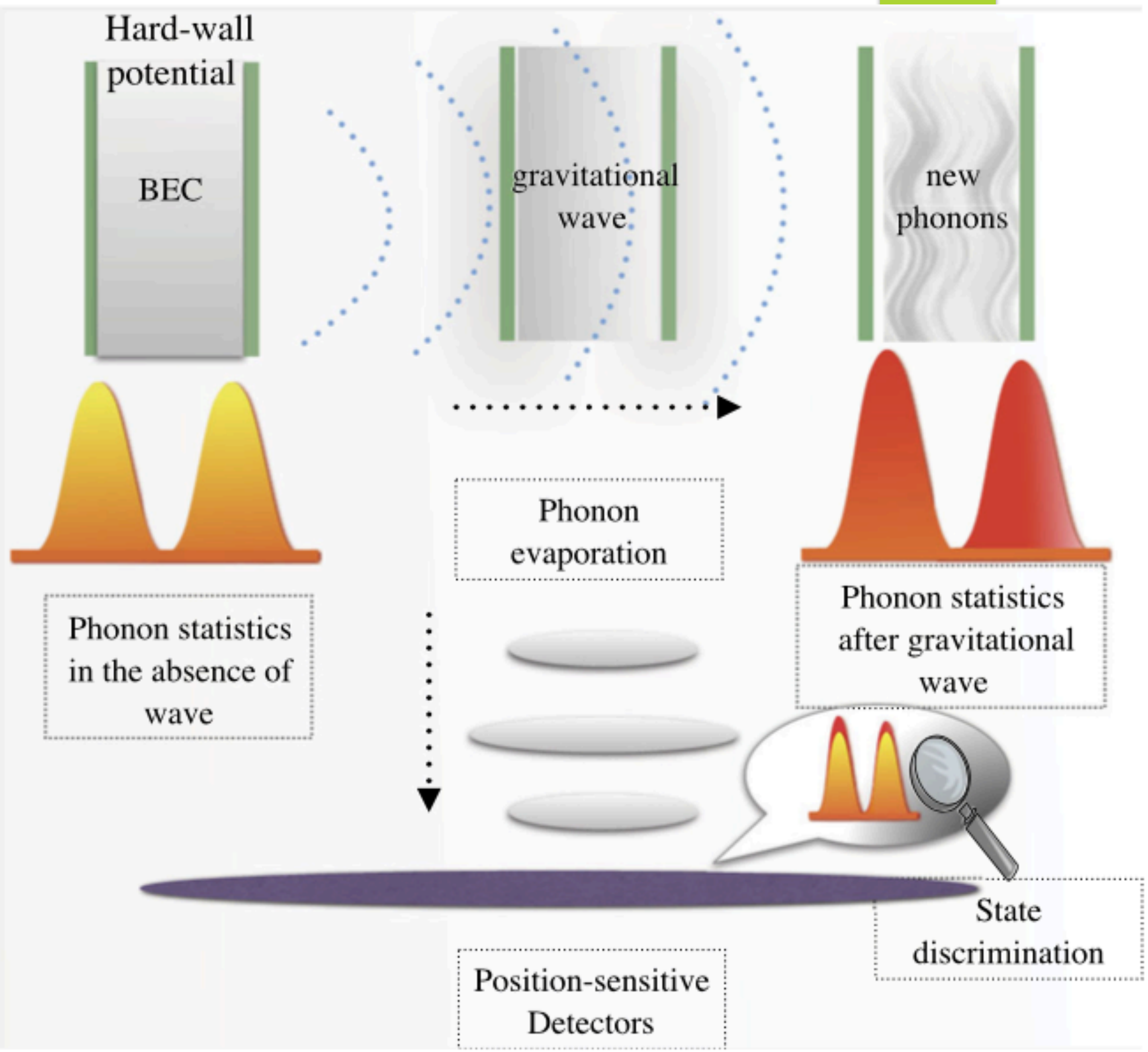
Abstract
We show that gravitational waves create phonons in a Bose–Einstein condensate (BEC). A traveling spacetime distortion produces particle creation resonances

Abstract

- A new type GW detector using a Bose-Einstein condensate was proposed
- As a consequence of relativistic quantum field theory, phonons are excited by GWs in a BEC trapped in a box-like potential(cavity)

Sketch of setup

- BEC in a cavity
- phonons are excited by GWs
- measure the final state and estimate the GW amplitude



BEC phononic field

- The phononic field of BEC obeys a massless Klein-Gordon equation:

$$\Box \hat{\Pi} = 0$$

here, d'Alembertian operator $\Box$ is :

$$\Box = 1/\sqrt{-\mathfrak{g}}\,\partial_a\left(\sqrt{-\mathfrak{g}}\,\mathfrak{g}^{ab}\partial_b\right)$$

$$\mathfrak{g}_{ab} = \left(\frac{n_0^2\, c_s^{-1}}{\rho_0 + p_0}\right)\begin{pmatrix} -c_s^2 & 0 & 0 & 0 \\ 0 & 1+h_+(t) & h_\times(t) & 0 \\ 0 & h_\times(t) & 1-h_+(t) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

: Effective spacetime metric

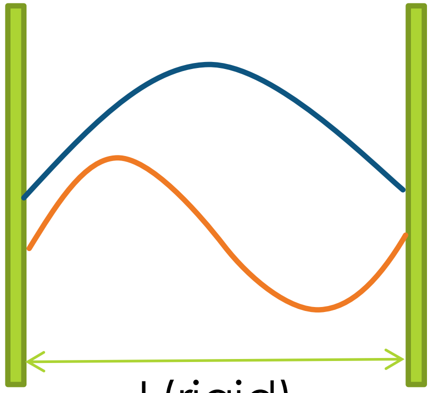
Solution

- The solution of Klein-Gordon equation(in flat spacetime)

$$\Pi(t,x) = \sum_k \left(\phi_k(t,x)\, a_k + \phi_k^*(t,x)\, a_k^\dagger \right)$$

$$\left(\phi_n = \frac{1}{\sqrt{n\,\pi}} \sin \frac{n\pi(x - x_L)}{L} e^{-i\,\omega_n\,t}, \qquad \omega_n = \frac{n\,\pi\,c_s}{L} \right)$$

a_k , $a_k^\dagger$: annihilation&creation operator



Particle creation by spacetime distorsion

- effects of spacetime distorsion :

$$\hat{\phi}_m = \sum_n (\alpha_{mn}\, \phi_n - \beta_{mn}\, \phi_n^*)$$

$$\hat{a}_m = \sum_n (\alpha_{mn}^*\, a_n + \beta_{mn}^*\, a_n^\dagger)$$

$$|\alpha_{mn}|^2 - |\beta_{mn}|^2 = 1$$

(Bogoliubov transformation)

- the coefficient β_{mn} is associated to particle(phonon) creation :

$$\boxed{N_n = \sum_m |\beta_{mn}|^2}$$

Effect of gravitational waves

- the effect of sinusoidal GW : $h_+(t) = \epsilon \sin \Omega t$

$$\beta_{mn}(t) = -i\frac{(-1)^{m+n}\sqrt{mn}}{2(m+n)}\epsilon(\omega_m+\omega_n)\underline{\int_0^t e^{-i(\omega_m+\omega_n)t'}\sin(\Omega t')dt'}$$

(m≠n)

resonate at $\Omega=\omega_m+\omega_n$

- at resonance ($\Omega=\omega_m+\omega_n$), after enough long duration ($\omega t >> 1$),

$$\boxed{\beta_{mn}(t) \simeq -(-1)^{m+n}\frac{\epsilon}{4}\sqrt{\frac{n}{m}}\,\omega_m t}$$

⇒ phonon creation associated with ϵ, t, ω_m

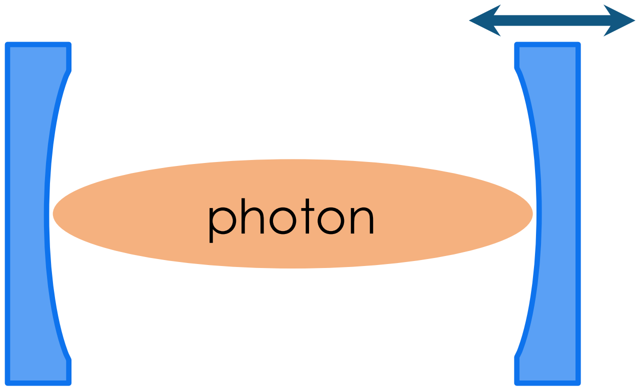
Similarity to dynamical Casimir effect

$$\boxed{\beta_{mn}(t) \simeq -(-1)^{m+n}\frac{\epsilon}{4}\sqrt{\frac{n}{m}}\,\omega_m t}$$

- effect of GWs coincides with those of a cavity with sinusoidally varying length in flat spacetime

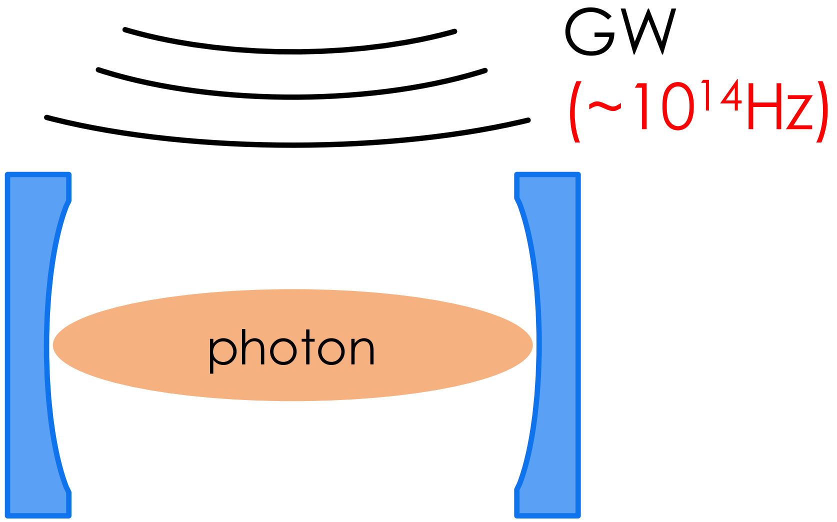
$$L(t) = L(0)(1 + \epsilon\,\sin(\Omega\,t))$$

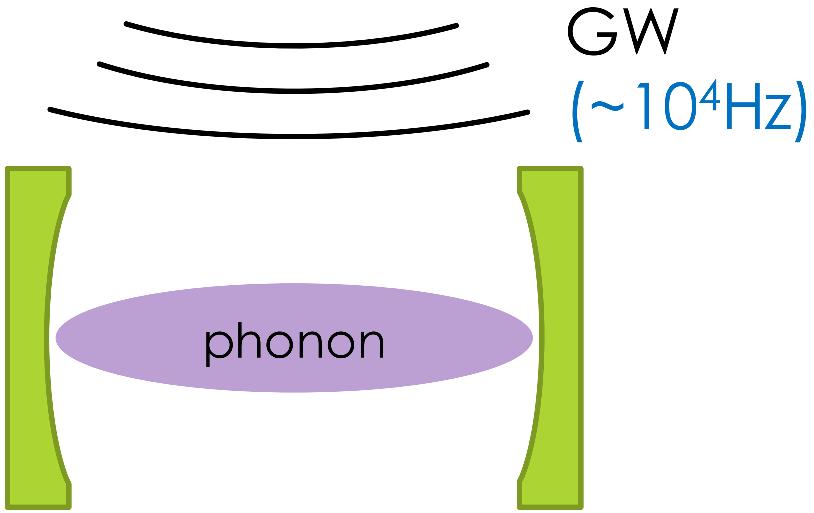
- similar to dynamical Cassimir efect
  - a cavity with moving walls creates photon



Compare with an optical cavity

- the same effect appears in an optical cavity but at high frequency(~PHz)





~10mm/s

$$\mathfrak{g}_{ab} = \left(\frac{n_0^2\, c_s^{-1}}{\rho_0 + p_0}\right) \begin{pmatrix} -c_s^2 & 0 & 0 & 0 \\ 0 & 1 + h_+(t) & h_\times(t) & 0 \\ 0 & h_\times(t) & 1 - h_+(t) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Estimation error

- Cramer-Rao bound (quantum measurement limit):

$$\langle(\Delta\hat{\epsilon})^2\rangle \geq \frac{1}{MH_\epsilon}$$

ε : GW amplitude
M : number of probes ($\sim 10^{14}$)
H : Quantum Fisher Information

quantum Fisher Information H_ε

$$H_\epsilon = \frac{n}{4m}\omega_m^2 t^2 \left(8 - 4\cosh^4(r) + 2\sinh^2(2r)\right)$$

r : two-mode squeezing parameter
between mode m,n

Optimal bound of the strain sensitivity(1)

$$\langle(\Delta\hat{\epsilon})^2\rangle \geq \frac{1}{MH_\epsilon}$$

$\Delta\varepsilon \propto 1/\sqrt{mn}$

$$H_\epsilon = \frac{n}{4m}\omega_m^2 t^2 \left(8 - 4\cosh^4(r) + 2\sinh^2(2r)\right)$$

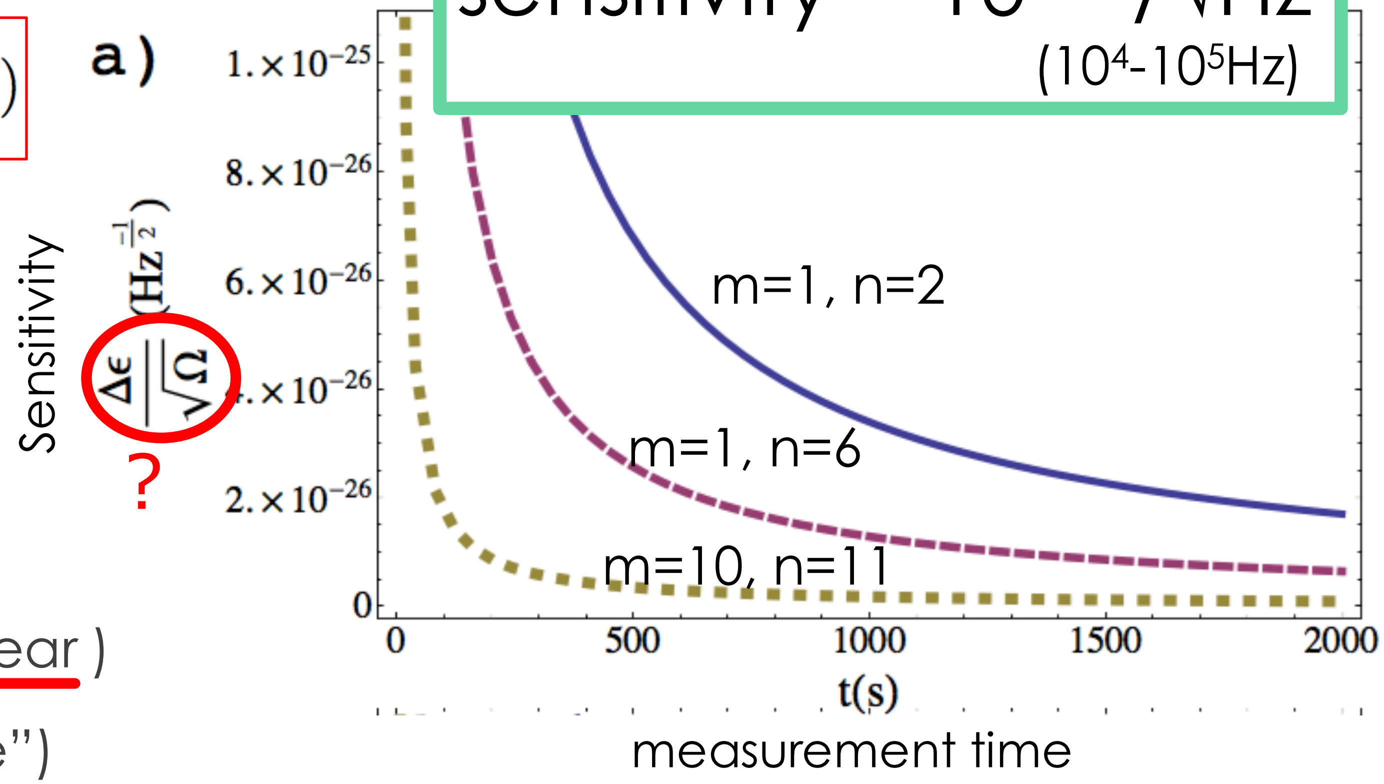
- mode dependence
  - L=1μm
  - c_s=10mm/s
  - M=10^{14}

    ?

    (= 10^6 atoms × 5Hz repetetion ×1year)
  - r=10 ("seem in principle achievable")

sensitivity ~ 10^{-26} /√Hz

(10^4-10^5Hz)



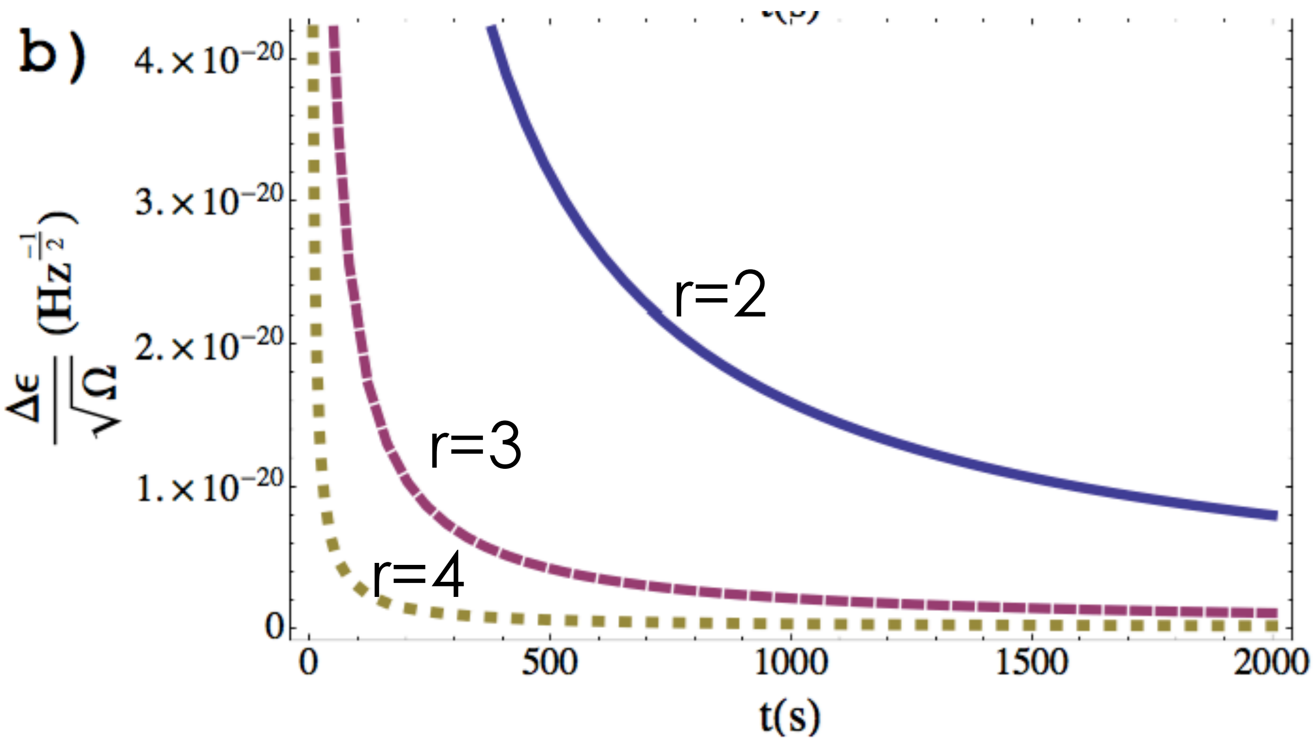
Optimal bound of the strain sensitivity(2)

$$\langle(\Delta\hat{\epsilon})^2\rangle \geq \frac{1}{MH_\epsilon}$$

$\Delta\varepsilon \propto e^{-2r}$

$$H_\epsilon = \frac{n}{4m}\omega_m^2\, t^2 \,\underline{\left(8 - 4\cosh^4(r) + 2\sinh^2(2r)\right)}$$

- squeezing dependence
  - L=1μm,
  - c_s=10mm/s
  - M=10^{14}
  - (m,n)=(10,11)

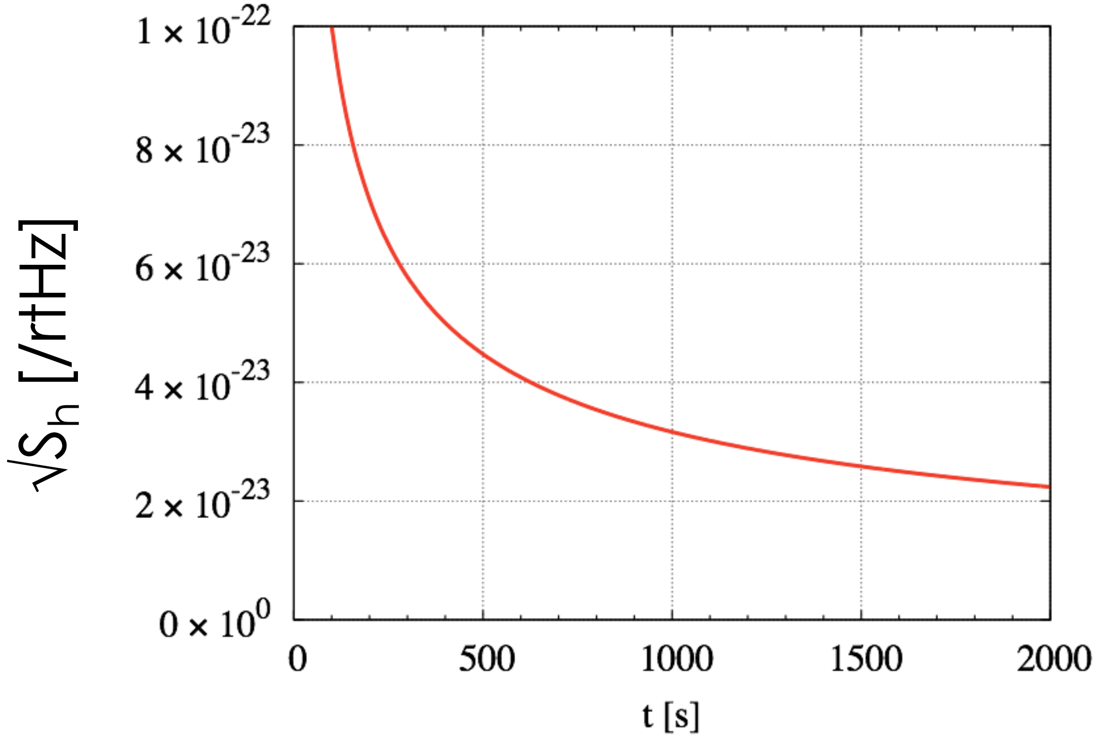


$\Delta\varepsilon/\sqrt{\Omega}$??

- A sensitivity to continuous GWs ($h_0 \sin\omega t$):

$$\sqrt{S_h} = \frac{2}{5} h_0 \sqrt{\frac{T_{obs}}{(\mathrm{SNR})}}$$

Karl Watte, Phys. Rev. D 85,042003(2012)



in our familiar sensitivity,

$$\boxed{\sqrt{S_h} \sim 10^{-23}\ /\sqrt{\mathrm{Hz}}\ ?}$$

Noise sources in phonon creation

- thermal phonon excitation
  - negligible at achievable temperature

@10nK (normal) : $N_{phonon} = 10^{-31}$

@0.5nK (best *) : $N_{phonon} = 10^{-625}$

$\Leftrightarrow N_{phonon}(\varepsilon=10^{-26},\ t=1000s) \sim \underline{10^{-36}}$

(* A. E. Leanhardt et. al., 2003 / ※achieved for 2,500 atoms)

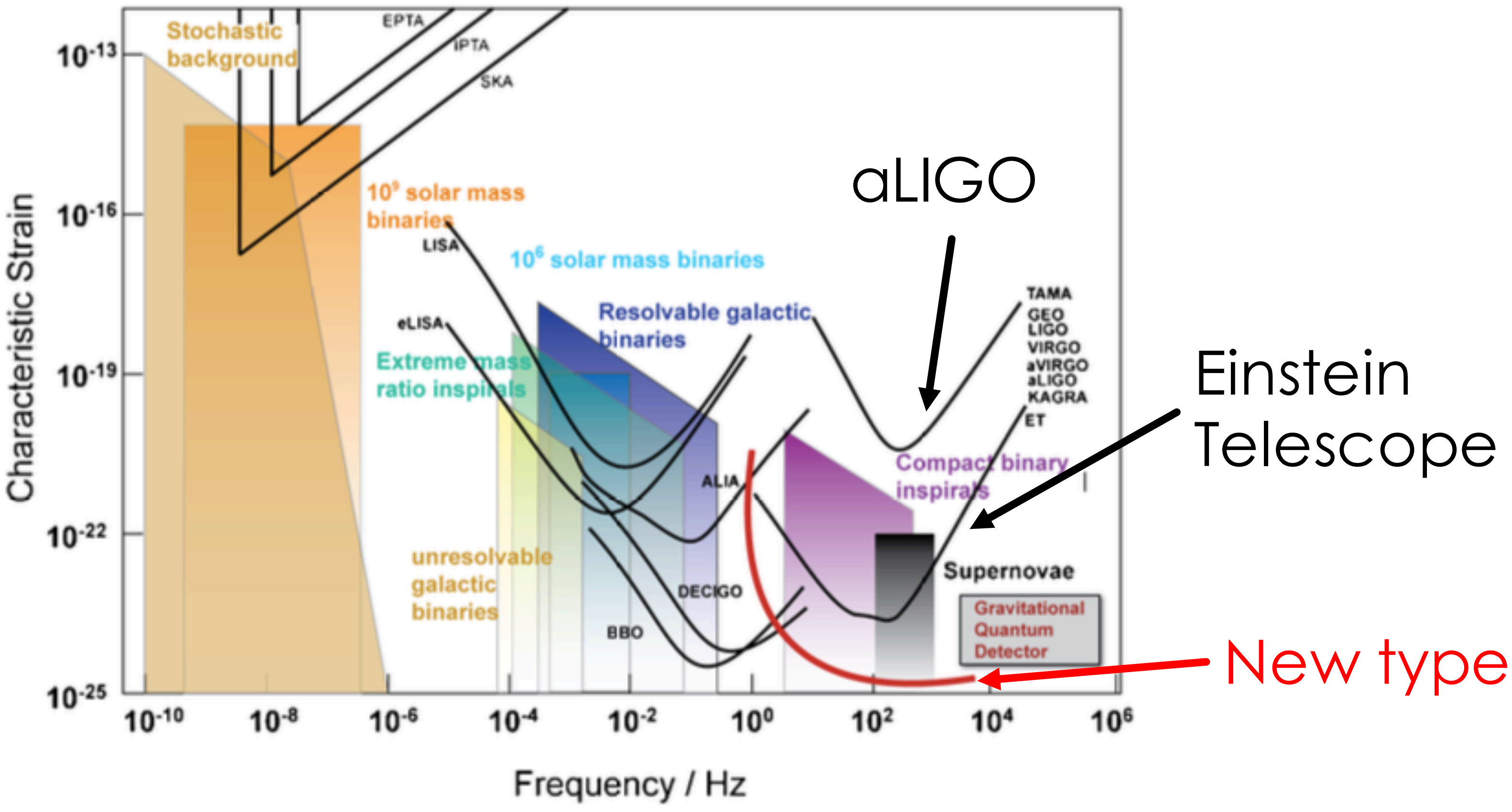
How to measure phonons?

- Measure the momentum of atoms
  - release the condensate trapping potential
  - each phonon state is mapped into the state of an atom
  - measurement with position-sensitive single-atom detector
- Non-destructive method
  - using atomic quantum dots interacting with the BEC *

* C. Sabin et. al., Sci. Rep. 4, 6436 (2014)

Sensitivity curve (???)

- detailed information about this plot was not described in the paper



Summary

- The effect based on the relativistic quantum field theory was calculated
- Spacetime distorsions of GWs can create phonons in a BEC
- There are resonances at $\Omega=\omega_m+\omega_n$ (Ω:GW freq. / ω_n:mode freq.)
- At resonance, sensitivity $\Delta\varepsilon/\sqrt{\Omega}\sim 10^{-26}$ /√Hz is assumed to be achieved (?)
  - (In our familiar sensitivity, $\sqrt{S_h}\sim 10^{-23}/\sqrt{\mathrm{Hz}}$?)

- Concrete configuration remains to be considered