

Angular Sensor for TOBA and my future plans

M2 Yuki Miyazaki

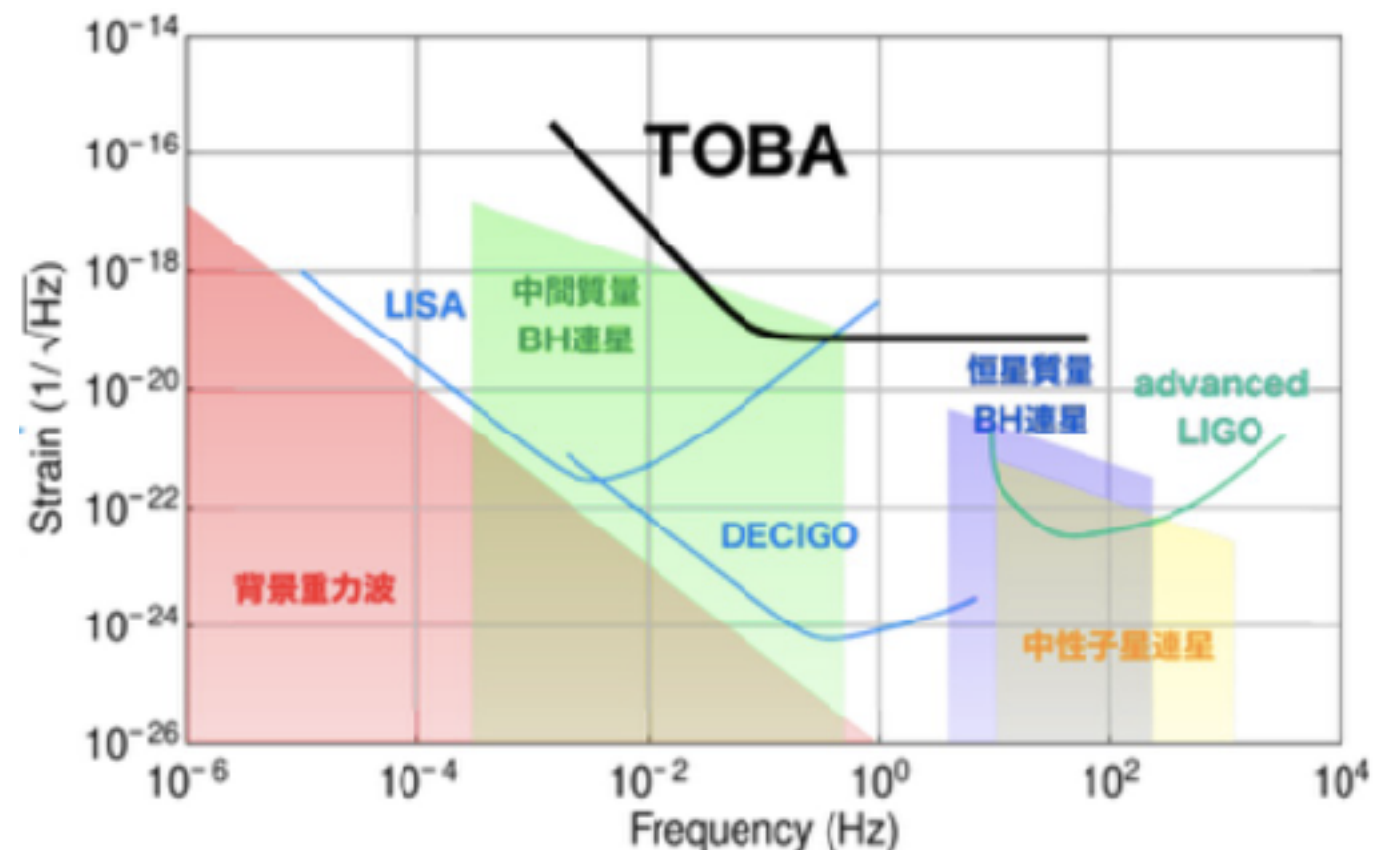
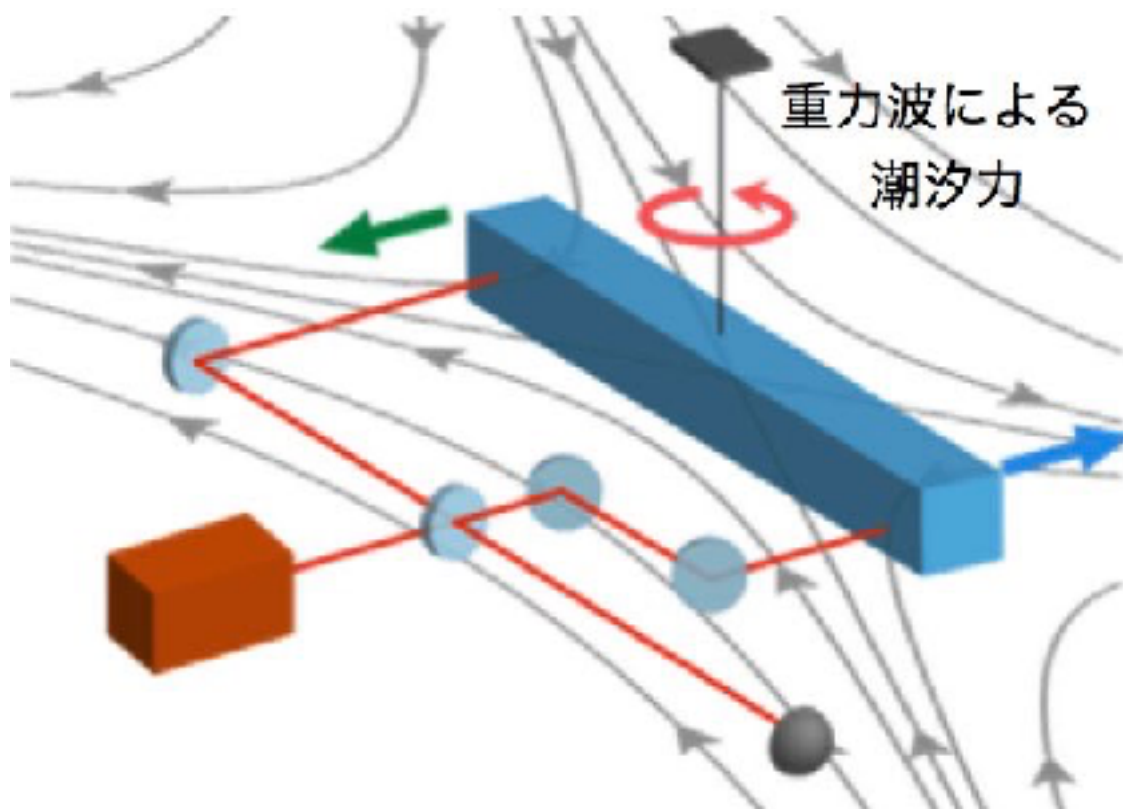
Contents

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 - merits and demerits
- What is done so far
- What I want to study with the coupled WFS (For M Thesis)
 - demonstration test (confirm its principle)
 - check its characteristics
 - obtain good sensitivity
- Summary

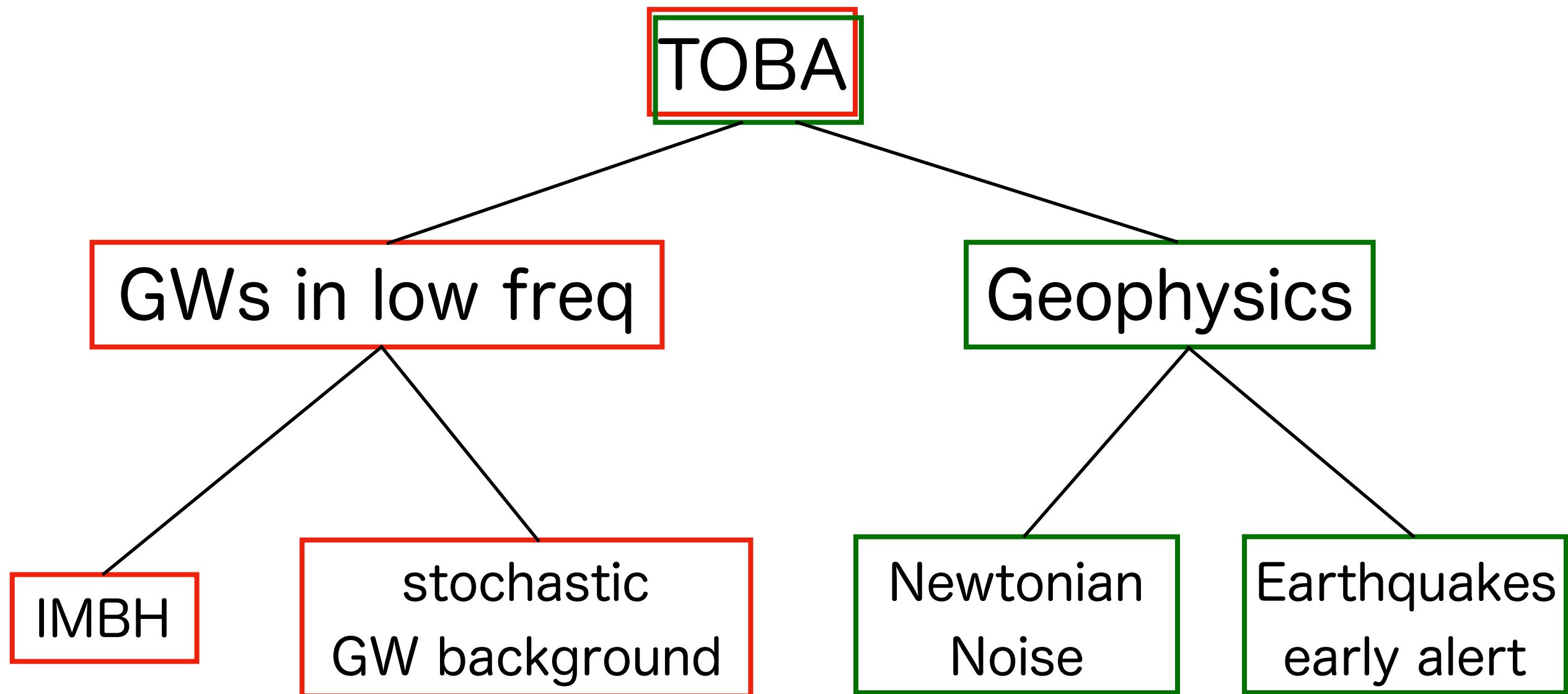
I made too much slides..
Sorry for fast speaking..

TOBA (TOrsion-Bar Antenna)

- TOBA: a gravitational wave detector using torsion pendulums
- It rotates when GWs come
- Its resonant freq of Yaw rotation: ~mHz
 - It has good sensitivity at low freq
- Goal sensitivity: $h \sim 10^{-19} / \sqrt{\text{Hz}}$ @ 0.1 Hz (Final TOBA)



Targets of TOBA

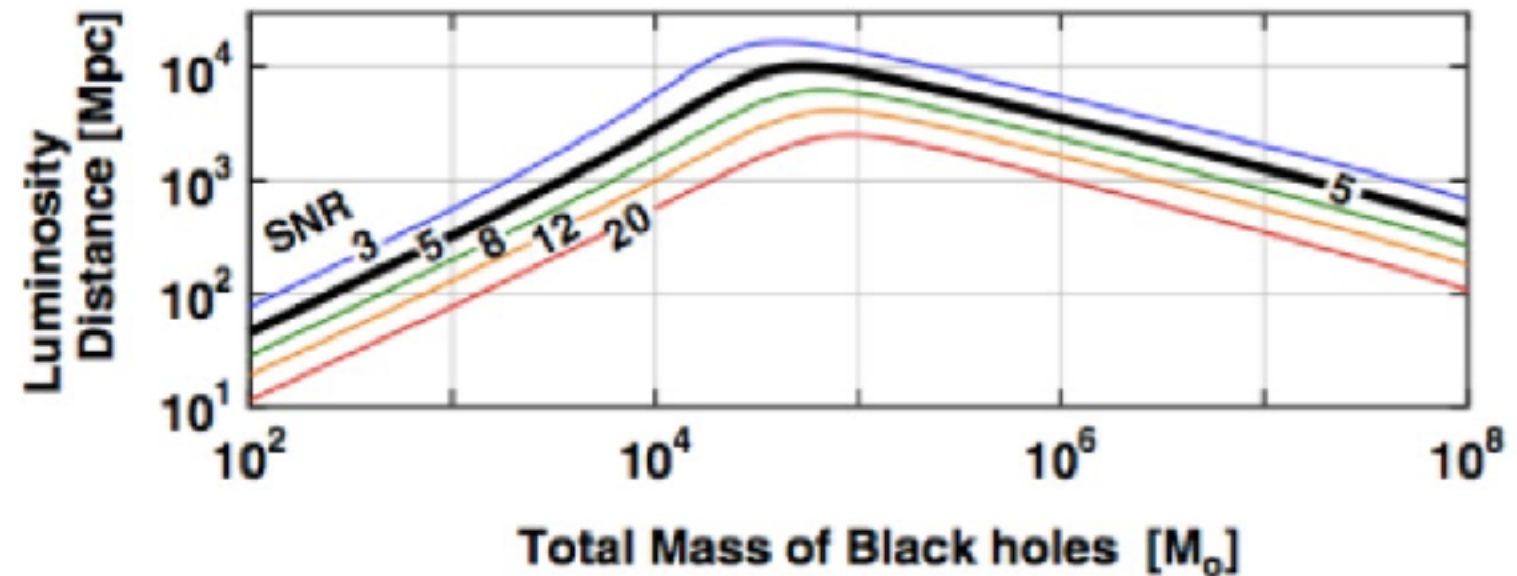


Targets of TOBA

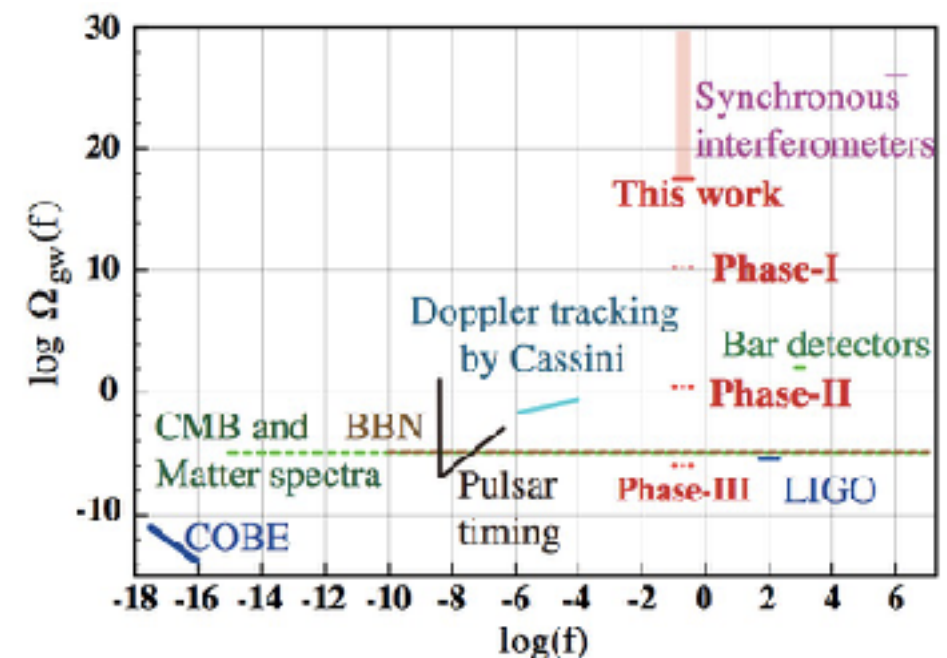
GWs in low freq

- Intermediate mass blackholes ($\sim 10^5 M_{\text{sun}}$)
- Stochastic GW background
 - Big Bang Nucleosynthesis
~information for element ratio
 - give knowledge for early universe

M. Ando et al., PRL, 105, 161101(2010)



K. Ishidoshiro et al., PRL, 106, 161101(2011)

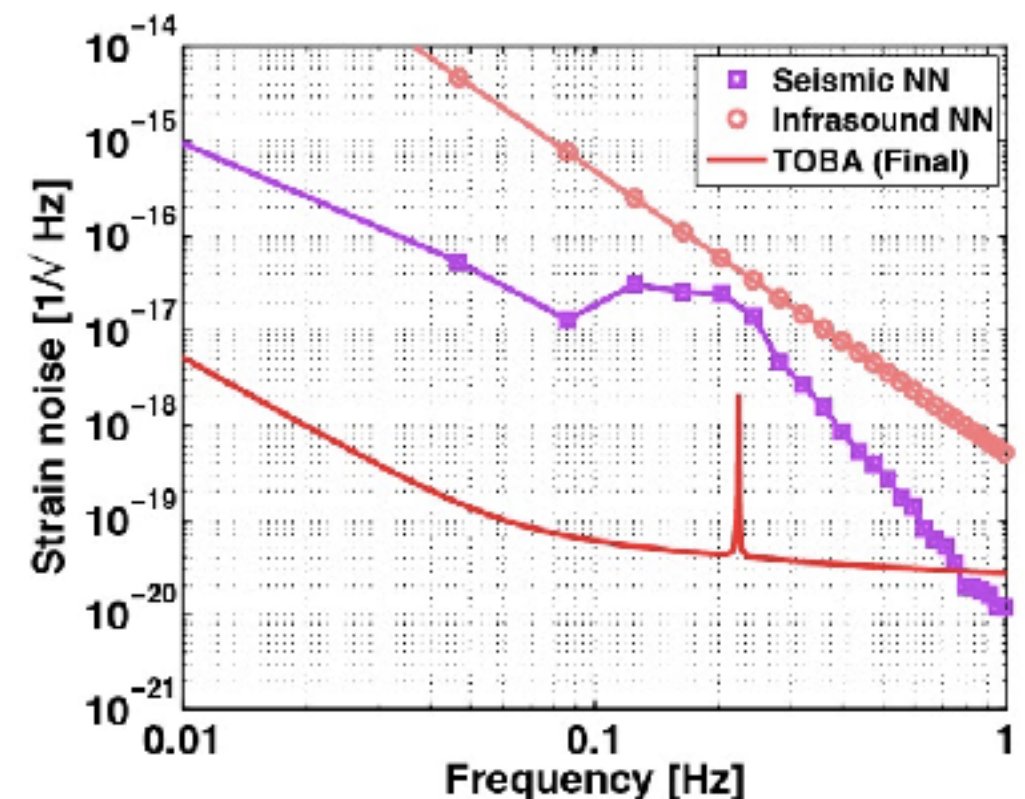


Targets of TOBA

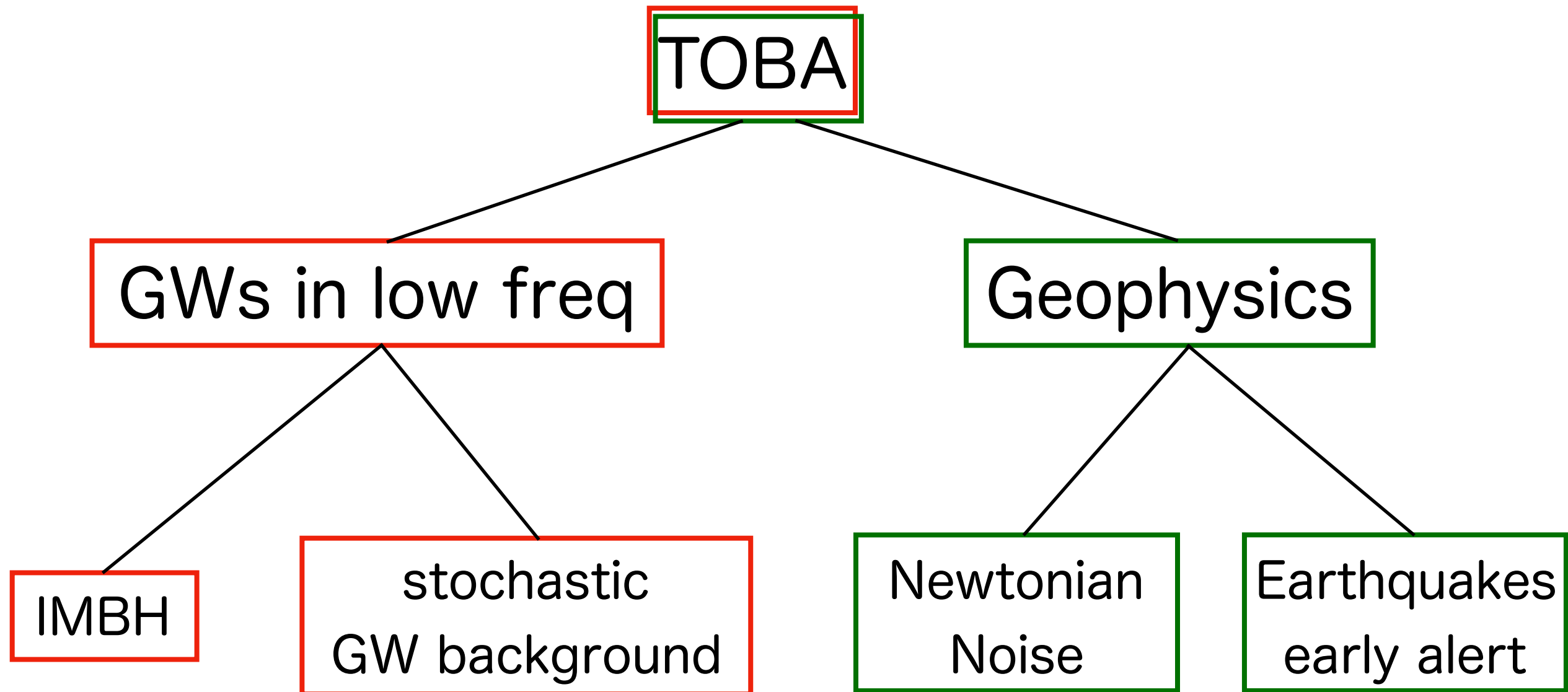
Geophysics

- Newtonian noise
 - density perturbation of the ground and the atmosphere
 - It may limit sensitivity of 3G detectors
- Early alert for earthquakes
 - M6.0 earthquakes are detectable in 10sec from 100km away

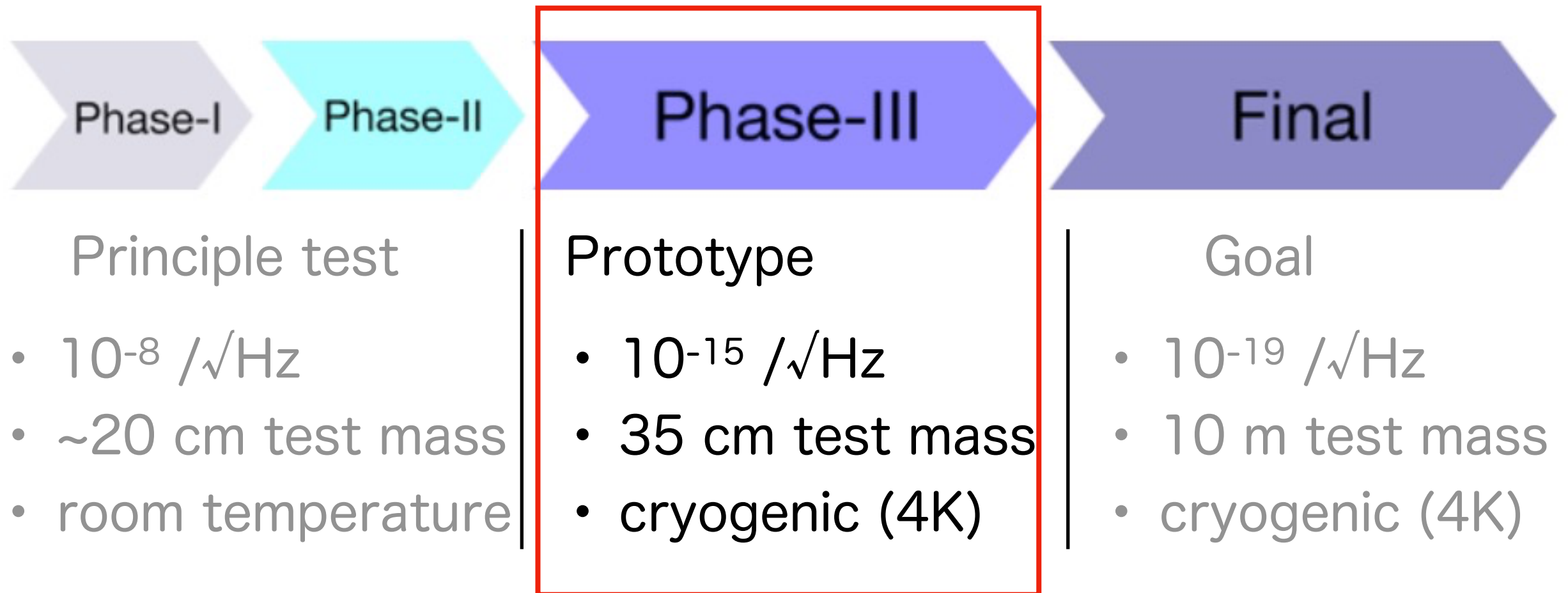
J.Harms et al., PRD, 88, 122003(2013)



Targets of TOBA

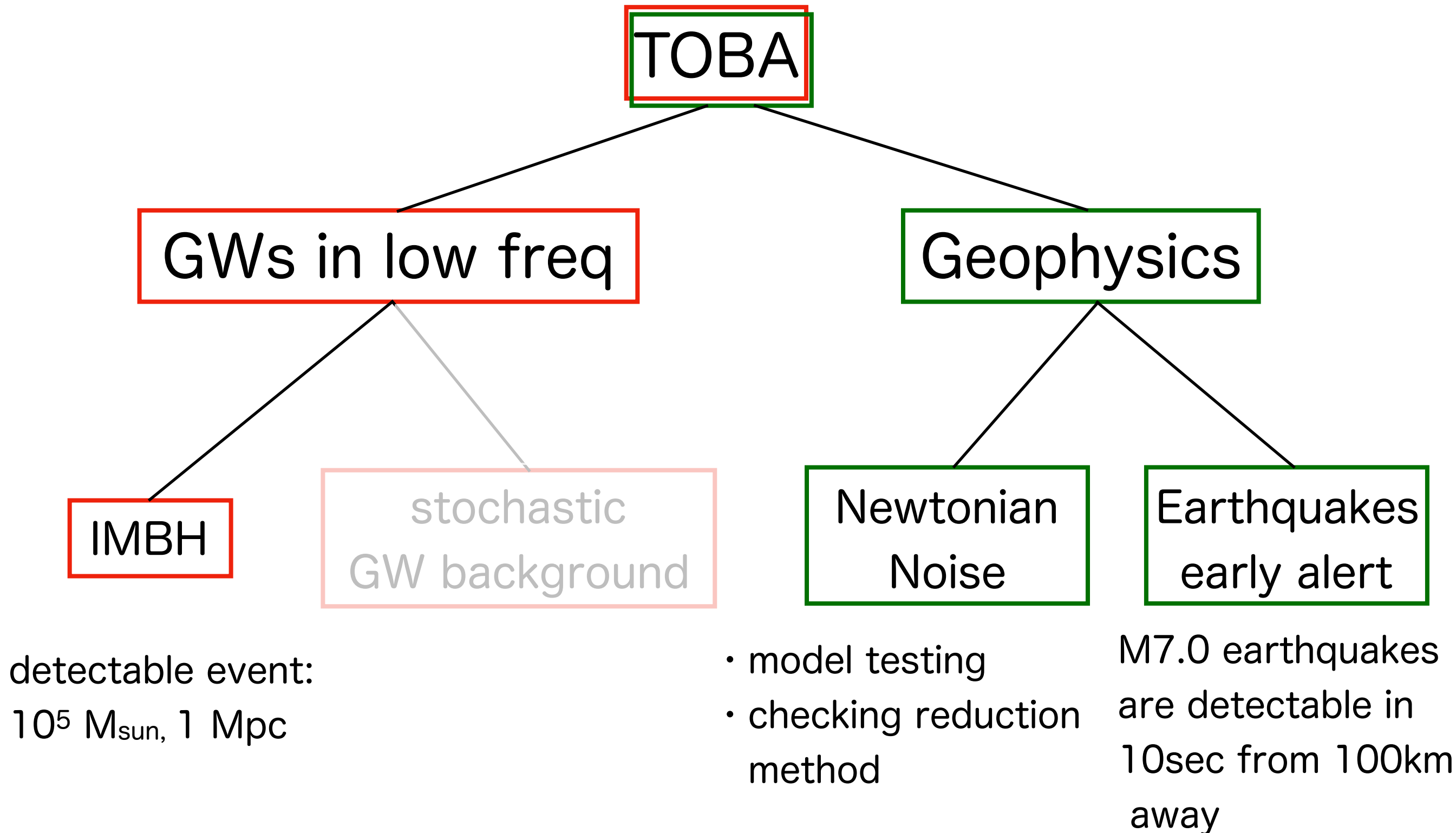


R&D plan



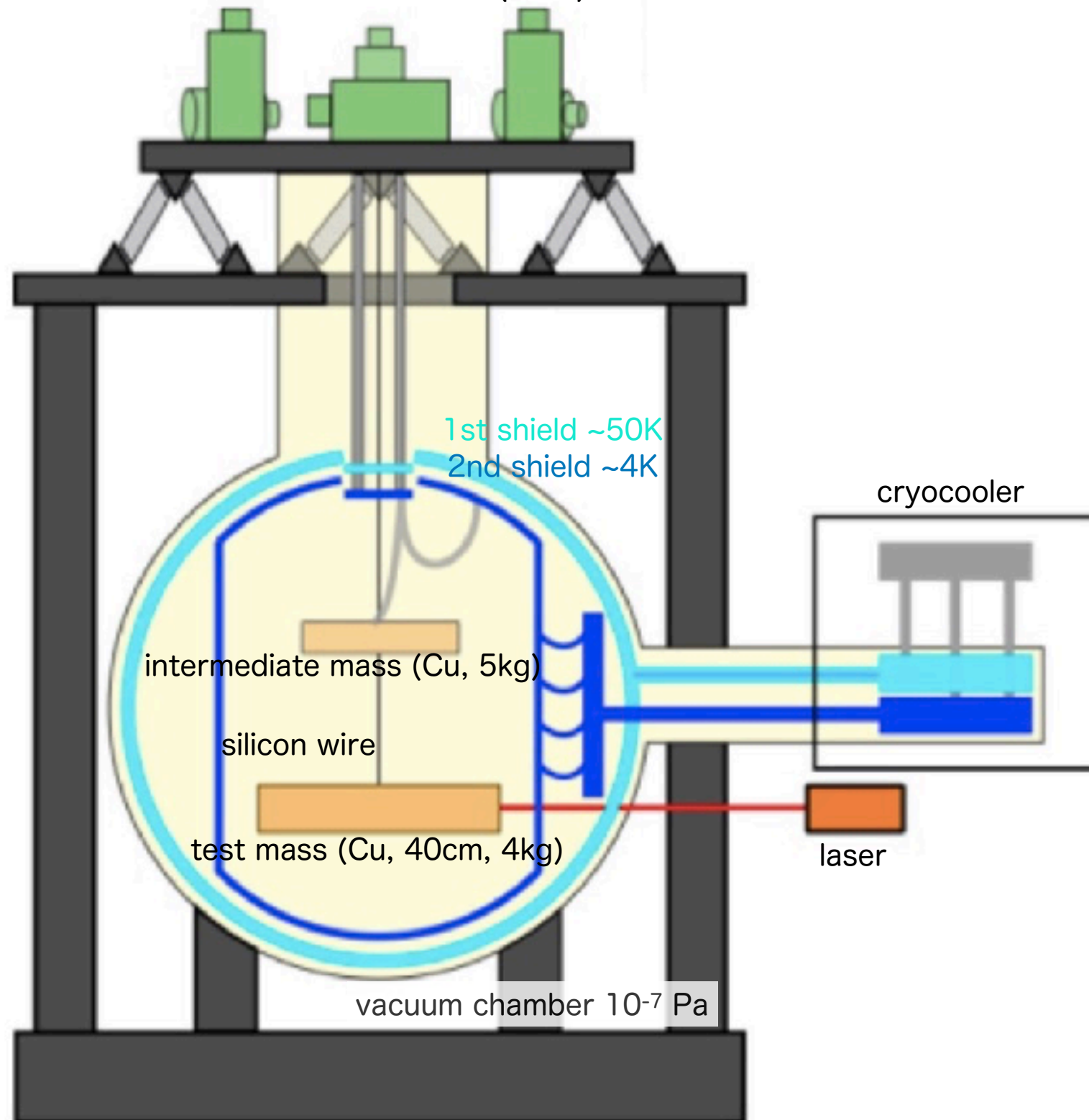
Now: Development for Phase-III TOBA

Targets of Phase-III TOBA



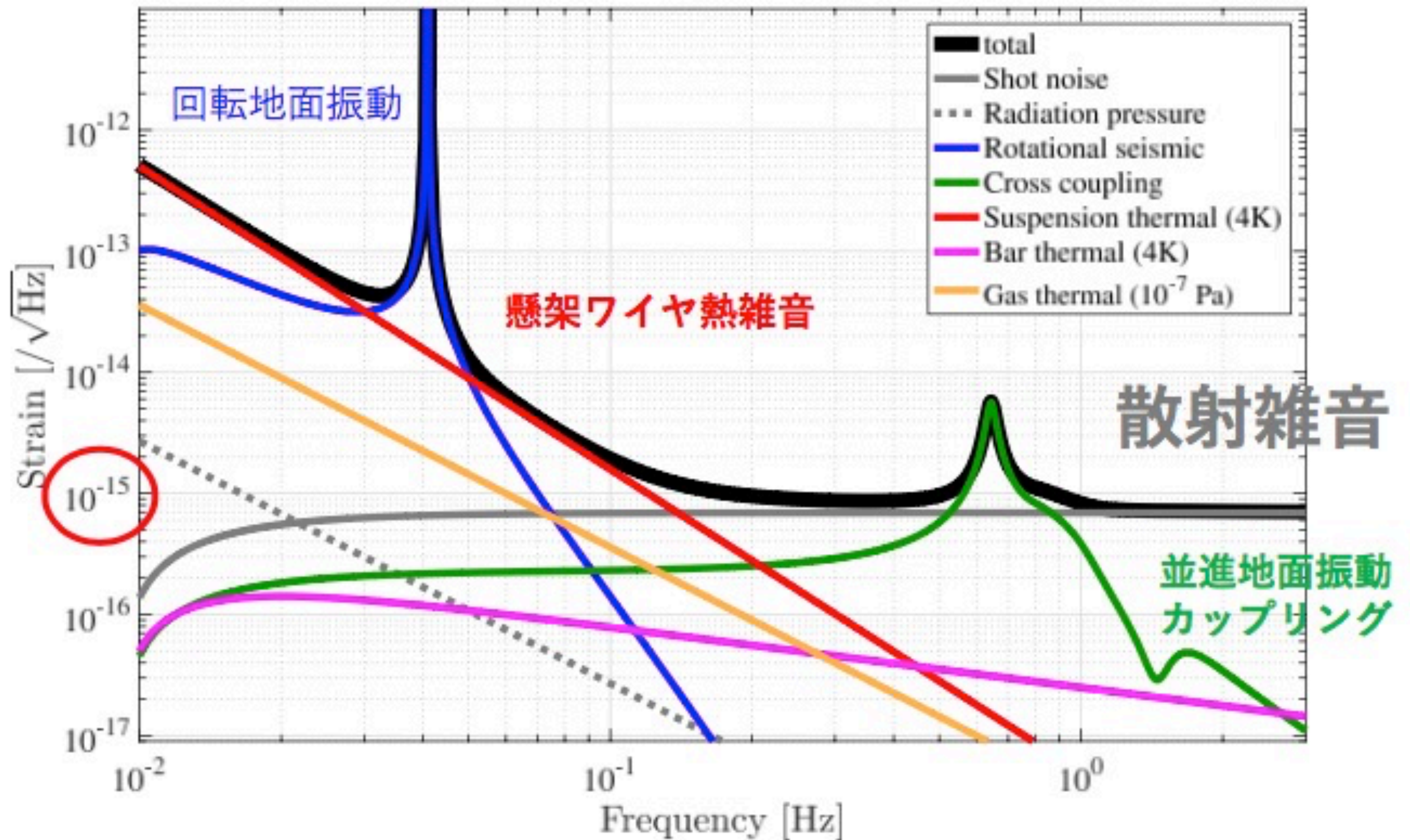
Setup of Phase-III TOBA

Active Vibration Isolation Table (AVIT)

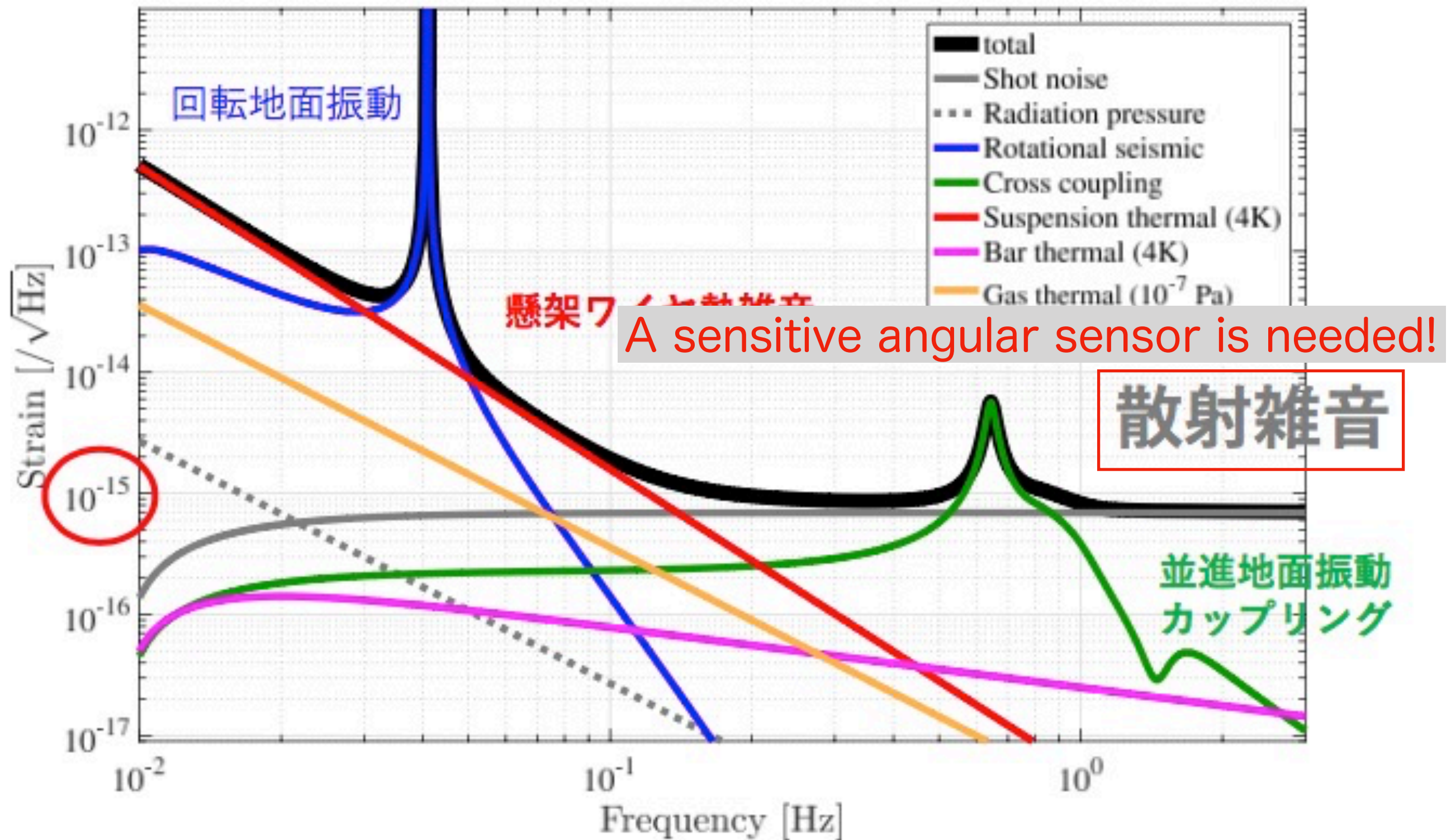


- 40 cm test mass
- cooling to 4K
- active vibration isolation
- new readout system

Design sensitivity

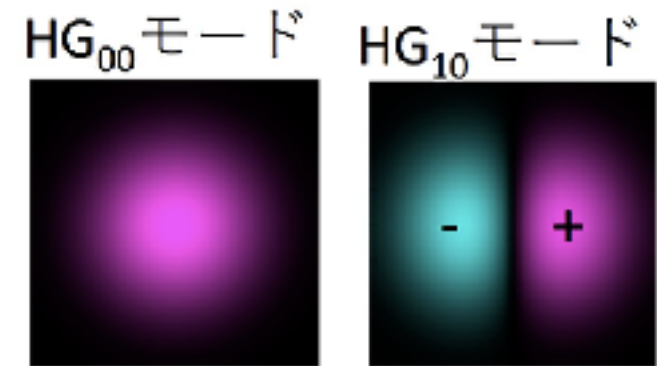
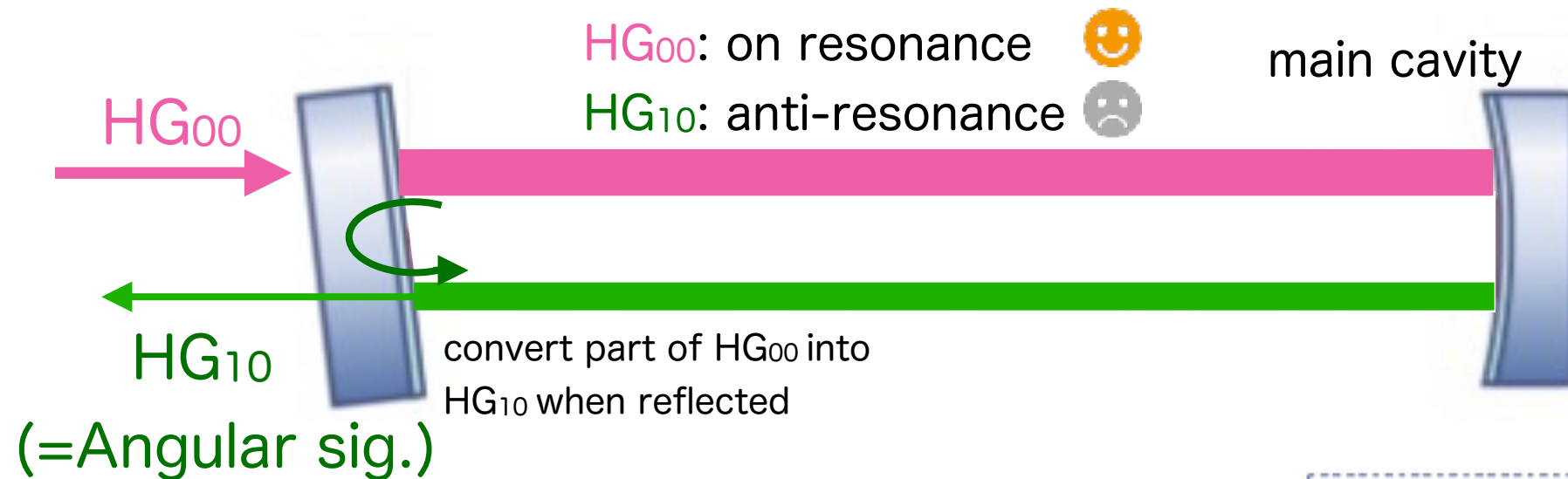


Design sensitivity

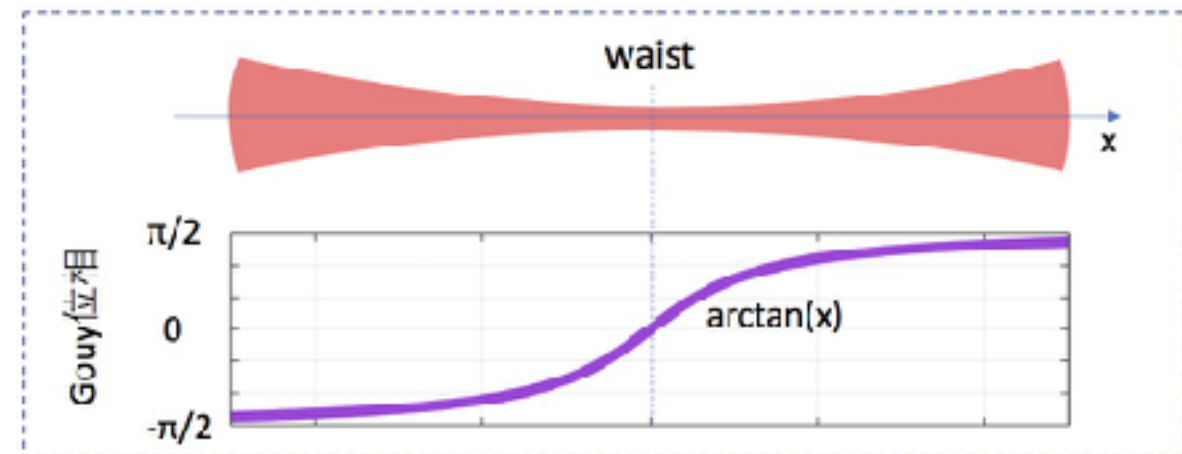


Conventional Wave Front Sensor

Angular sig. = HG₁₀ mode created by tilted mirror



The phase HG_{00} and HG_{10} receives in a round trip differs. The phase shift is called **Gouy phase**.



In an ordinary cavity,
 $0 < (\text{round trip Gouy phase}) < 2\pi$

→ conventional one:

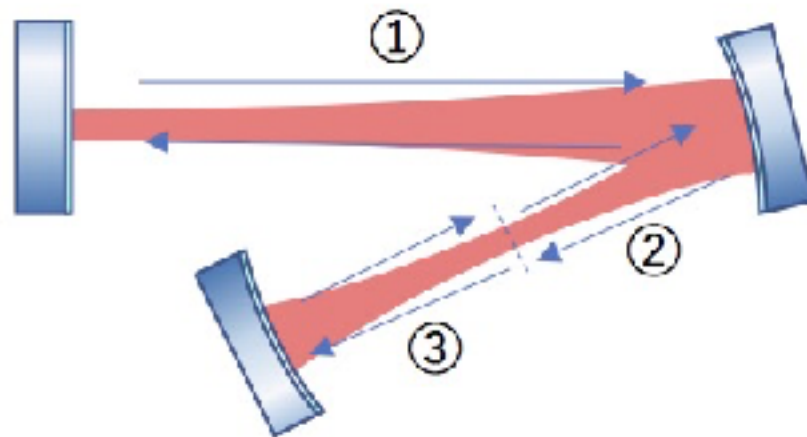
HG_{00} and HG_{10} cannot resonate simultaneously

Idea of sensitive sensors

00 and 10 can resonate simultaneously in the below sensors

① folded cavity (demonstration test was done by Shimoda-san)

- make round Gouy phase 2π using curved mirrors



round Gouy phase
 $(\textcircled{1} + \textcircled{2} + \textcircled{3}) \times 2 = 2\pi$

② coupled cavity

- reflective phase compensates Gouy phase



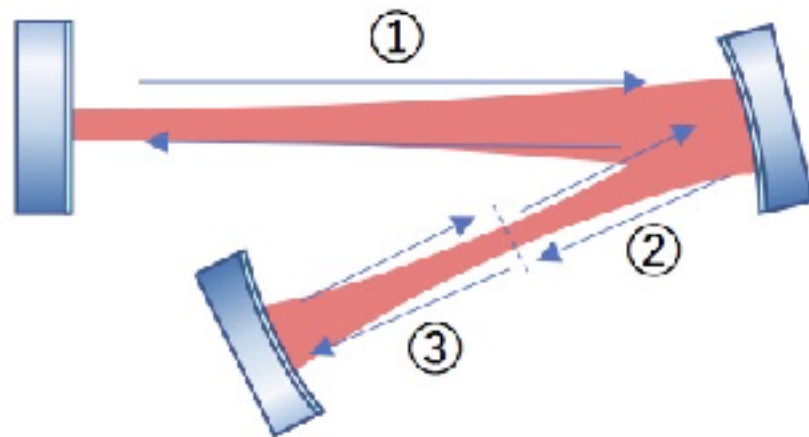
round Gouy phase
+ reflective phase = 0

Idea of sensitive sensors

00 and 10 can resonate simultaneously in the below sensors

① folded cavity (demonstration test was done by Shimoda-san)

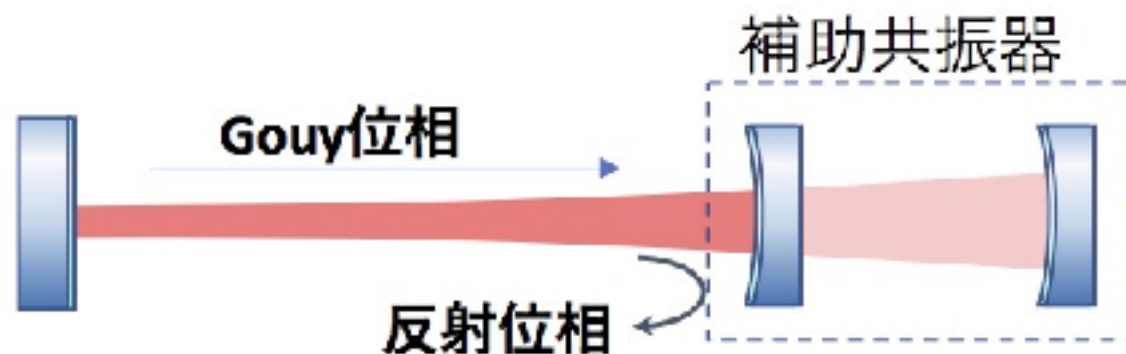
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round Gouy phase
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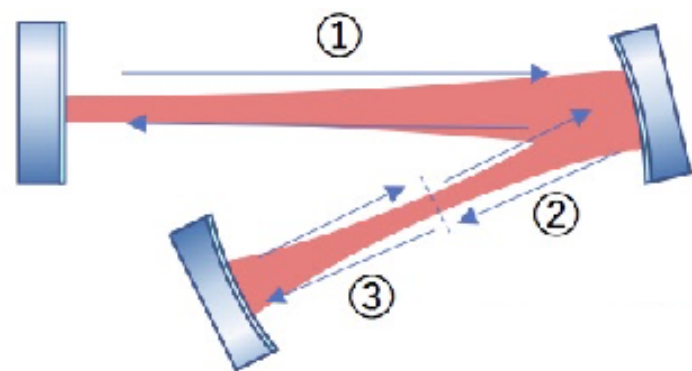
round Gouy phase
+ reflective phase = 0

Folded cavity

① folded cavity (demonstration test was done by Shimoda-san)

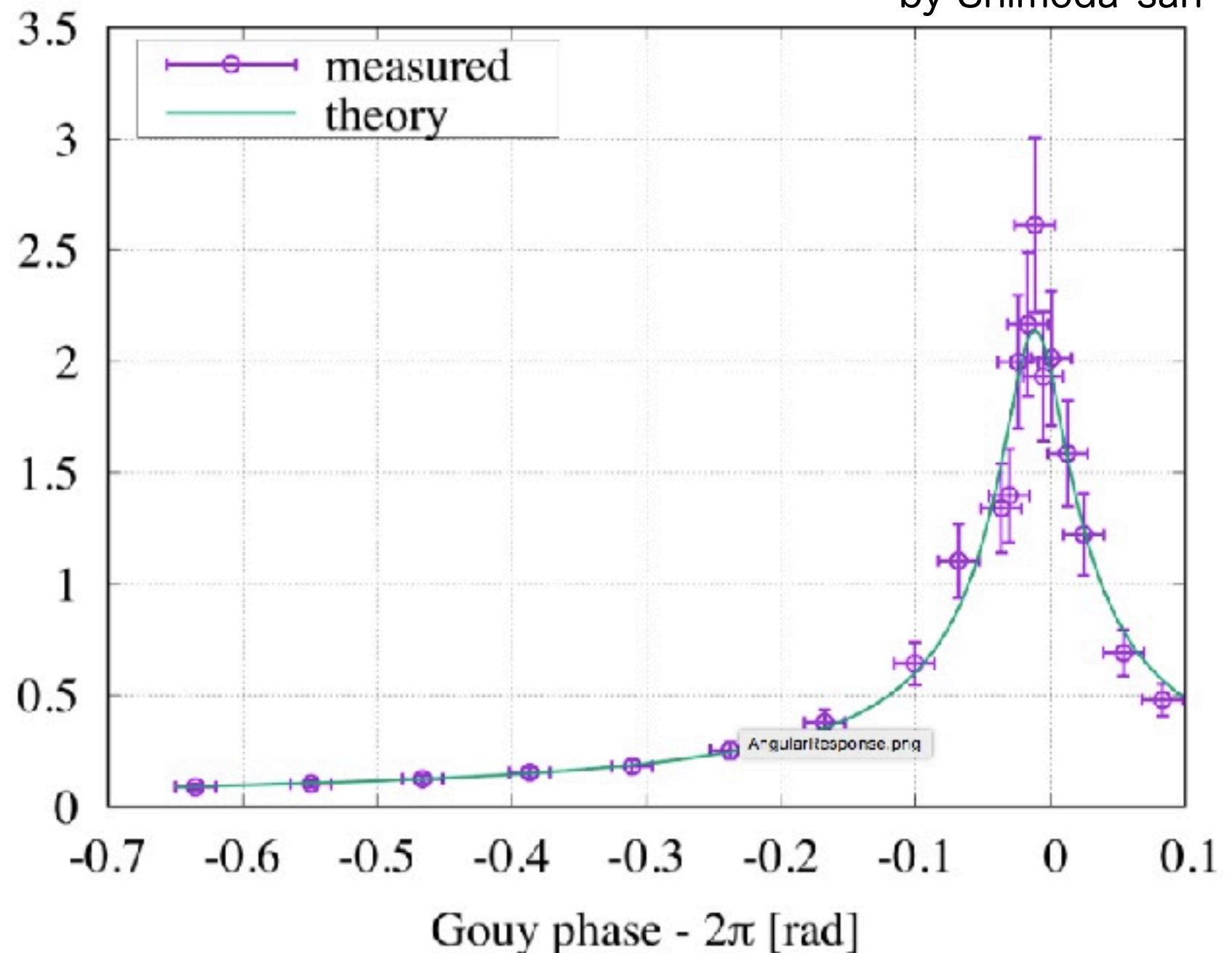
- make round Gouy phase 2π using curved mirrors

by Shimoda-san



round Gouy phase
 $(\textcircled{1} + \textcircled{2} + \textcircled{3}) \times 2 = 2\pi$

Angular response [W/rad]



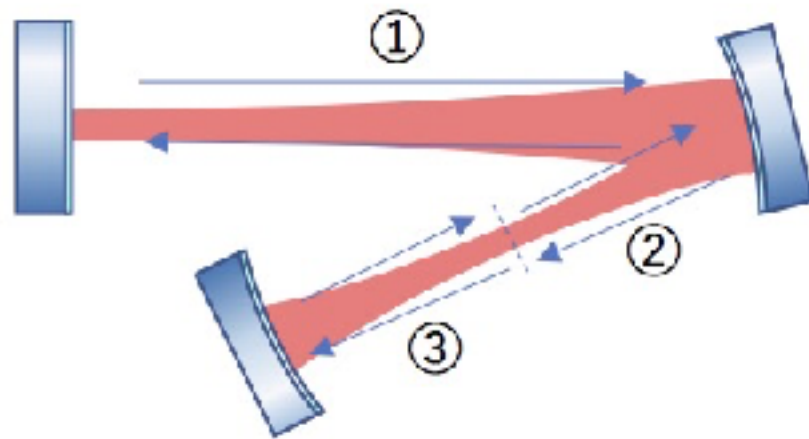
Changing cav length,
confirmed the angular
signal is amplified when
00 and 10 modes resonate
simultaneously.

Idea of sensitive sensors

00 and 10 can resonate simultaneously in the below sensors

① folded cavity (demonstration test was done by Shimoda-san)

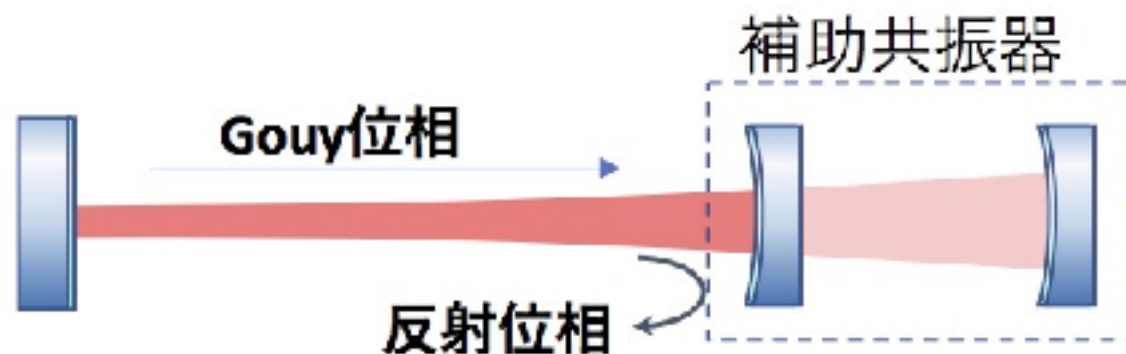
- make round Gouy phase 2π using curved mirrors



round Gouy phase
 $(\textcircled{1} + \textcircled{2} + \textcircled{3}) \times 2 = 2\pi$

② coupled cavity

- reflective phase compensates Gouy phase

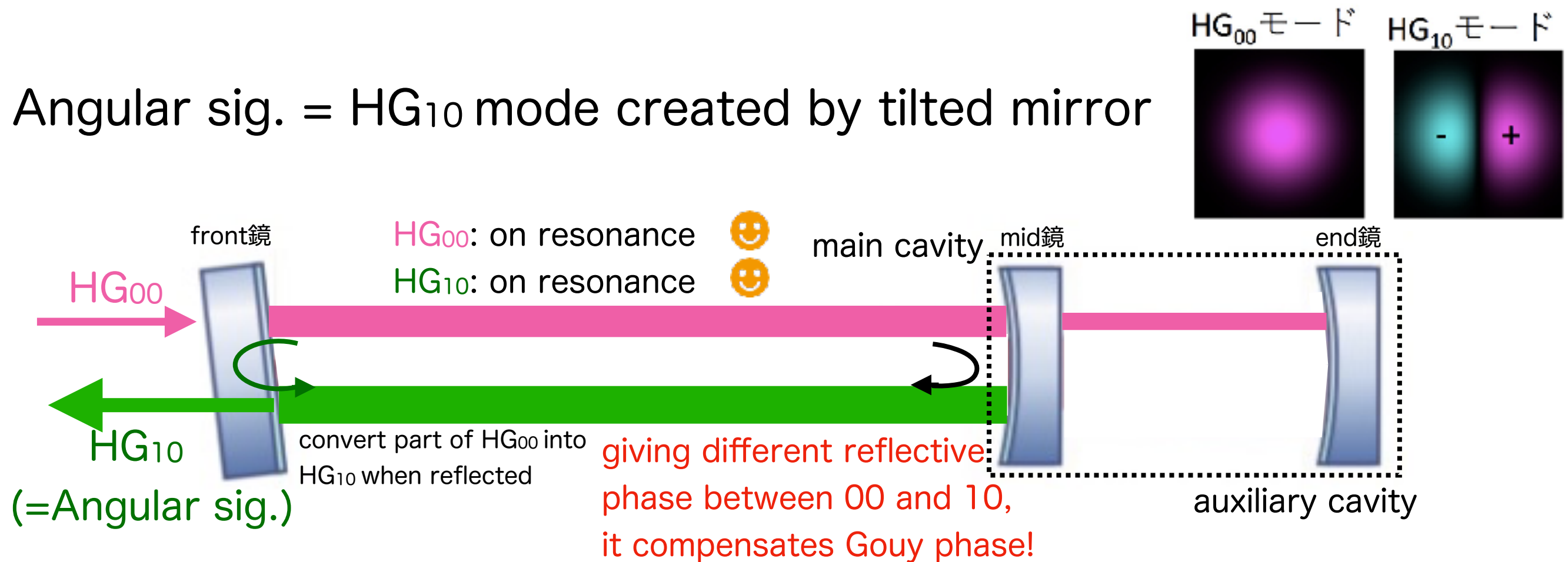


On going

round Gouy phase
+ reflective phase = 0

Coupled Wave Front Sensor

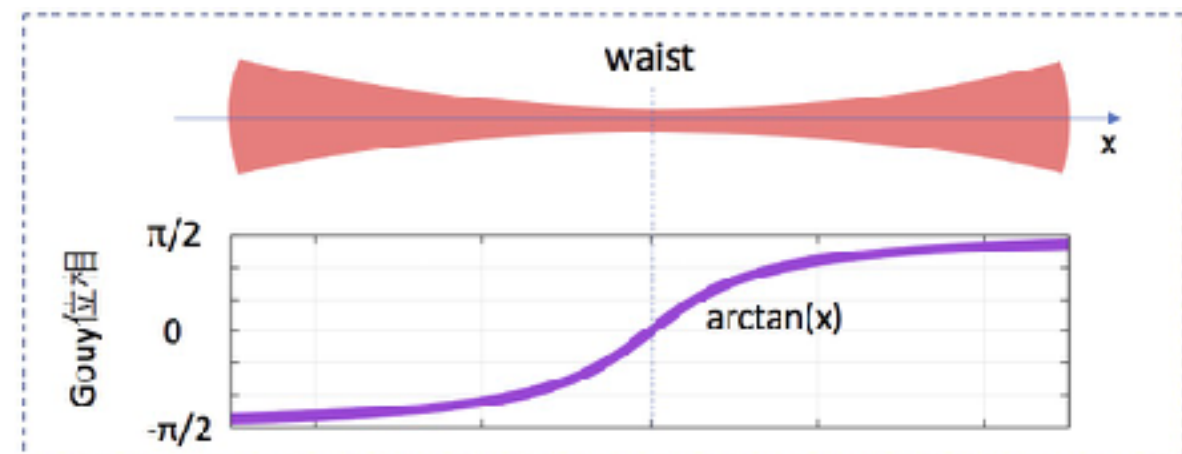
Angular sig. = HG₁₀ mode created by tilted mirror



The phase HG₀₀ and HG₁₀ receives in a round trip differs. The phase shift is called **Gouy phase**.

→ improved one:

HG₀₀ and HG₁₀ CAN resonate simultaneously

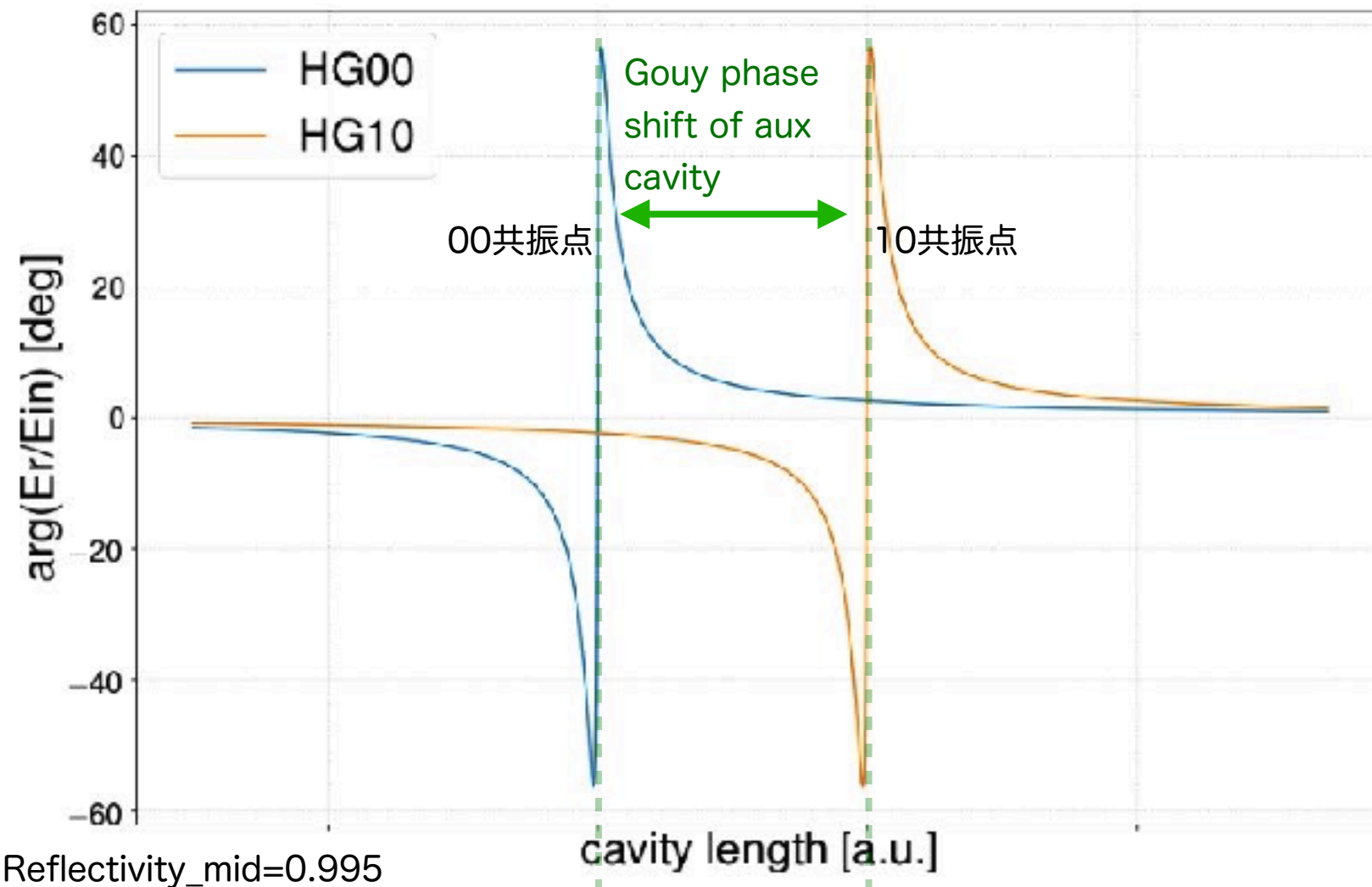
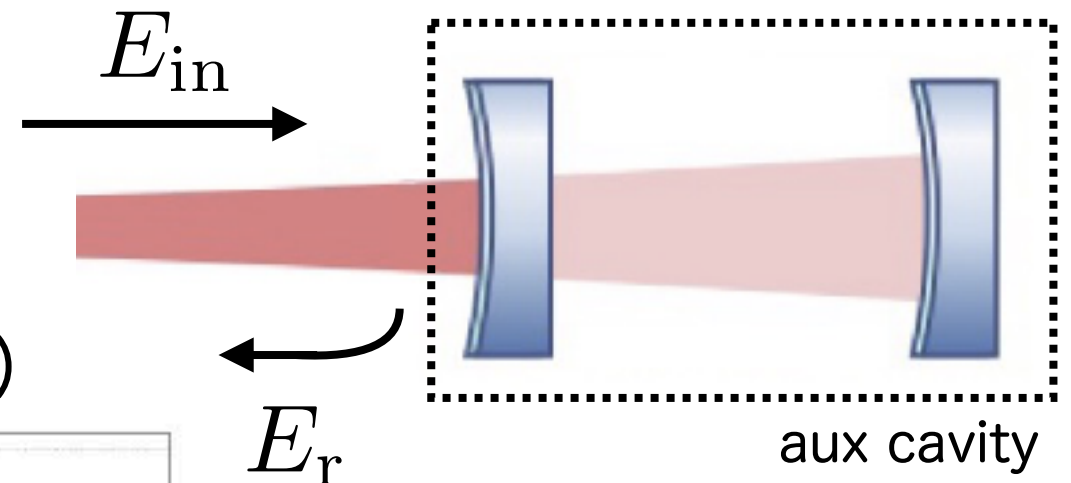


In an ordinary cavity,
 $0 < (\text{round trip Gouy phase}) < 2\pi$

phase compensation by using a cavity

(complex) reflectivity of a cavity
$$= E_r / E_{in}$$

$\arg(E_r / E_{in})$ (=reflective phase)



give different reflective phase between HG₀₀ and HG₁₀ when reflected by the auxiliary cavity

Reflectivity_{mid}=0.995

Reflectivity_{end}=0.994

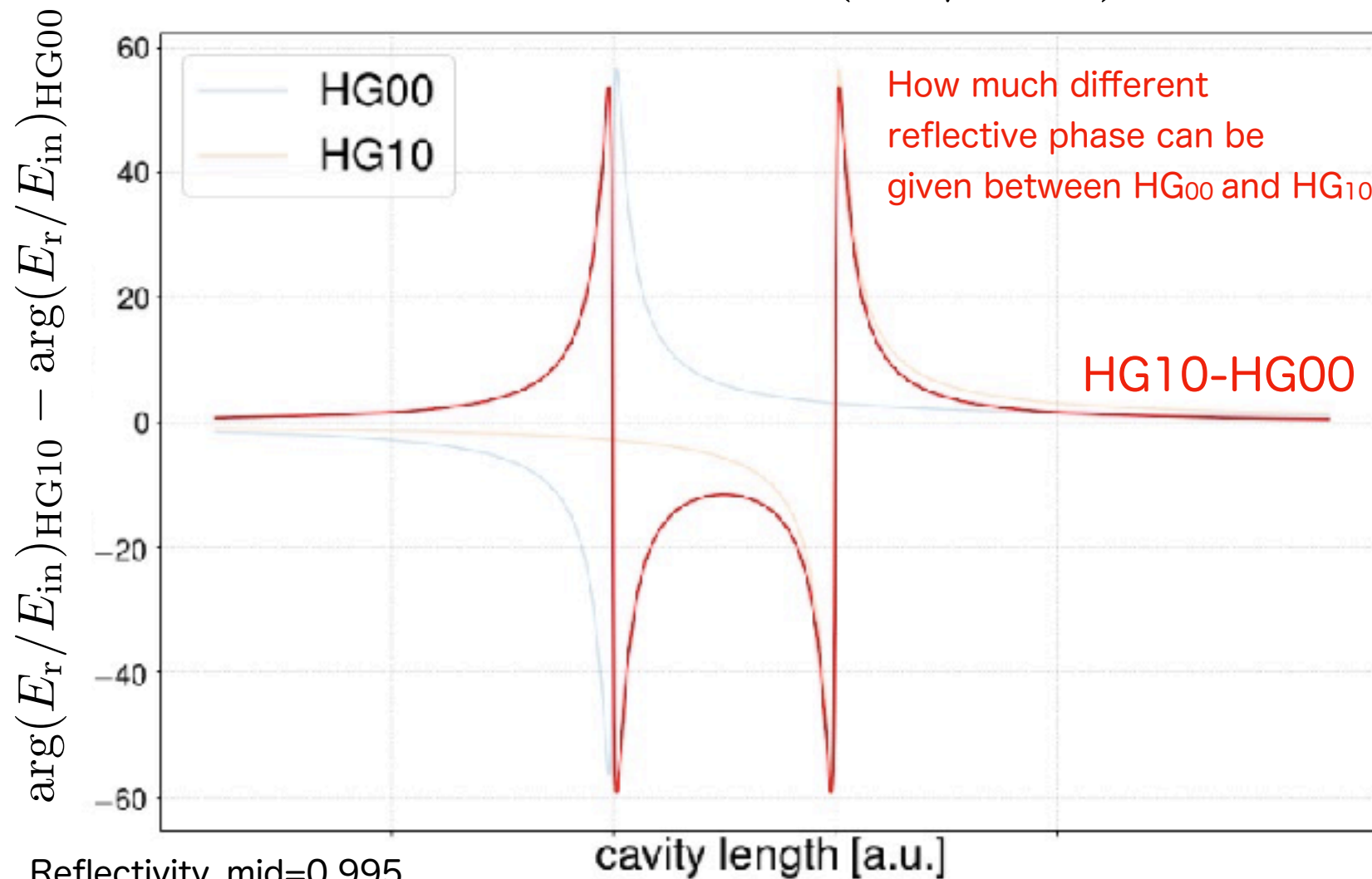
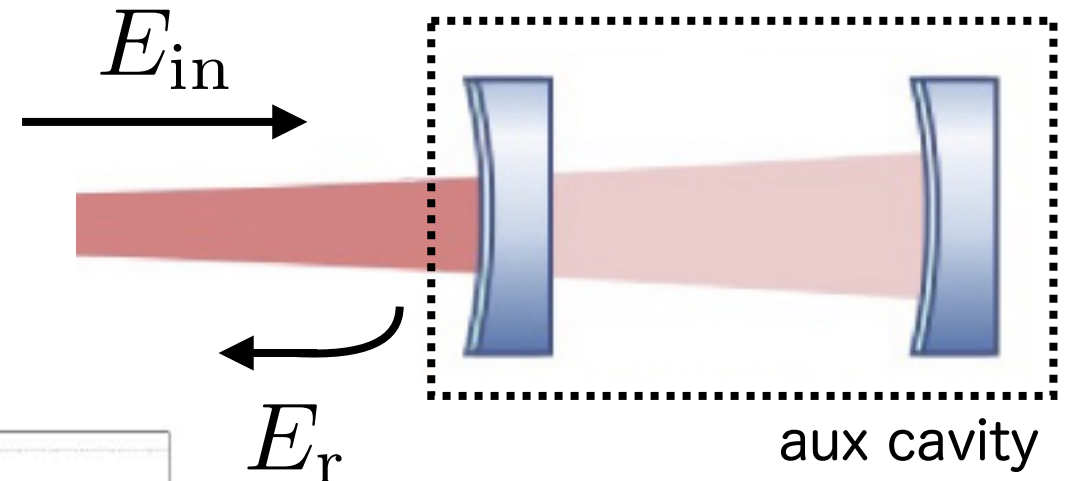
Gouy_{main}=20deg, Gouy_{auxiliary}=5.6deg

phase compensation by using a cavity

(complex) reflectivity of a cavity

$$= E_r / E_{in}$$

difference of $\arg(E_r / E_{in})$



give different reflective phase between HG₀₀ and HG₁₀ when reflected by the auxiliary cavity

Reflectivity_{mid}=0.995

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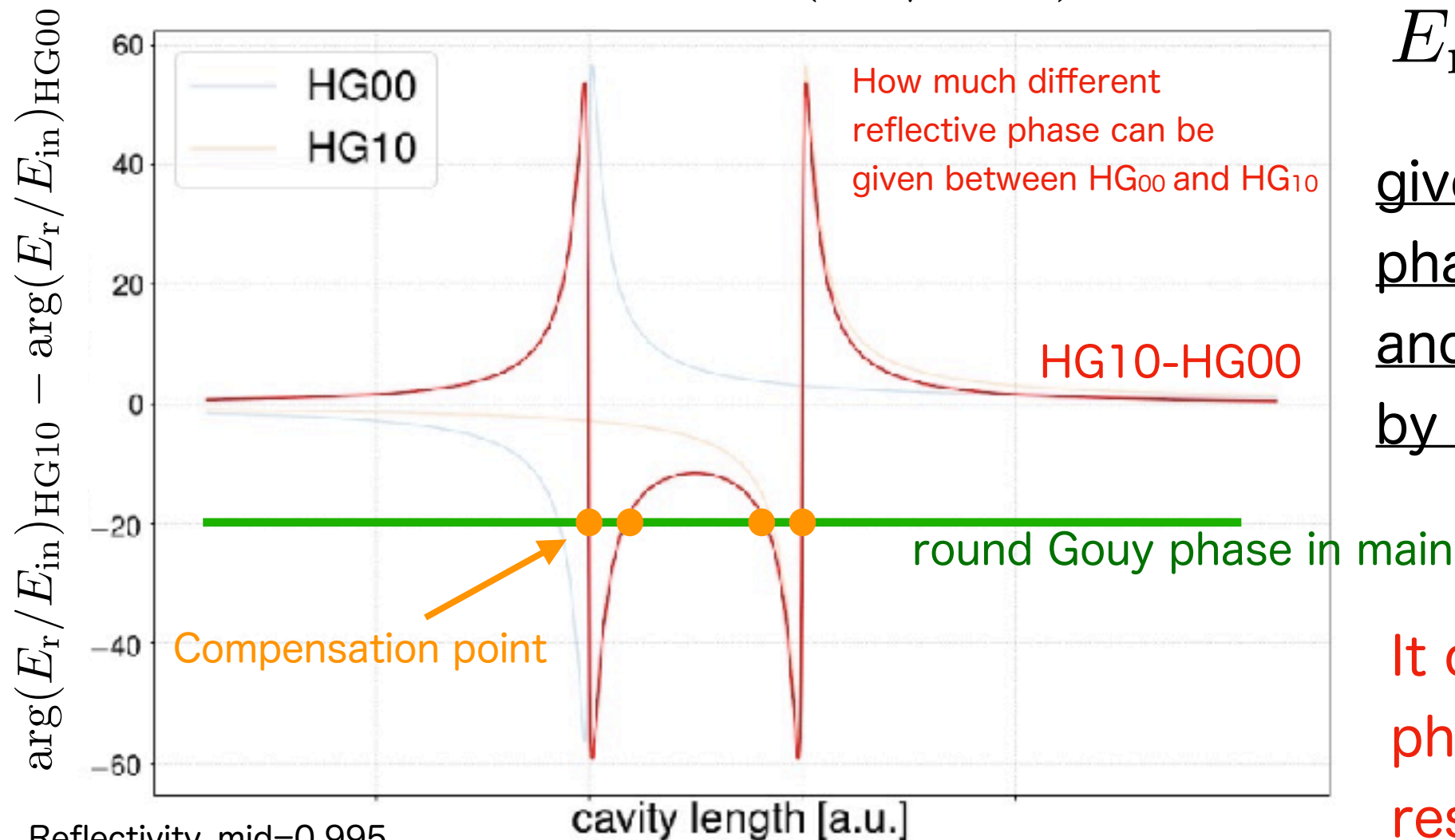
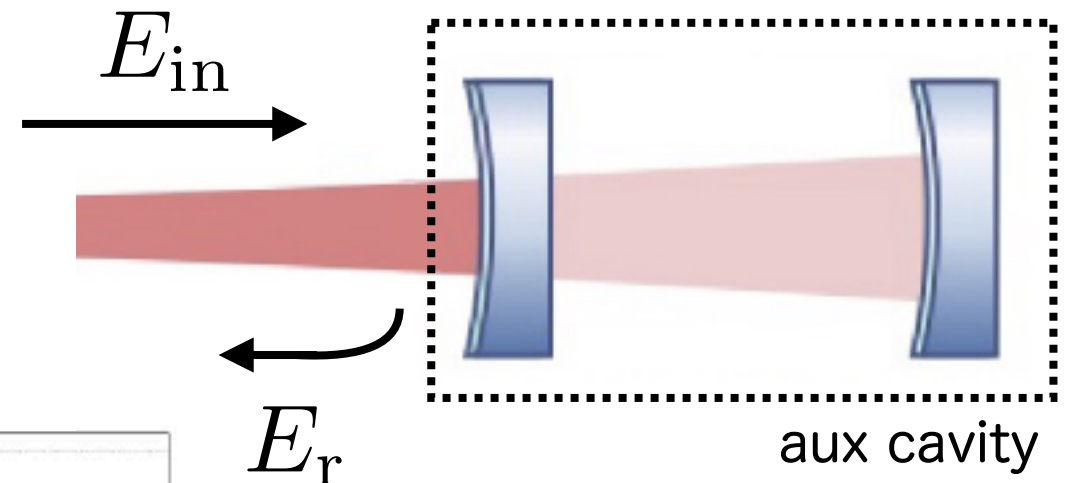
Gouy_{main}=20deg, Gouy_{auxiliary}=5.6deg

phase compensation by using a cavity

(complex) reflectivity of a cavity

$$= E_r / E_{in}$$

difference of $\arg(E_r / E_{in})$



give different reflective phase between HG₀₀ and HG₁₀ when reflected by the auxiliary cavity

It can compensate Gouy phase shift and HG₁₀ can be resonated simultaneously

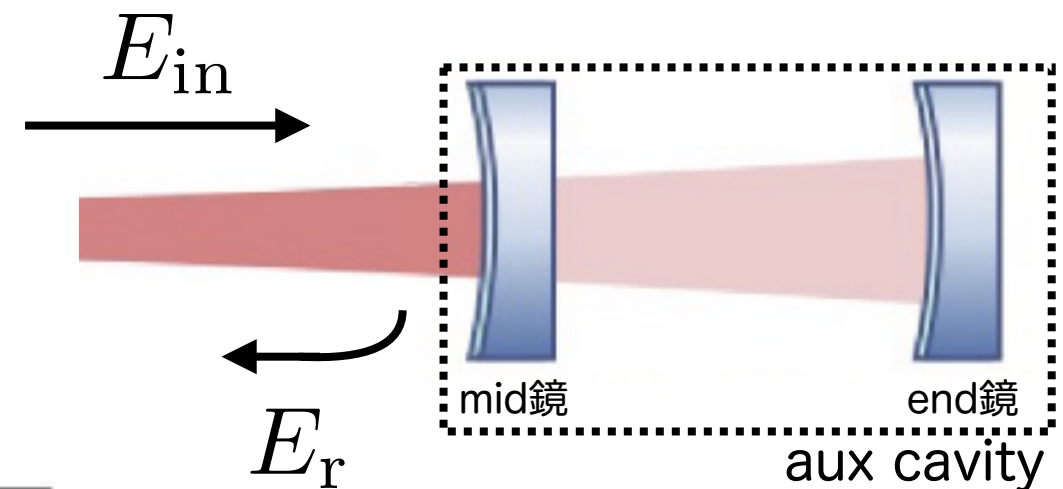
Reflectivity_{mid}=0.995

Reflectivity_{end}=0.994

Gouy_{main}=20deg, Gouy_{auxiliary}=5.6deg

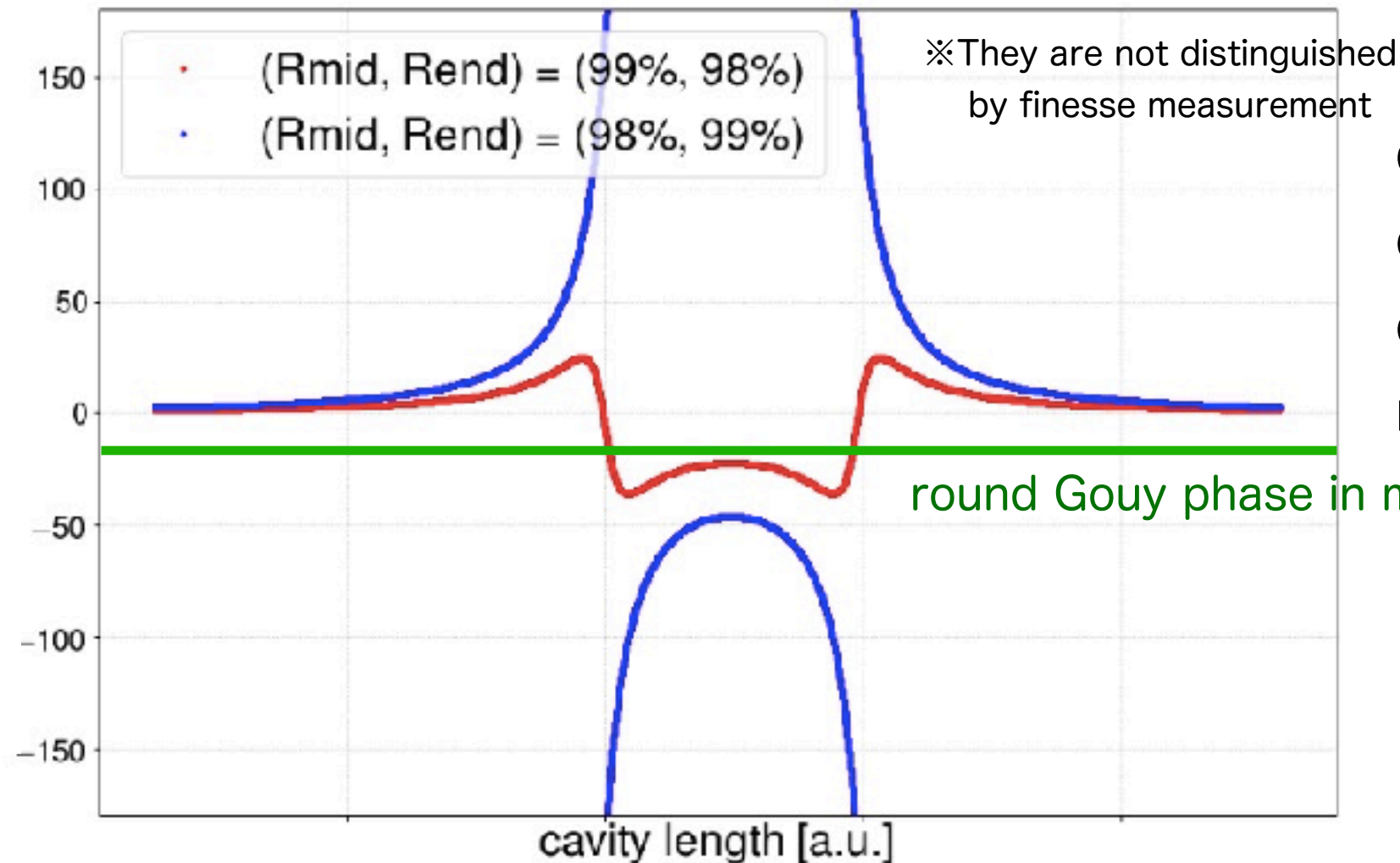
Bad case: cannot compensate Gouy phase

(complex) reflectivity of a cavity
 $= E_r / E_{in}$



difference of $\arg(E_r / E_{in})$

$\arg(E_r / E_{in})_{HG10} - \arg(E_r / E_{in})_{HG00}$



depending on the reflectivity
of 2 mirrors which composed
of aux cavity, Gouy phase is
not compensated.

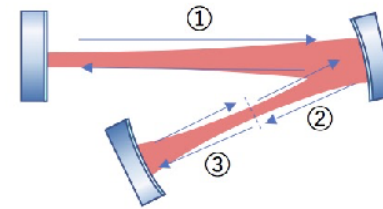
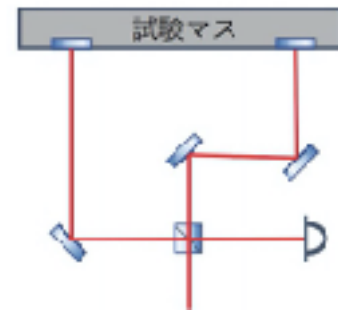
Angular signal (10mode)
is not amplified well

Gouy_main=6.7deg, Gouy_auxiliary=5.6deg

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- about TOBA
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 - merits and demerits (overview)
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merits and demerits



	Michelson interferometer	folded WFS	coupled WFS
shot noise	😊 30cm bar	😊 1mm beam, finesse 300	
frequency noise	😞	😊	😊
trans-coupling	😞	😊	😊
thermal noise	😊	😞	😞
linear range	😞	😊	😊
beam jitter	😐 ?	😊 → 😐 ?	😊 → 😐 ?
shrink in cryo	😊	😞	😊
control/operation	😊	😊	😞

I will talk this
issue later

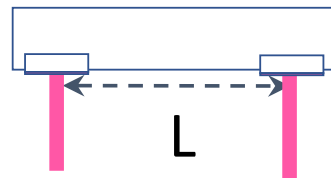
low shot noise



- \Leftrightarrow high angular response (almost same as MI)

Michelson (P_{in} : input power, L : bar length)

$$\sim P_{in} \frac{L}{\lambda} \text{ [W/rad]}$$



new WFS (w : beam radius, F : finesse)

$$\sim P_{in} \frac{w}{\lambda} F \text{ [W/rad]}$$

new WFS with 1mm beam
& finesse 300

\Leftrightarrow MI with 30cm bar

- same as MI shot noise level

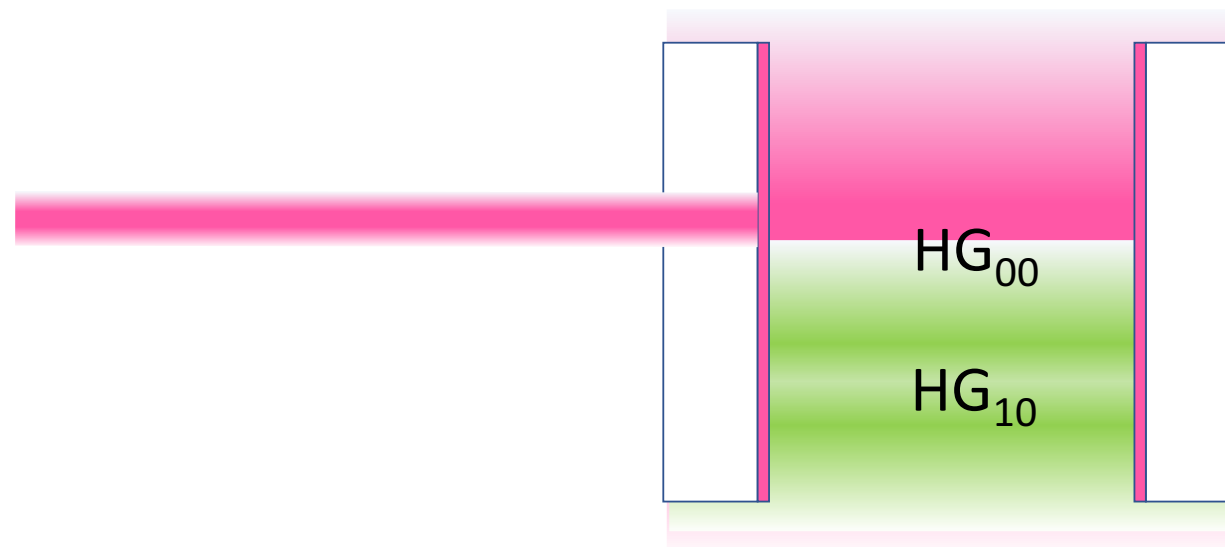
$$\sim 10^{-15} \text{ rad/rtHz}$$

(from Shimoda-san's slides)

no(low) frequency noise



- no frequency noise in wave front sensor



- (spatial nonuniformity of frequency fluctuation can be noise ...?)

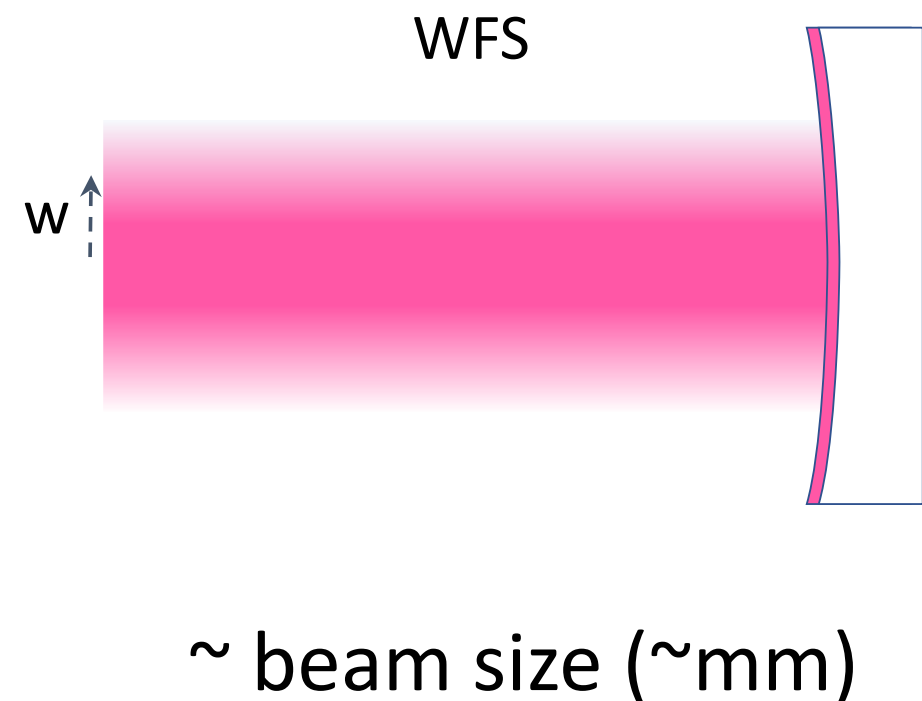
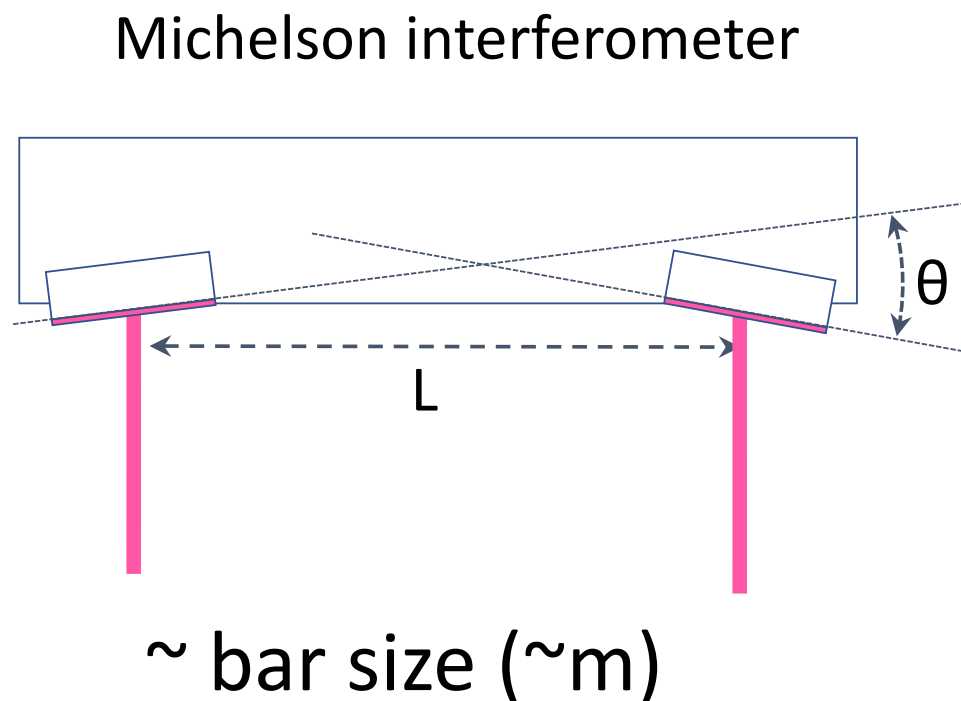
(from Shimoda-san's slides)

small trans-coupling(?)

than Michelson interferometers



- making flatness is easier in smaller scale
- especially this is good point for large bars ($L > 50\text{cm}$) which are difficult to make flatness between both ends



(from Shimoda-san's slides)

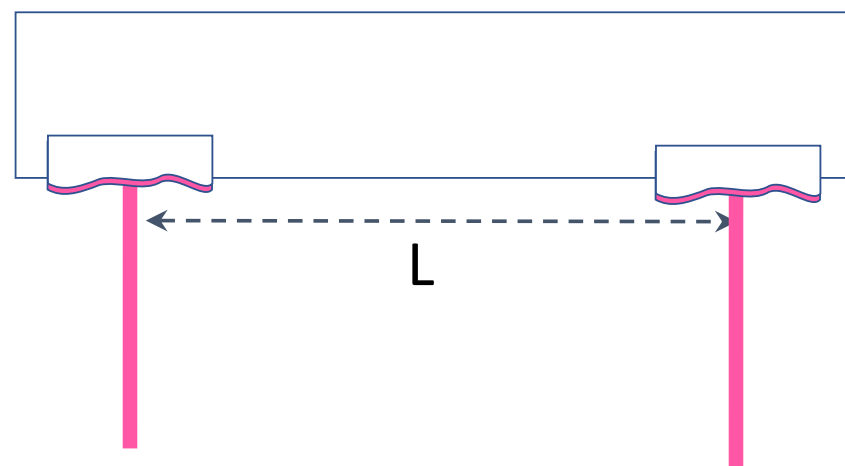
higher thermal brownian

than Michelson interferometers



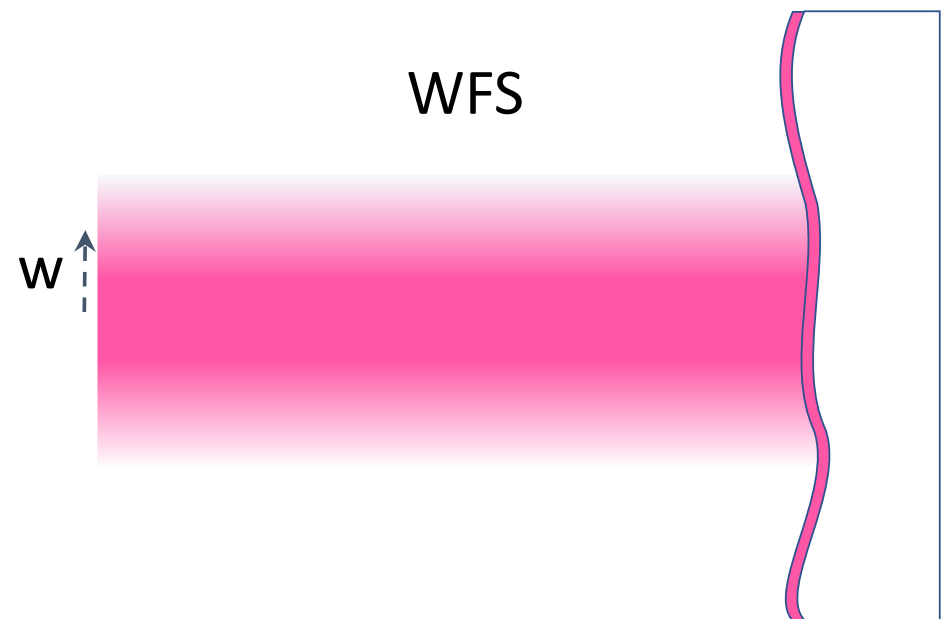
- higher by (bar length) / (beam diameter)

Michelson interferometer



noise \sim (fluctuation) / L

WFS



noise \sim (fluctuation) / w

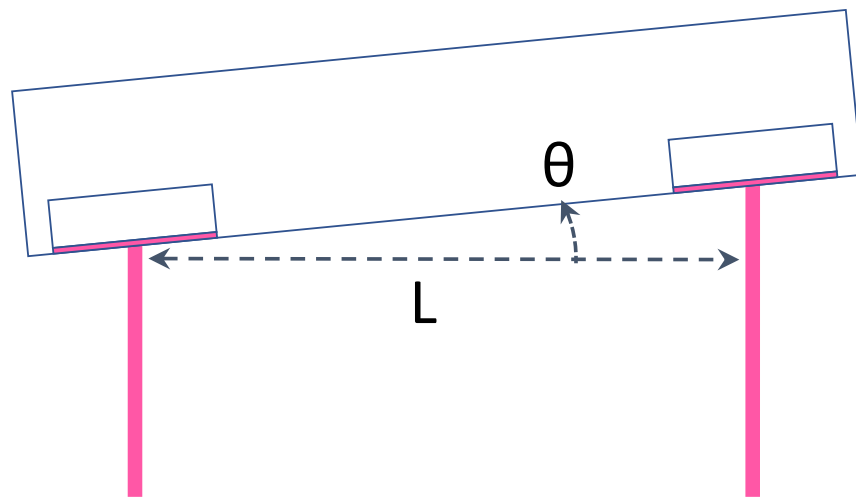
(from Shimoda-san's slides)

large linear range

than Michelson interferometers



- roughly larger by (bar length) / (beam diameter)



$$\theta < \lambda/L \sim 1 \mu\text{rad}$$



$$\theta < \lambda/w \sim 0.1 \text{ mrad}$$

- less(no) angular control is required \Leftrightarrow less(no) actuator noise
- (practically, range of PD should be considered)

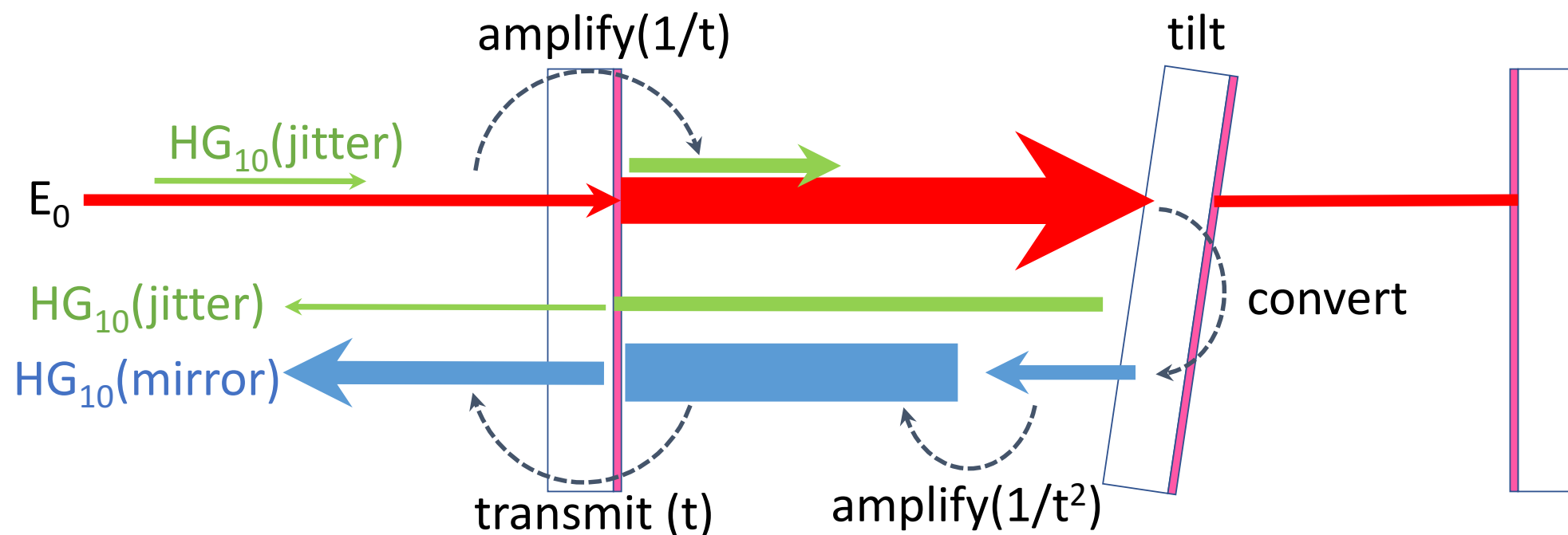
(from Shimoda-san's slides)

low beam jitter noise

than conventional WFSs



- suppressed by finesse of 00 mode



beam jitter θ_{jitter} : generate HG_{10} outside the cavity

$$\Rightarrow E_{10,\text{jitter}} = E_0 \times \theta_{\text{jitter}} \times \underset{\text{(amplify)}}{1/t} \times \underset{\text{(transmit)}}{t}$$

cavity mirror tilt θ_{mir} : generate HG_{10} inside the cavity

$$\Rightarrow E_{10,\text{mirror}} = E_0 \times \underset{\text{(amplify)}}{1/t} \times \theta_{\text{mir}} \times \underset{\text{(amplify)}}{1/t^2} \times \underset{\text{(transmit)}}{t}$$

$$E_{10,\text{jitter}} \sim t^2 \times E_{10,\text{mirror}} \quad (\sim E_{10,\text{mirror}} / F)$$

(from Shimoda-san's slides)

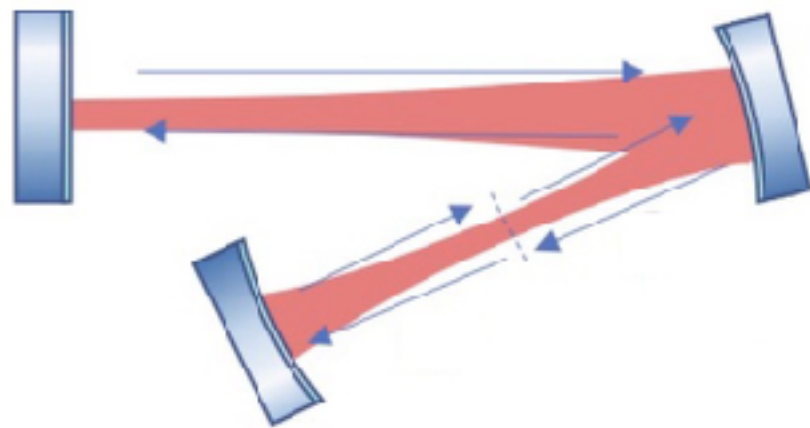
shrink in cryogenic



When putting the WFS into a cryogenic, the system may shrink. (I don't know how much)

→ initial adjustment in cryogenic is needed

folded cavity



- need to adjust the cavity length about Rayleigh range (~ 1m)



coupled cavity



- need to adjust the cavity length about wave length (~ 1 μ m)



difficulty of control/operation ☹️

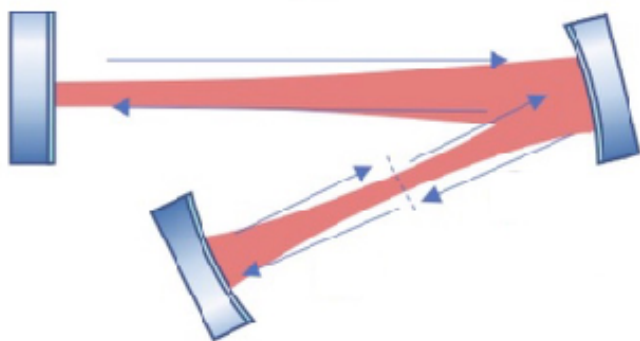


signal separation

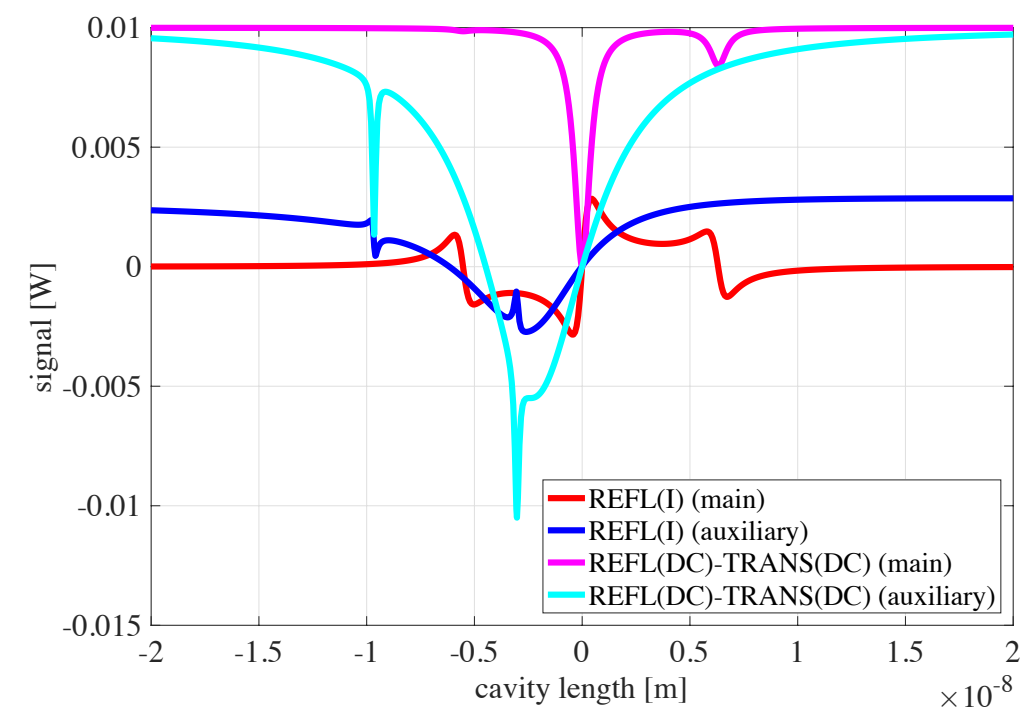
- control of 2 DoF is much difficult

- not bad
($R_{\text{front}} = 99\%$,
 $R_{\text{mid}} = 99.95\%$,
 $R_{\text{end}} = 99.9\%$)

On the other hand,



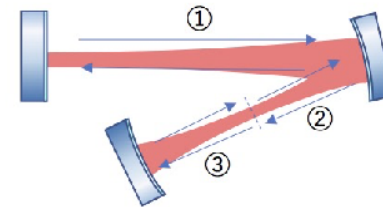
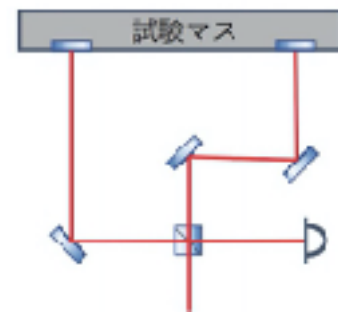
- folded cavity has only 1 DoF.



sensing matrix	main cavity	auxiliary cavity
PDH signal (refl)	1 —	0.09 —
DC signal (refl-trans)	0 —	1 —

(from Shimoda-san's slides)

merits and demerits



	Michelson interferometer	folded WFS	coupled WFS
shot noise	😊 30cm bar	😊 1mm beam, finesse 300	
frequency noise	😞	😊	😊
trans-coupling	😞	😊	😊
thermal noise	😊	😞	😞
linear range	😞	😊	😊
beam jitter	😐 ?	😊 → 😐 ?	😊 → 😐 ?
shrink in cryo	😊	😞	😊
control/operation	😊	😊	😞

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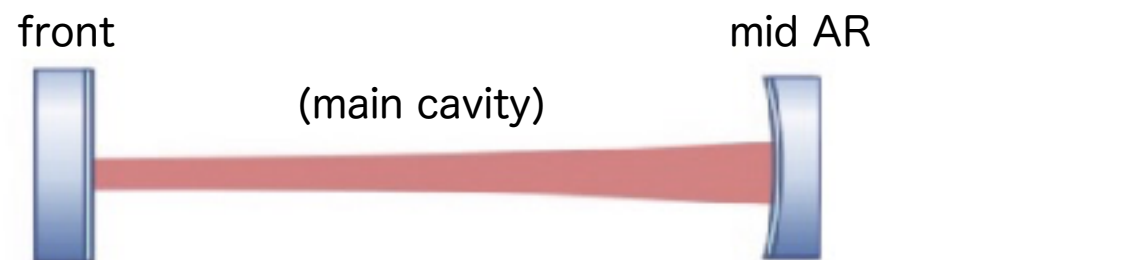
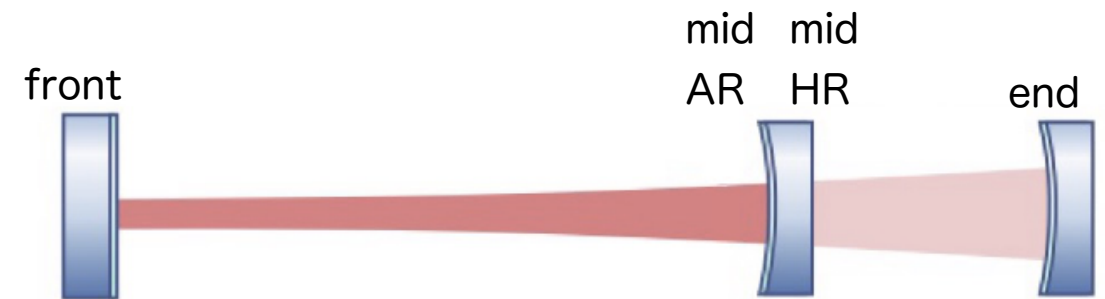
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reflectivity measurement

reflective phase and finesse are important

→ reflectivity is to be measured value

there are 4 reflectivity



finesse

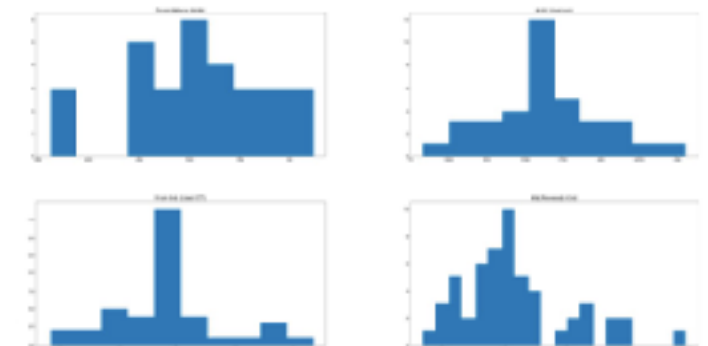
506 +/- 68
(503)

1625 +/- 346
(4189)

481 +/- 94
(571)

2177 +/- 875
(1795)

histogram of measured finesse



(see log p=2303)

Results:

$R_{\text{front}} = 98.87 \pm 0.16 \%$ (99%)

$R_{\text{end}} = 99.82 \pm 0.16 \%$ (99.9%)

$R_{\text{midAR}} = 99.88 \pm 0.16 \%$ (99.75%)

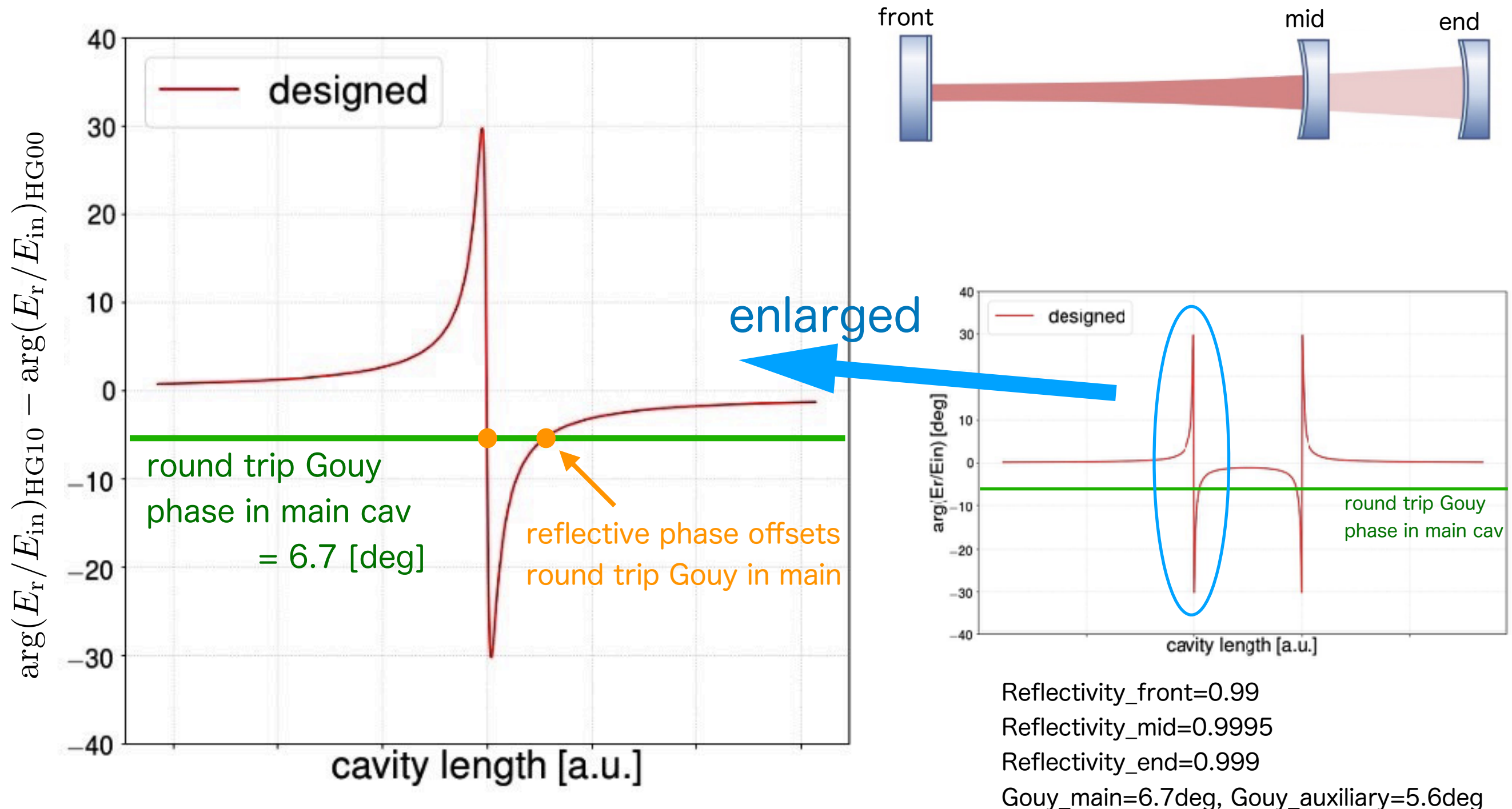
$R_{\text{midHR}} = 99.79 \pm 0.18 \%$ (99.95%)

$\text{ARloss} = -0.09 \pm 0.31 \%$

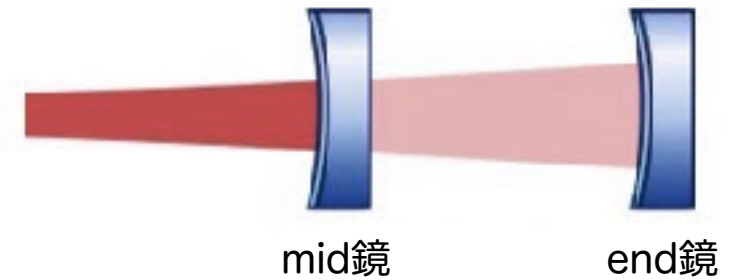
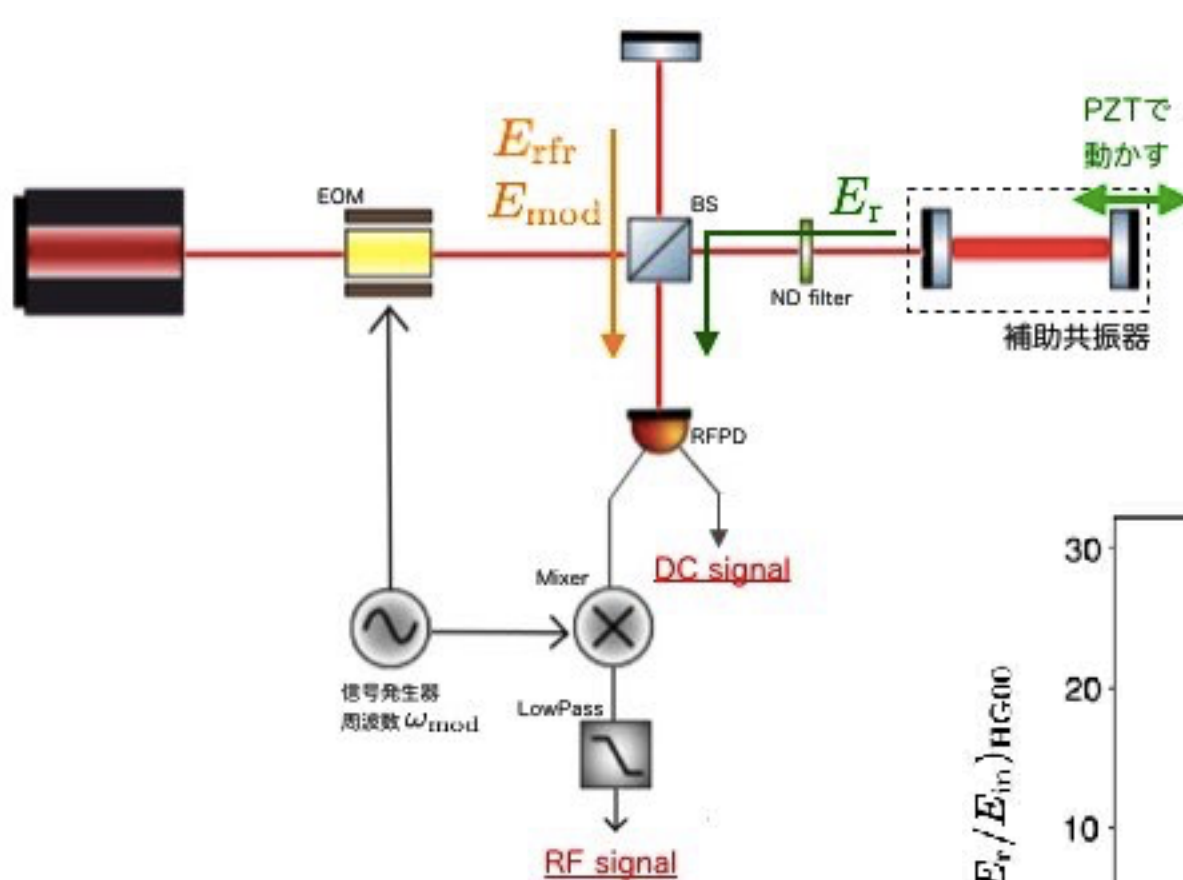
statistical error is big

reflectivity measurement

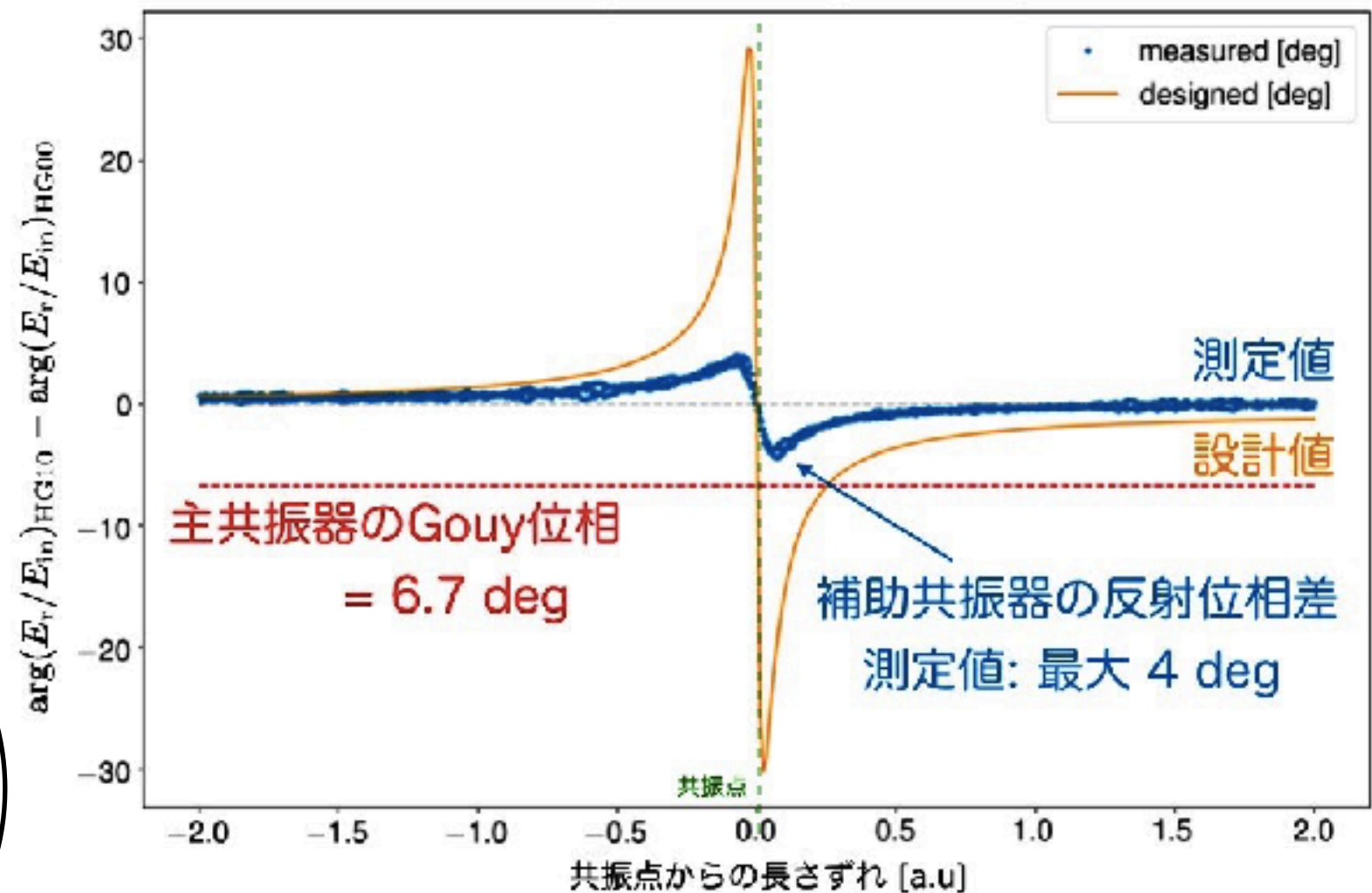
Designed reflectivity:



reflectivity measurement



補助共振器の反射位相差 (HG10 - HG00)



$$R_{\text{mid}} = 99.97(2)\%$$

$$R_{\text{end}} = 99.5(2)\%$$

小

The result is consistent with
finesse measurement

$$2 - (R_{\text{mid}} + R_{\text{end}}) = 0.39(8)\% \\ (\text{design: } 0.15\%)$$

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Future Plans

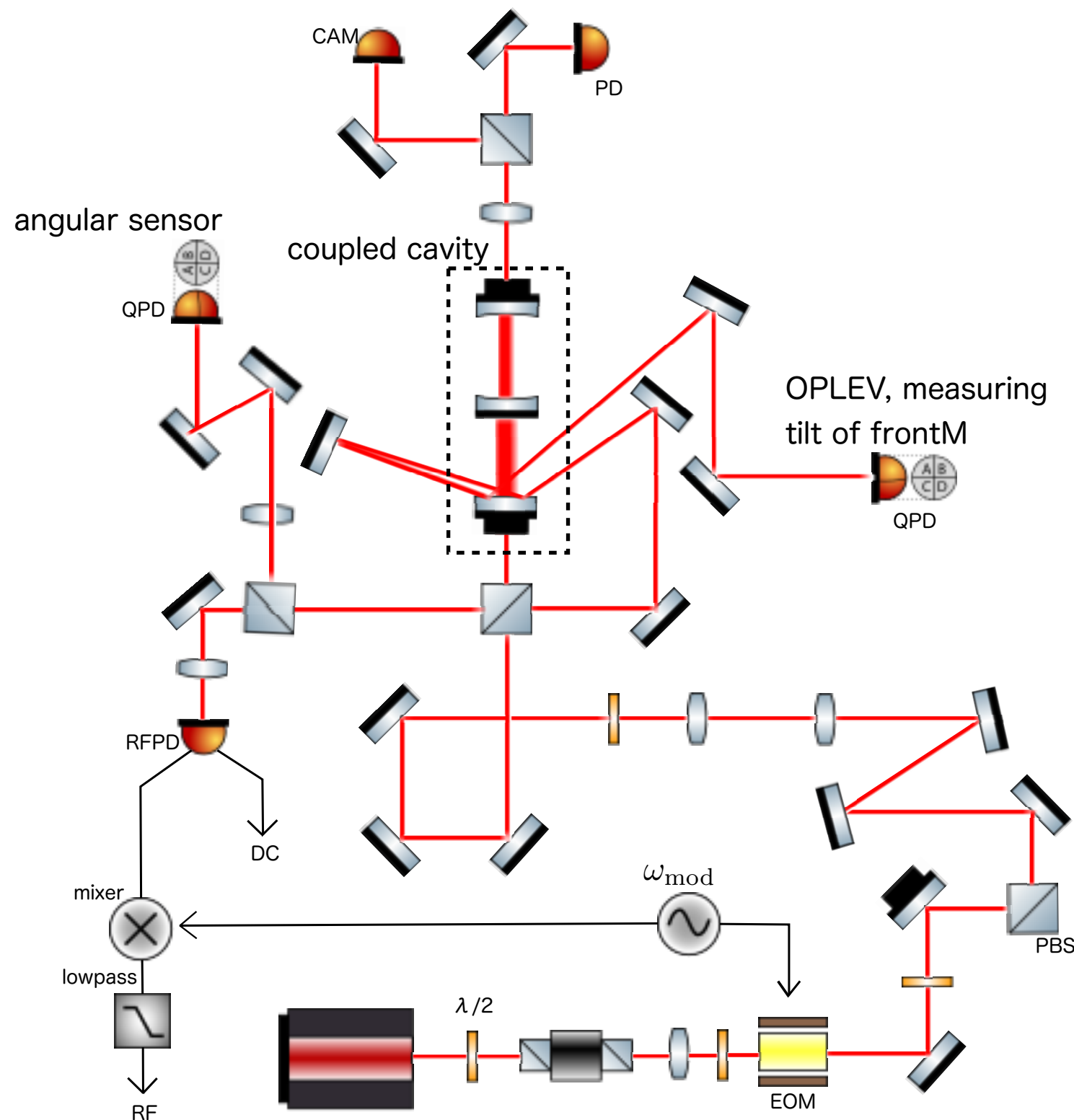
- demonstration test (confirm its principle)
 - confirm the amplification of angular signal
 - compare with theoretical one
 - check its characteristics
 - frequency noise
 - beam jitter
- construct a new (semi-monolithic) system
 - trans-rotational coupling
 - performance test in cryogenic
 - obtain good sensitivity
 - reduce jitter noise
 - other technical noises

Future Plans

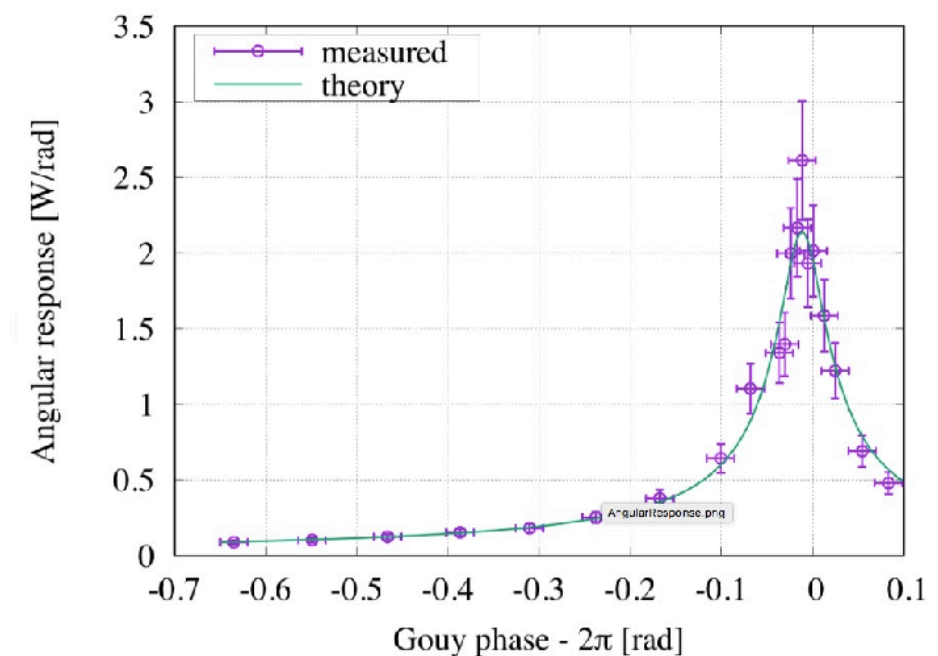
- demonstration test (confirm its principle)
 - confirm the amplification of angular signal
 - compare with theoretical one
 - check its characteristics
 - frequency noise
 - beam jitter
- construct a new (semi-monolithic) system
 - trans-rotational coupling
 - performance test in cryogenic
 - obtain good sensitivity
 - reduce jitter noise
 - other technical noises

demonstration test

- confirm the amplification of angular signal
- compare with theoretical one



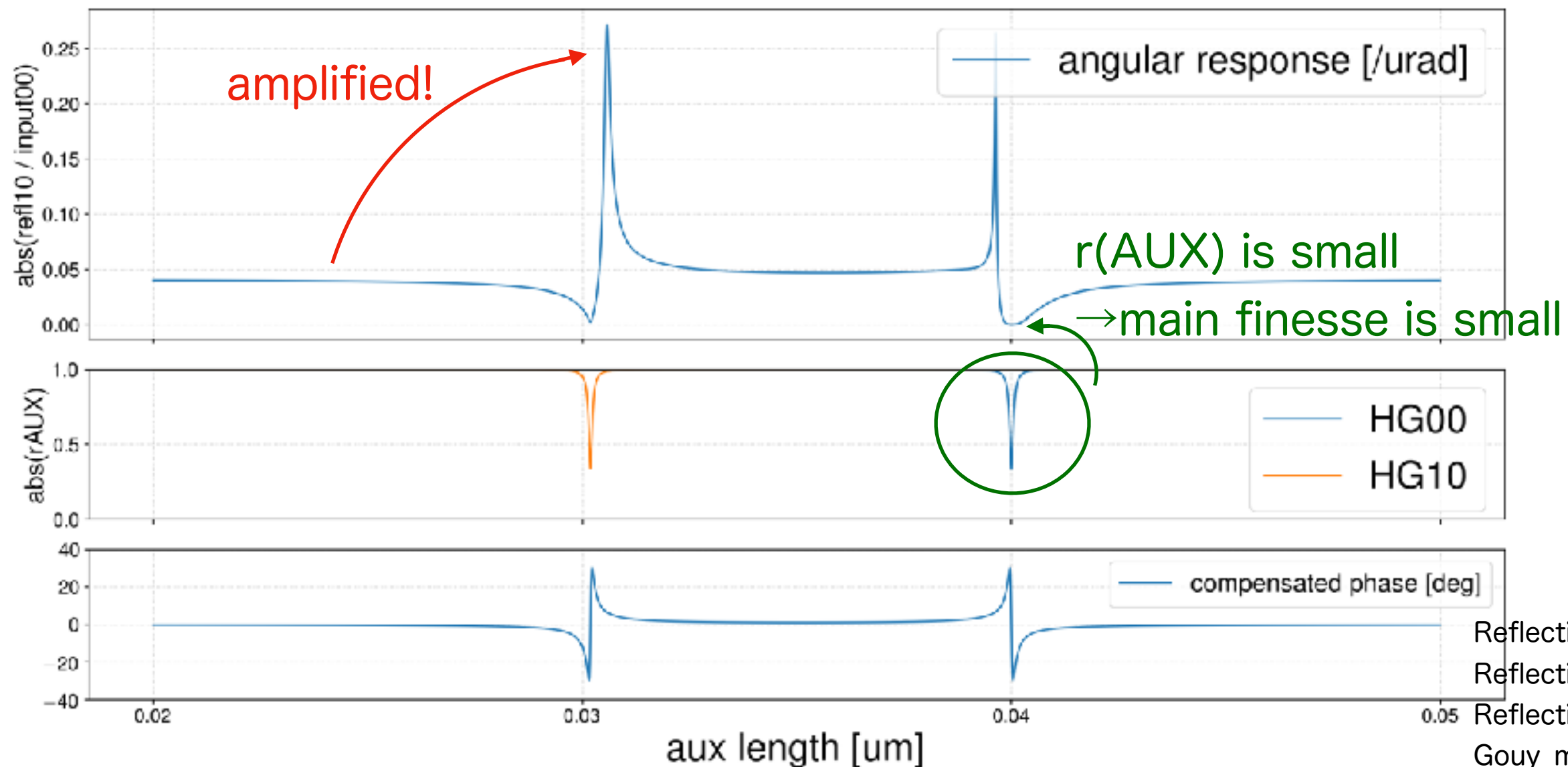
Example:
confirming the amplification of
Folded cavity by Shimoda-san



demonstration test

- confirm the amplification of angular signal
- compare with theoretical one

How much the angular signal(HG10) is generated. (main:locked, front:tilted(1 urad), end:sweep)



Reflectivity_front=0.99
Reflectivity_mid=0.9995
Reflectivity_end=0.999
Gouy_main=6.7deg
Gouy_auxiliary=5.6deg

demonstration test

- confirm the amplification of angular signal
- compare with theoretical one

What to do to confirm the amplification of angular signal:



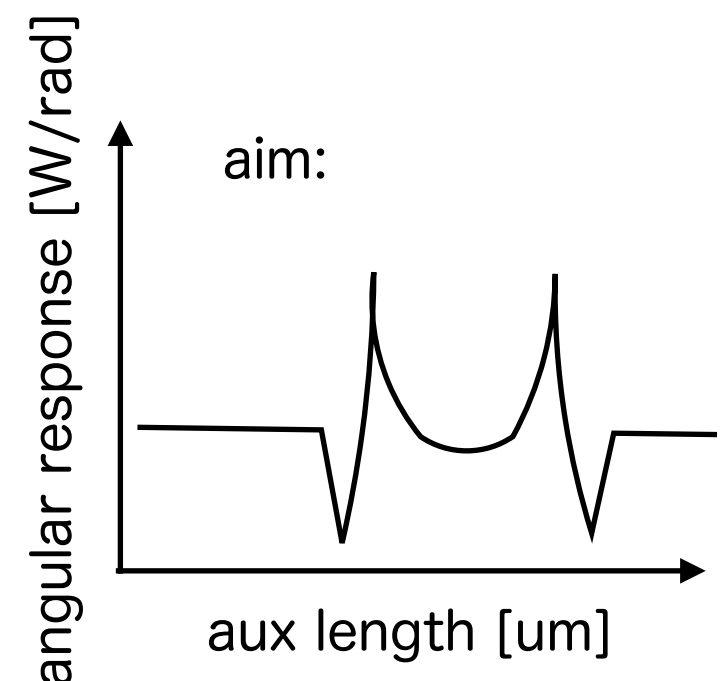
- increase reflectivity of (end) mirror
- control 3 mirrors
- sweep endM while main cav locked
- calibrate efficiency of PZT on endM
- calibrate OPLEV [rad/V]
- measure DC/RF efficiency of QPD
→ obtain angular response [W/rad]

QPD receives

$$E_{00} + (\alpha + i\beta)E_{10}$$

α : translational → DC

β : rotational → RF



demonstration test

- test merits of the coupled cavity

- no/low frequency noise

injecting signal to EOM,

check angular response doesn't change

- suppress beam jitter

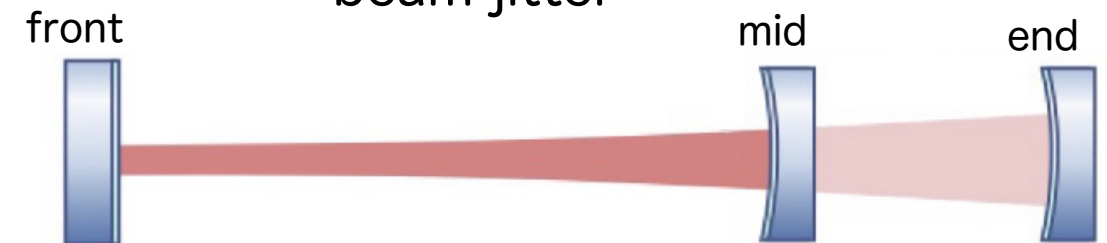
shaking PZT on a mirror
in front of the cavity,

check angular response
doesn't change

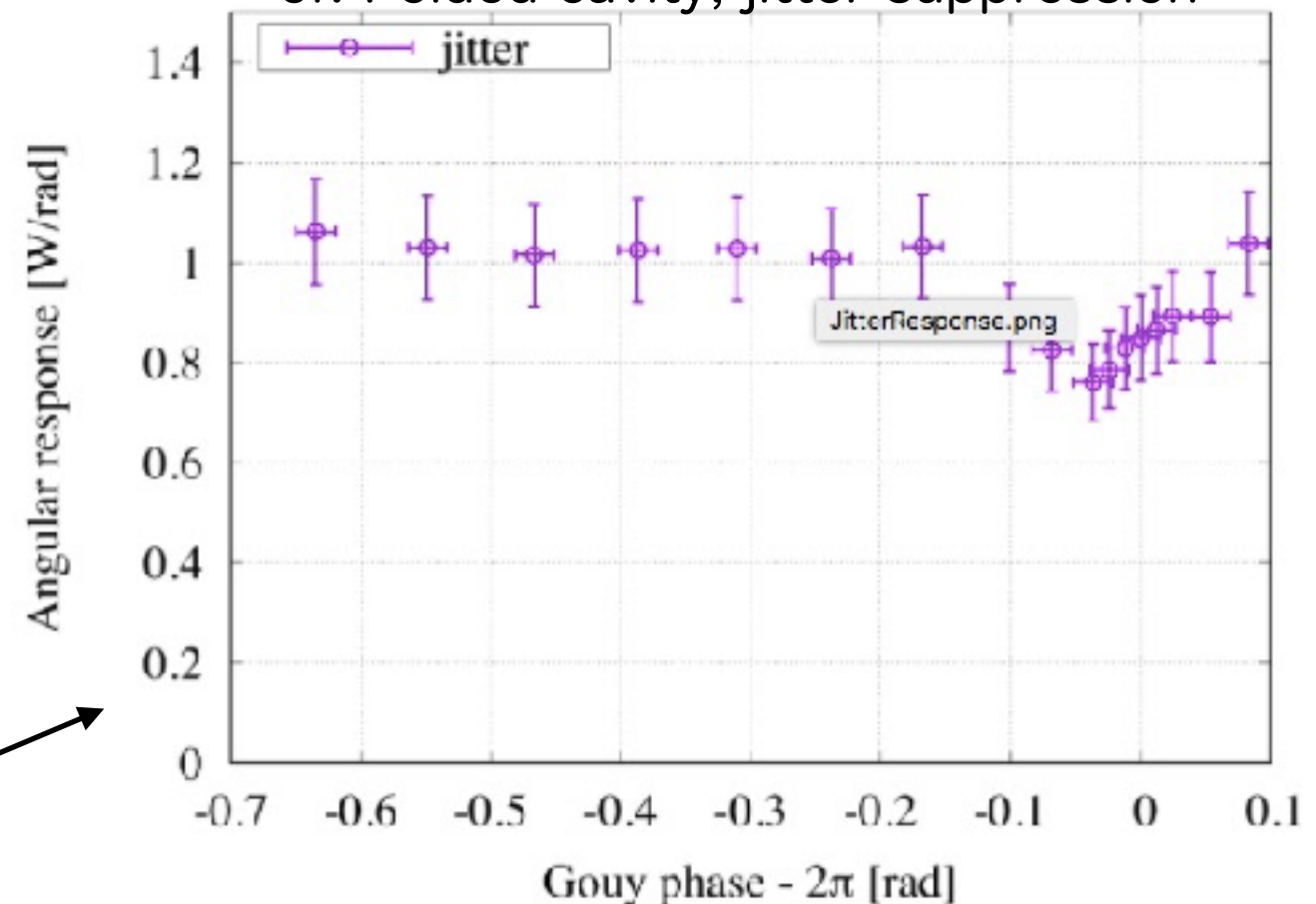
- check its characteristics

- frequency noise

- beam jitter



cf. Folded cavity, jitter suppression



ref: response to beam jitter for Folded cavity
by Shimoda-san

Future Plans

- demonstration test (confirm its principle)
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 - frequency noise
 - beam jitter
- construct a new (semi-monolithic) system
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 - other technical noises

Future Plans

I also want to check/do

- trans-rotational coupling
- performance in cryogenic
- obtain good sensitivity

put the frontM on a stage and translate it

construct a small system and put it into cryogenic

	Michelson interferometer	folded WFS	coupled WFS
shot noise	😊 30cm bar	😊 1mm beam, finesse 300	
frequency noise	😞	😊	😊
<u>trans-coupling</u>	😞	😊	😊
thermal noise	😊	😞	😞
linear range	😞	😊	😊
beam jitter	😞?	😊 → 😞?	😊 → 😞?
<u>stability in cryo</u>	😊	😞	😊
control/operation	😊	😊	😞

I will talk this issue later

Future Plans

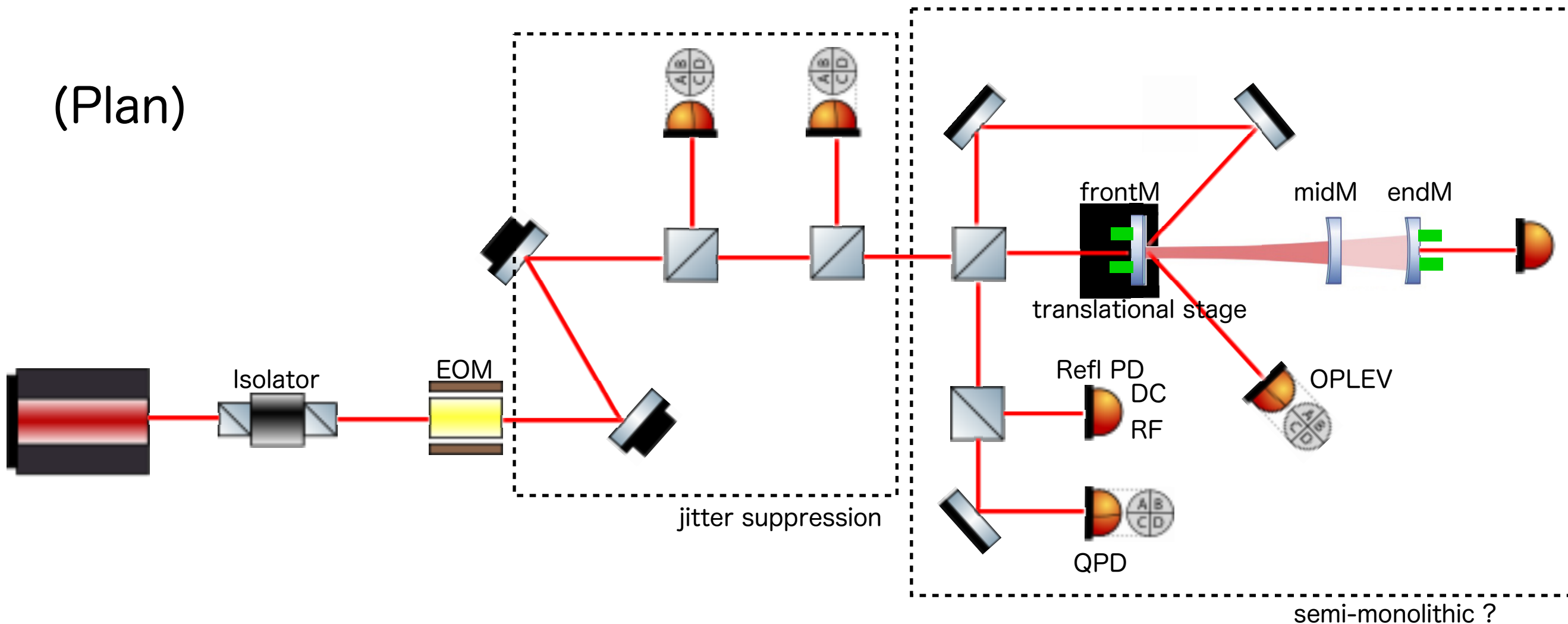
I also want to check/do

- trans-rotational coupling
- performance in cryogenic
- obtain good sensitivity

put the frontM on a stage and translate it

construct a small system and put it into cryogenic

(Plan)



Future Plans

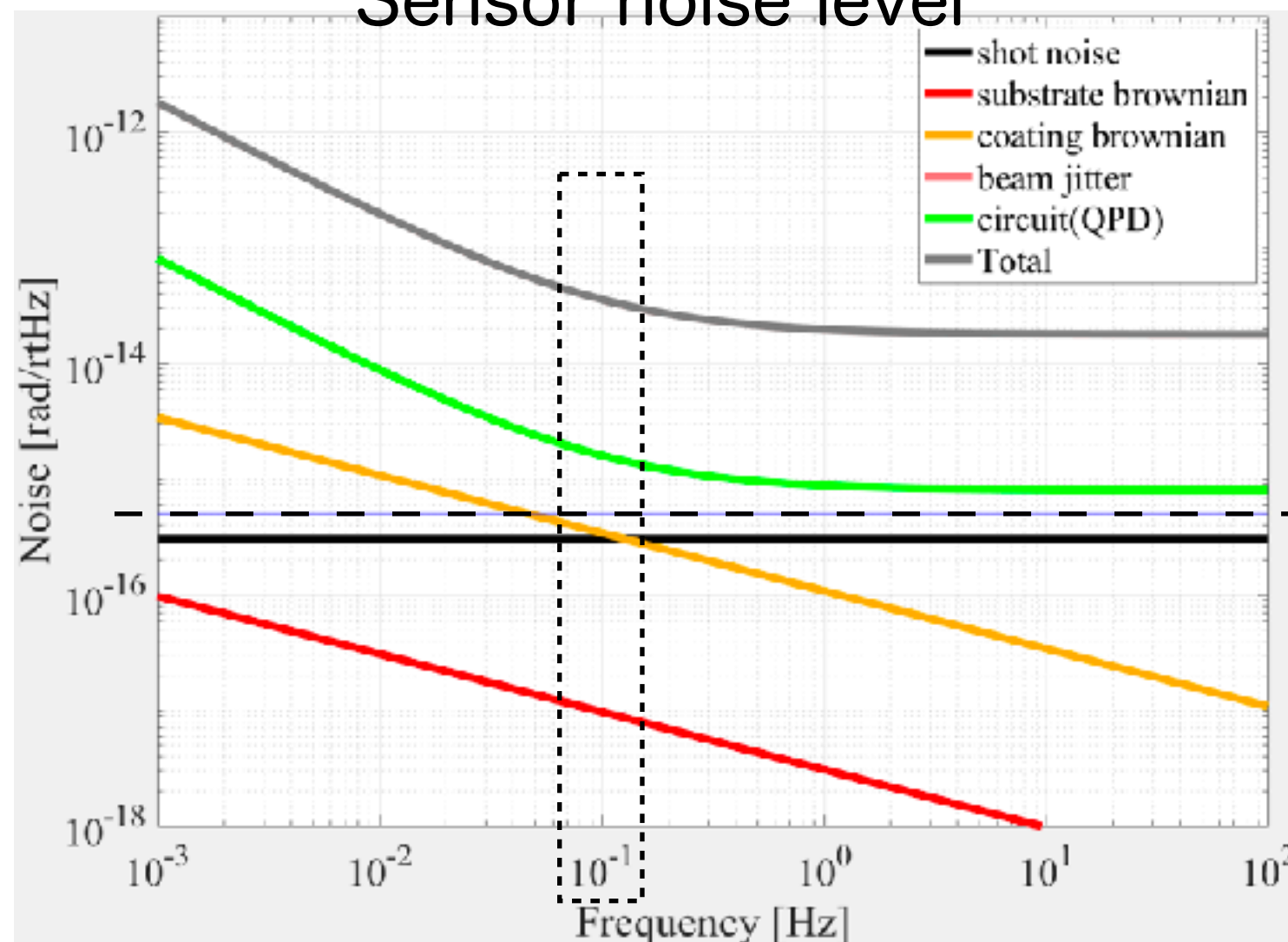
I also want to check/do

- trans-rotational coupling
- performance in cryogenic
- obtain good sensitivity

put the frontM on a stage and translate it

construct a small system and put it into cryogenic

Sensor noise level



wavelength: 1064nm

beam radius: 3.5mm

beam power: 50mW

cavity length: ~10cm

finesse main cavity:

620 (00 mode)

210 (10 mode)

temperature: 4K

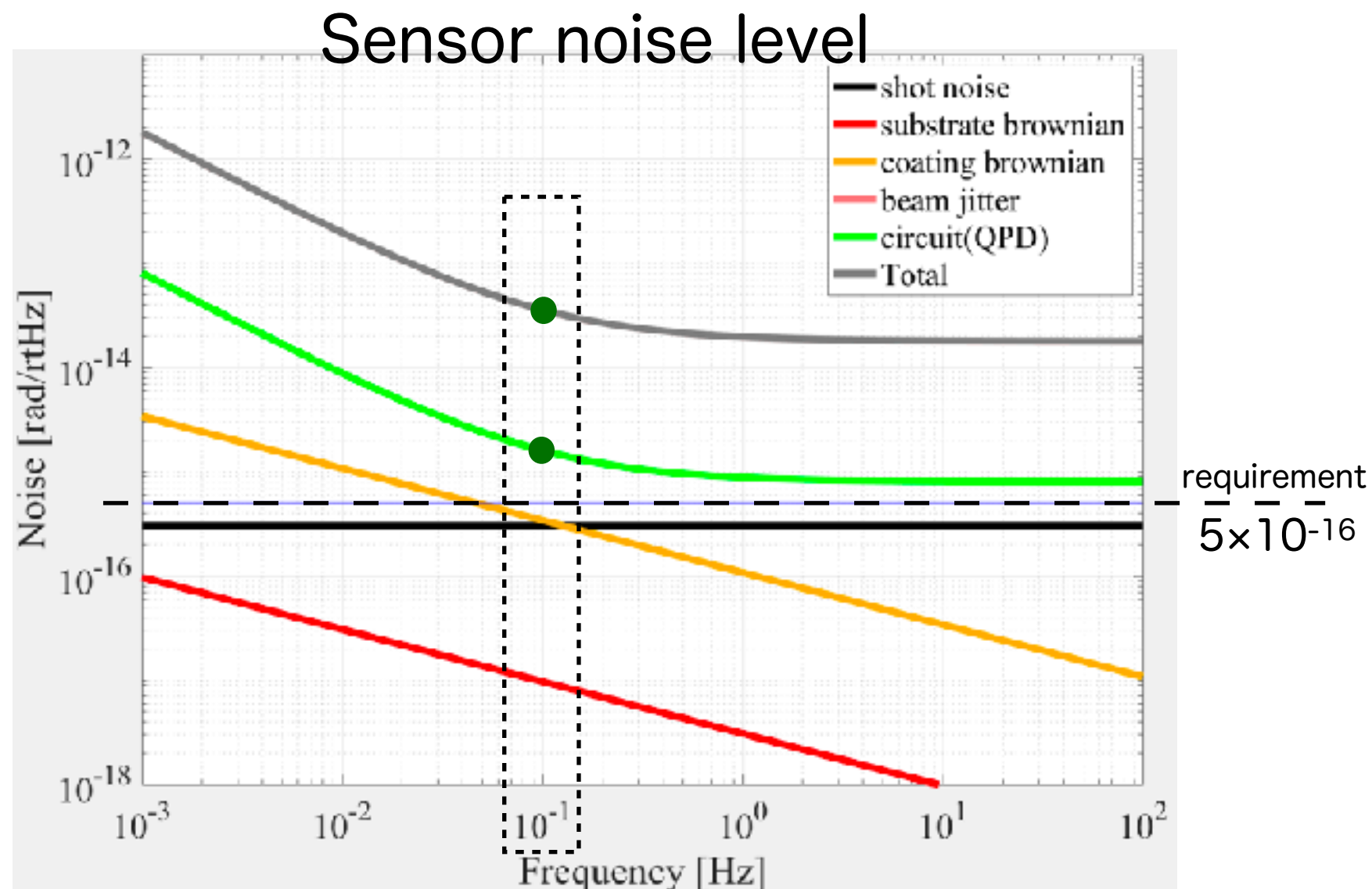
substrate: sapphire

substrate loss: 10^{-8}

coating loss: 10^{-3}

To achieve good sensitivity

- shot noise OK
- thermal noise OK
- beam jitter noise factor: 100
- QPD noise factor: 3



wavelength: 1064nm
beam radius: 3.5mm
beam power: 50mW
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finesse main cavity:
620 (00 mode)
210 (10 mode)
temperature: 4K
substrate: sapphire
substrate loss: 10^{-8}
coating loss: 10^{-3}

beam jitter noise suppression

- beam jitter noise factor: 100

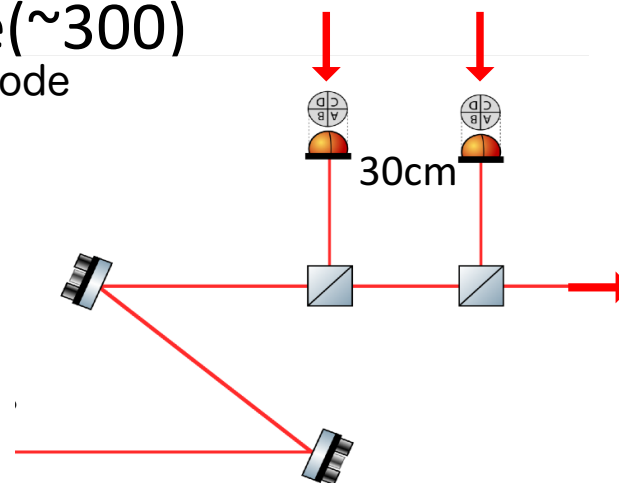
(from Shimoda-san)

jitter suppression

- QPD noise : $\sim 10^{-7}$ V/rtHz @0.1Hz
- position sensitivity of QPD ($w=0.4\text{mm}$) : ~ 30 V/mm
 - position sensing noise : 3×10^{-12} m/rtHz
- measure beam center at two points (separated by 30cm)
 - angular jitter : 10^{-11} rad/rtHz
- jitter noise is suppressed by finesse(~ 300) of 00mode
 - 3×10^{-14} rad/rtHz

How to suppress:

1. set distance between QPDs much bigger ($\sim 1\text{m}$?)



beam jitter noise suppression

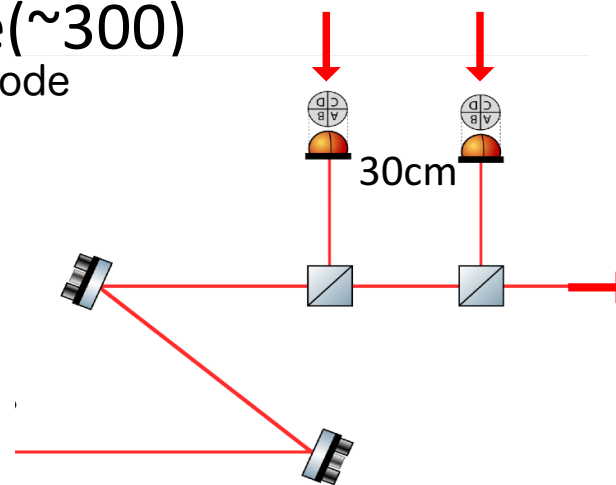
- beam jitter noise factor: 100

(from Shimoda-san)

jitter suppression

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→ 3×10^{-14} rad/rtHz

of 00mode



How to suppress:

1. set distance between QPDs much bigger ($\sim 1\text{m}$?)
2. suppress QPD noise
current problem:
 - cannot increase sensing gain due to saturation of OPamp
solution:
 - use RF (not DC) ?
 - take difference between output electricity (not voltage) of QPD devices ?

beam jitter noise suppression

- beam jitter noise factor: 100

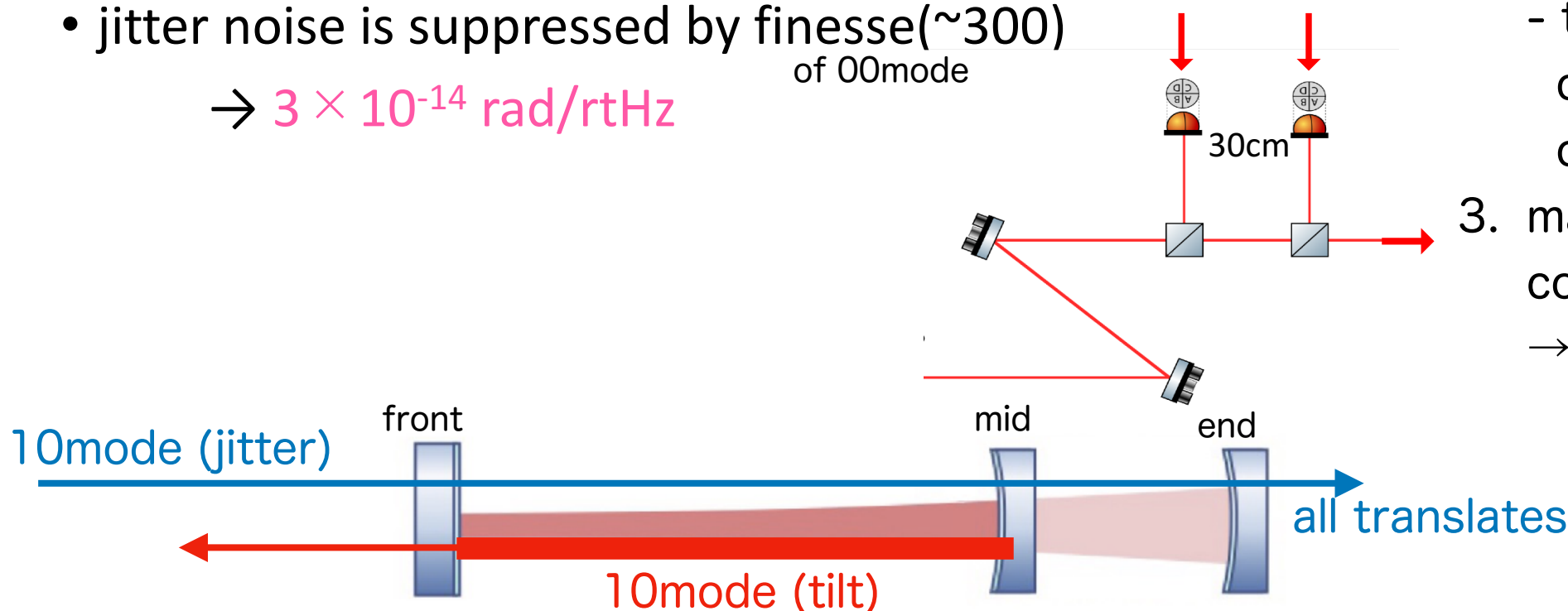
(from Shimoda-san)

jitter suppression

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- position sensitivity of QPD ($w=0.4\text{mm}$) : ~ 30 V/mm
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How to suppress:

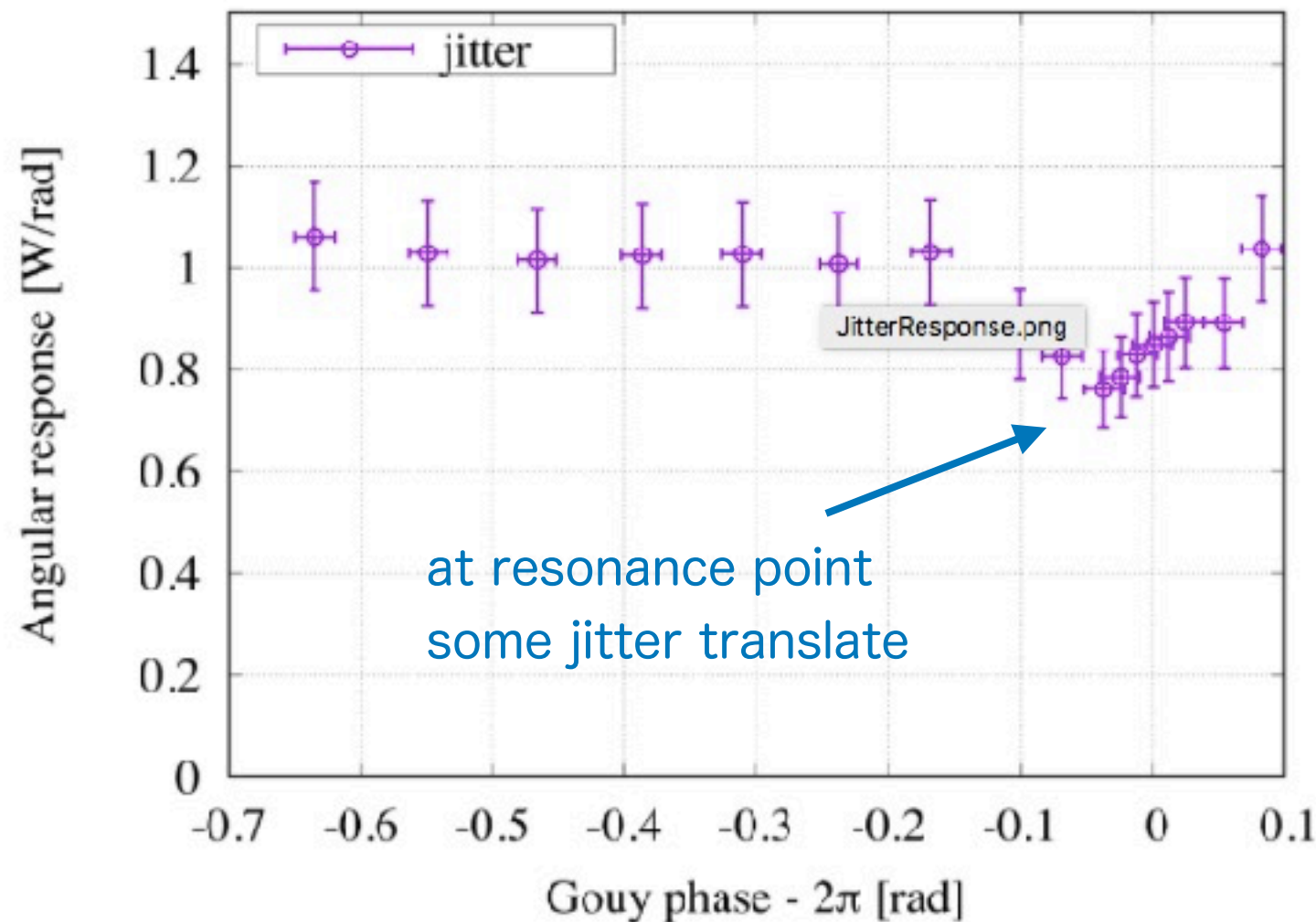
1. set distance between QPDs much bigger ($\sim 1\text{m}$?)
2. suppress QPD noise
current problem:
 - cannot increase sensing gain due to saturation of OPampsolution:
 - use RF (not DC) ?
 - take difference between output electricity (not voltage) of QPD devices ?
3. make the cavity critical coupling for 10mode
→ input 10mode does not reflect



beam jitter noise suppression

- beam jitter noise factor: 100

Folded cavity demonstration by Shimoda-san
Angular response while making beam jitter



How to suppress:

1. set distance between QPDs
much bigger ($\sim 1\text{m}$?)

2. suppress QPD noise

current problem:

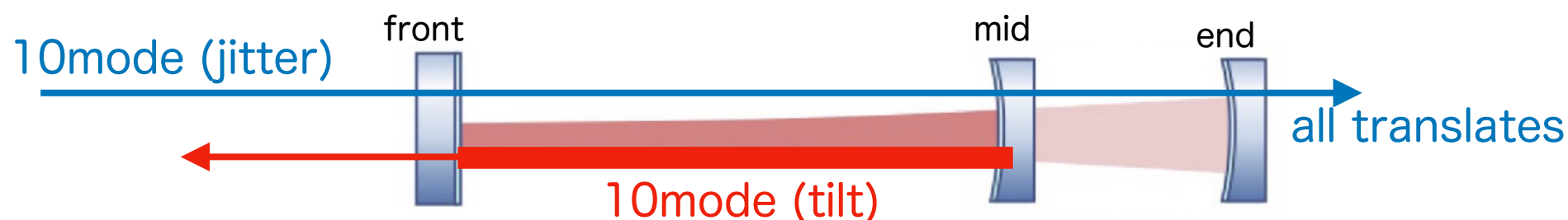
- cannot increase sensing gain
due to saturation of OPamp

solution:

- use RF (not DC) ?

- take difference between
output electricity (not voltage)
of QPD devices ?

3. make main-cavity critical
coupling for 10mode
→ input 10mode does not
reflect



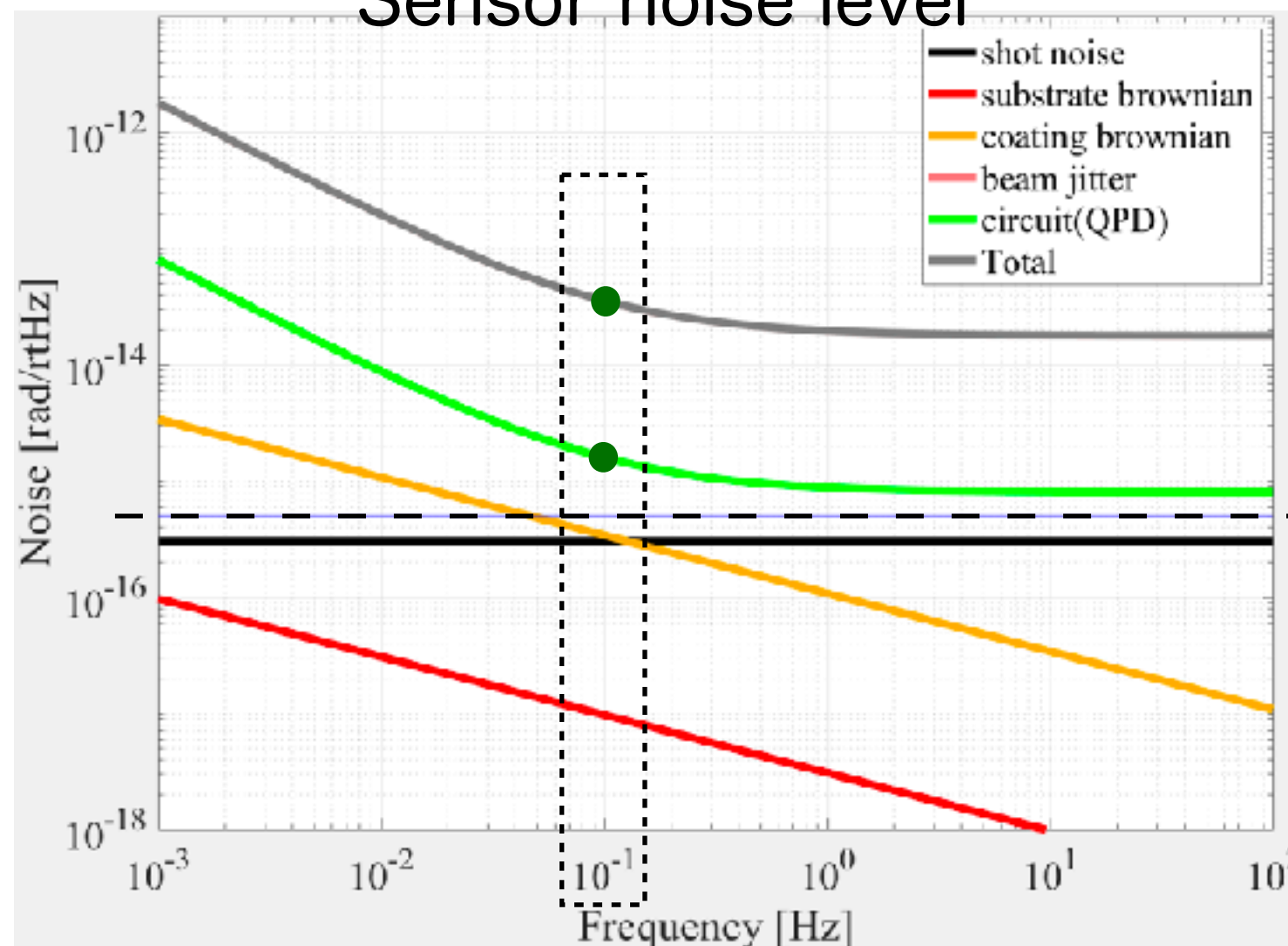
➡ Need fine tune

To achieve good sensitivity

- shot noise OK
- thermal noise OK
- beam jitter noise factor: 100
- QPD noise factor: 3

As I told in about beam jitter noise, "taking difference between output electricity (not voltage) of QPD devices" and increase QPD gain may solve the problem.

Sensor noise level



wavelength: 1064nm
 beam radius: 3.5mm
 beam power: 50mW
 cavity length: ~10cm
 finesse main cavity:
 620 (00 mode)
 210 (10 mode)
 temperature: 4K
 substrate: sapphire
 substrate loss: 10^{-8}
 coating loss: 10^{-3}

Future Plans

- demonstration test (confirm its principle)
 - confirm the amplification of angular signal
 - compare with theoretical one
 - check its characteristics
 - frequency noise
 - beam jitter
- construct a new (semi-monolithic) system
 - trans-rotational coupling
 - performance test in cryogenic
 - obtain good sensitivity
 - reduce jitter noise
 - other technical noises

schedule

I took too much time
to deal with trivial things

until
today

4

5

6

7

8

9

10

11

12

1

2

3

demonstration test

- 10 amplification
- check its characteristics

write thesis

design & order

construct the setup

- jitter suppression system
 - 10 amplification
 - check its characteristics
 - improve sensitivity
 - cryo test?



Contents

- about TOBA
- coupled wave front sensor
 - merits and demerits
- What is done so far
- What I want to study with the coupled WFS (For M Thesis)
 - demonstration test (confirm its principle)
 - check its characteristics
 - obtain good sensitivity
- Summary

Summary

- Phase-III TOBA is on going
- a sensitive angular sensor "coupled cavity" is designed
- its demonstration and operation are have to be checked
- What I want to study with the coupled WFS (For M Thesis)
 - demonstration test (confirm its principle)
 - check its characteristics
 - obtain good sensitivity

(The preparation of this seminar was good opportunity for me to find what I have/want to do. Although the way is tough, I will do my best.)

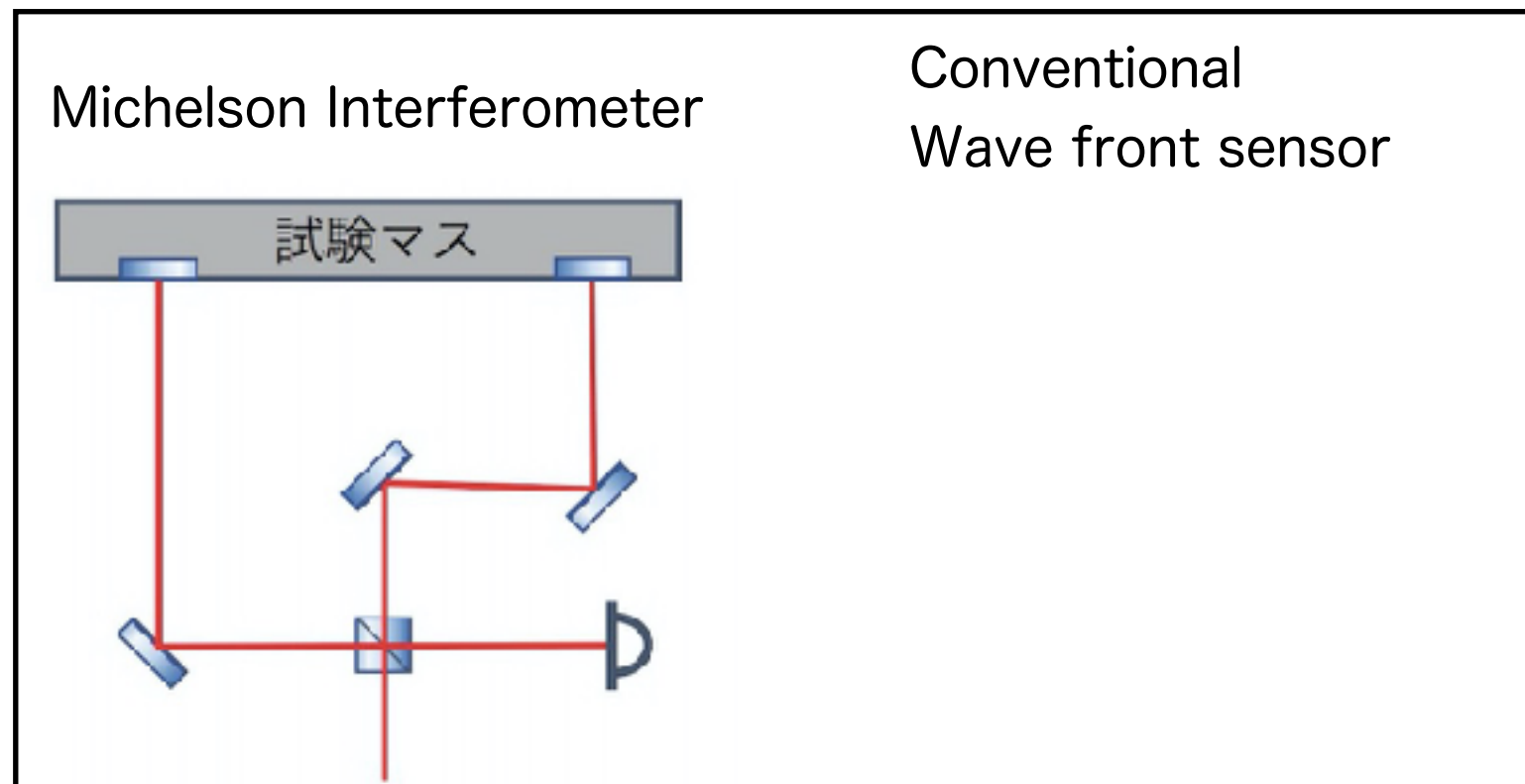
End.

ゴミ箱

Angular sensors for TOBA

Required sensitivity: 5×10^{-16} rad/ $\sqrt{\text{Hz}}$ @0.1Hz (phase-III)

Previous study:



New sensor:

Improved wave front sensor

Sensitivity



Angular signal
is small



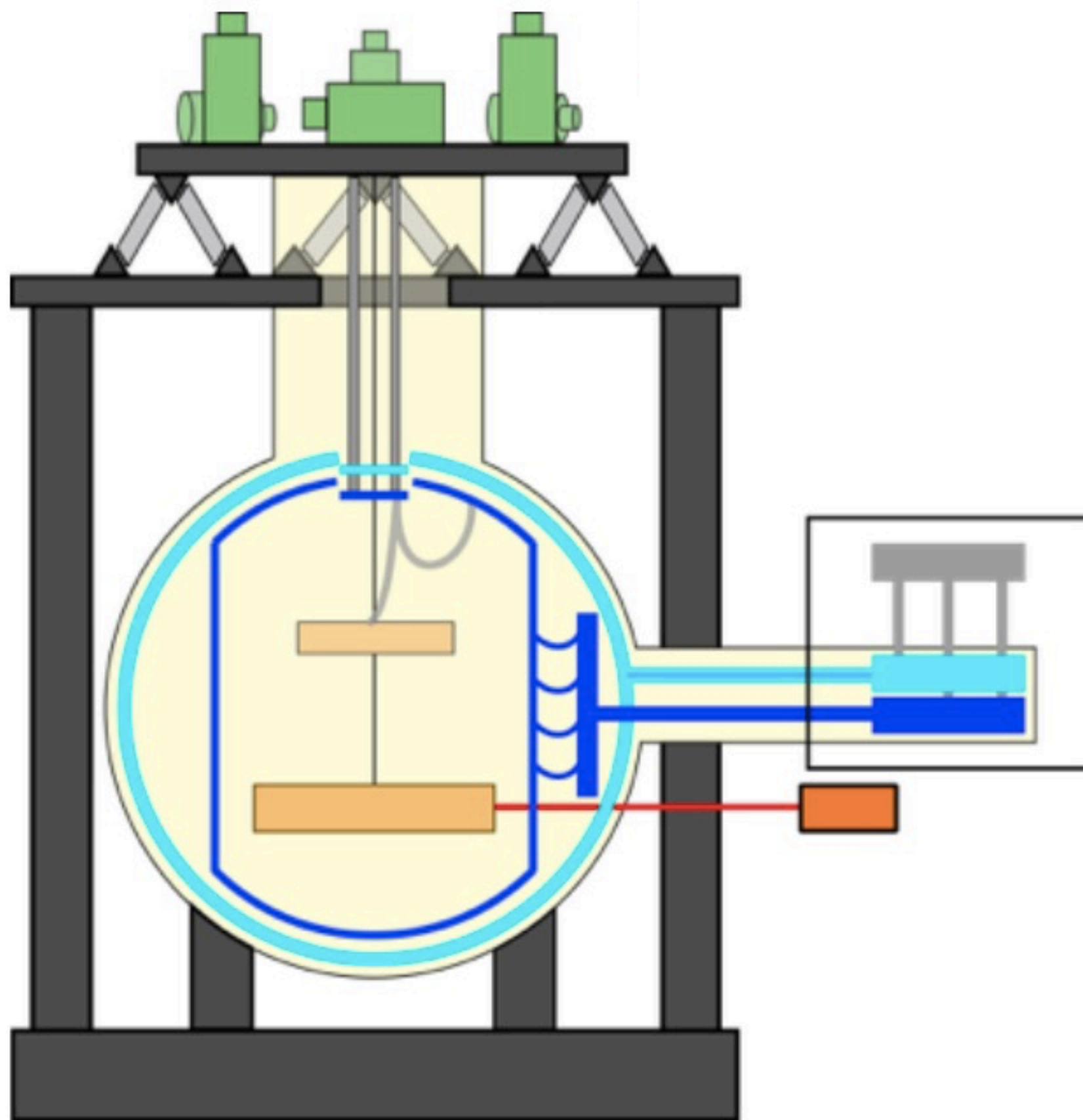
Angular signal
is amplified

Trans-Rotation
coupling



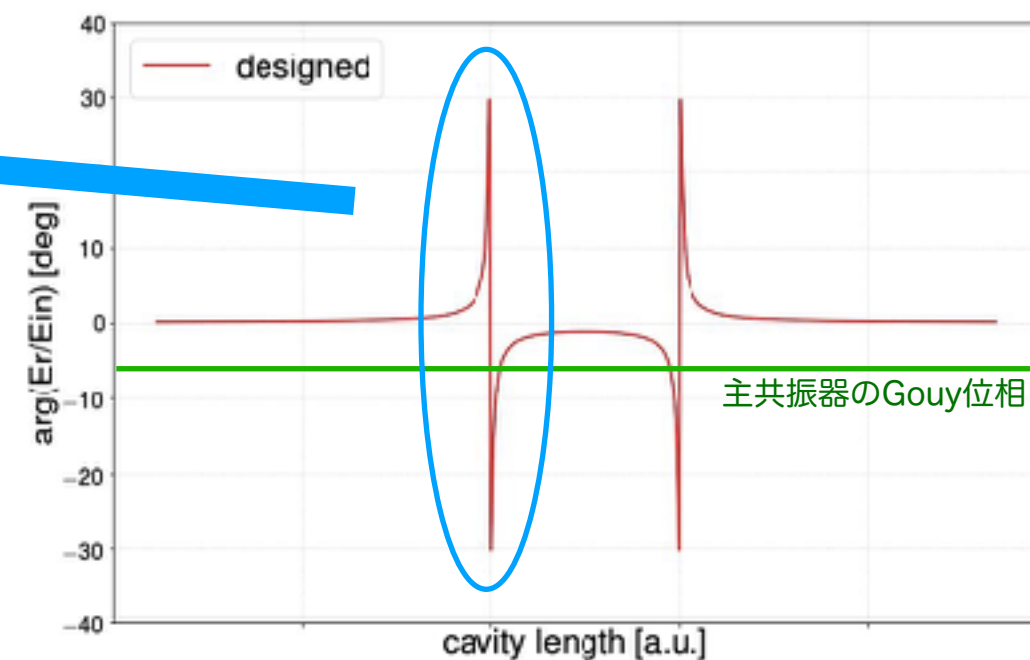
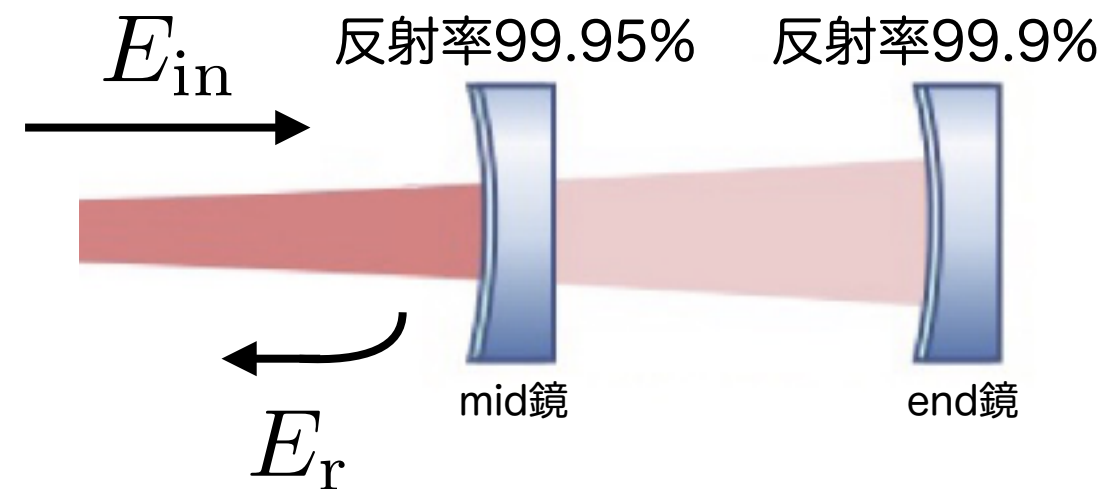
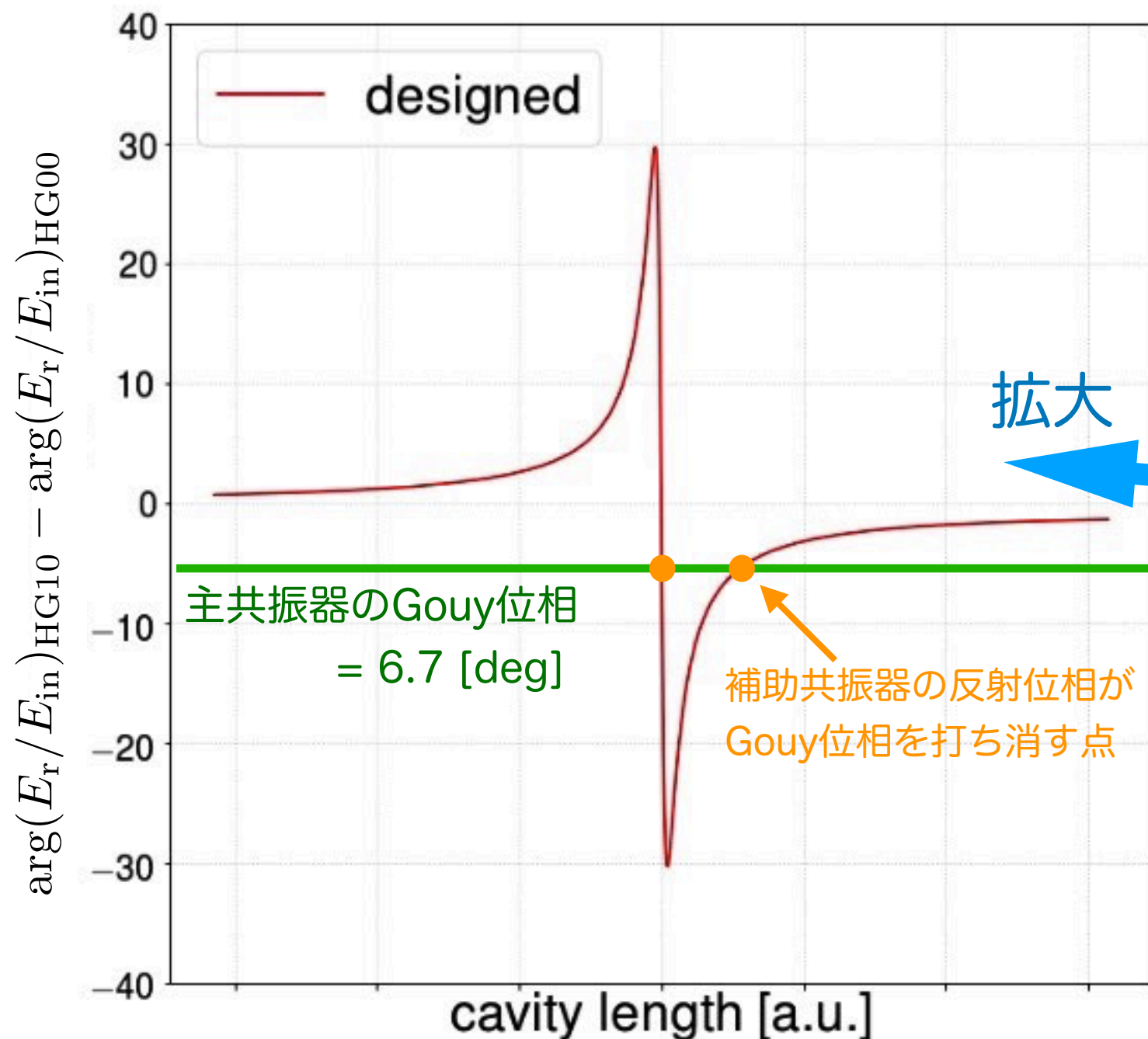
Difficult to put distant
mirrors in parallel





反射位相の設計値

$\arg(E_r/E_{in})$ の差 (=反射位相の差)



Reflectivity_mid=0.9995
 Reflectivity_end=0.999
 Gouy_main=6.7deg, Gouy_auxiliary=5.6deg

実際に反射位相を測定して、角度信号を増幅できるか確認

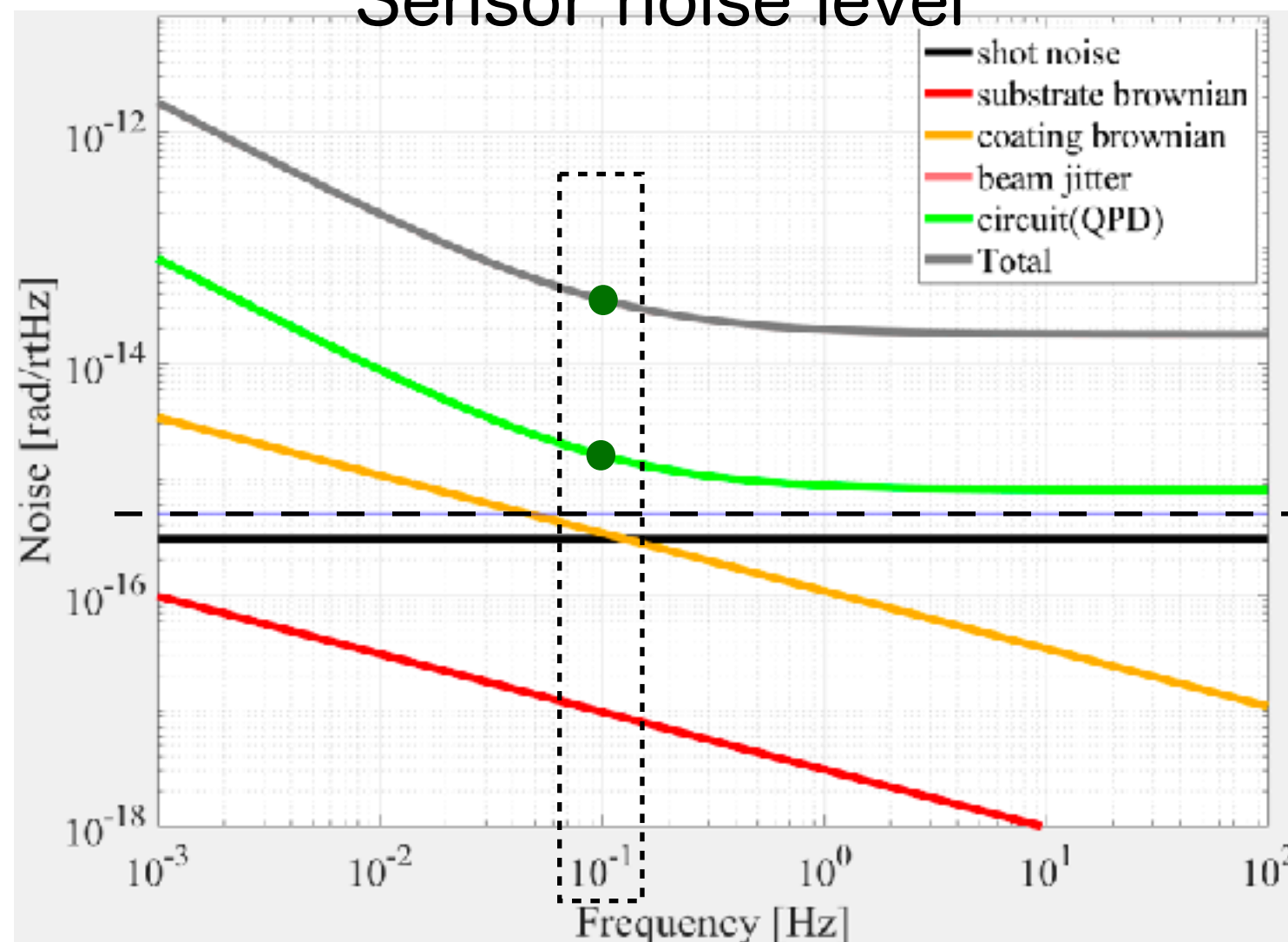
To achieve good sensitivity

- shot noise OK
- beam jitter noise factor: 100
- QPD noise factor: 3
- thermal noise OK

$$\propto \omega^{-1}$$

Angular response is proportional to beam power and beam radius. So setting beam radius to 3.5mm seems good, but the change also affects Rayleigh range (and Gouy phase), so fine tune is needed. (Not done yet)

Sensor noise level



wavelength: 1064nm
 beam radius: 3.5mm
 beam power: 50mW
 cavity length: ~10cm
 finesse main cavity:
 620 (00 mode)
 210 (10 mode)
 temperature: 4K
 substrate: sapphire
 substrate loss: 10^{-8}
 coating loss: 10^{-3}

requirement
 5×10^{-16}