Ando lab. Intermediate Seminar
(May 9th, 2018)

Title :

Compact Stars

Yuki Miyazaki (M1)

About myself

2014 :

Undergraduate student of Univ. of Tokyo

2018 :

Graduate student

↑ My photo of license.
Appear to criminals ...

About myself

2014 :

Undergraduate student of Univ. of Tokyo

2017 :

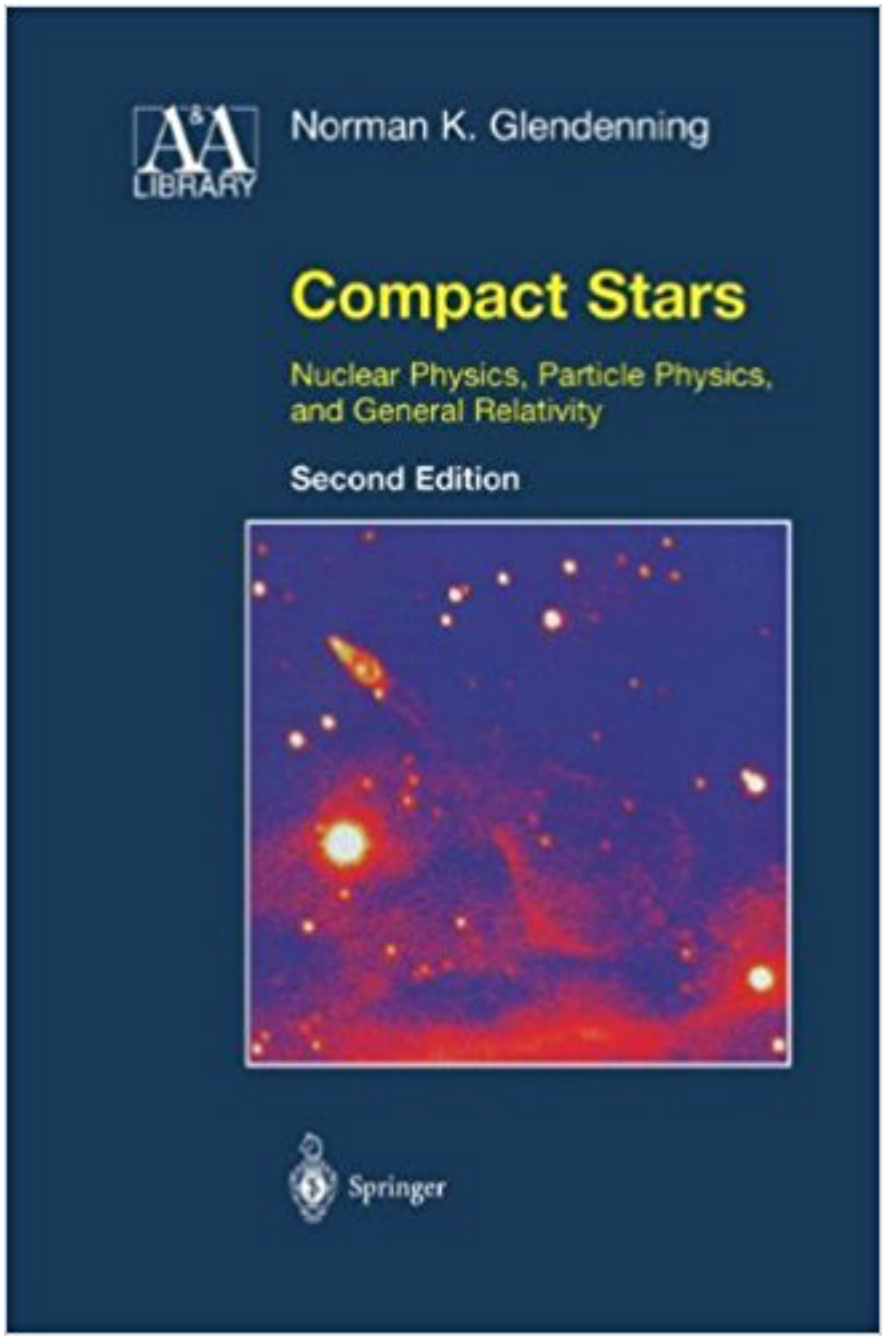
Class of reading in turn

2018 :

Graduate student

↑ My photo of license.
Appear to criminals ...

"Compact Stars"



- About compact stars
  - White Dwarf
  - Neutron Stars
- You can buy this with $100 (Amazon)

Table of contents

Table of contents

1. Some assumption we will use
2. Derive Oppenheimer-Volkoff eq. (dp/dr)
3. Derive Equation of State
   (eq. of energy density & pressure)
4. Properties of Compact stars
   (mass of stars)

1: Some assumption

Compact stars are

- static
- charge neutral
- zero temperature (= degenerate)
- composed of "p", "e", "n" ideal gas in its lowest energy state

Charge neutral

The repulsive Coulomb force < Gravitational attraction

$$\frac{(Z_{net}e)e}{R^2} \leq \frac{GMm}{R^2} < \frac{G(Am)m}{R^2}$$

$$\therefore \frac{Z_{net}}{A} < \left(\frac{m}{e}\right)^2 \sim 10^{-36}$$

The net charge per nucleon is very small.

Znet : net charge
A : number of baryons
M : mass of stars
R : radius of stars
m : mass of particles

Zero temperature

(Gravitational Units : G=c=hbar=kb=1)

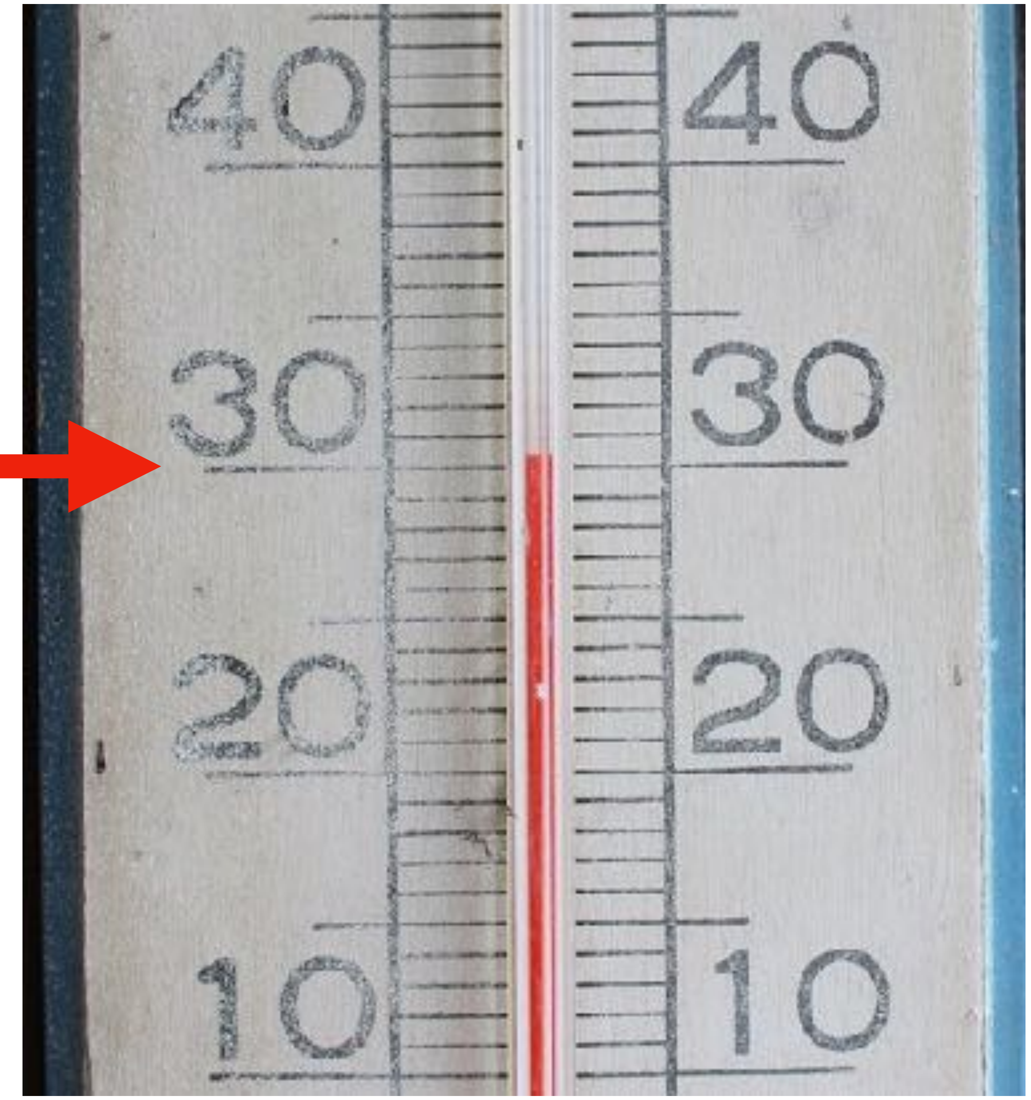
$$E_F > m_e = 0.511\mathrm{MeV} \sim 6 \times 10^9 \mathrm{K}$$
$$T_{star} \sim 10^{6\sim 7}\ \mathrm{K}$$
$$\therefore\ T_{star} << E_F = \sqrt{k_F^2 + m^2}$$

(degeneracy condition)

Compact stars satisfy the degeneracy condition.
We consider "cold stars" in this sense.

Zero temperature

30℃ !!!! →

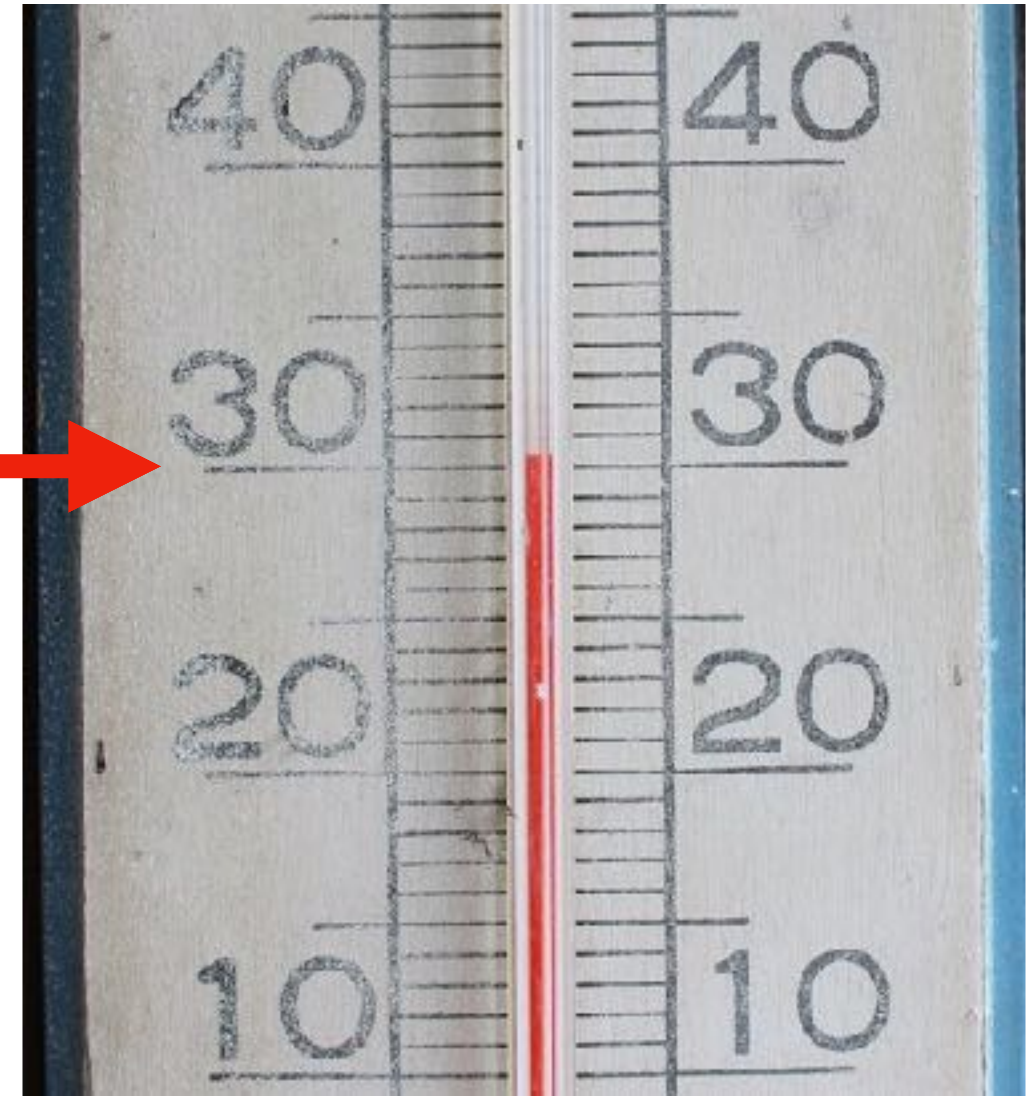


Compact stars satisfy the degeneracy condition.

We consider "cold stars" in this sense.

Zero temperature

30℃ !!!!



It's cold today.

Compact stars satisfy the degeneracy condition.

We consider "cold stars" in this sense.

Table of contents

Table of contents

2: Oppenheimer-Volkoff eq

Assume static and isotropic universe,

$$d\tau^2 = e^{2\nu(r)}dt^2 - e^{2\lambda(r)}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

and solve Einstein eq,

$$G^{\mu\nu} = -8\pi G T^{\mu\nu}$$

$$T^{\mu\nu} = \mathrm{diag}(\epsilon, p, p, p)$$

ε : energy density

p : pressure

(in explicitly writing, ...)

2: Oppenheimer-Volkoff eq

In explicitly writing,

$$G^0_0 = e^{-2\lambda}\left(\frac{1}{r^2} - \frac{2\lambda'}{r}\right) - \frac{1}{r^2} = -8\pi G\epsilon(r)$$

$$G^1_1 = e^{-2\lambda}\left(\frac{1}{r^2} + \frac{2\nu'}{r}\right) - \frac{1}{r^2} = 8\pi G p(r)$$

$$G^2_2 = e^{-2\lambda}\left(\nu'' + \nu'^2 - \lambda'\nu' + \frac{\nu' - \lambda'}{r}\right) = 8\pi G p(r)$$

$$G^3_3 = G^2_2 = 8\pi G p(r)$$

$$M(r) = 4\pi \int_0^r \epsilon(r) r^2 dr$$

$$e^{-2\lambda} = \left(1 - \frac{2GM(r)}{r}\right)^{-1}$$

ε : energy density

p : pressure

2: Oppenheimer-Volkoff eq

vanishing λ, ν, then we derive O-V eq,

$$\frac{dp}{dr} = -\frac{[\epsilon(r) + p(r)]\left[M(r) + 4\pi r^3 p(r)\right]}{r\left[r - 2M(r)\right]}$$

ε : energy density

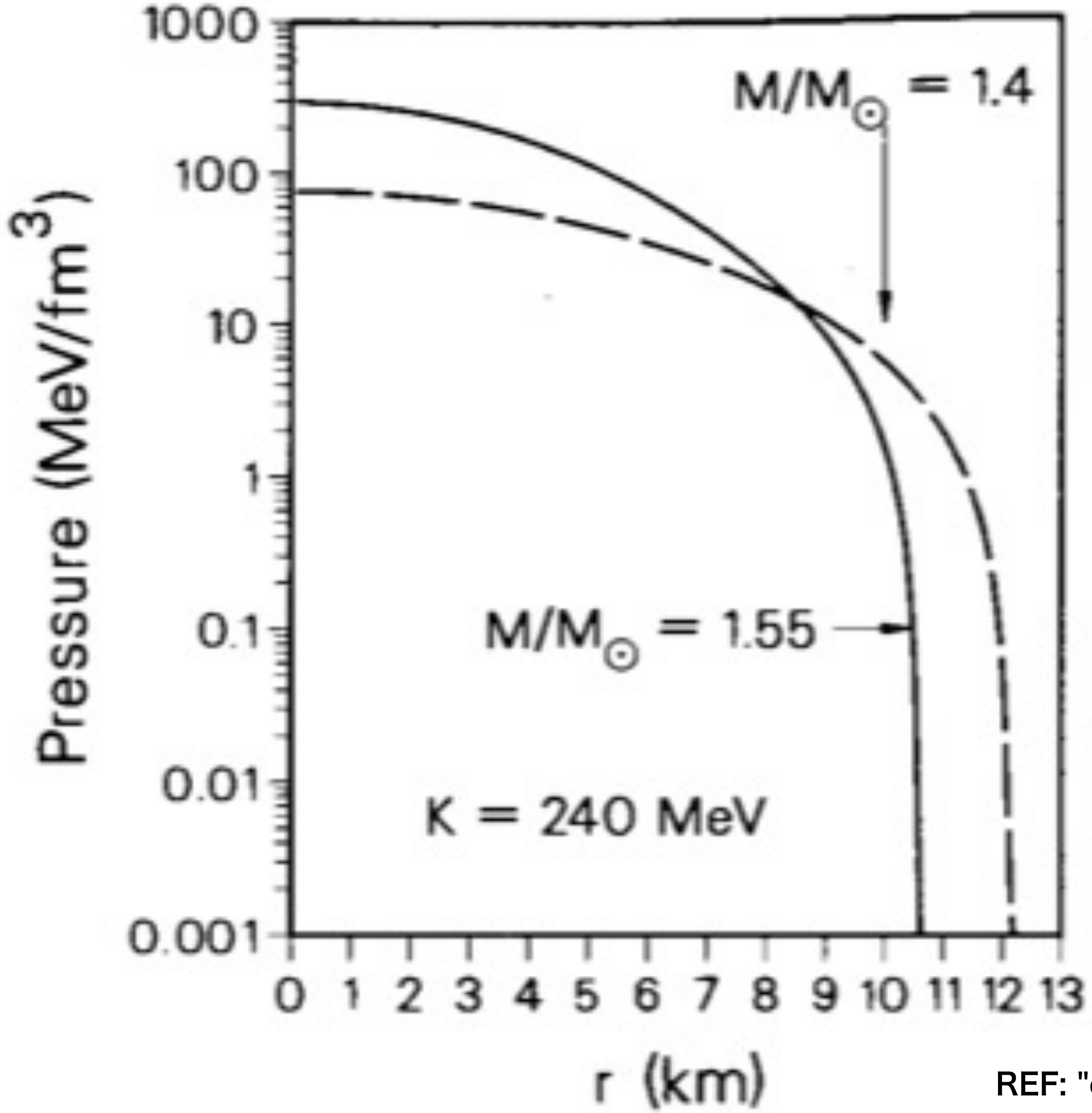
p : pressure

dp/dr < 0

⇔

the amount of overlaying material decreases with the radial coordinate.

2: Oppenheimer-Volkoff eq



$$\frac{dp}{dr} = -\frac{[\epsilon(r) + p(r)]\,[M(r) + 4\pi r^3 p(r)]}{r\,[r - 2M(r)]}$$

REF: "compact stars" Norman K. Glendenning p79

Table of contents

Table of contents

Equation of State

Here we use the degenerate condition.

$$\frac{1}{L^3}\sum_k \cdots \to \int \frac{d^3k}{(2\pi)^3}\cdots = \frac{1}{2\pi^2}\int_0^{k_{\mathrm{F}}} k^2 dk \cdots$$

Then we can write

$$\text{energy density} \quad \epsilon = \frac{\gamma}{2\pi^2}\int_0^k \sqrt{k^2+m^2}\; k^2 dk$$

$$\text{number density} \quad \rho = \frac{\gamma}{2\pi^2}\int_0^k k^2 dk$$

$$\text{pressure} \quad p = \frac{1}{3}\frac{\gamma}{2\pi^2}\int_0^k \frac{k^2}{\sqrt{k^2+m^2}}\; k^2 dk$$

Equation of State

About pressure, let us recall a thermodynamic relationship,

$$p = -\left(\frac{\partial E}{\partial V}\right)_S = -\frac{\partial(\epsilon/\rho)}{\partial(1/\rho)} = \rho^2 \frac{\partial}{\partial \rho}\left(\frac{\epsilon}{\rho}\right)$$

$$\text{energy density} \quad \epsilon = \frac{\gamma}{2\pi^2}\int_0^k \sqrt{k^2+m^2}\; k^2 dk$$

$$\text{number density} \quad \rho = \frac{\gamma}{2\pi^2}\int_0^k k^2 dk$$

$$\text{pressure} \quad p = \frac{1}{3}\frac{\gamma}{2\pi^2}\int_0^k \frac{k^2}{\sqrt{k^2+m^2}}\; k^2 dk$$

Equation of State

In explicitly writing,

$$\epsilon = \frac{1}{4\pi^2}\left[\mu k\left(\mu^2 - \frac{1}{2}m^2\right) - \frac{1}{2}m^4 \ln\left(\frac{\mu + k}{m}\right)\right]$$

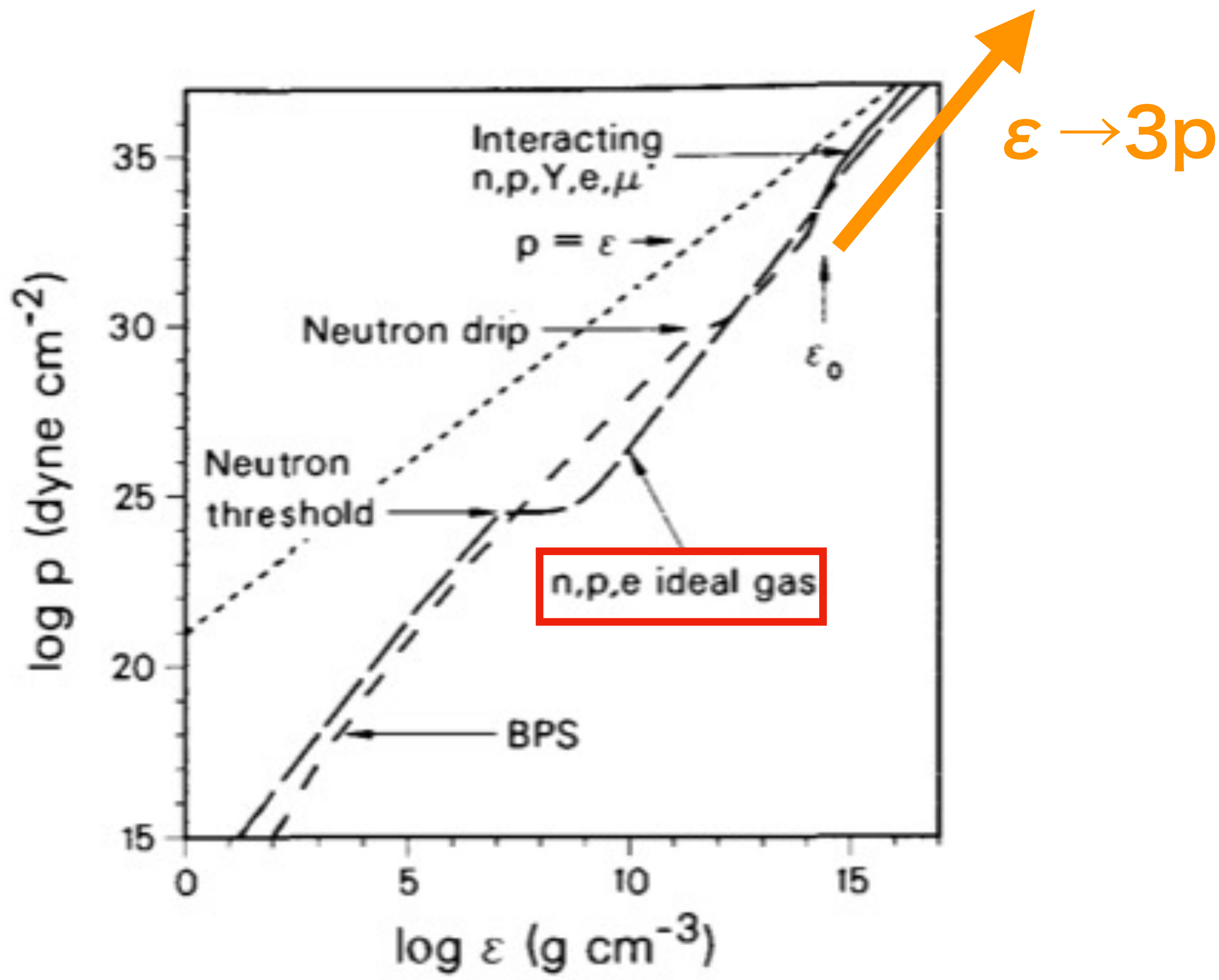
$$\rho = \frac{k^3}{3\pi^2}$$

$$p = \frac{1}{12\pi^2}\left[\mu k\left(\mu^2 - \frac{5}{2}m^2\right) + \frac{3}{2}m^4 \ln\left(\frac{\mu + k}{m}\right)\right]$$

with

Fermi energy $\quad \mu = \sqrt{k^2 + m^2}$

p - ε relation



REF: "compact stars" Norman K. Glendenning p99

Table of contents

Table of contents

4. Properties of Compact stars

O-V eq :

$$\frac{dp}{dr} = -\frac{[\epsilon(r) + p(r)]\left[M(r) + 4\pi r^3 p(r)\right]}{r\left[r - 2M(r)\right]}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r)$$

EoS :

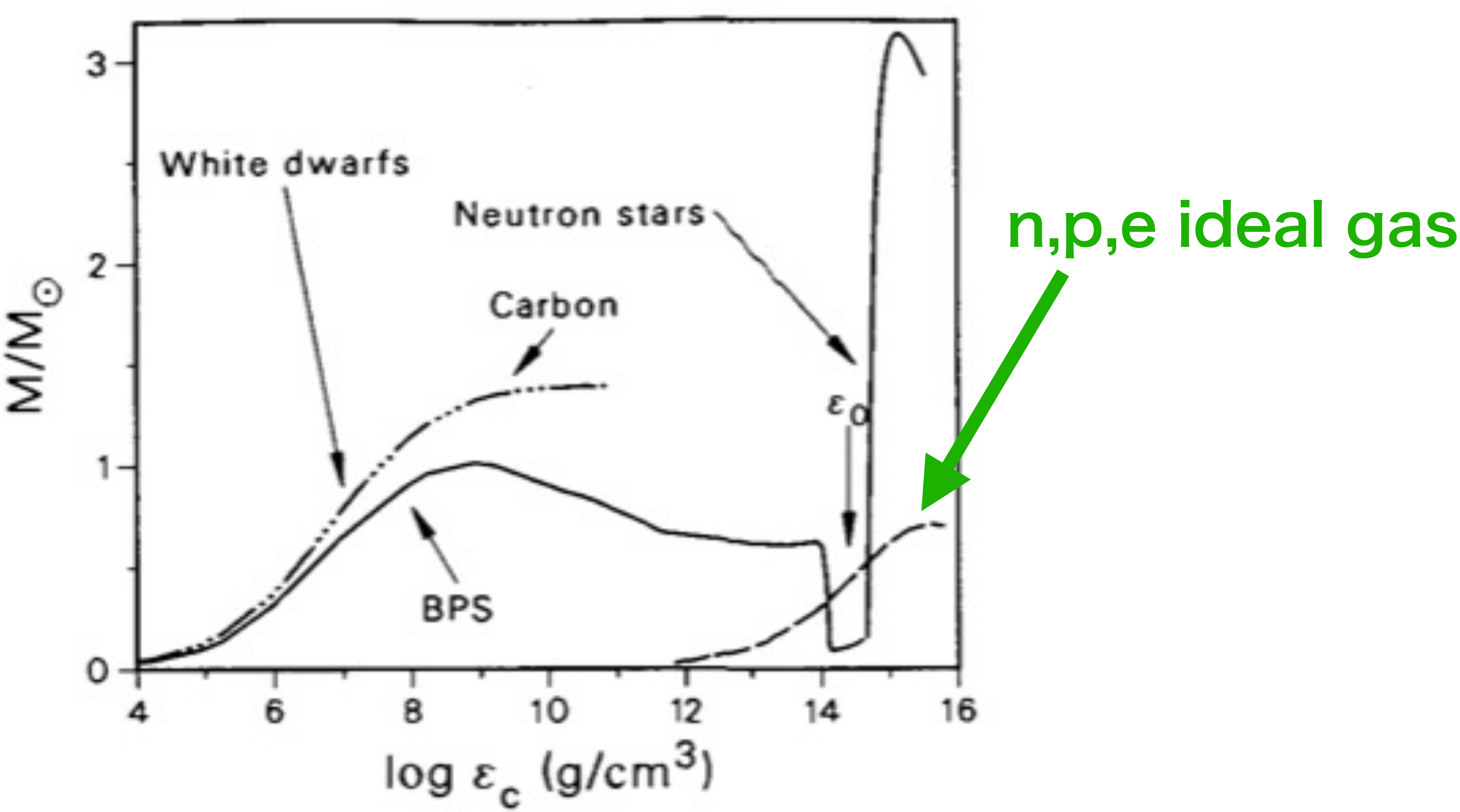
$$\epsilon = \epsilon(p)$$

We can solve them by numerical analysis with initial conditions at r=0

$$M(0) = 0, \quad \epsilon(0) \equiv \epsilon_c, \quad p(0) = p(\epsilon = \epsilon_c)$$

4. Properties of Compact stars

Solution over a broad range of central densities



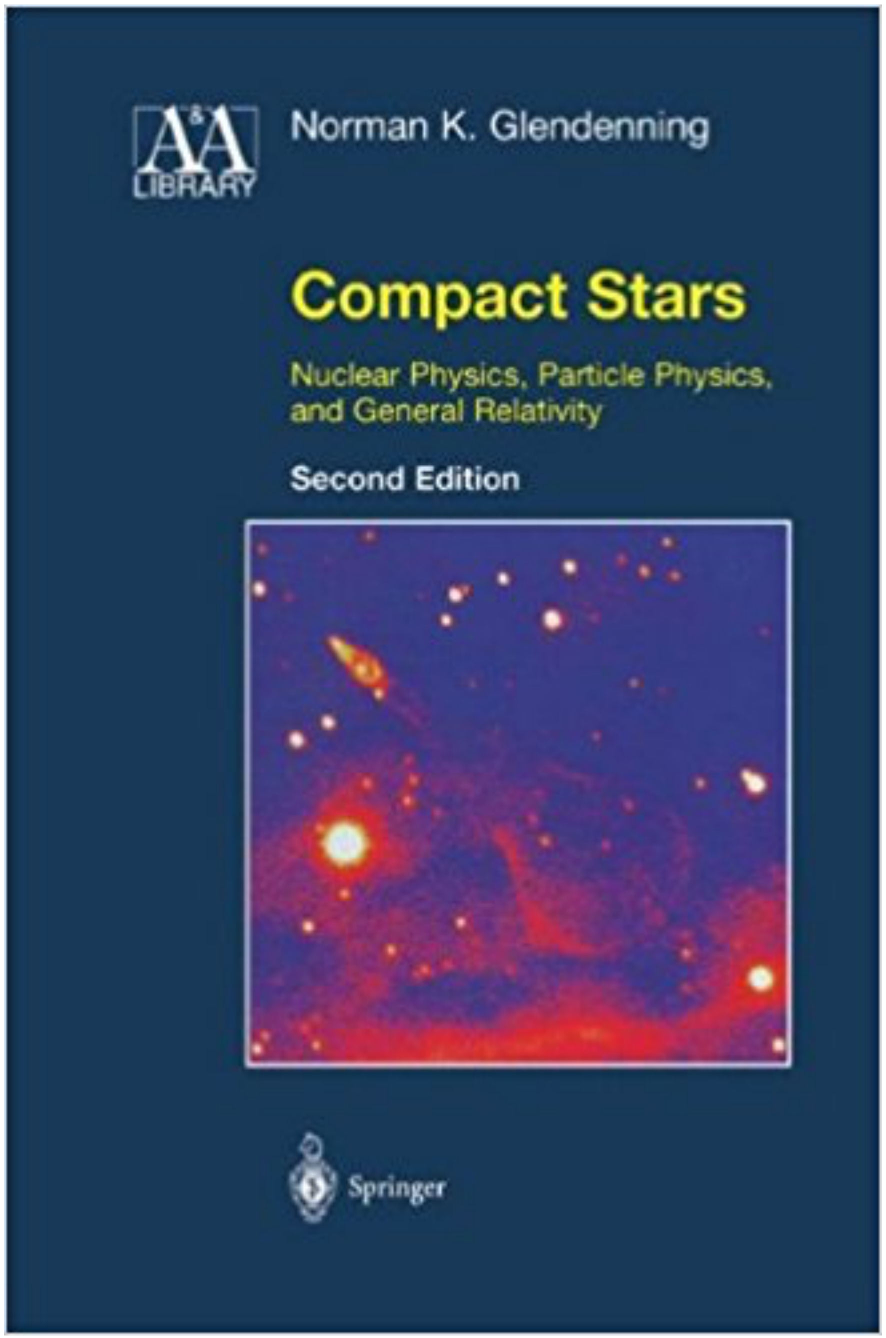
REF: "compact stars" Norman K. Glendenning p106

Table of contents

and summary

1. Some assumption we will use
2. Derive Oppenheimer-Volkoff eq. (dp/dr)
3. Derive Equation of State
   (energy density & pressure)
4. Properties of Compact stars
   (mass of stars)

Thank you for listening !



If you are interested in compact stars, buy and read it.

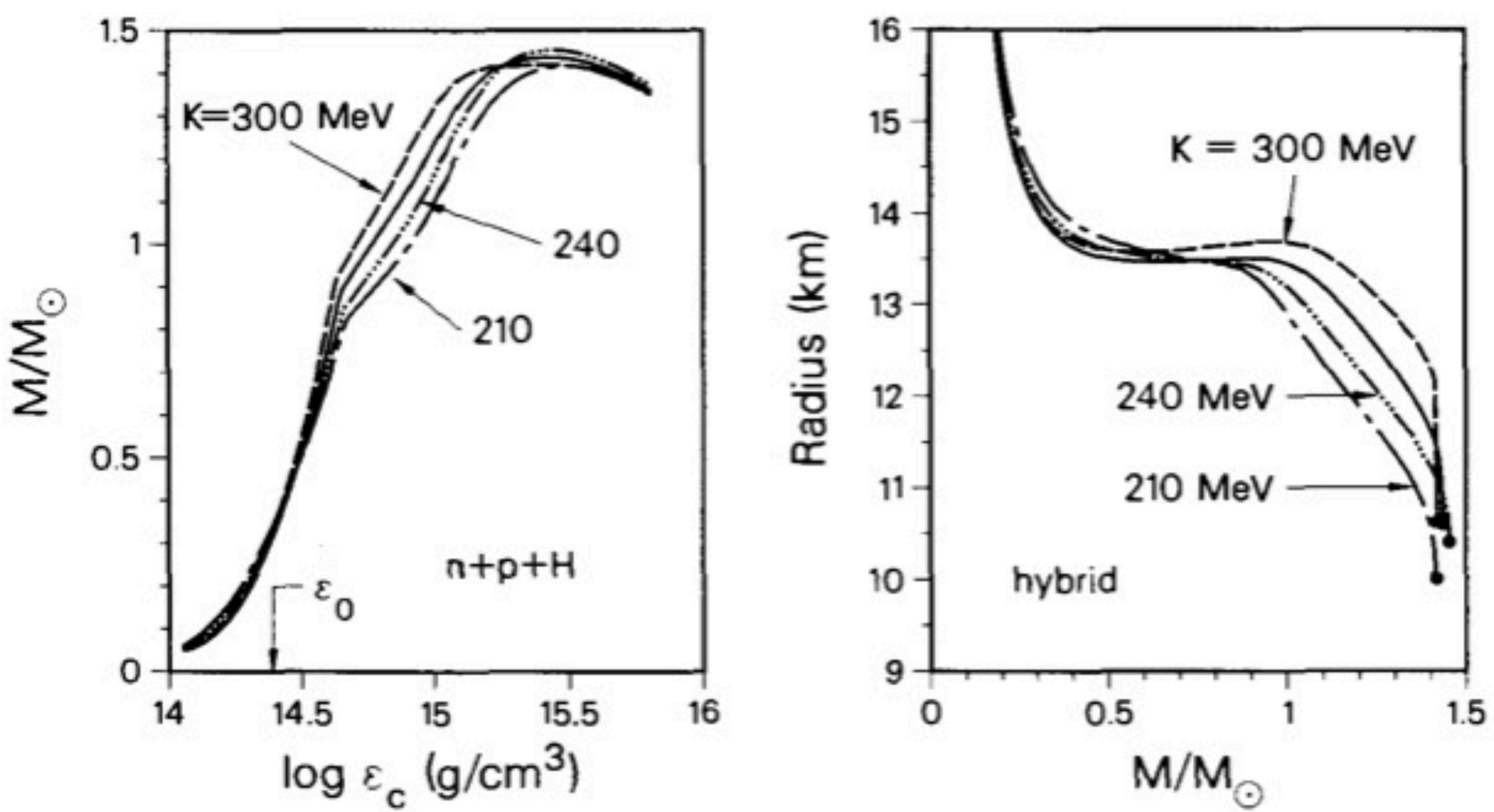
For accurate calculation

- finite temperature (adds Fermi distribution)
- strong interaction term (makes EoS stiff)
- another baryon species (hyperon)
- property of nuclear matter (isospin-sym.)
- the rotation of stars (changes the metric)
- quark deconfinement

For accurate calculation

- finite temperature (adds Fermi distribution)
- strong interaction term (makes EoS stiff)
- another baryon species (hyperon)
- property of nuclear matter (isospin-sym.)
- the rotation of stars (changes the metric)
- quark deconfinement

M-R relation

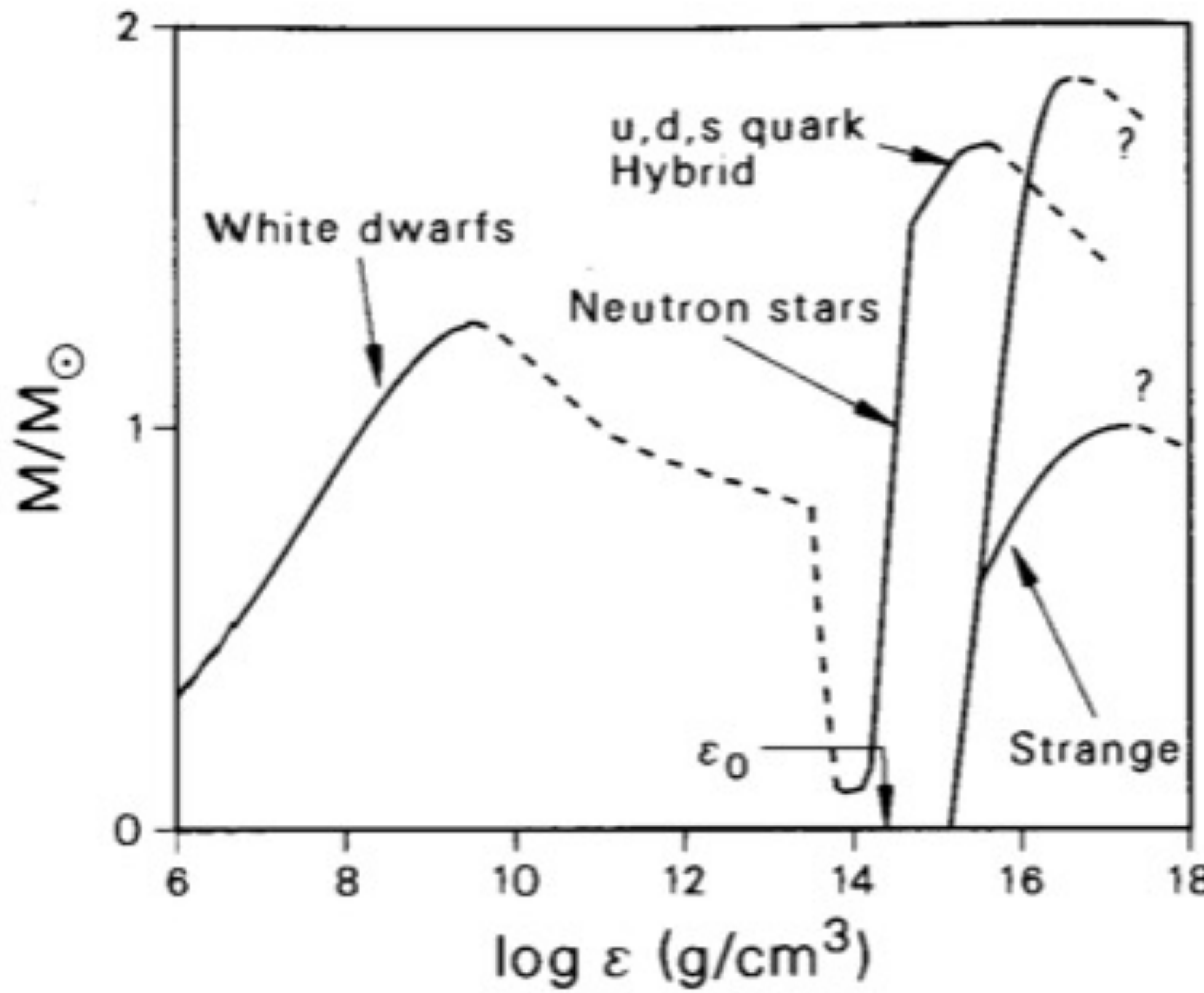


hybrid stars : n,p,e + quark **K is parameter of stiffness**

REF: "compact stars" Norman K. Glendenning p339

4. Properties of Compact stars

Solution over a broad range of central densities

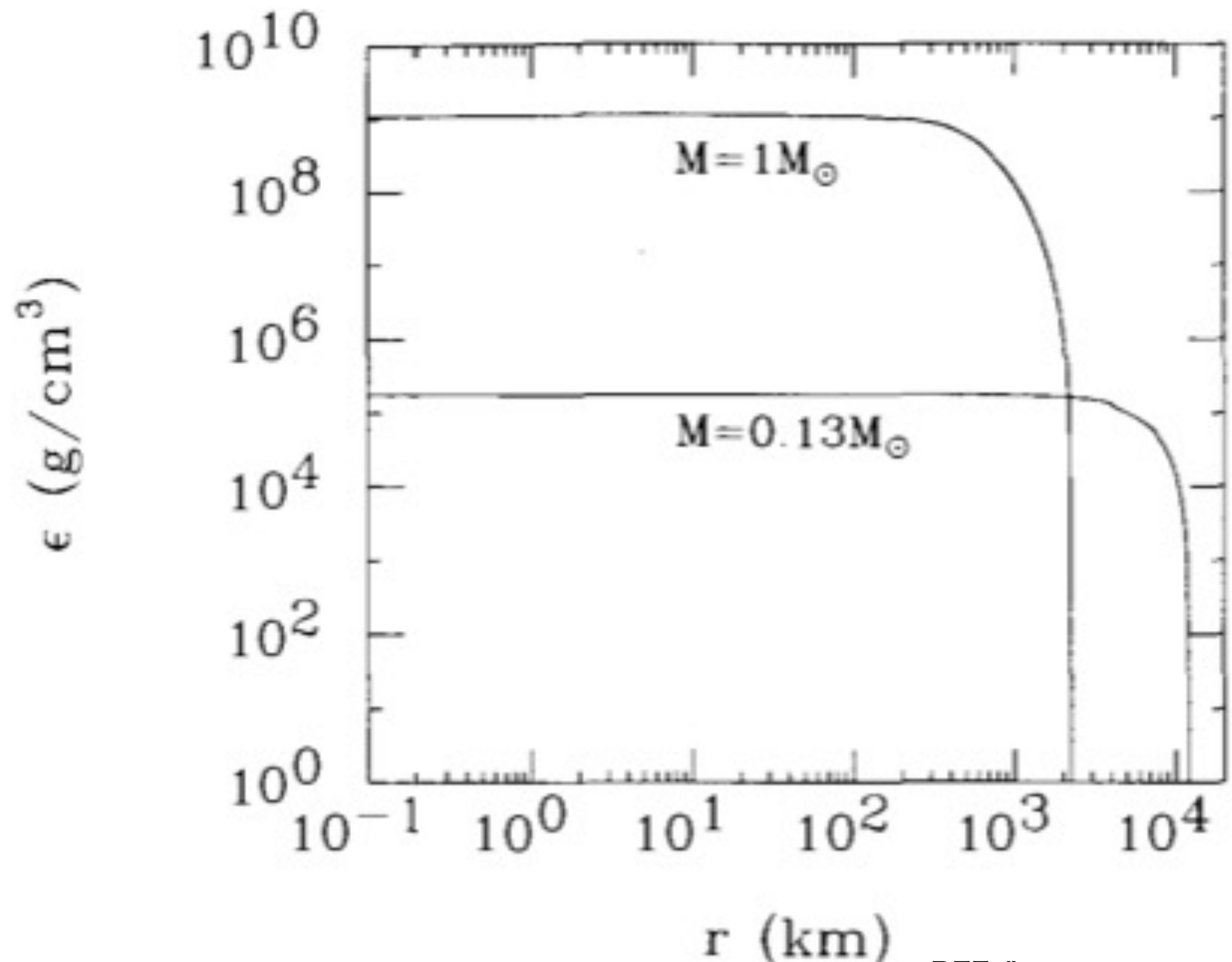


What EoS?

REF: "compact stars" Norman K. Glendenning p81

4. Properties of Compact stars

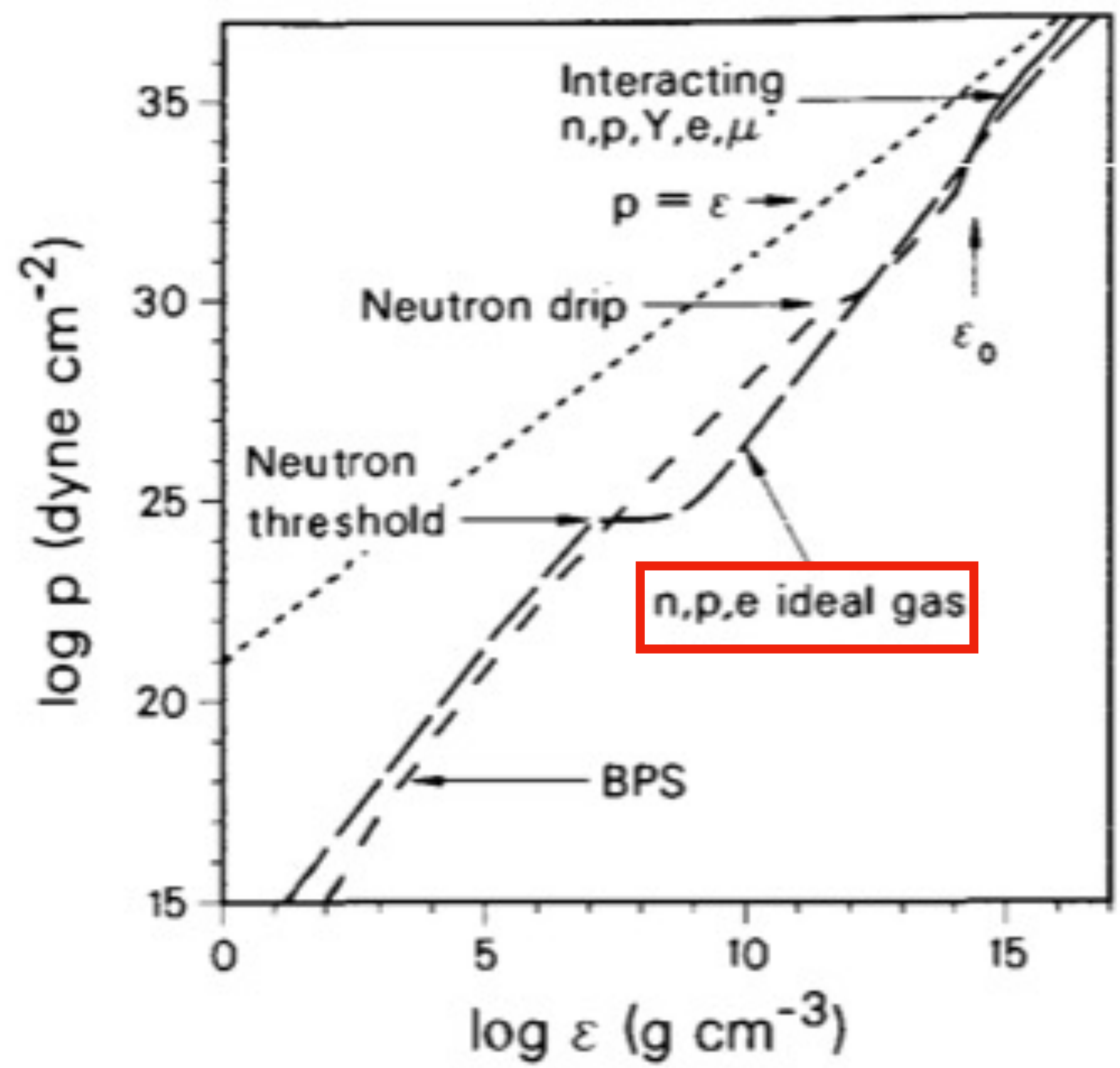
mass-energy distribution in two WD



BPS

REF: "compact stars" Norman K. Glendenning p119

p - ε relation



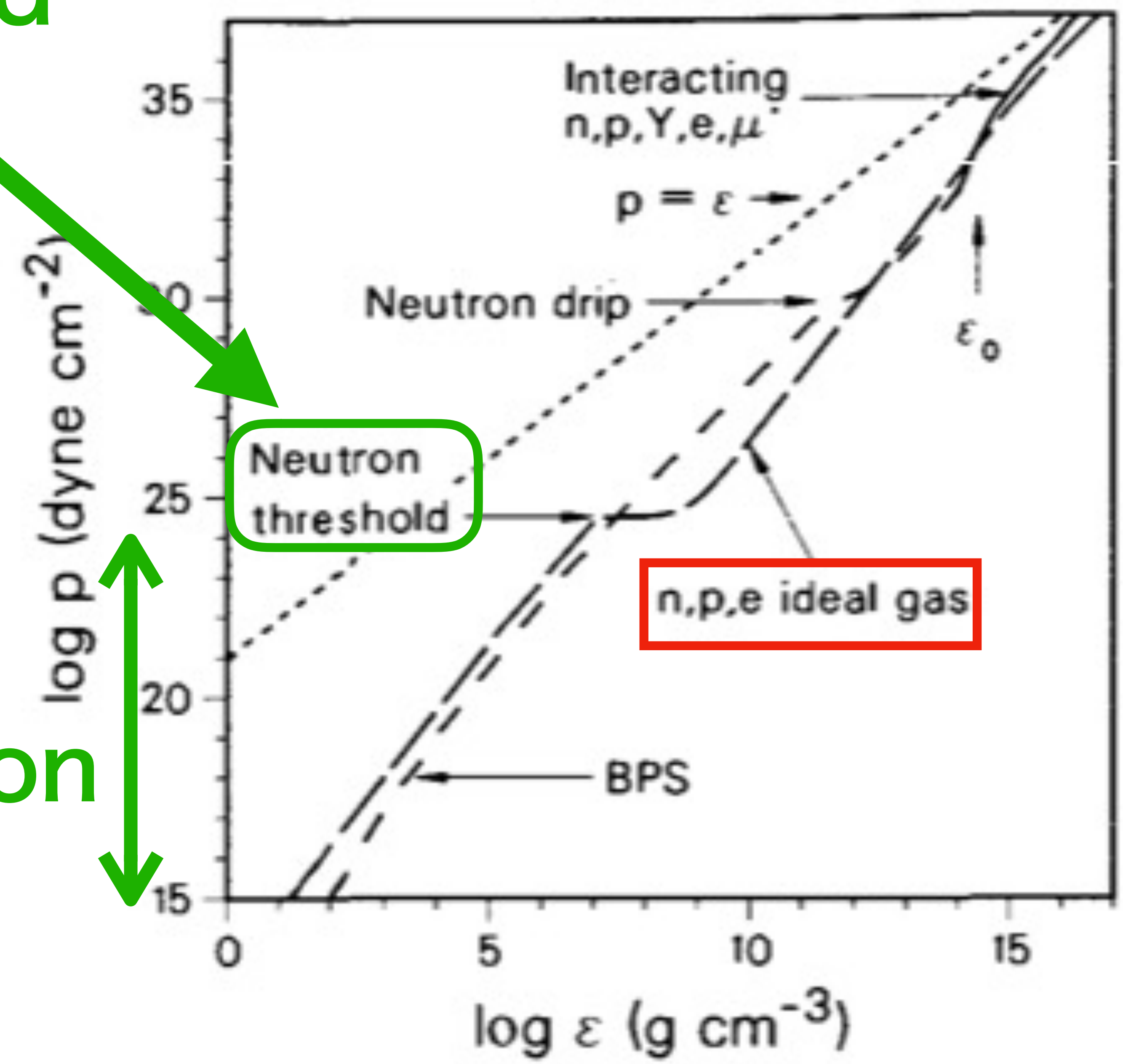
REF: "compact stars" Norman K. Glendenning p99

p - ε relation

Neutron threshold

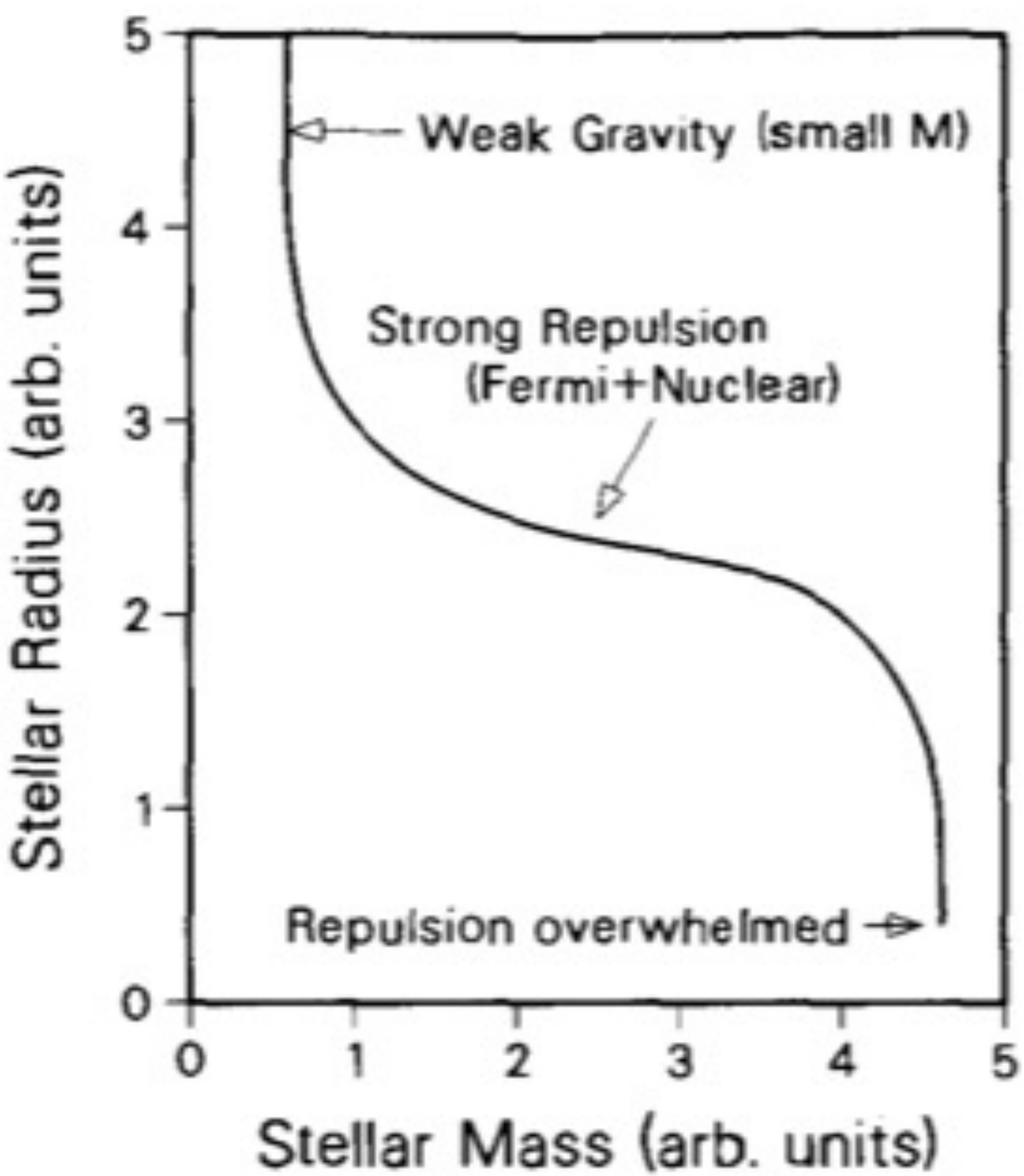
the threshold of neutron existing

No neutron region



REF: "compact stars" Norman K. Glendenning p99

M-R relation



qualitative explanation

REF: "compact stars" Norman K. Glendenning p80