安東研輪講

2015年4月30日

万有引力定数の測定

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万有引力定数

- 重力相互作用の強さを表す物理定数

$$F = \underline{G}\frac{m_1 m_2}{r^2}$$

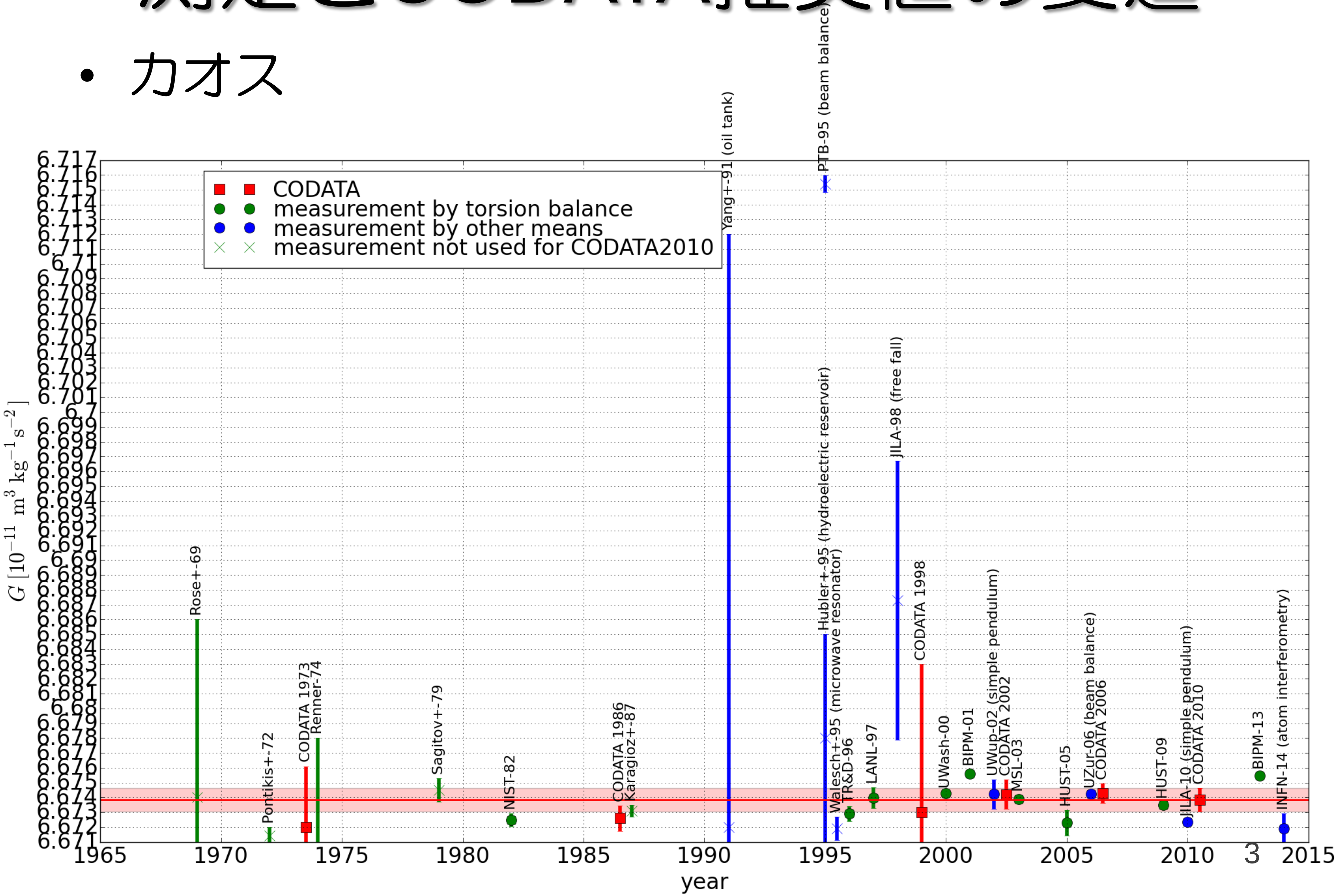
- 測定精度が低い(相対標準不確かさ 1.2e-4)

$$G = 6.67384(80) \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$$

ボルツマン定数: 9.1e-7
プランク定数: 4.4e-8
アボガドロ定数: 4.4e-8
微細構造定数: 3.3e-10

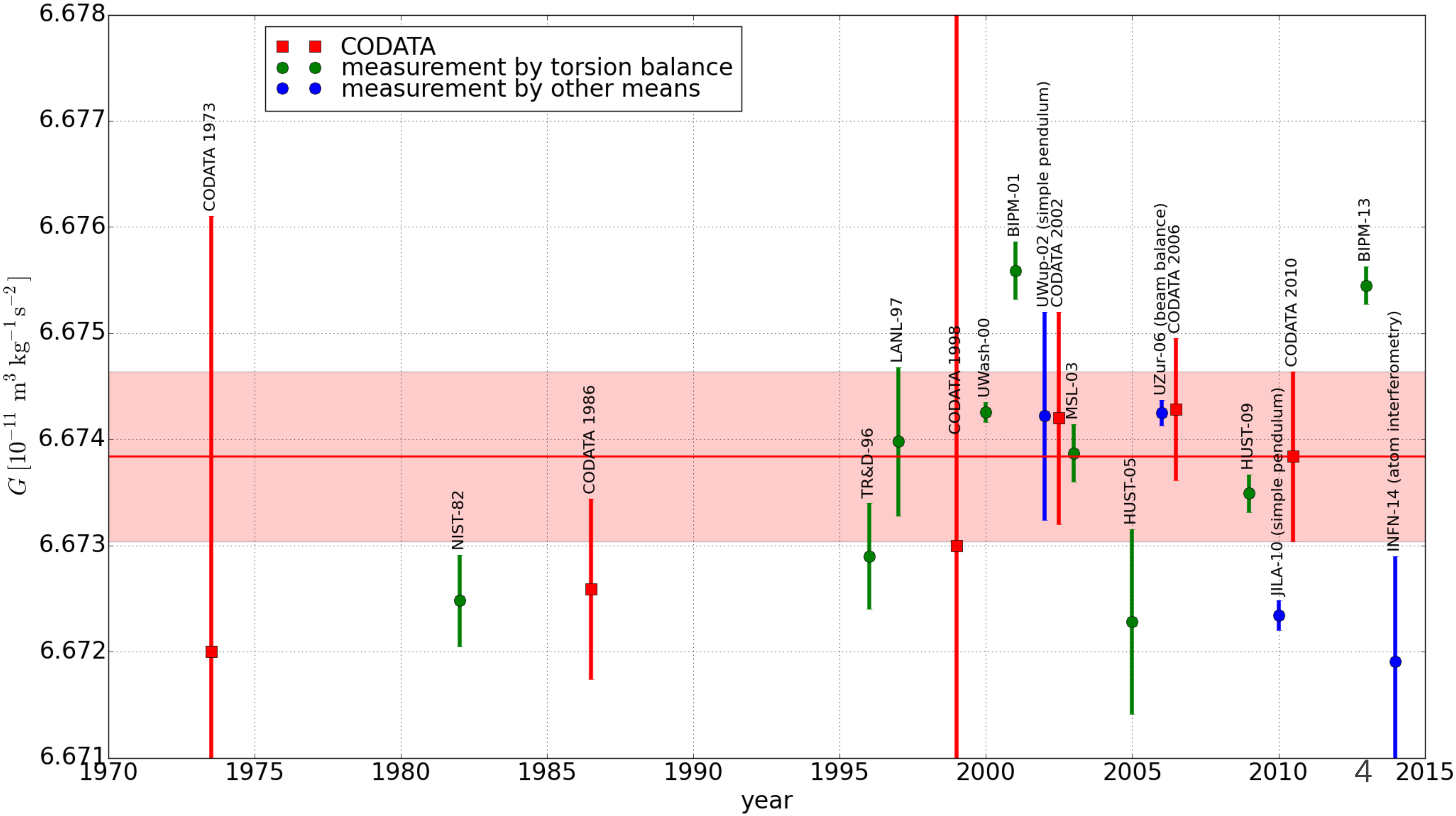
測定とCODATA推奨値の変遷

- カオス



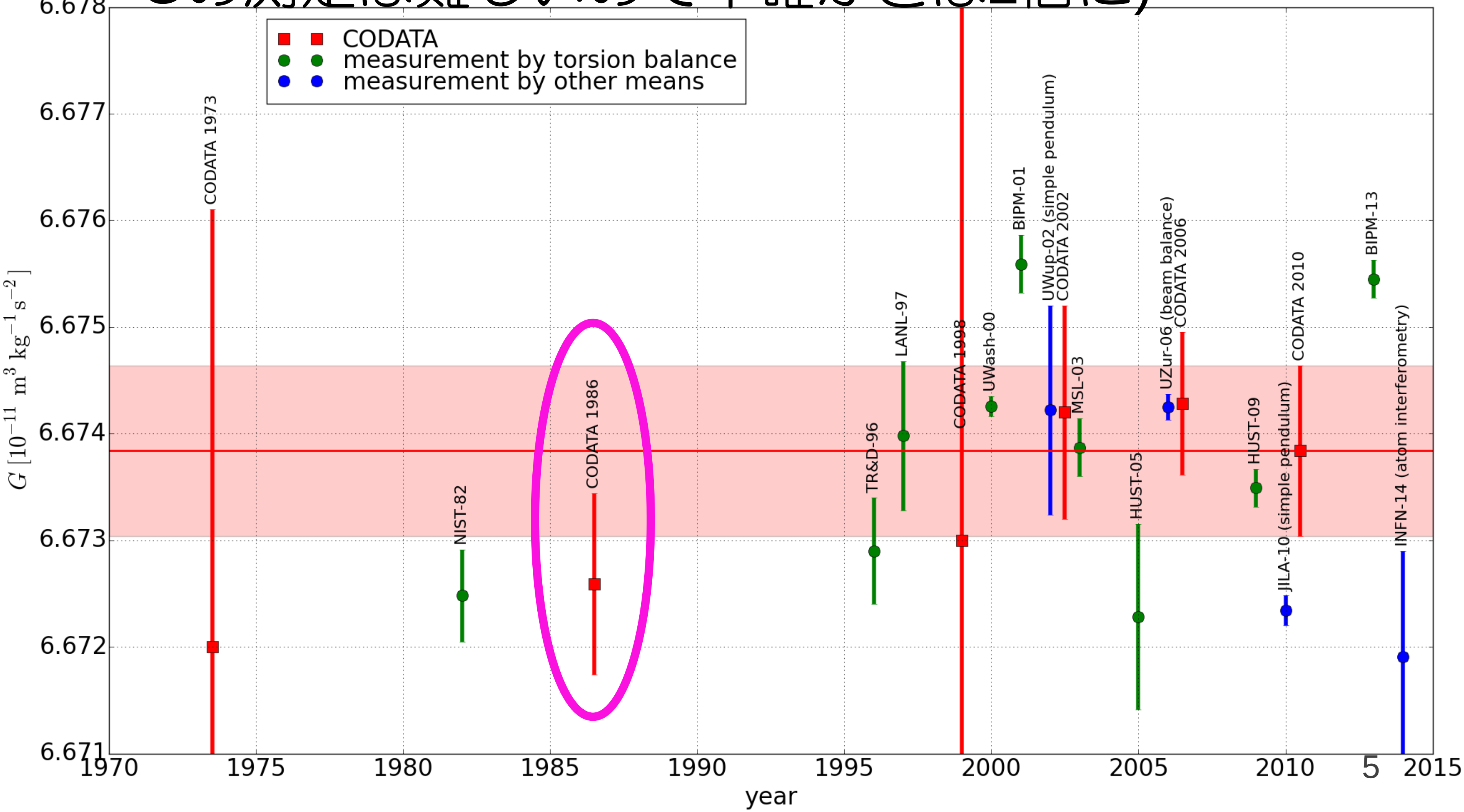
CODATA2010に使われたものだけ

- ばらつきが大きい(10σ)



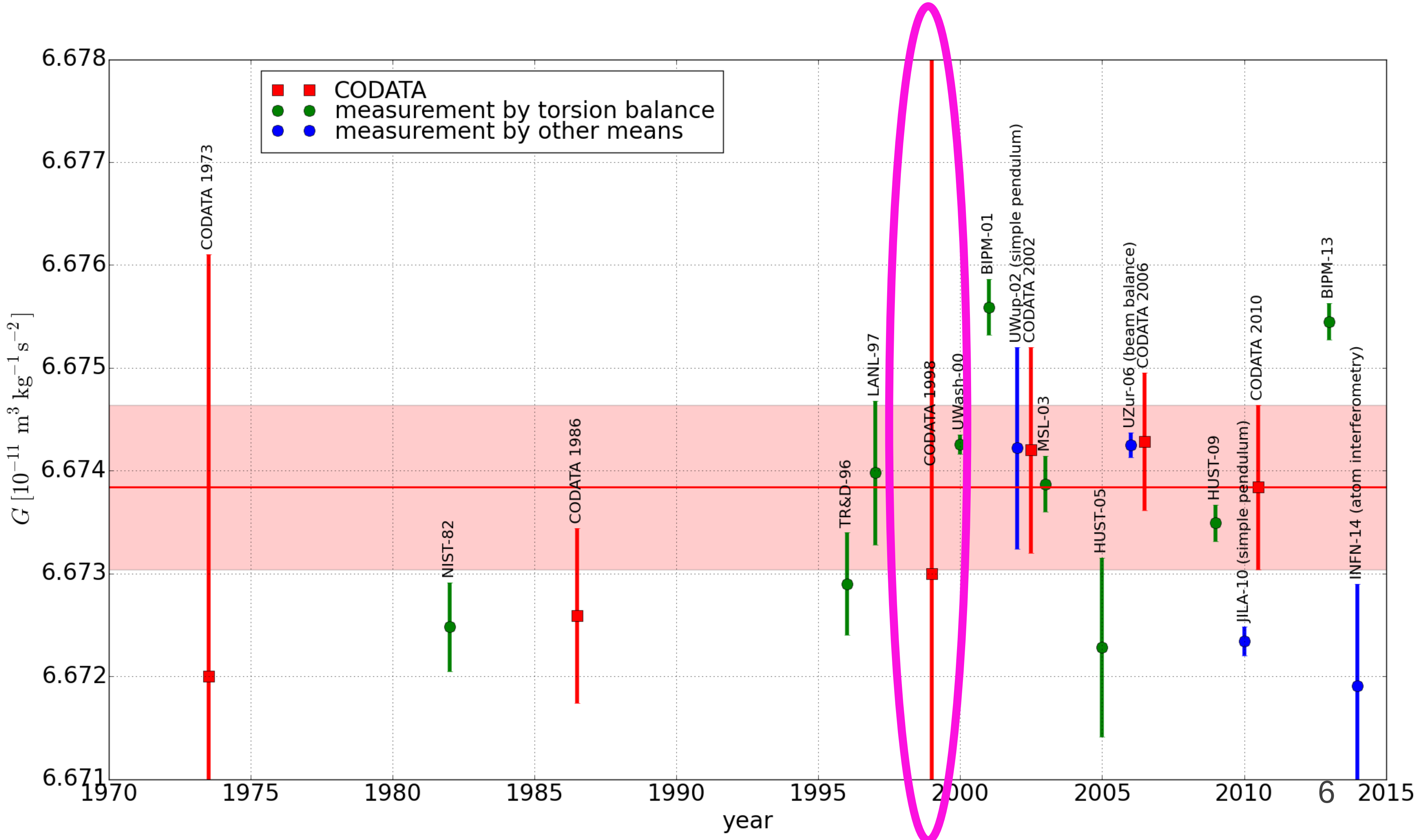
CODATA1986

- ほぼNIST-82によって決まった(ただし、歴史的にGの測定は難しいので不確かさは2倍に)



CODATA1998

- 標準不確かさを12倍上げた



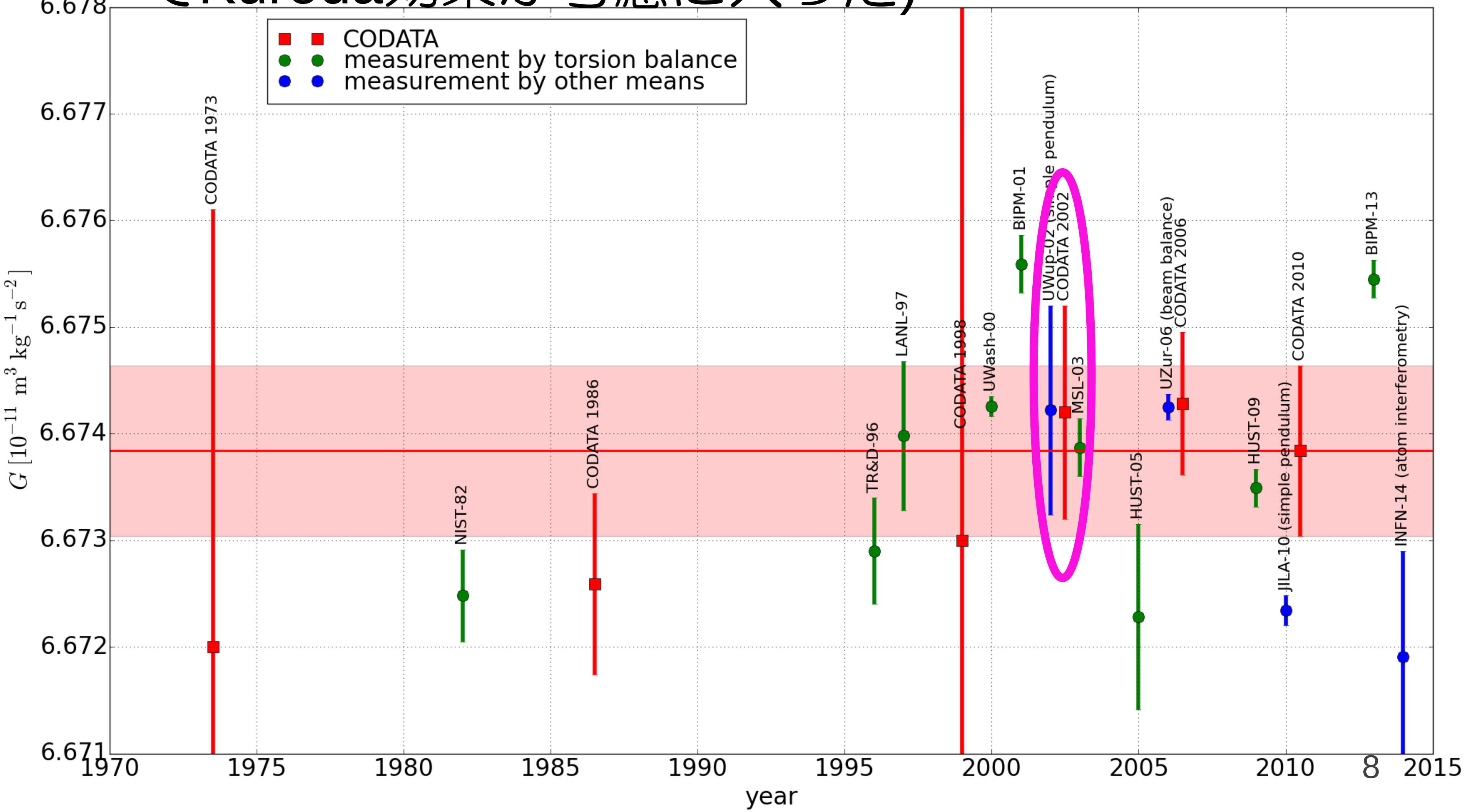
CODATA 1998の精度悪化理由

- NIST-82などでの、ファイバーの非弾性の効果について未決着(Kuroda pointed out)
  K. Kuroda, PRL **75**, 2796 (1995)
- 出た実験もまだpreliminaryで、数年の間に1e-5の精度で実験ができるはず
- PTB-95を無視できない
- 歴史的にGの測定は難しい

黒田和明先生

CODATA2002

- 標準不確かさを戻した(PTB-95は棄却。他の実験でKuroda効果が考慮に入った)



NIST-82

- G. G. Luther & W. R. Towler, PRL **48**, 121 (1982)
- 重力源の有無でねじれ振り子の共振周波数が代わることを利用(time-of-swing method)

VOLUME 48, NUMBER 3 PHYSICAL REVIEW LETTERS 18 JANUARY 1982

Redetermination of the Newtonian Gravitational Constant *G*

Gabriel G. Luther
Center for Absolute Physical Quantities, National Bureau of Standards, Washington, D. C. 20234

and

William R. Towler
Department of Nuclear Engineering and Engineering Physics, University of Virginia, Charlottesville, Virginia 22901
(Received 17 November 1981)

The universal Newtonian gravitational constant is being redetermined at the National Bureau of Standards with use of the method of Boyes in which the period of a torsion pendulum is altered by the presence of two 10.5-kg tungsten balls. The difference in the squares of the frequencies with and without the balls is proportional to G. The resulting value of G is $(6.6726 \pm 0.0005) \times 10^{-11}\ \mathrm{m^3 \cdot sec^{-2} \cdot kg^{-1}}$.

NIST-82の装置

- 長さ40cm、太さ12umの石英繊維(クロム-金コーティングで導電性確保)
- Q = 2e4
  (2010のprivate communication; だいぶ怪しい)
- ダンベル長28.5 mm
- 光てこ
- 1e-5 torr
- 温度安定化で±0.1 K以下
- 重力源との距離は
  14.059454(30) cm

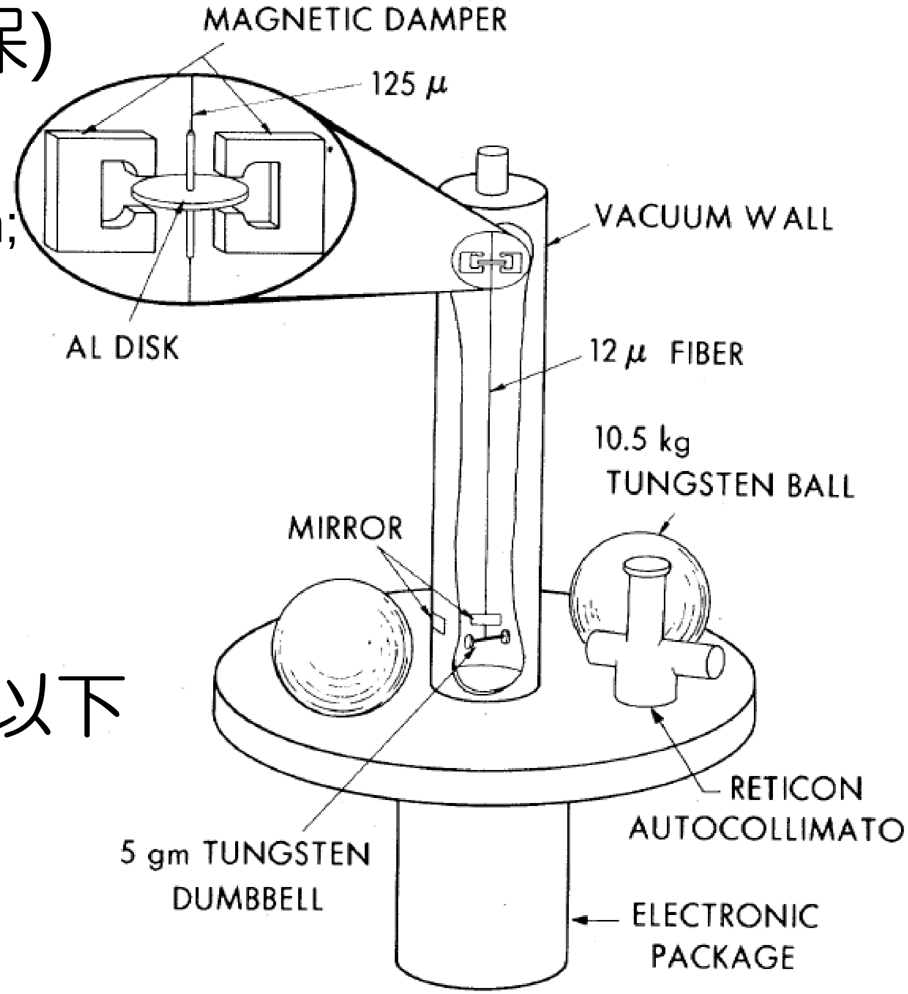


FIG. 1. Diagram of the apparatus with inset showing detail of the damper.

NIST-82の結果

- 共振周波数差は0.5 mHz程度

  (実際には角速度を測定)

  これを40ppmの精度で測定

TABLE II. Error budget.

Source of uncertainty	Uncertainty in ppm
Position of the large masses	10
Mass of the large masses	1
Length of the small mass	22
Thickness of the small mass	36
Density of the small mass	6
Moment of inertia of mirror	23
$\Delta(\omega^2)$	40
Total	64

- 1982年当初の結果は

  G = (6.6726 ± 0.0005) e-11 $m^3kg^{-1}s^{-2}$

- Kuroda効果の見積もり(2010)

  G = (6.67248 ± 0.00043) e-11 $m^3kg^{-1}s^{-2}$

- Kuroda効果が指摘され、CODATA2002とCODATA2006では使われなかったが、Kuroda効果が見積もられてCODATA2010には使われた

Kuroda効果

- K. Kuroda, PRL **75**, 2796 (1995)
- ばね定数の周波数非依存の虚数成分がある場合は

$$G = \frac{G(\mathrm{obs})}{1 + \pi Q}$$

の補正が必要(速度に比例したdampingのみの場合は補正不要)

VOLUME 75, NUMBER 15 PHYSICAL REVIEW LETTERS 9 OCTOBER 1995

Does the Time-of-Swing Method Give a Correct Value of the Newtonian Gravitational Constant?

Kazuaki Kuroda

Institute for Cosmic Ray Research, University of Tokyo, 3-2-1, Midoricho, Tanashi, Tokyo 188, Japan

(Received 12 June 1995)

A standard way of measuring the Newtonian gravitational constant has been the time-of-swing method using a torsion pendulum. A key assumption is that the spring constant of the torsion fiber is independent of frequency. This is likely to be true to a good approximation if any damping present is proportional to velocity. However, recent work on the elasticity of flexure hinges suggests that typically the damping at low frequency is best modeled by including a frequency-independent imaginary component in the spring constant. In this case, the real part of the spring constant must vary, leading to an upward bias in a measurement of G.

どういうこと？

- ばね定数の実部に周波数依存性が出る
  (structure dampingの場合; viscousの場合は関係ない)

represents damping. A simple system that shows anelasticity is the spring and dash-pot system of Fig. 2, which has the following relationship between stress σ and strain ϵ:

$$\epsilon E_R + \dot{\epsilon}(E_R + \delta E)\tau = \sigma + \dot{\sigma}\tau . \qquad (1)$$

E_R is the relaxed Young's modulus, i.e., the effective value in the limit of low frequencies, and δE is the difference $E_U - E_R$ between the relaxed modulus and the high frequency, unrelaxed modulus. The relaxation time constant τ is the ratio of the dash-pot viscosity ν and δE. Since we are interested in the frequency response we take the Fourier transform of Eq. (1),

$$E(\omega) = \frac{\sigma(\omega)}{\epsilon(\omega)} = E_R + \delta E\left(\frac{\omega^2\tau^2}{1 + \omega^2\tau^2} + \frac{i\omega\tau}{1 + \omega^2\tau^2}\right). \qquad (2)$$

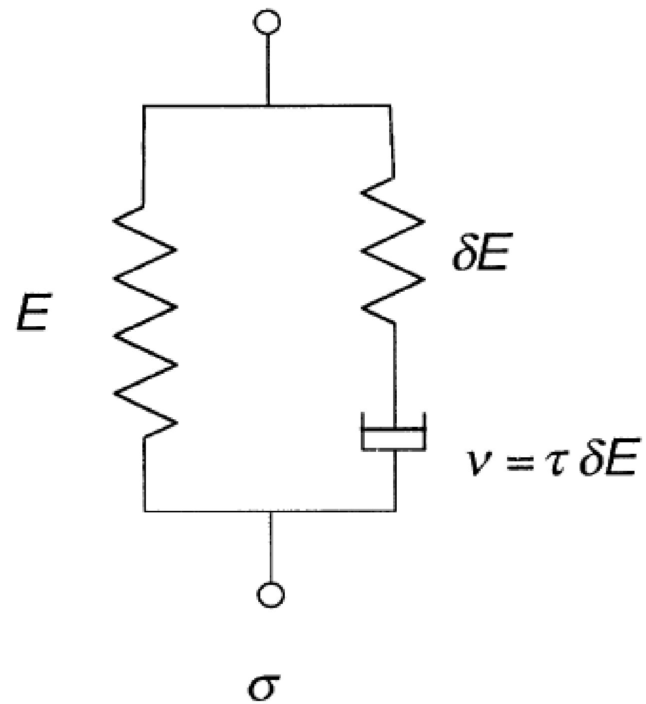


FIG. 2. An ideal spring in parallel with a Maxwell unit represents a single relaxation process. The materials considered in this paper are modeled as an infinite number of microdomains, each behaving as a Maxwell unit. The units have a common relaxation amplitude but a continuum of time constants

BIPM-01, BIPM-13

- T. J. Quinn+, PRL **87**, 111101 (2001)
  T. J. Quinn+, PRL **111**, 101102 (2013)
  同様の測定方法で精度を41ppmから27ppmに
- 重力によるねじれ角を測定(Cavendish type)
- 重力によるねじれトルクを制御により測定

VOLUME 87, NUMBER 11 PHYSICAL REVIEW LETTERS 10 SEPTEMBER 2001

A New Determination of *G* Using Two Methods

T. J. Quinn,[1,*] C. C. Speake,[2] S. J. Richman,[1,†] R. S. Davis,[1] and A. Picard[1]
[1]*Bureau International des Poids et Mesures, Pavillon de Breteuil, F-92312 Sèvres Cedex, France*
[2]*School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom*
(Received 12 February 2001; published 27 August 2001)

We present the results of a measurement of G made with a torsion-strip balance used in two substantially independent ways. The two results agree to within their respective uncertainties; the correlation coefficient of the two methods is -0.18. The result is $G = 6.67559(27) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ with a standard uncertainty of 4.1 parts in 10^5. Our result is 2 parts in 10^4 higher than the recent result of Gundlach and Merkowitz.

DOI: 10.1103/PhysRevLett.87.111101 PACS numbers: 04.80.Cc, 06.20.Jr

BIPM-01, BIPM-13の装置

- 長さ160 mm、太さ30 umのCu-1.8%Be線
- Q=3e5、共振周波数8 mHz
- 光てこ

test mass: Cu-0.7%Te 1.2 kg
source mass: Cu-0.7%Te 12 kg
test-source distance: 94 mmc

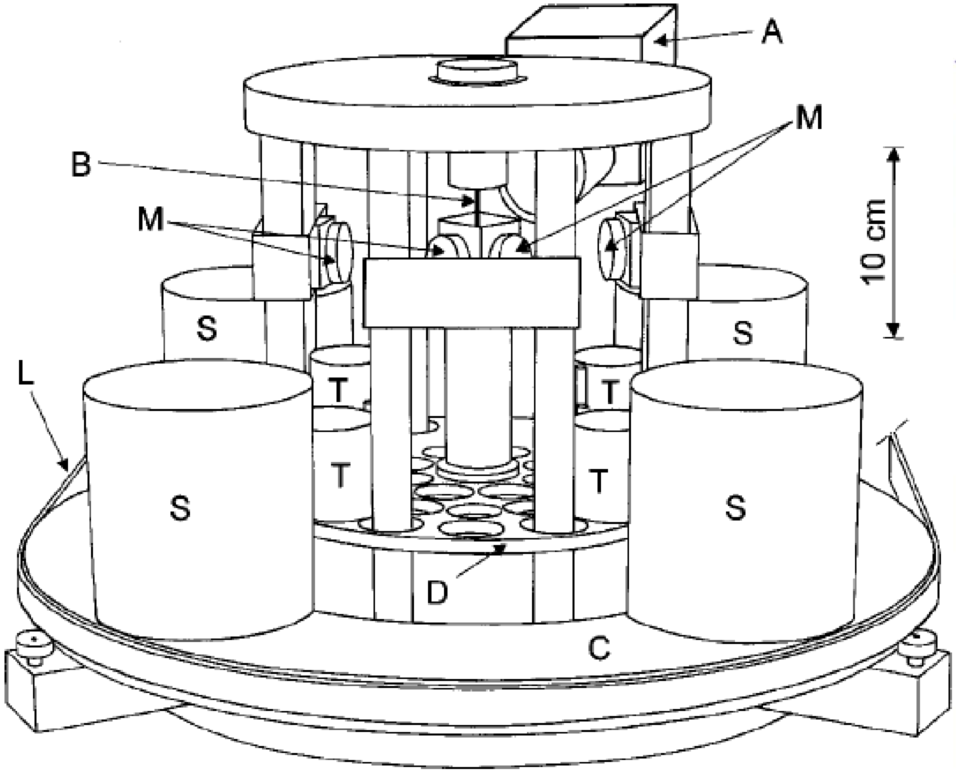


FIG. 1. Outline of apparatus: T, test masses; S, source masses; D, torsion balance disk; B, torsion strip; C, carousel; L, drive belt; M, mirrors for sixfold multiplying optics; A, autocollimator.

C. Speake & T. Quinn, Physics Today **67**, 27 (2014)

BIPM-13の結果

- 2つの方法の平均を取って

  $G = (6.67545 \pm 0.00018)\ \mathrm{e}{-11}\ m^3 kg^{-1} s^{-2}$

type A: 統計的手法によって評価される不確かさ
type B: それ以外の手法で評価される不確かさ

TABLE I. Uncertainty budget (types A and B in ppm of G), with correlation coefficients.

Parameter	Servo	Cavendish	Correlation coefficients
Source mass value (A)	1	1	3×10^{-4}
Test mass value (A)	1	0	0
Source mass position (A)	15	15	0
Test mass position (A)	18	9	5×10^{-2}
Dimensional metrology (B)	4	3	-3×10^{-3}
Moment of inertia (A)	0	13	0
Capacitance (B)	6	0	0
Voltages (B)	12	0	0
Balance period (B)	0	1	0
Angle (B)	47	47	−0.64
Anelasticity (B)	0	4	0
Torque (A)	30	19	0
Totals	61	56	−0.59

JILA-10

- H. V. Parks & J. E. Faller, PRL **105**, 110801 (2010)
- 重力によって変化する懸架されたFabry-Perot共振器の長さを測定

PRL **105,** 110801 (2010) PHYSICAL REVIEW LETTERS week ending 10 SEPTEMBER 2010

Simple Pendulum Determination of the Gravitational Constant

Harold V. Parks[1,2,*] and James E. Faller[1]

[1]*JILA, University of Colorado and National Institute of Standards and Technology, Boulder, Colorado 80309, USA*
[2]*Sandia National Laboratories, Albuquerque, New Mexico 87185, USA*

(Received 22 June 2010; published 7 September 2010)

We determined the Newtonian constant of gravitation G by interferometrically measuring the change in spacing between two free-hanging pendulum masses caused by the gravitational field from large tungsten source masses. We find a value for G of $(6.672\,34 \pm 0.000\,14) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$. This value is in good agreement with the 1986 Committee on Data for Science and Technology (CODATA) value of $(6.672\,59 \pm 0.000\,85) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ [Rev. Mod. Phys. **59**, 1121 (1987)] but differs from some more recent determinations as well as the latest CODATA recommendation of $(6.674\,28 \pm 0.000\,67) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ [Rev. Mod. Phys. **80**, 633 (2008)].

DOI: 10.1103/PhysRevLett.105.110801 PACS numbers: 06.20.Jr, 04.80.Cc

JILA-10の装置

- 長さ72 cmの振り子
- 共振器長 34 cm
- test mass: Cu 780g
- source mass: W 120 kg
- He-Neレーザー
- フィネス 4000
- ねじれに比べて
  - ほぼ地球重力でトラップされるので理想的なばね
  - 共振周波数は高いが干渉計のおかげで測定精度は高い

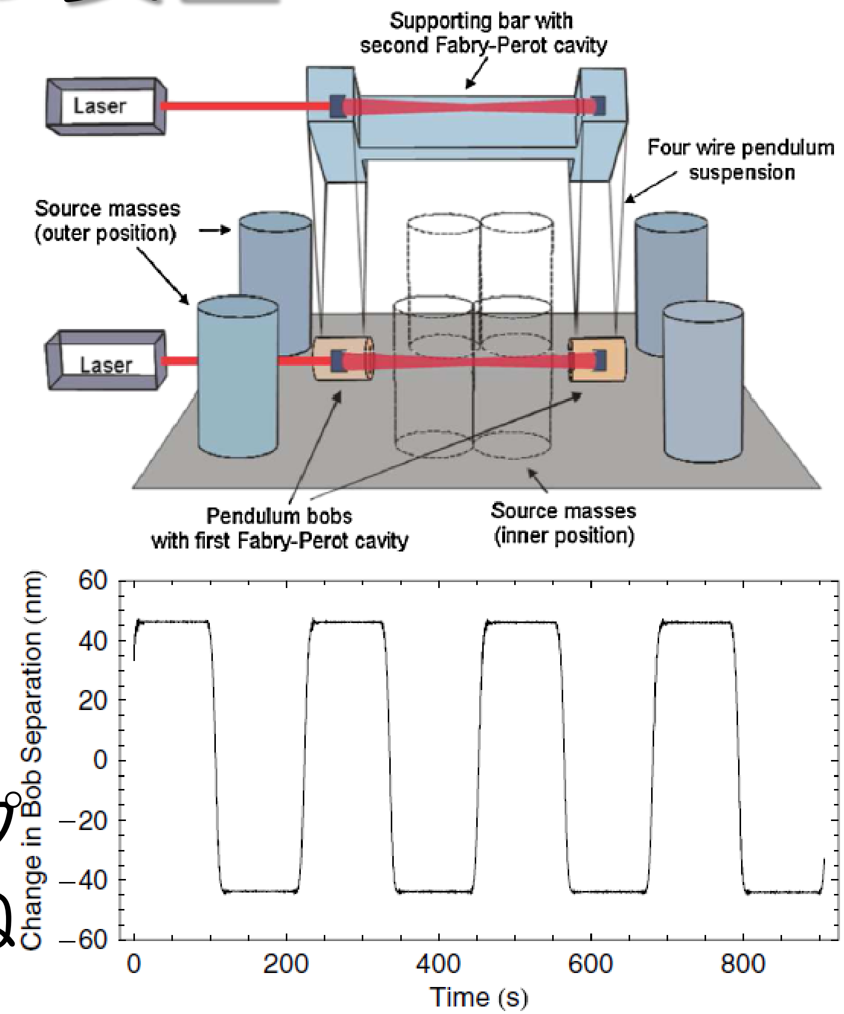


FIG. 1 (color online). A schematic of the apparatus is shown on top. A Fabry-Perot interferometer measures the spacing between the two pendulum bobs with respect to a suspension-point-located reference cavity. The bobs are made of oxygen-free copper and have a mass of 780 g. The pendulum length is 72 cm, and the spacing between the bob centers is 34 cm. When

JILA-10の結果

- $G = (6.67234 \pm 0.00014)$ e-11 $m^3 kg^{-1} s^{-2}$
- "Interferometer"はソース質量の動きによるミスアラインメントからくる不確かさ

TABLE I. The major components of uncertainty are listed here expressed in terms of each contribution to $\delta G/G$ in parts in 10^5 at the 1σ level. The uncertainties in this table, along with all other uncertainties in this Letter, are expressed as standard (1σ) uncertainties.

Uncertainty component	$\delta G/G(\times 10^{-5})$
Six critical dimensions	1.4
All other dimensions	0.8
Source mass density inhomogeneities	0.8
Pendulum spring constants	0.7
Total mass measurement	0.6
Interferometer	0.6
Tilt due to source mass motion	0.1
Day-to-day scatter	0.4
Combined uncertainty	2.1

- "The apparatus is very simple" (Cavendish, 1798)
- "The measurement is very hard"

INFN-14

- G. Rosi +, [Nature 510, 518 (2014)]
- 原子干渉計を利用
  - S. Schlamminger, [Nature 510, 478 (2014)]
  - 測定精度は他より5倍くらい悪いが、全くの新手法

FUNDAMENTAL CONSTANTS

A cool way to measure big *G*

Published results of the gravitational constant, a measure of the strength of gravity, have failed to converge. An approach that uses cold atoms provides a new data point in the quest to determine this fundamental constant. SEE LETTER P.518

STEPHAN SCHLAMMINGER

In our daily lives, we can see the effect of the gravitational force between Earth and an object, say an apple. However, the this cloud in free fall probes gravity, quantum mechanics is needed. For simplicity, consider that the atoms in the cloud can be in two different atomic states, A and B. At the beginning, all atoms are in state A. By exposing an atom to an

LETTER

Precision measurement of the Newtonian gravitational constant using cold atoms

G. Rosi[1], F. Sorrentino[1], L. Cacciapuoti[2], M. Prevedelli[3] & G. M. Tino[1]

About 300 experiments have tried to determine the value of the Newtonian gravitational constant, *G*, so far, but large discrepancies in the results have made it impossible to know its value precisely[1]. The weakness of the gravitational interaction and the impossibility of shielding the effects of gravity make it very difficult to measure *G* while keeping systematic effects under control. Most previous experiments performed were based on the torsion pendulum or torsion balance scheme as in the experiment by Cavendish[2] in 1798, and in all cases macroscopic masses were used. Here we report the precise determination of *G* using laser-cooled atoms and quantum interferometry. We obtain the value $G = 6.67191(99) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ with a relative uncertainty of 150 parts per million (the combined standard

the relevant gravitational signal. An additional cancellation of common-mode spurious effects was obtained by reversing the direction of the two-photon recoil used to split and recombine the wave packets in the interferometer[18]. Efforts were devoted to the control of systematics related to atomic trajectories, the positioning of the atoms and effects due to stray fields. The high density of tungsten was instrumental in maximizing the signal and in compensating for the Earth's gravitational gradient in the region containing the atom interferometers, thus reducing the sensitivity of the experiment to the vertical position and size of the atomic probes.

The atom interferometer is realized using light pulses to stimulate ^{87}Rb atoms at the two-photon Raman transition between the hyperfine

INFN-14の装置

- test mass: 1e9個のRb原子(磁気光学トラップ)
- source mass: W 516 kg
- 2つの原子雲を投げ上げ、自由落下中させて重力加速度を測定(その差からGを測定)

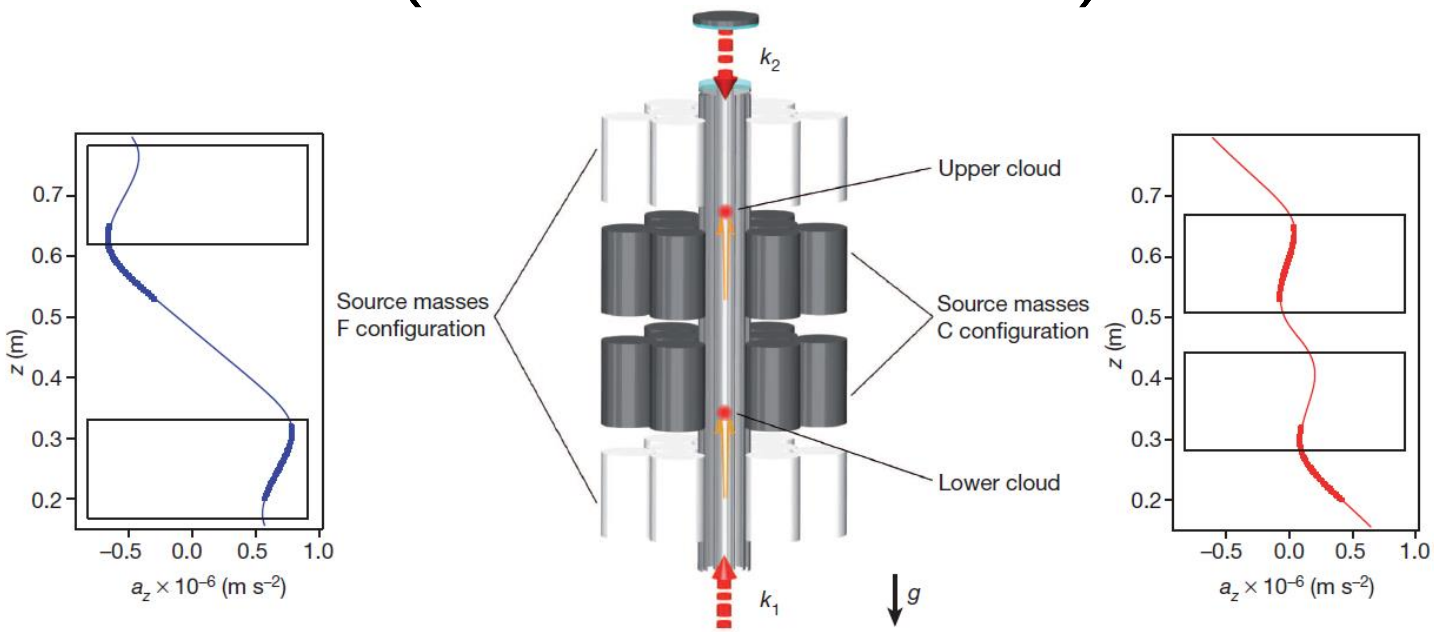


Figure 1 | **Sketch of the experiment.** The Rb atom interferometer operates as a gravity gradiometer and the W masses are used as the source of the gravitational field. For the measurement of *G*, the position of the source masses is alternated between configurations F and C. Plots of gravitational acceleration (a_z) produced along the symmetry axis by the source masses are also shown for each configuration; a constant value for Earth's gravity was subtracted. The spatial regions of the upper and lower atom interferometers are indicated by the thick lines. The vertical acceleration plots show the effect of source mass in cancelling the local gravity gradient at the positions of the atomic apogees.

原理

- Rb原子の2準位系(F=1とF=2)で、各準位にある原子数の割合の変化を測定する
- F=2の方が運動量が大きいのでF=1とは重力加速度の存在下で運動が異なる
→F=1とF=2の割合の変化量は重力加速度に依存
- 例えば質量Mが原子雲のzだけ上/下にあるとき

$$g_{\mathrm{ab}} = -g + \frac{GM}{z^2}$$

$$\rightarrow G = \frac{z^2}{2M}(g_{\mathrm{ab}} - g_{\mathrm{bel}})$$

$$g_{\mathrm{bel}} = -g - \frac{GM}{z^2}$$

(環境重力場の影響をキャンセルできる)

- 2つの原子雲を同時に投げ上げることで実現
- 測定精度としては一秒で3e-9*g

INFN-14の結果

- G = (6.67191 ± 0.00099) e-11 $m^3kg^{-1}s^{-2}$
- 原子雲の位置やサイズ、ソース質量の位置の不確かさがやっぱり大きい
  →Sr原子だともっと位置精度上げられるかもと言っている

Table 1 | Effects, relative corrections and uncertainties considered in our determination of *G*

Parameter	Uncertainty in parameter	Relative correction to G (p.p.m.)	Relative uncertainty in G (p.p.m.)
Air density	10%	60	6
Apogee time	30 μs	—	6
Atomic cloud horizontal size	0.5 mm	—	24
Atomic cloud vertical size	0.1 mm	—	56
Atomic cloud horizontal position	1 mm	—	37
Atomic cloud vertical position	0.1 mm	—	5
Atom launch direction change C/F	8 μrad	—	36
Cylinder density homogeneity	10^{-4}	91	18
Cylinder radial position	10 μm	—	38
Ellipse fit	—	−13	4
Size of detection region	1 mm	—	13
Support platform mass	10 g	—	5
Translation stage position	0.5 mm	—	6
Other effects	—	<2	1
Total systematic uncertainty	—	—	92
Statistical uncertainty	—	—	116
Total	—	137	148

Uncertainties are quoted as one standard deviation. The third column contains the corrections we applied to account for effects not included in the Monte Carlo simulation. The bias and systematic error from ellipse fitting are evaluated by a numerical simulation on synthetic data. Other effects include cylinder mass, cylinder vertical position, gravity gradient, gravity acceleration, Raman mirror tilt, Raman *k* vector and timing.

Sr原子による重力加速度測定

- N. Poli+, [PRL **106**, 038501 (2011)]
- 光格子にトラップしたSr原子で重力加速度をBloch振動を利用して測定
- 1e-7*gの精度
- 地球表面からの高低差に換算すると0.3 m相当
  (地球半径6371 km)
- INFN-14の3e-9*gは1 cm相当
- 重力赤方偏移で1 cm測定には1e-18必要

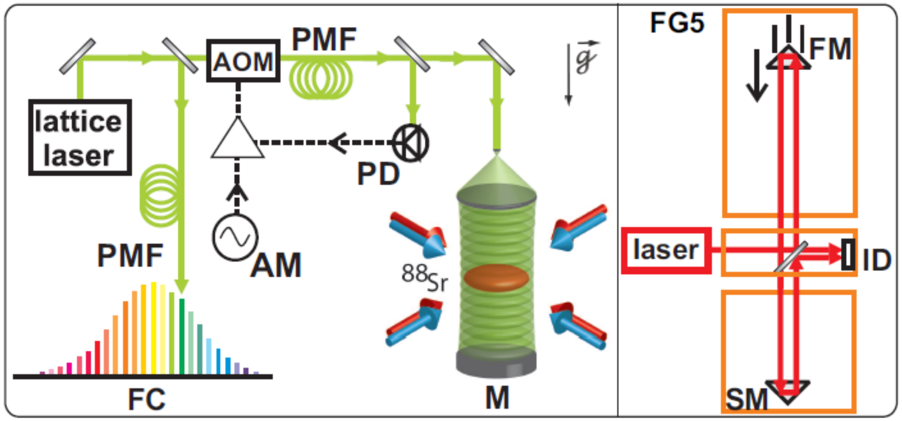
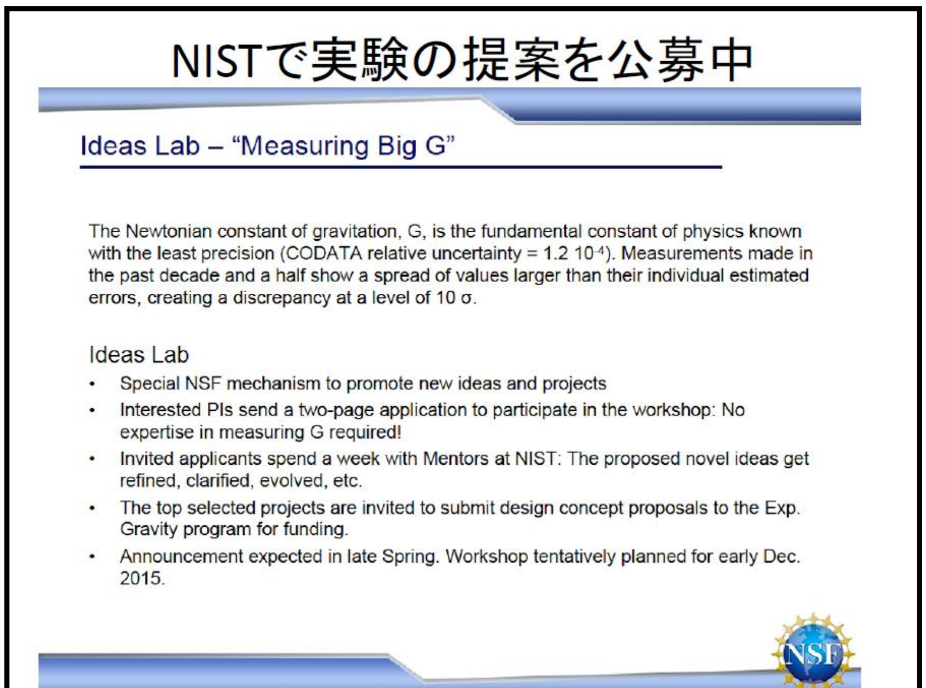
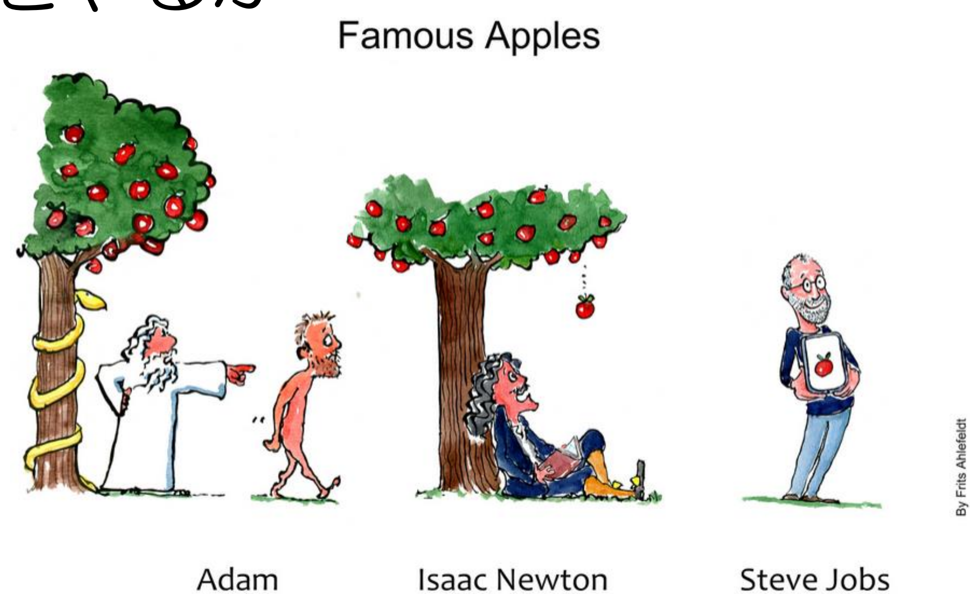


FIG. 1: Experimental setup for the measurement of gravity with ^{88}Sr atoms trapped in a vertical optical lattice and the comparison with a classical absolute gravimeter (FG5). M: lattice mirror; FC: frequency comb; FM: freely-falling mirror; SM: stationary mirror; ID: interference detector; AOM: acousto-optical modulator; PMF: polarization maintaining fiber; AM: RF generator for amplitude modulation of the laser producing the lattice; PD: photodiode.

まとめ

- 装置の原理は単純でも、不確かさの低減は難しい
- 同様の手法でやるなら、先行研究が未評価の不確かさを指摘してからでないとあまり意味なさそう
- それか全く新しい手法をやるか





http://piximggif.com/newton-apple

黒田先生最終講義スライドより
(NSFの元のスライドは見つけられなかった)

その他

- 質量をどうやって測っているの？
  キログラム原器と天秤で測定できるのでGを知らなくても質量は測れる
- 逆二乗則を4桁以上の精度で測定できるの？
  標準的な重力との相対的な大きさを比較してるだけだから、できる
- 万有引力定数の恒常性
  面白そう

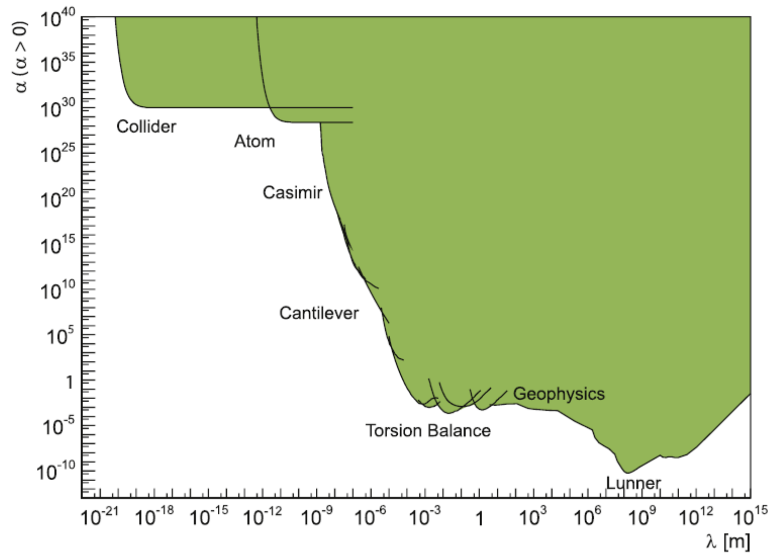


Figure 1. Experimental constraints on the parameters α (coupling strength) and λ (range) of the Yukawa interaction for α (>0). The shaded area indicates the excluded area at a 95% confidence level. The constraint curves for over km scales are taken from Fischbach and Talmadge (1999) and Adelberger *et al* (2009). See section 3 for short-range tests at below laboratory scale.

J. Murata and S. Tanaka, [CQG 32, 033001 (2015)]