Laser Interferometry for Gravitational Wave Observations

3. Sensitivity Design

Yuta Michimura

Department of Physics, University of Tokyo

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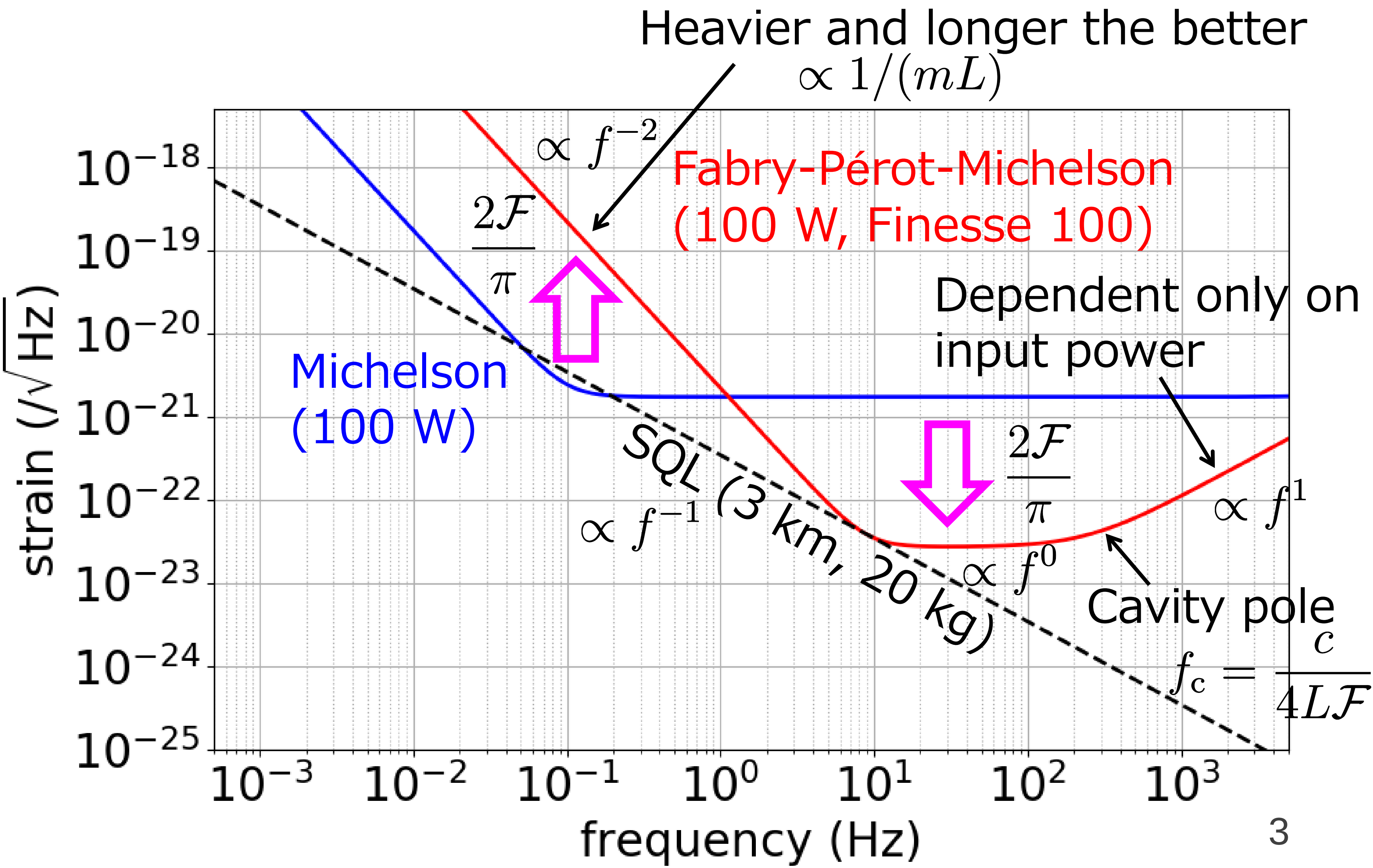
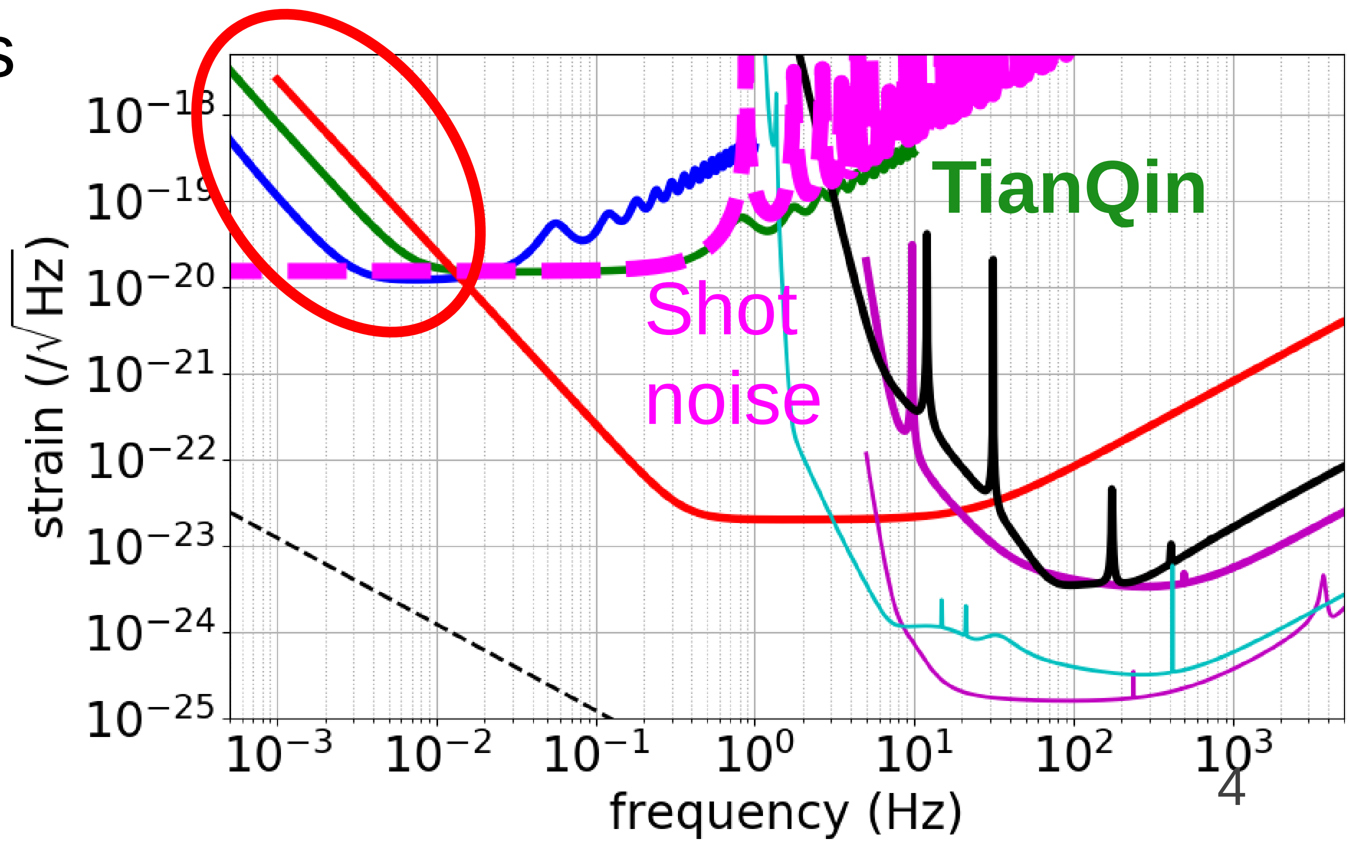
   Future prospects

Review on Quantum Noise



Low Frequency Noises

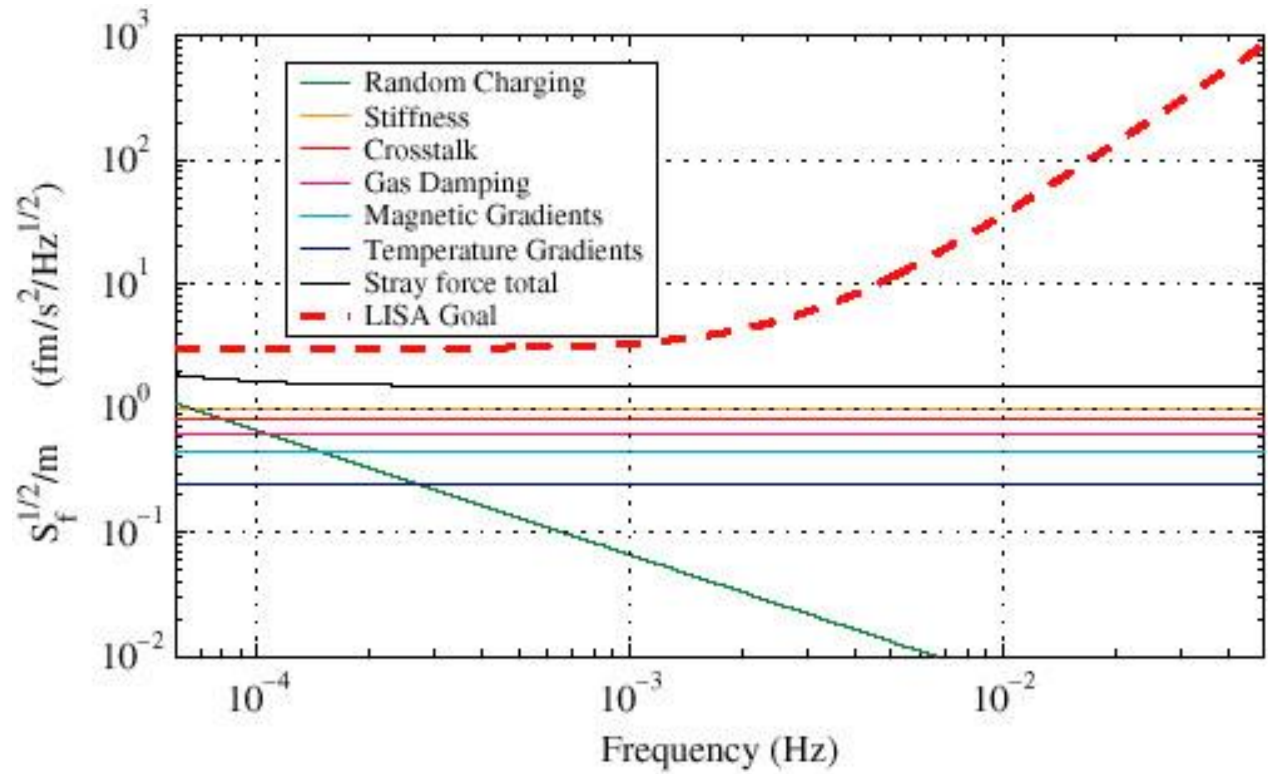
- We have shown that the designed sensitivity of current/proposed GW detectors are mostly determined by quantum noise
- But there are other classical noises at low frequencies



Force Noises

- There are many kinds of force noises (acceleration noises)
- Contribution of force noises to strain sensitivity

$$h_{\mathrm{f}} = \frac{n_f}{m\omega^2 L}$$

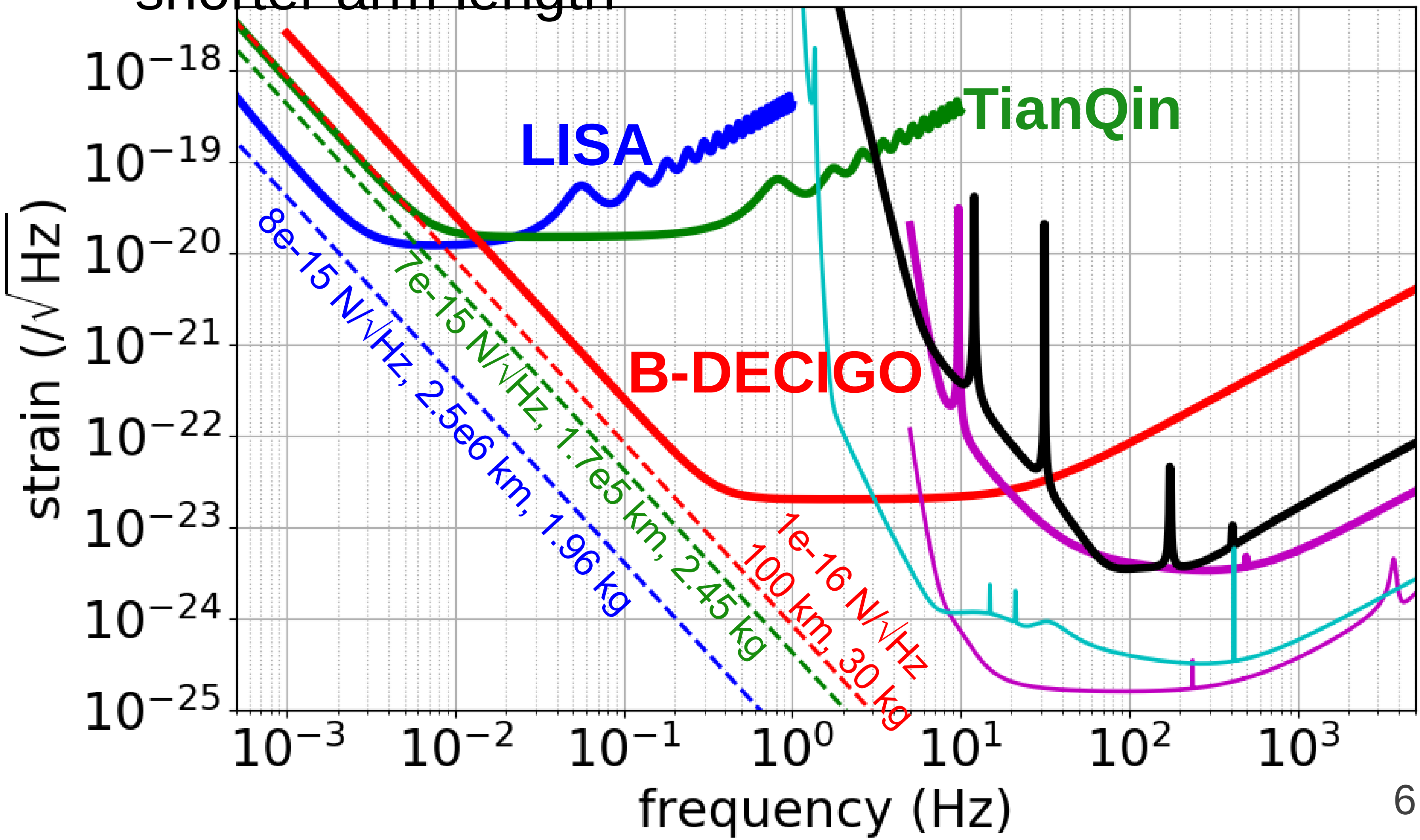


- Longer arm is better

D. Bortoluzzi+, [CQG 21, S573 (2004)]

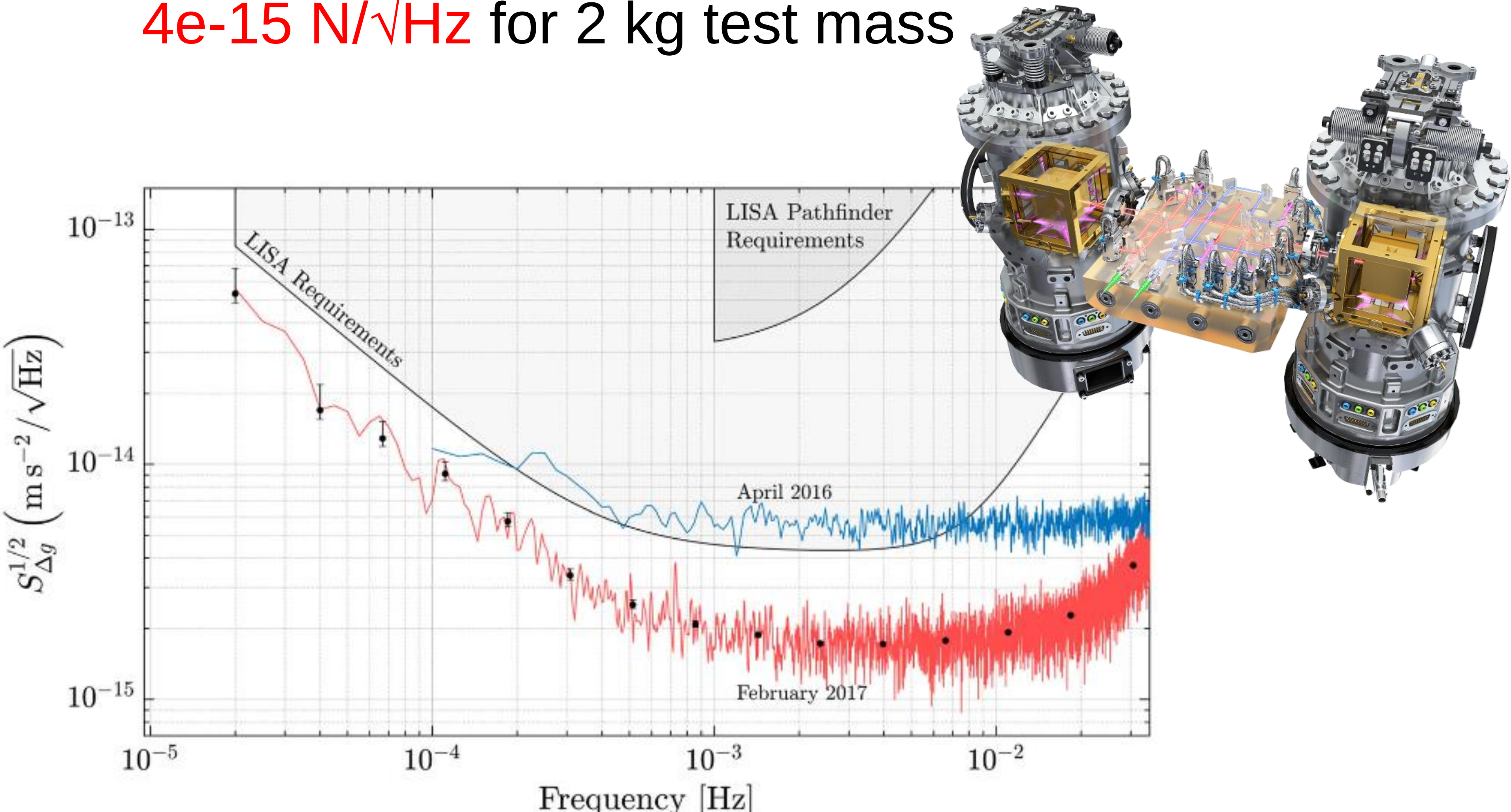
Force Noise Requirements

- B-DECIGO requirement is most stringent due to shorter arm length



LISA Pathfinder Acceleration Noise

- 2e-15 m/sec²/√Hz achieved, which correspond to 4e-15 N/√Hz for 2 kg test mass



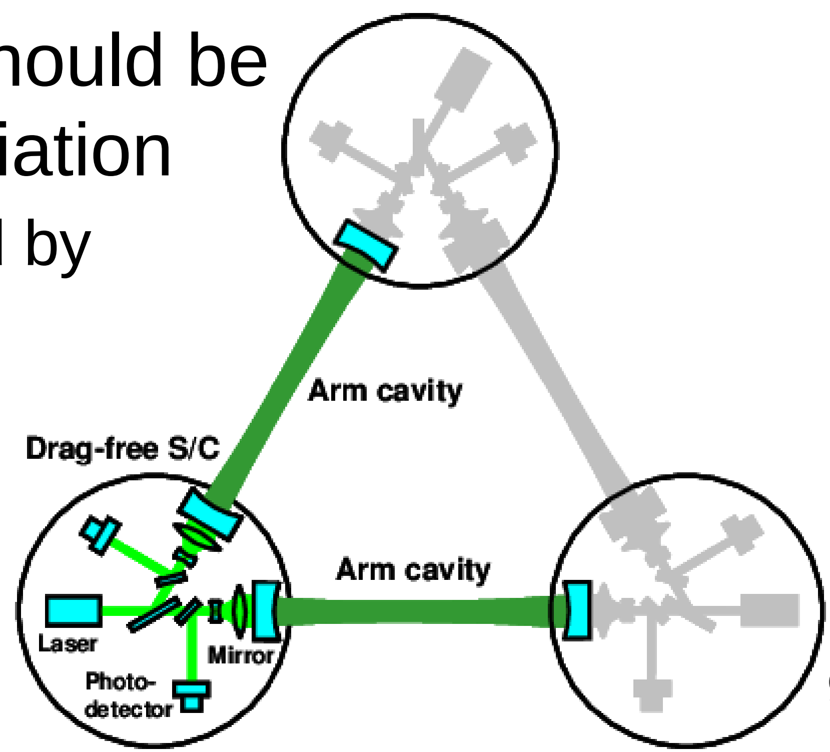
M. Armano+, [PRL 120, 061101 (2018)]

Design Comparison

	LISA	TianQin	B-DECIGO
Arm length	2.5e6 km	1.7e5 km	100 km
Interferometry	Optical transponder	Optical transponder	Fabry-Pérot cavity
Laser frequency stabilization	Reference cavity, 1064 nm	Reference cavity, 1064 nm	Iodine, 515 nm
Orbit	Heliocentric	Geocentric, facing J0806.3+1527	Geocentric (TBD)
Flight configuration	Constellation flight	Constellation flight	Formation flight
Test mass	1.96 kg	2.45 kg	30 kg
Force noise req.	8e-15 N/rtHz Achieved PRL 120, 061101 (2018)	7e-15 N/rtHz CQG 33, 035010 (2016)	1e-16 N/rtHz

DECIGO Design

- To be sensitive to 0.1-10 Hz band, shorter arm is required
- DECIGO chose to do Fabry-Pérot to be sensitive in 0.1-10 Hz band, which requires large mirrors (and short arm) to form a cavity
- Low frequency noise should be limited by quantum radiation pressure noise (if limited by classical force noise, higher power would be preferable in terms of horizon distance)



Horizon Distance

- One of the figures of merit of gravitational wave detectors is detectable distance of compact binary coalescences
- Maximum signal to noise ratio of inspiral signal can be calculated with

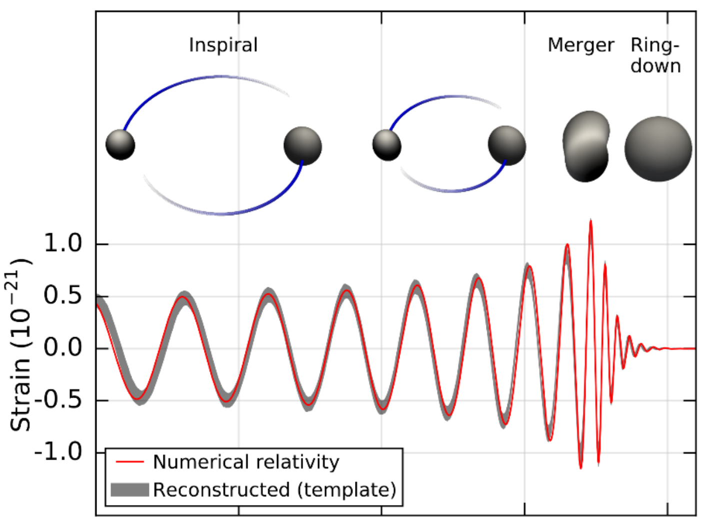
$$\rho_{\mathrm{max}} = \frac{1}{d_{\mathrm{L}}} \left(\frac{5}{6}\right)^{1/2} \frac{c}{\pi^{2/3}} \left(\frac{G\mathcal{M}_{\mathrm{c}}}{c^3}\right)^{5/6} \left[\int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} \frac{f^{-7/3}}{S_{\mathrm{n}}(f)} \mathrm{d}f\right]^{1/2}$$

Luminosity distance

Detector sensitivity

Chirp mass

$$\mathcal{M}_{\mathrm{c}} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$



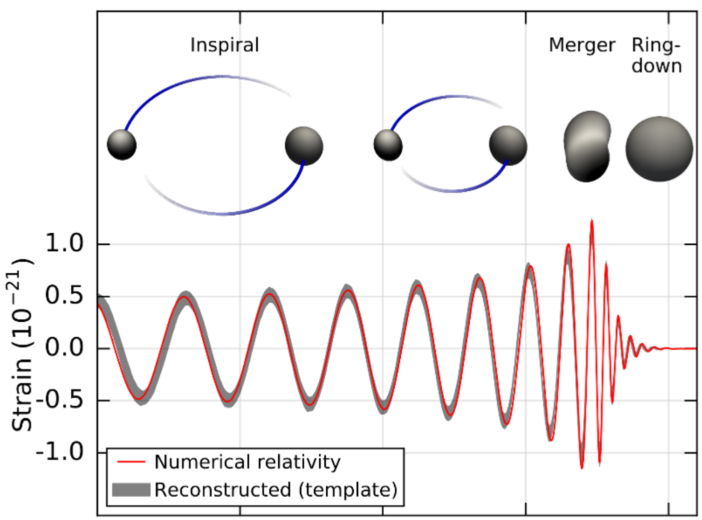
Horizon Distance

- If we set the SNR threshold, maximum luminosity distance can be calculated for given chirp mass
- We usually set $\rho_{\mathrm{th}} = 8$
- Chirp mass used below is detector frame mass

$$\rho_{\mathrm{max}} = \frac{1}{d_{\mathrm{L}}}\left(\frac{5}{6}\right)^{1/2} \frac{c}{\pi^{2/3}} \left(\frac{G\mathcal{M}_{\mathrm{c}}}{c^3}\right)^{5/6} \left[\int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} \frac{f^{-7/3}}{S_{\mathrm{n}}(f)}\mathrm{d}f\right]^{1/2}$$

Luminosity distance

Detector sensitivity



Chirp mass

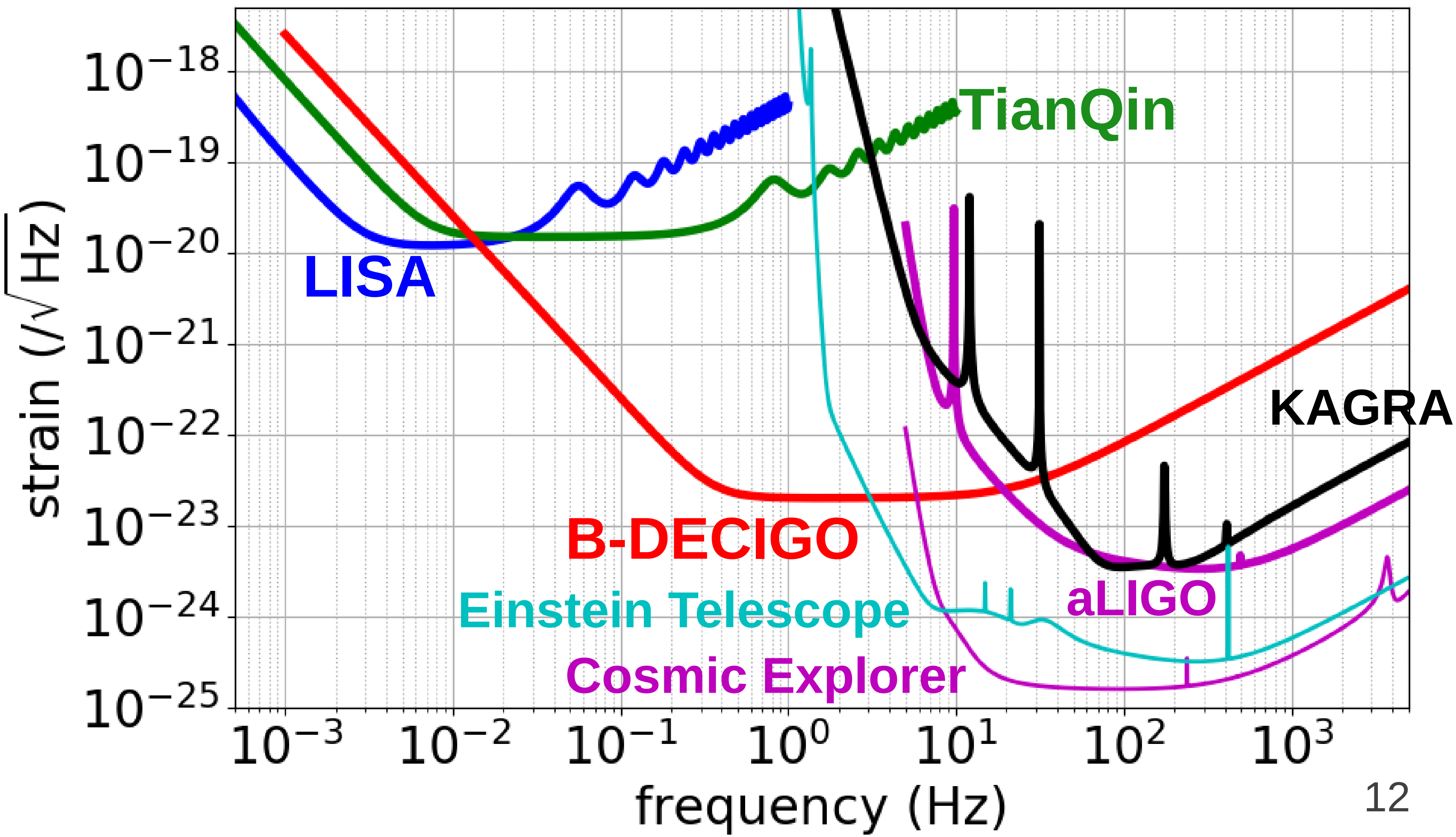
$$\mathcal{M}_{\mathrm{c}} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$\mathcal{M}_{\mathrm{c}}^{\mathrm{source}} = \frac{\mathcal{M}_{\mathrm{c}}}{1+z}$$

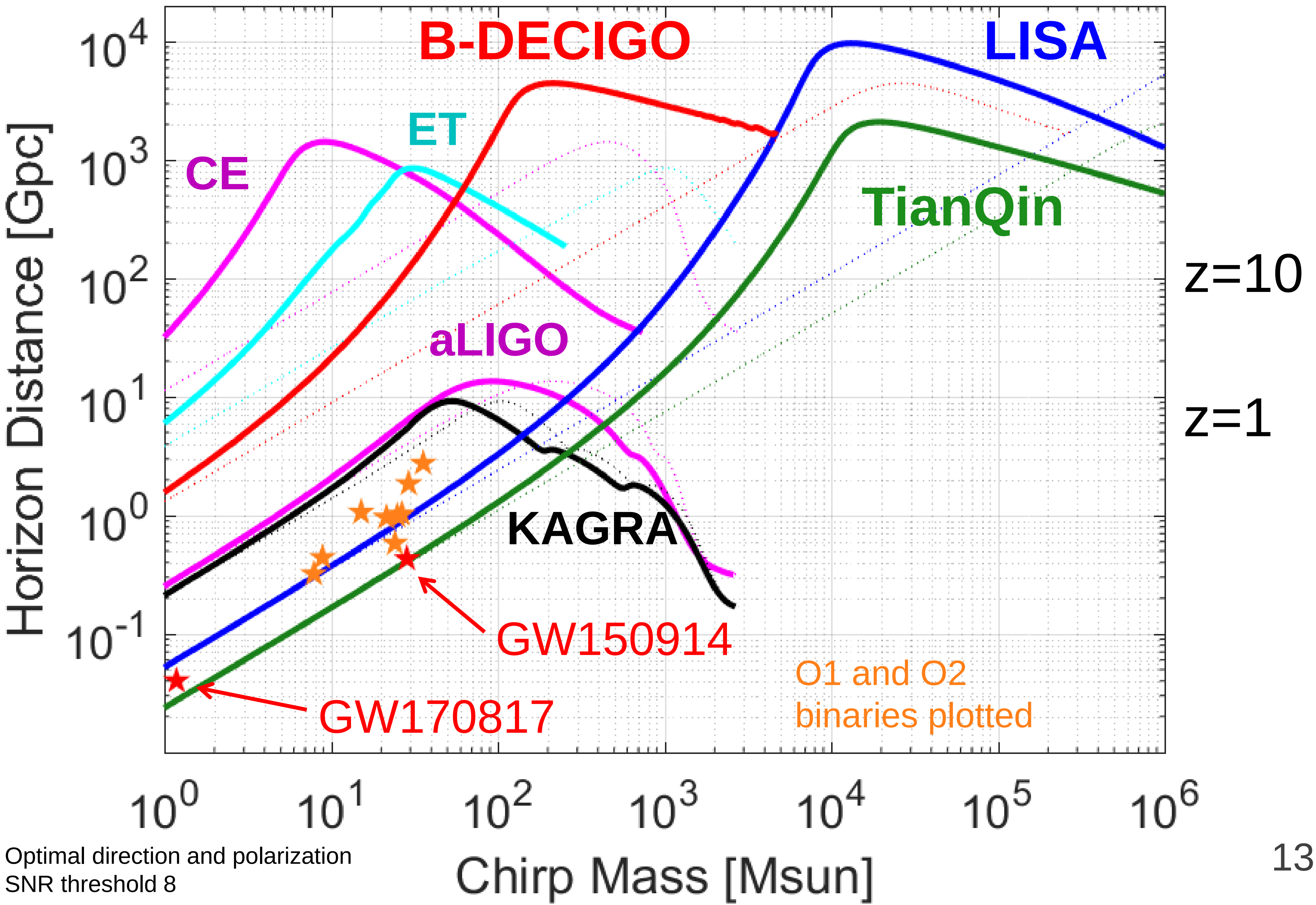
Sensitivity Curves of GW Detectors

LISA: https://perf-lisa.in2p3.fr/
TianQin: arXiv:1902.04423 (from Yi-Ming Hu)
B-DECIGO: PTEP 2016, 093E01 (2016)

KAGRA: PRD 97, 122003 (2018)
aLIGO: LIGO-T1800044
ET: http://www.et-gw.eu/index.php/etdsdocument
CE: CQG 34, 044001 (2017)

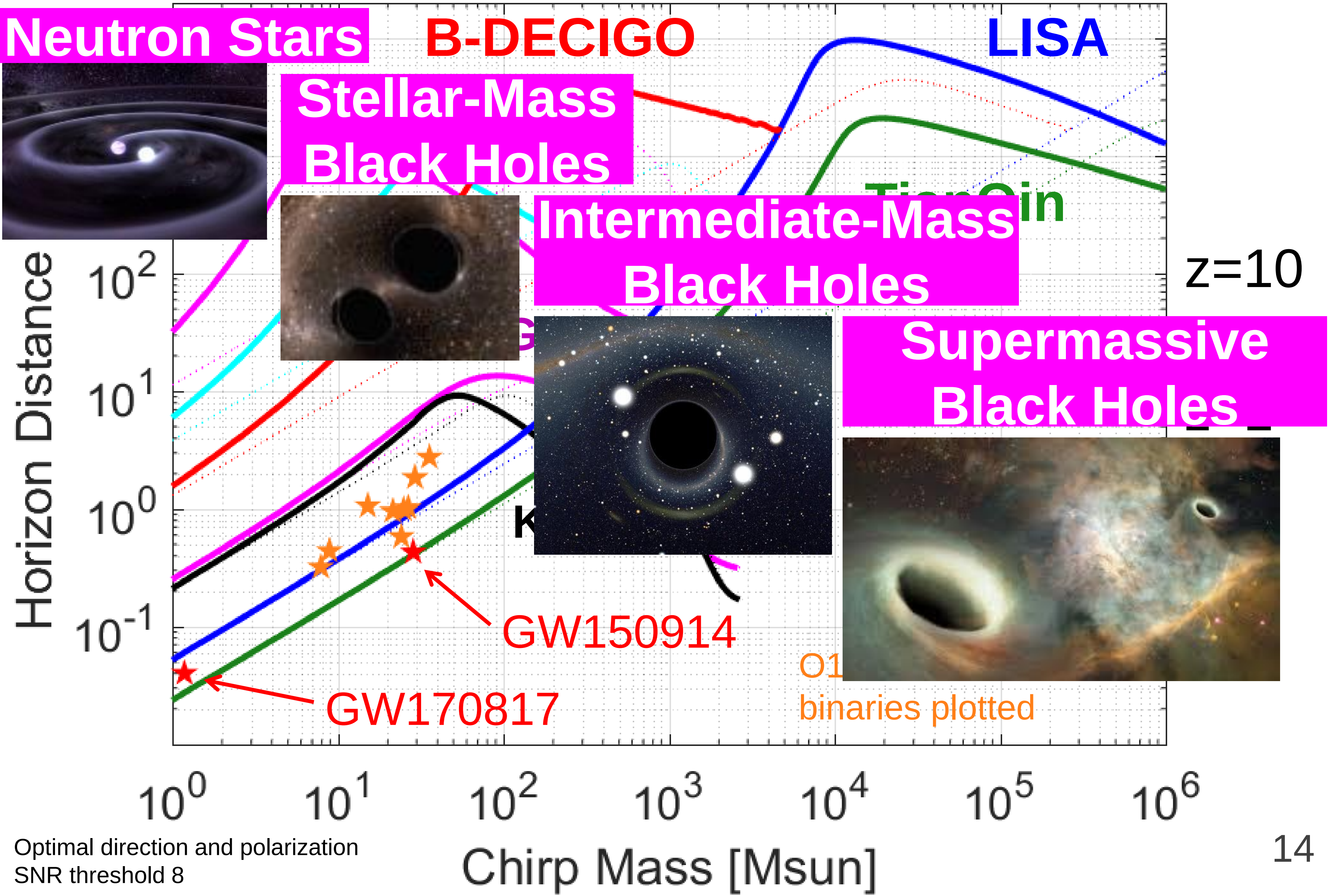


Horizon Distance Comparison



Optimal direction and polarization
SNR threshold 8

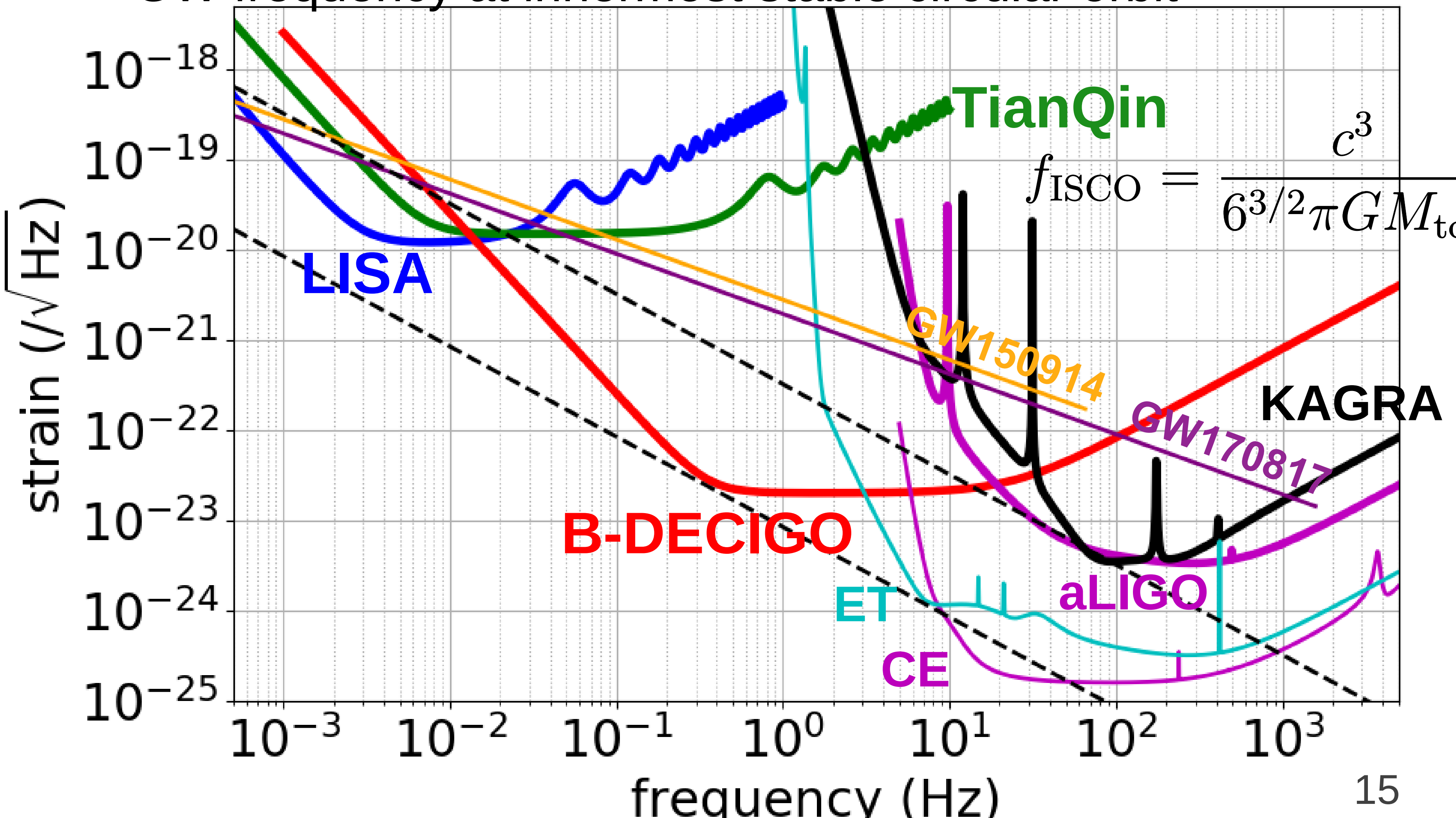
Horizon Distance Comparison



Optimal direction and polarization
SNR threshold 8

Inspiral Signal

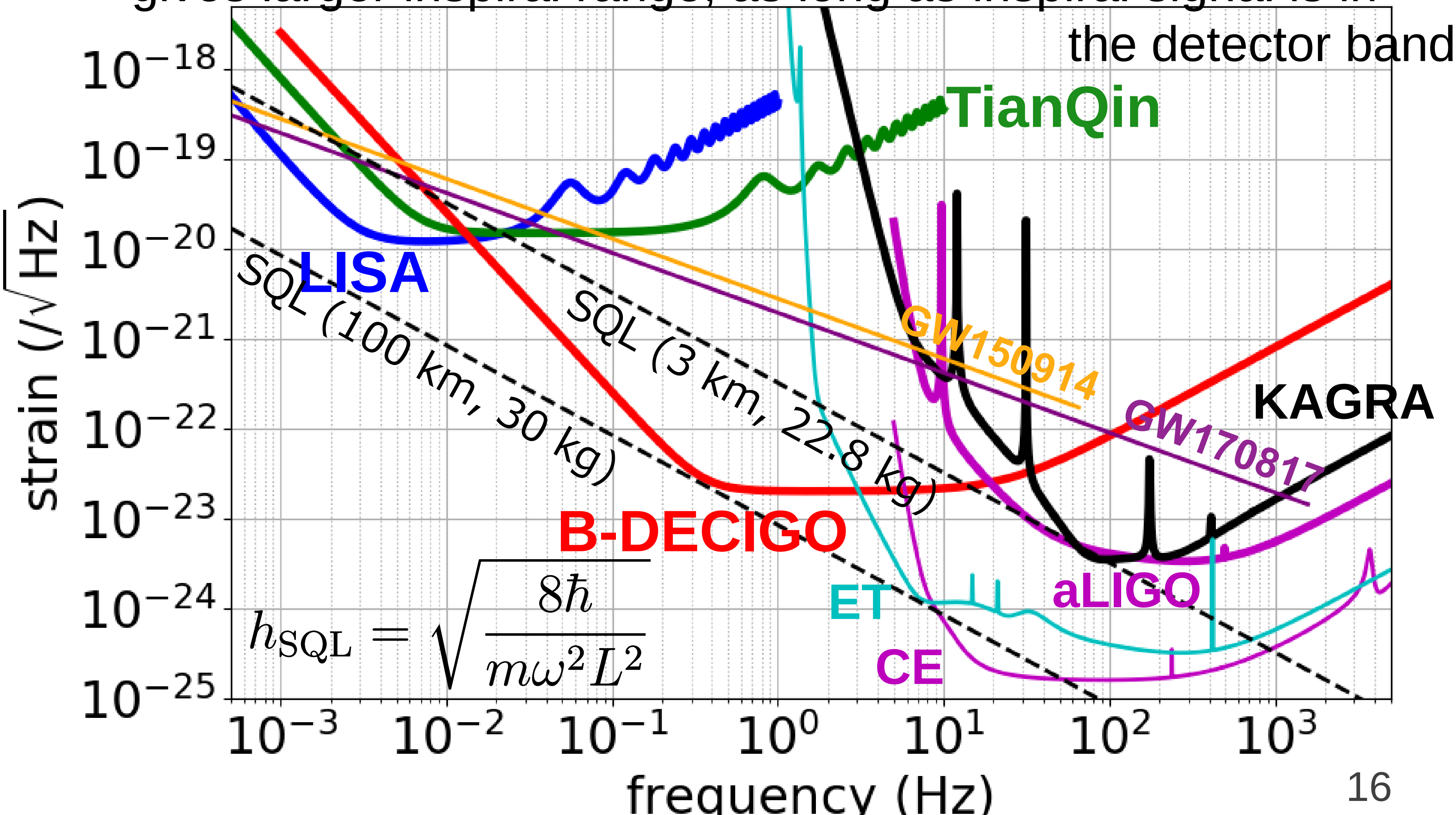
- $f^{-2/3}$ in amplitude spectrum density, upto around GW frequency at innermost stable circular orbit
  -7/3*1/2+1/2=-2/3



$$f_{\mathrm{ISCO}} = \frac{c^3}{6^{3/2}\pi G M_{\mathrm{tot}}}$$

Inspiral Signal and SQL

- SQL follows f^{-1} , so higher SQL touching frequency gives larger inspiral range, as long as inspiral signal is in the detector band



SQL Touching Frequency

- Quantum noise touches SQL where $h_{\rm shot} = h_{\rm rad}$

$$h_{\rm shot} = \frac{1}{2\pi L}\sqrt{\frac{hc\lambda}{2P_0}}\frac{\pi}{2\mathcal{F}}\sqrt{1+(\omega\tau)^2}$$

$$h_{\rm rad} = \frac{2}{m\omega^2 L}\sqrt{\frac{8hP_0}{c\lambda}}\frac{2\mathcal{F}}{\pi}\frac{1}{\sqrt{1+(\omega\tau)^2}}$$

- When $\omega \ll 1/\tau$ (frequency below cavity pole) ,

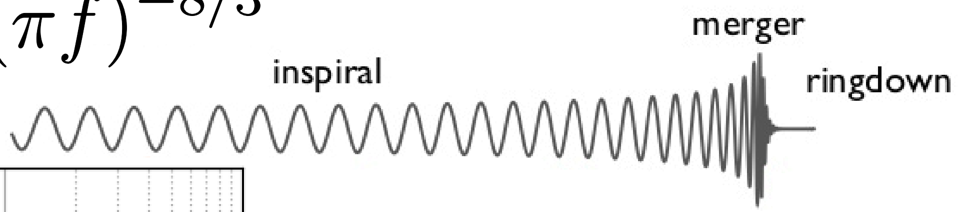
$$f_{\rm SQL} = \frac{1}{2\pi}\sqrt{\frac{16\pi P_0}{mc\lambda}}\frac{2\mathcal{F}}{\pi}$$

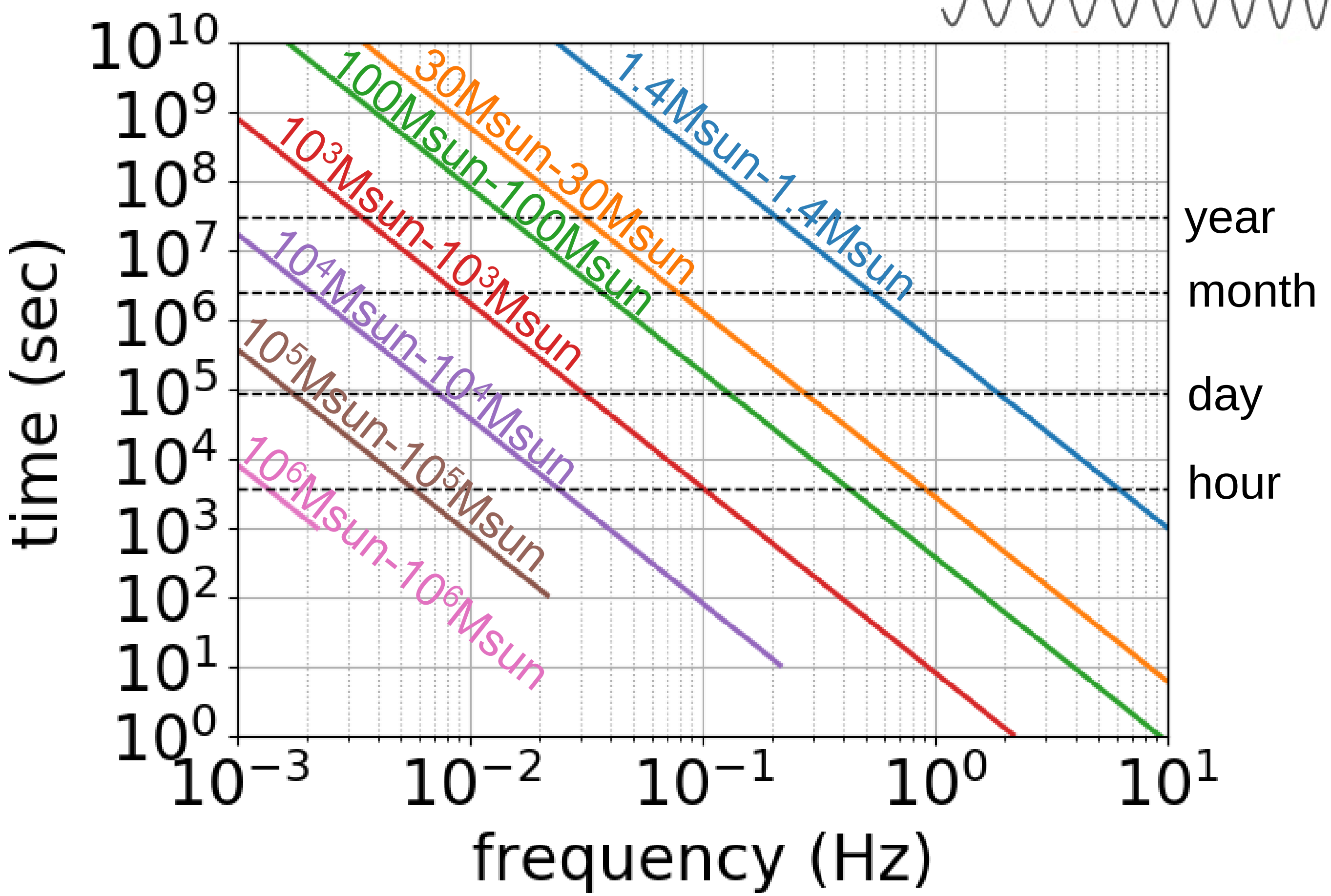
- Higher power is required for higher SQL touching frequency

Time to Merger

- Time it takes to merger from a certain GW frequency

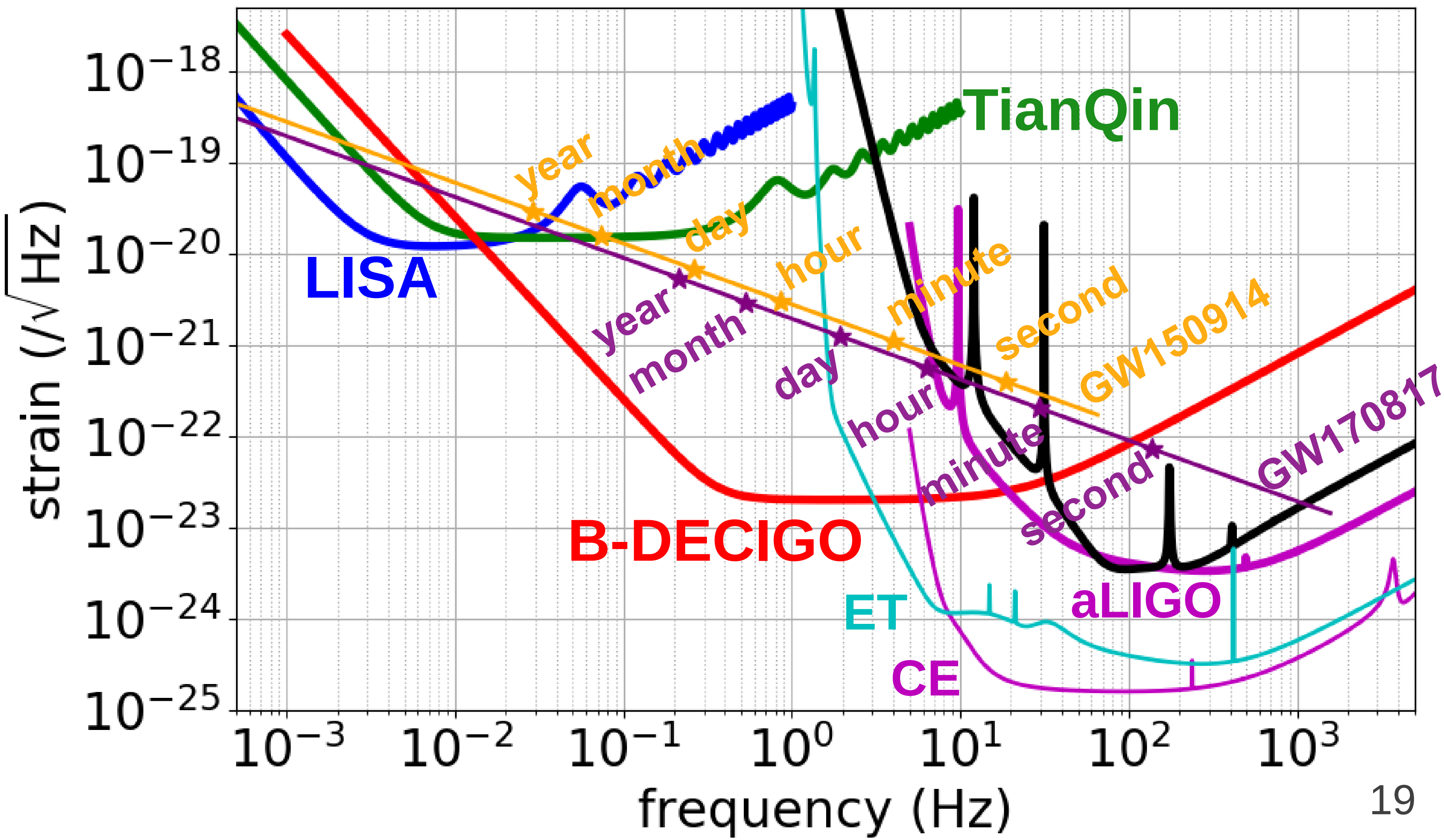
$$t = \frac{5}{256}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{-5/3} (\pi f)^{-8/3}$$





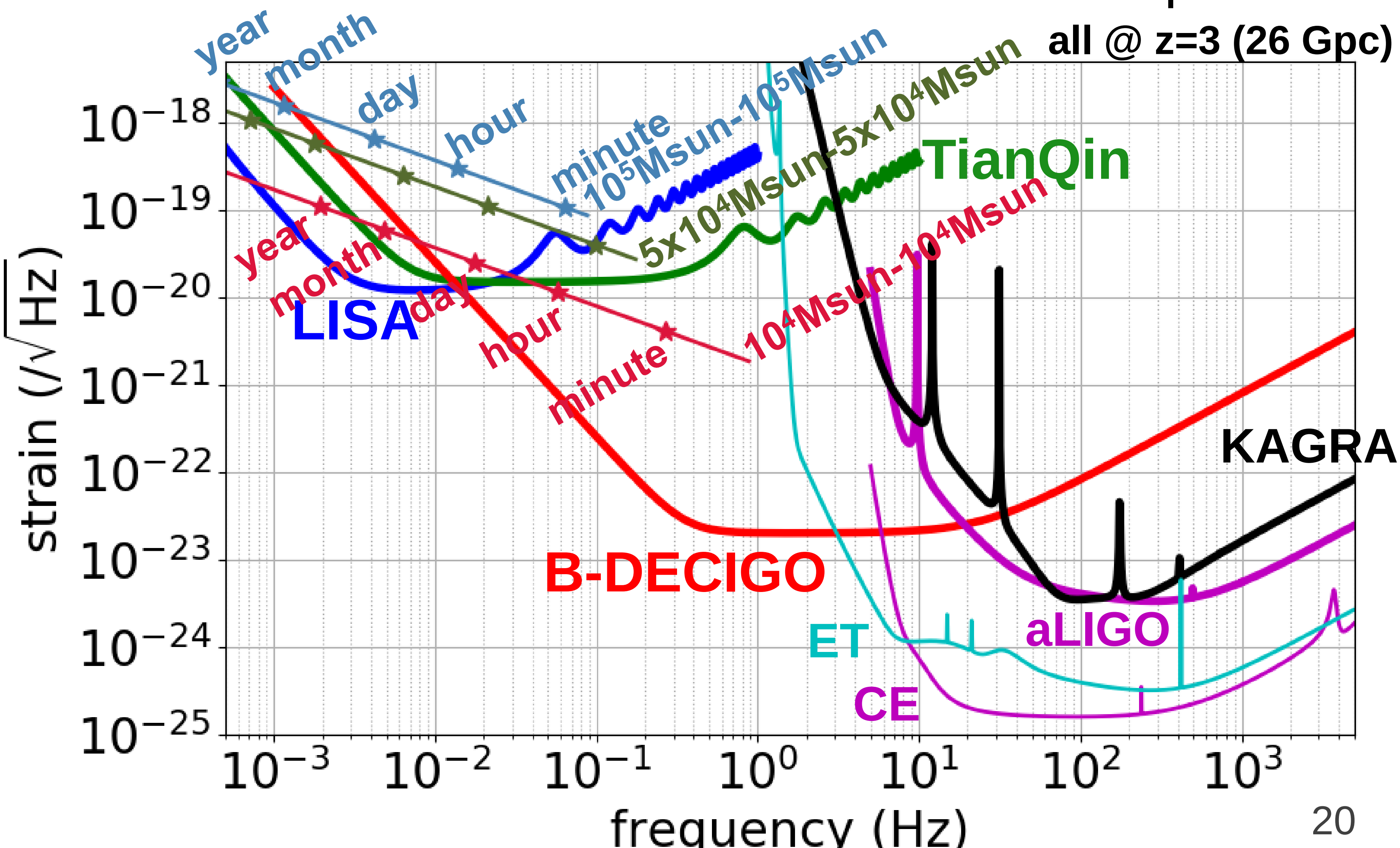
Time to Merger and Detector Band

- Different observation band sees different phases



Time to Merger and Detector Band

- Different observation band sees different phases



Detector Design Example 1

- What will be the detector design if you want to detect 10^5Msun-10^5Msun merger at the highest signal to noise ratio?
- Assume 100 km arms with 30 kg mirrors

Detector Design Example 1

- What will be the detector design if you want to detect 10^5Msun-10^5Msun merger at the highest signal to noise ratio?
- Assume 100 km arms with 30 kg mirrors

These give you SQL

$$h_{\mathrm{SQL}} = \sqrt{\frac{8\hbar}{m\omega^2 L^2}}$$

Let's simplify and consider detector frame mass
Also, consider SNR at ISCO frequency

$$f_{\mathrm{ISCO}} = \frac{c^3}{6^{3/2}\pi G M_{\mathrm{tot}}} = 0.02\,\mathrm{Hz}$$

Detector Design Example 1

- What will be the detector design if you want to detect 10^5Msun-10^5Msun merger at the highest signal to noise ratio?
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These give you SQL

$$h_{\rm SQL} = \sqrt{\frac{8\hbar}{m\omega^2 L^2}}$$

Let's simplify and consider detector frame mass

Also, consider SNR at ISCO frequency

$$f_{\rm ISCO} = \frac{c^3}{6^{3/2}\pi G M_{\rm tot}} = 0.02\,{\rm Hz}$$

FP case

We want to reach SQL at ISCO frequency

$$f_{\rm SQL} = \frac{1}{2\pi}\sqrt{\frac{16\pi P_0}{mc\lambda}\frac{2\mathcal{F}}{\pi}} = f_{\rm ISCO} \rightarrow P_0\mathcal{F}^2 = 9\,{\rm W}$$

(1064 nm)

Detector Design Example 1

- What will be the detector design if you want to detect 10^5Msun-10^5Msun merger at the highest signal to noise ratio?
- Assume 100 km arms with 30 kg mirrors

These give you SQL

$$h_{\rm SQL} = \sqrt{\frac{4\hbar}{m\omega^2 L^2}}$$

Let's simplify and consider detector frame mass

Also, consider SNR at ISCO frequency

$$f_{\rm ISCO} = \frac{c^3}{6^{3/2}\pi G M_{\rm tot}} = 0.02\,{\rm Hz}$$

Michelson case

We want to reach SQL at ISCO frequency

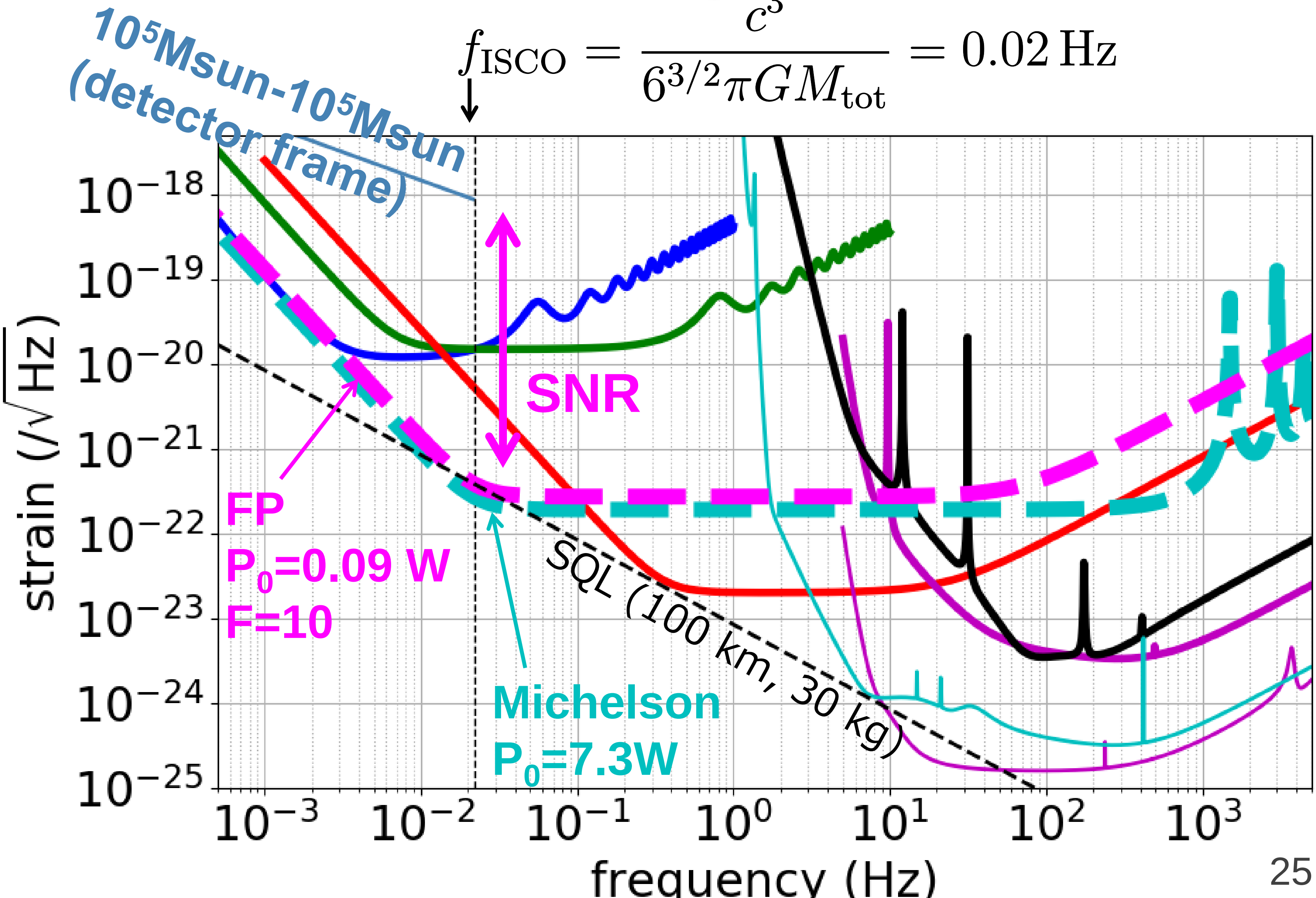
* Infinite mass BS assumed

$$f_{\rm SQL} = \frac{1}{2\pi}\sqrt{\frac{8\pi P_0}{mc\lambda}} = f_{\rm ISCO}$$

$\rightarrow P_0 = 7.3\,{\rm W}$ (1064 nm)

Detector Design Example 1

$$f_{\rm ISCO} = \frac{c^3}{6^{3/2}\pi G M_{\rm tot}} = 0.02\,{\rm Hz}$$



Detector Design Example 1

- What will be the detector design if you want to detect 10^5Msun-10^5Msun merger at the highest signal to noise ratio?
- Assume 100 km arms with 30 kg mirrors

Actually, force noise requirement will be quite severe with this configuration

$$h_{\mathrm{f}} = \frac{n_f}{m\omega^2 L} < 1 \times 10^{-25} \,/\sqrt{\mathrm{Hz}}(1\,\mathrm{Hz}/f)^2$$

$$\longrightarrow n_f < 1 \times 10^{-17}\,\mathrm{N}/\sqrt{\mathrm{Hz}}$$

To relax this requirement, it is better to increase the arm length at the cost of reducing the power

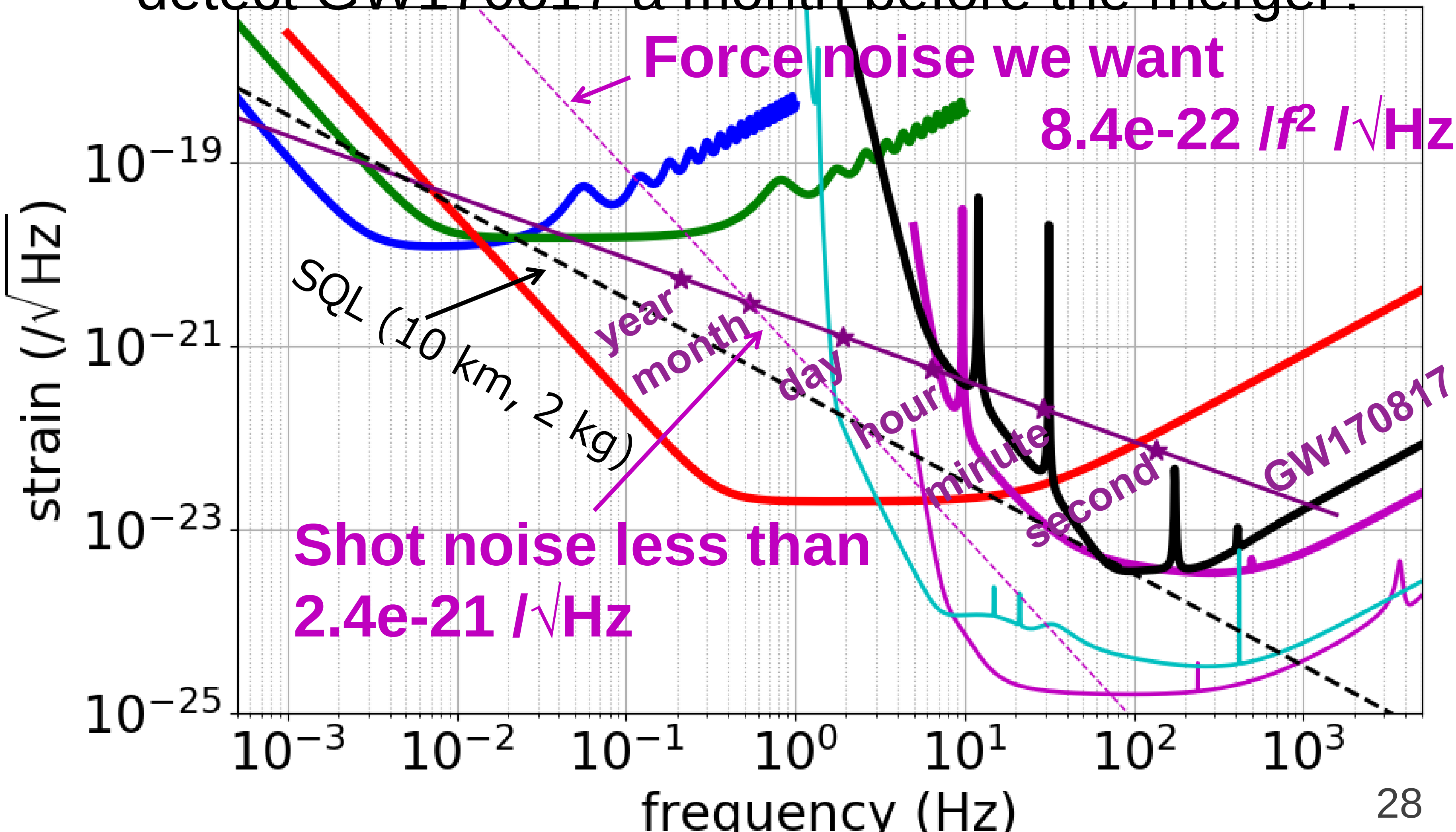
$\longrightarrow$ LISA and TianQin design

Detector Design Example 2

- What will be the detector design if you want to detect GW170817 a month before the merger?
- Assume 10 km arms with 2 kg mirrors

Detector Design Example 2

- What will be the detector design if you want to detect GW170817 a month before the merger?



Detector Design Example 2

- What will be the detector design if you want to detect GW170817 a month before the merger?

The force noise requirement will be

$$h_{\mathrm{f}} = \frac{n_f}{m\omega^2 L} < \frac{8.4 \times 10^{-22} \, /\sqrt{\mathrm{Hz}}}{(f/1\,\mathrm{Hz})^2}$$

$$\longrightarrow n_f < 6.6 \times 10^{-16}\,\mathrm{N}/\sqrt{\mathrm{Hz}}$$

With Michelson configuration, laser power requirement will be

$$h_{\mathrm{shot}} = \frac{1}{2\pi L}\sqrt{\frac{hc\lambda}{2P_0}} < 2.4 \times 10^{-21} \, /\sqrt{\mathrm{Hz}}$$

$$\longrightarrow P_0 > 6\,\mathrm{W}$$

Even larger power required for more SNR

Detector Design Example 2

- What will be the detector design if you want to detect GW170817 a month before the merger?

If FP configuration, power requirement will be relaxed

$$h_{\mathrm{rad}} = \frac{2}{m\omega^2 L}\sqrt{\frac{8hP_0}{c\lambda}\frac{2\mathcal{F}}{\pi}} < \frac{8.4\times 10^{-22}\,/\sqrt{\mathrm{Hz}}}{(f/1\,\mathrm{Hz})^2}$$

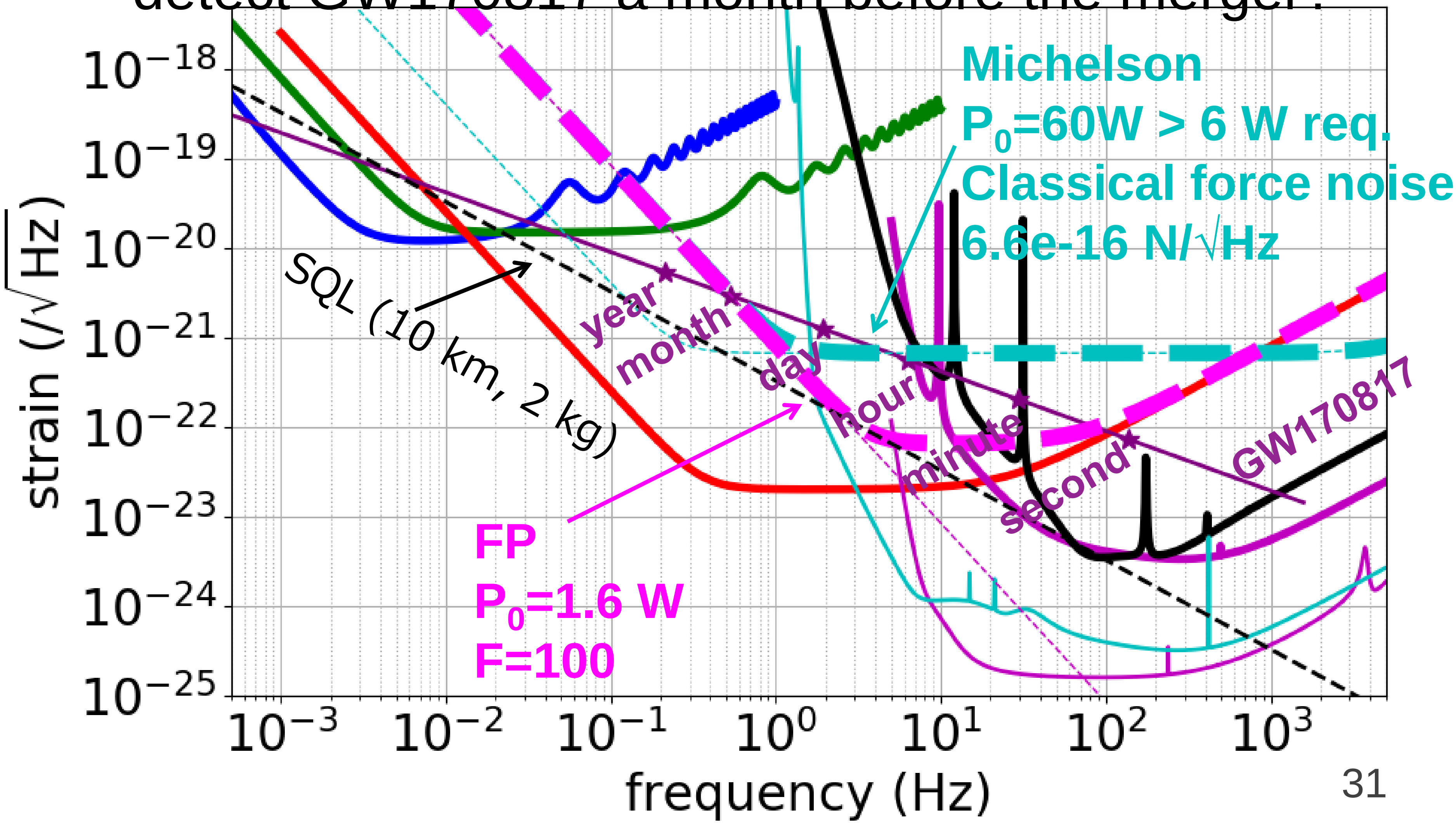
$$\longrightarrow P_0\mathcal{F}^2 < 1.6\times 10^4\,\mathrm{W} \qquad \omega \ll 1/\tau$$

Input power of 1.6 W
Finesse of 100 will do

Force noise requirement stays the same

Detector Design Example 2

- What will be the detector design if you want to detect GW170817 a month before the merger?

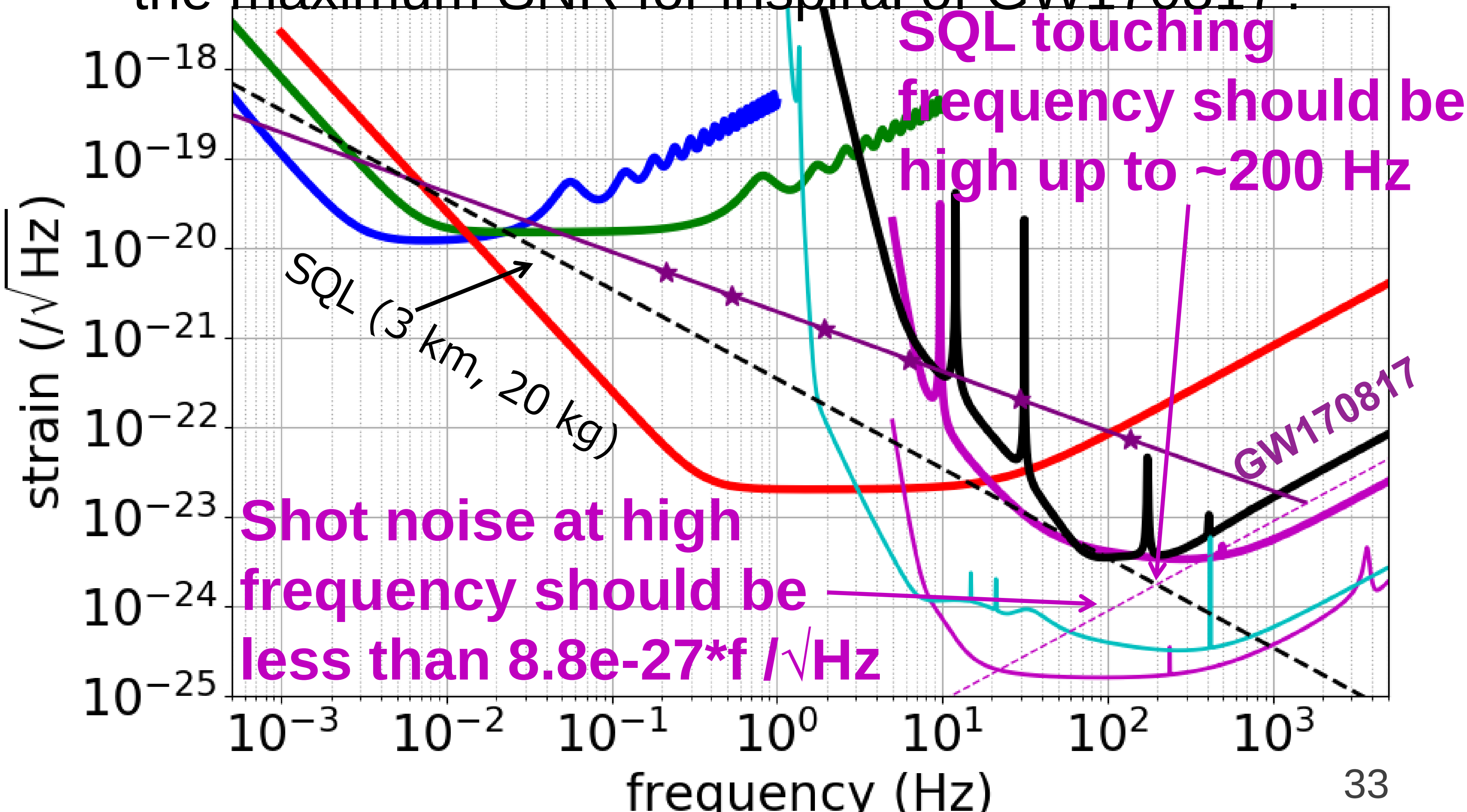


Detector Design Example 3

- What will be the detector design if you want to have the maximum SNR for inspiral of GW170817?
- Assume 3 km arms and 20 kg mirrors

Detector Design Example 3

- What will be the detector design if you want to have the maximum SNR for inspiral of GW170817?



Detector Design Example 3

- What will be the detector design if you want to have the maximum SNR for inspiral of GW170817?

If FP configuration, shot noise at high frequencies will be

$$h_{\rm shot}^{\rm HF} = \frac{1}{2\pi L}\sqrt{\frac{hc\lambda}{2P_0}\frac{\pi}{2\mathcal{F}}}\omega\tau \qquad \tau \equiv \frac{2L\mathcal{F}}{\pi c}$$

$$< 8.8 \times 10^{-27} /\sqrt{\rm Hz}(f/1\,{\rm Hz})$$

$$\longrightarrow P_0 > 1.5 \times 10^4\,{\rm W}$$ (Independent of L)

With this power and SQL touching frequency

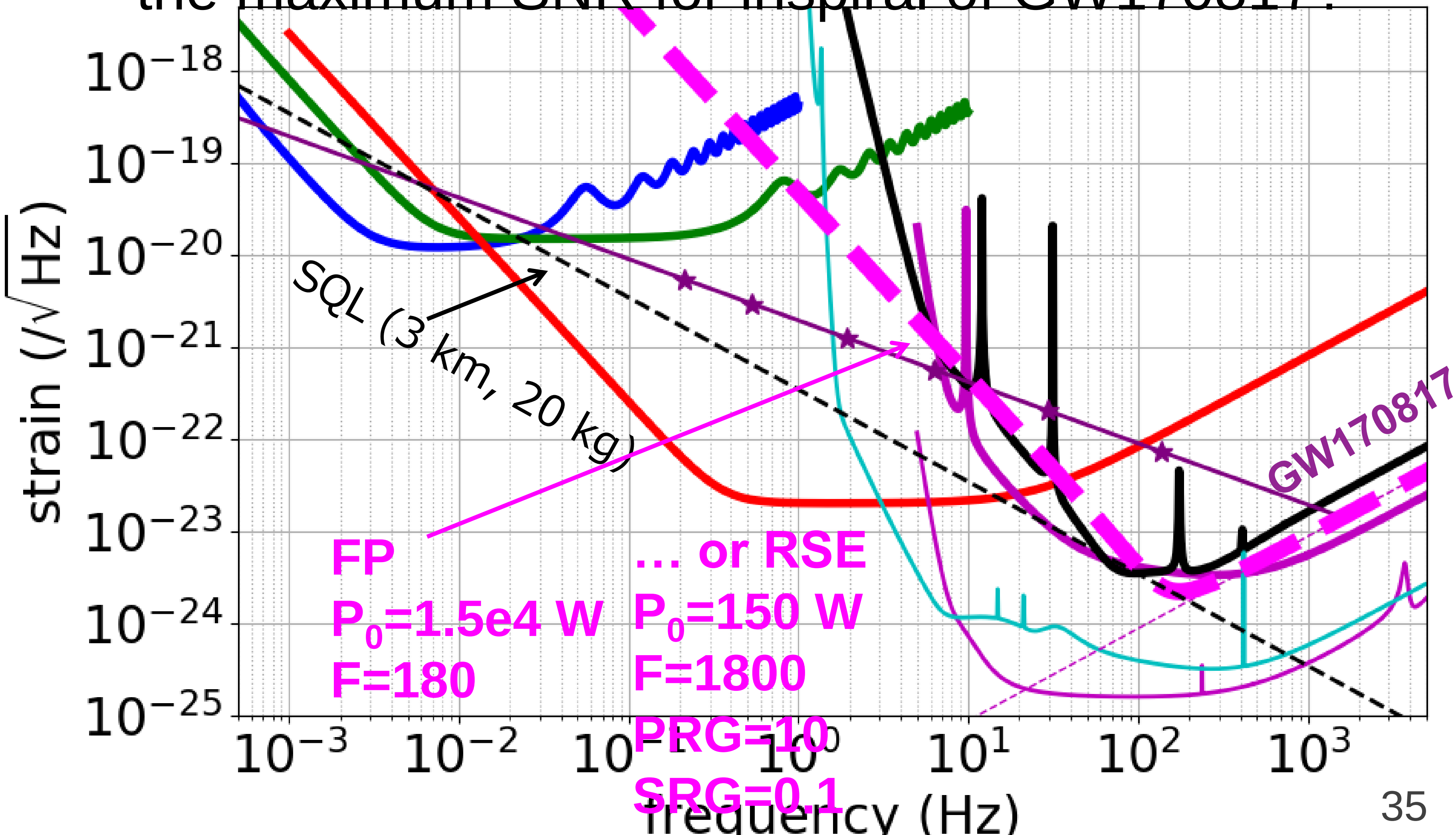
$$f_{\rm SQL} = \frac{1}{2\pi}\sqrt{\frac{16\pi P_0}{mc\lambda}\frac{2\mathcal{F}}{\pi}} = 200\,{\rm Hz}$$

$$\longrightarrow \mathcal{F} = 180$$

Not possible with Michelson

Detector Design Example 3

- What will be the detector design if you want to have the maximum SNR for inspiral of GW170817?

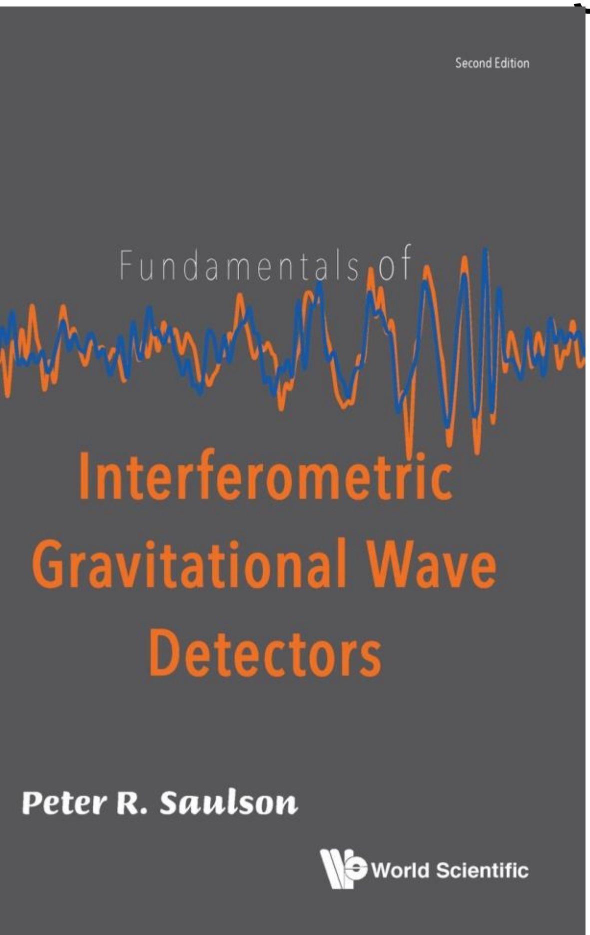


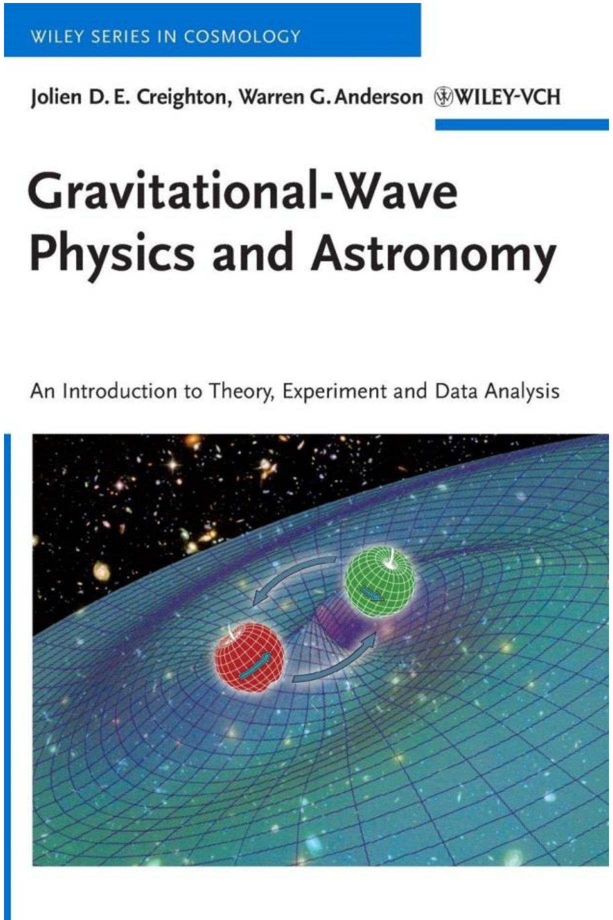
Summary

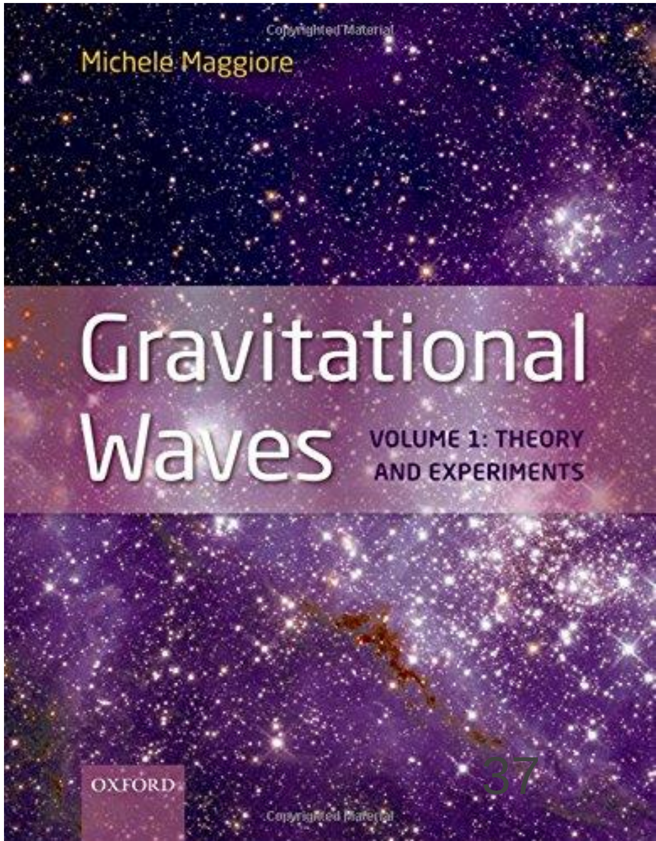
- Apart from quantum noise, classical force noise at low frequency have to be considered to design a gravitational wave detector
- Force noise requirement for Fabry-Pérot interferometers is severer compared with optical transponder due to shorter arm
- Different interferometer design is required for different observation bands
- Low frequency detectors can see heavier binary mergers and early phases of lighter binary inspiral signals

Books

- Peter R. Saulson
- Jolien D. E. Creighton, Warren G. Anderson
- Michele Maggoire







Further References

- E. D. Black & R. N. Gutenkunst, American Journal of Physics 71, 365 (2003)
An introduction to signal extraction in interferometric gravitational wave detectors
- A. Buonanno and Y. Chen, Phys. Rev. D 64, 042006 (2001)
Quantum noise in second generation, signal-recycled laser interferometric gravitational-wave detectors
- T. Nakamura+, Progress of Theoretical and Experimental Physics 2016, 093E01 (2016)
Pre-DECIGO can get the smoking gun to decide the astrophysical or cosmological origin of GW150914-like binary black holes
- Y. Michimura+, Phys. Rev. D 97, 122003 (2018)
Particle swarm optimization of the sensitivity of a cryogenic gravitational wave detector

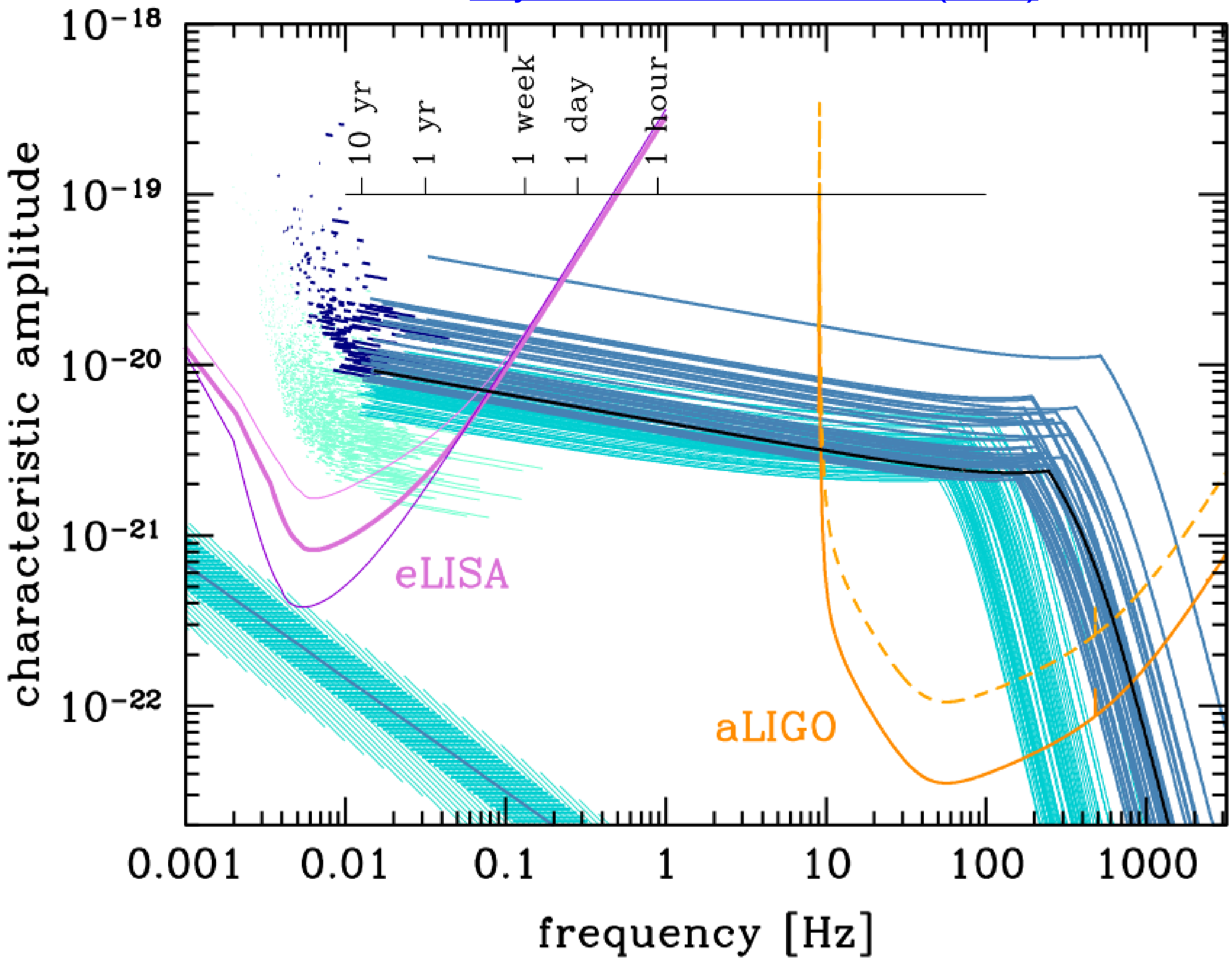
Slides Available Online

1. Laser Interferometers (July 25 PM)
   https://tinyurl.com/YM20190725-1
2. Quantum Noise (July 25 PM)
   https://tinyurl.com/YM20190725-2
3. Sensitivity Design (July 26 AM)
   https://tinyurl.com/YM20190725-3
4. Status of KAGRA (July 26 AM)
   https://tinyurl.com/YM20190725-4

Additional Slides

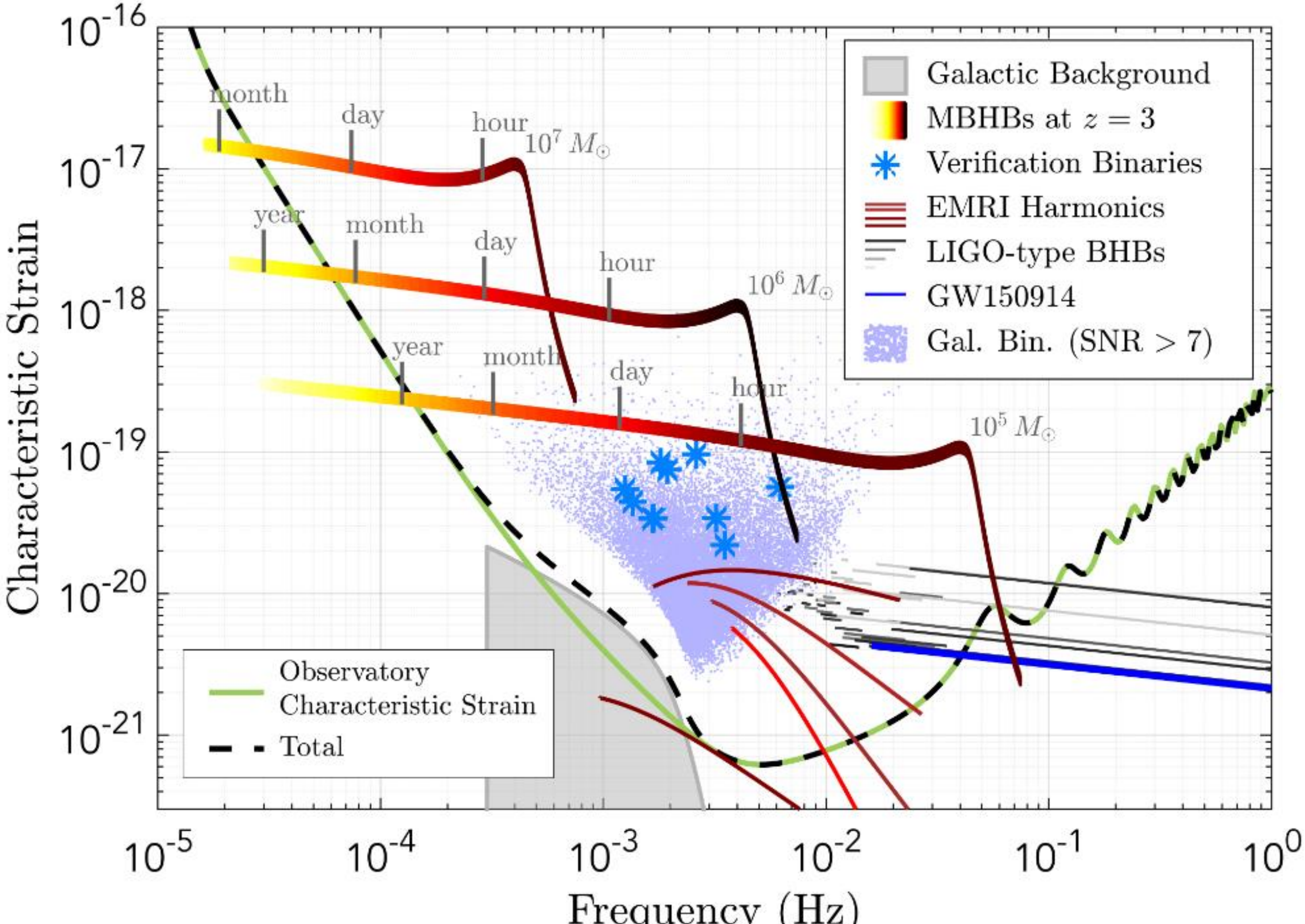
Multiband GW Observation

A. Sesana, [Phys. Rev. Lett. 116, 231102 (2016)]



Multiband GW Observation

LISA L3 proposal
https://www.elisascience.org/files/publications/LISA_L3_20170120.pdf



Mirror Mass and Arm Length (FP)

- Force noise requirement

$$h_{\rm f} = \frac{f}{m\omega^2 L} = \frac{1 \times 10^{-16}\,{\rm N}/\sqrt{\rm Hz}}{90\,{\rm kg}\cdot{\rm km}\,\omega^2}$$

- Radiation pressure noise

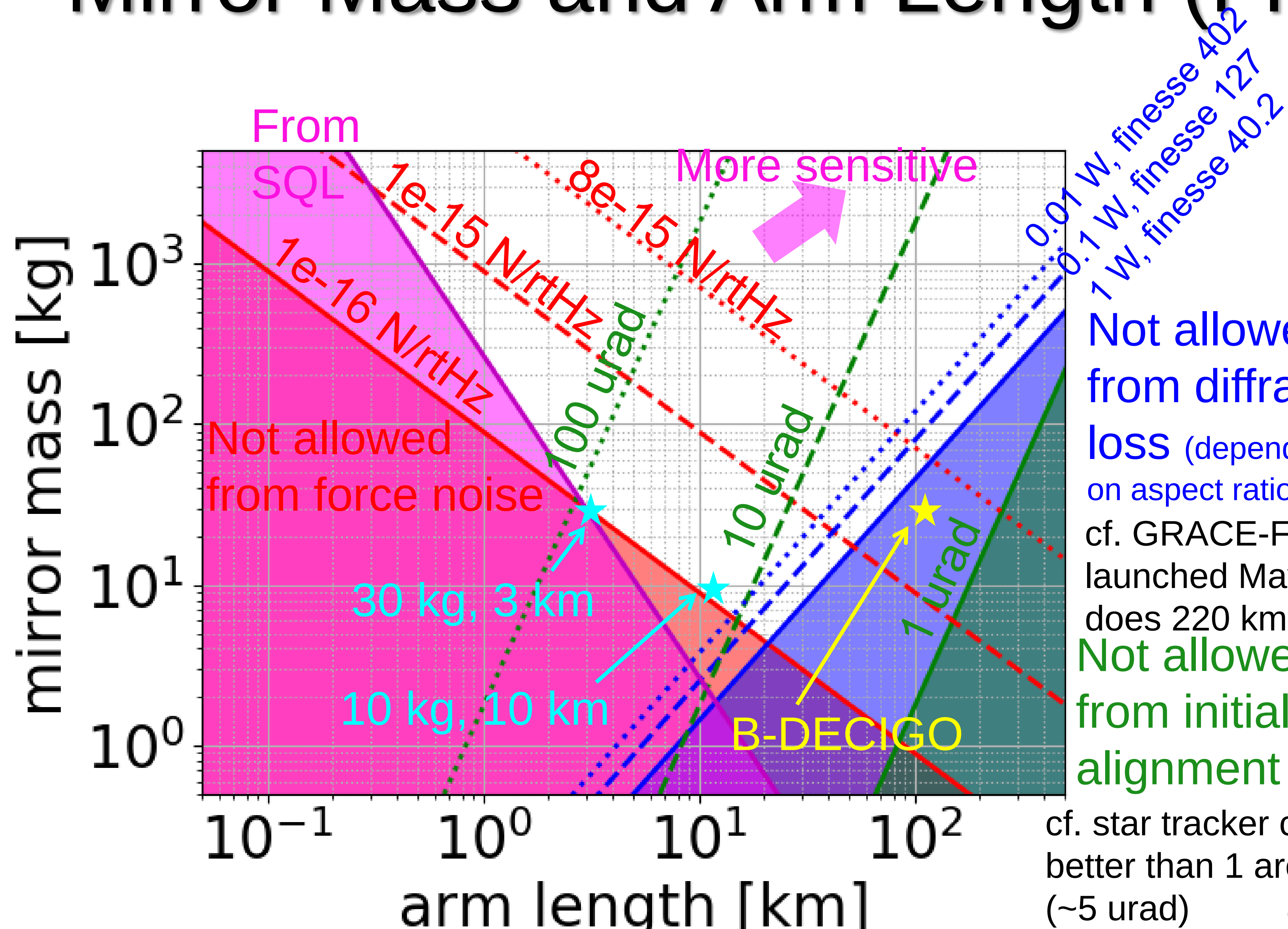
$$h_{\rm rp} = \frac{1}{m\omega^2 L}\frac{4\mathcal{F}}{\pi}\sqrt{\frac{16\pi\hbar P}{c\lambda}} = k_{\rm safe} h_{\rm f}$$

Say, this is 3

There's no point in reducing the finesse and input power if force noise is larger, in terms of sensitivity.

- If you fix requirement for f, requirement for mL is set
- If you fix P, finesse $\mathcal{F}$ is set
- Assuming g-factor g=0.3 and L, beam size is calculated
- This gives you the minimum mirror mass from diffraction loss (assume fused silica, aspect ratio t/d = 1)
- Also, if you fix initial alignment accuracy, minimum mirror diameter d is determined from d/L

Mirror Mass and Arm Length (FP)



Not allowed from diffraction loss (depends much on aspect ratio)

cf. GRACE-FO launched May 2018 does 220 km FF

Not allowed from initial alignment

cf. star tracker can do better than 1 arcsec (~5 urad)