TianQin Summer School 2019 @ Sun Yat-sen University

July 25, 2019

Laser Interferometry for Gravitational Wave Observations

2. Quantum Noise

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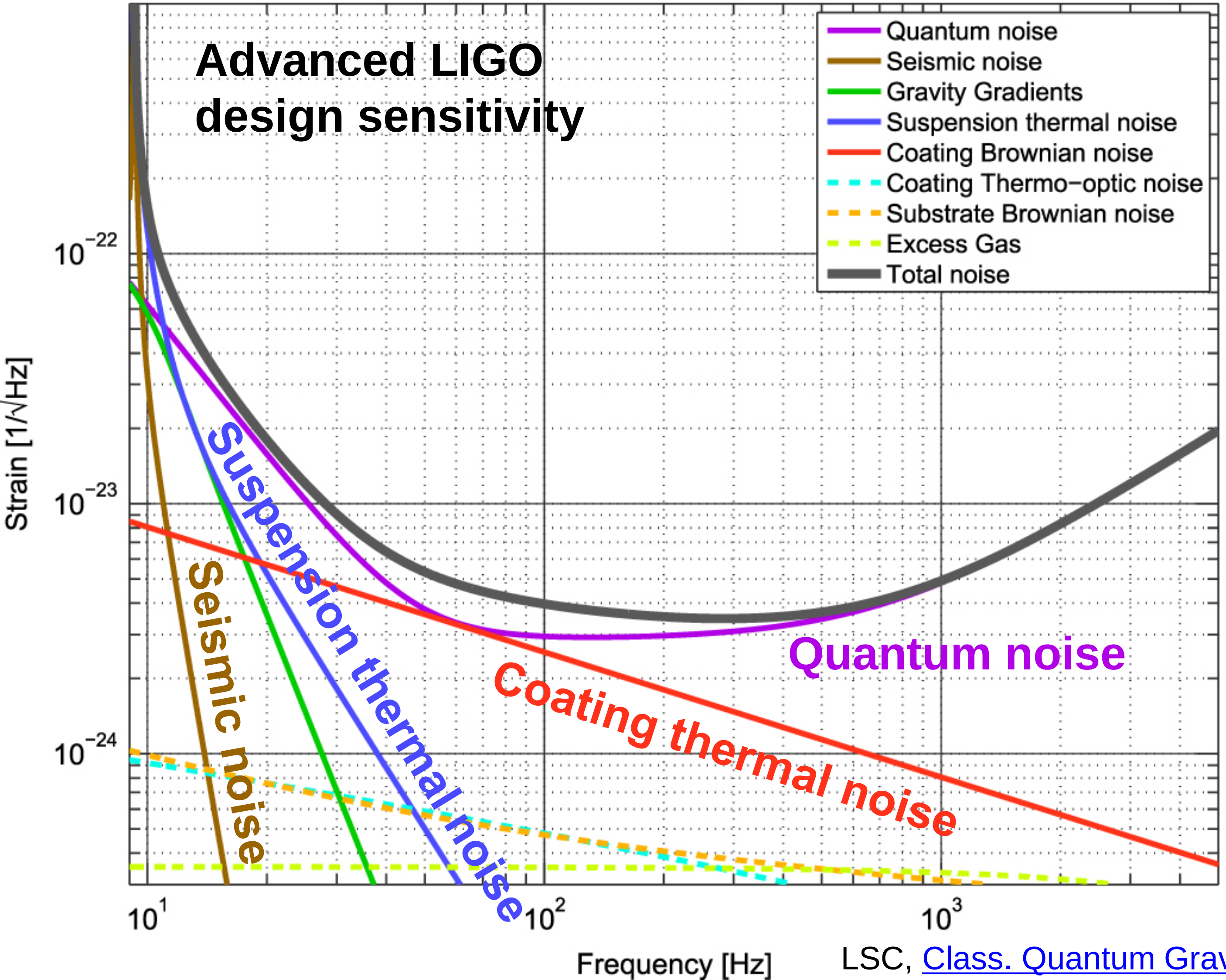
Strain Sensitivity of aLIGO

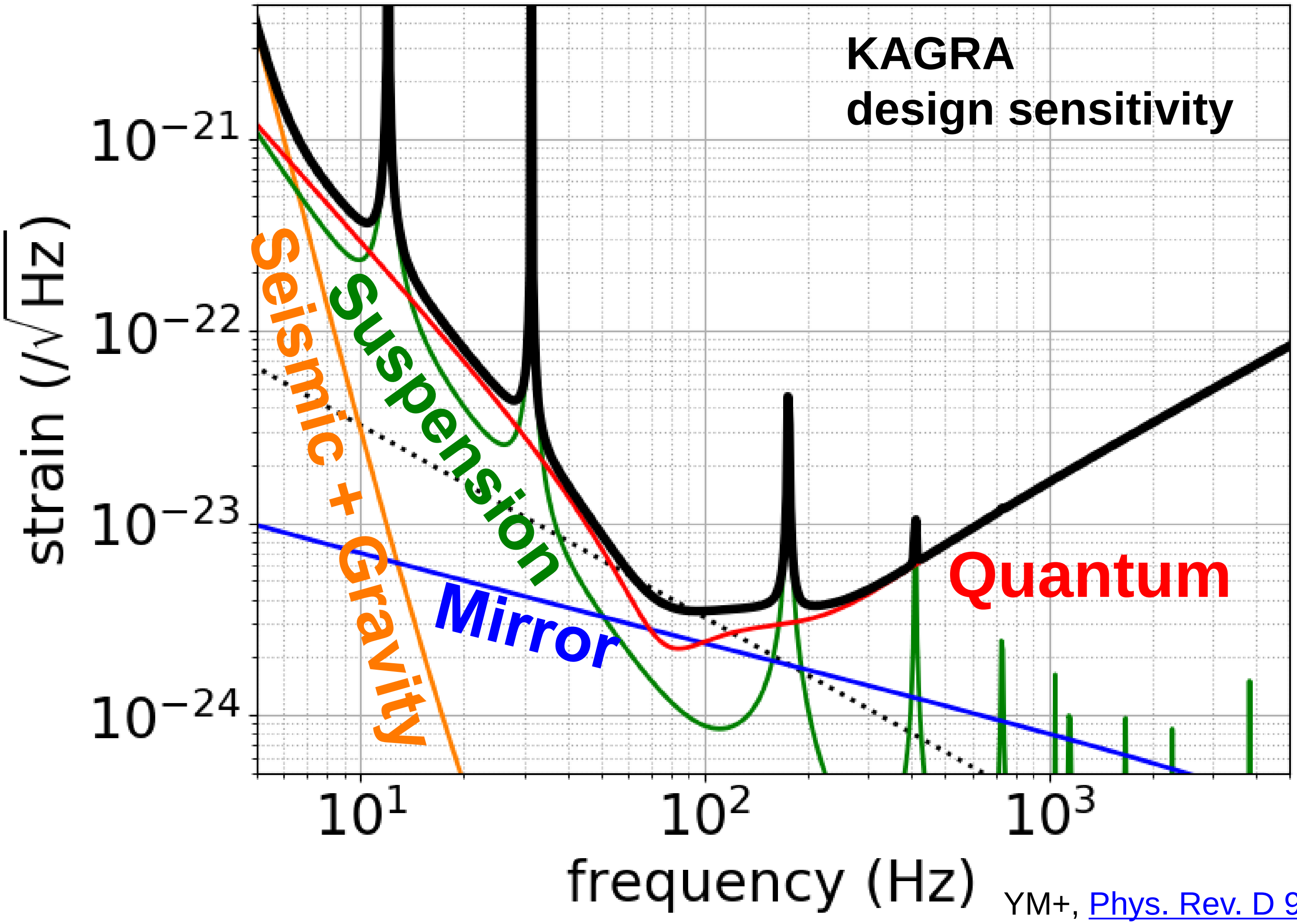
- Mostly limited by quantum noise



LSC, Class. Quantum Grav. 32, 074001 (2015)

Strain Sensitivity of KAGRA

- Mostly limited by quantum noise



YM+, Phys. Rev. D 97, 122003 (2018)

Quantum Noise

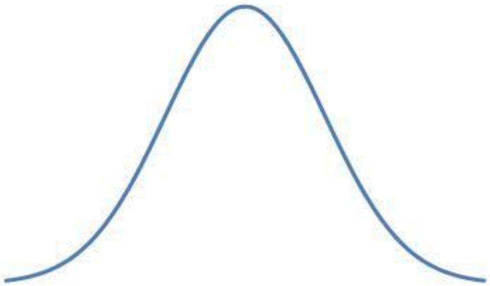
- Originated from quantum fluctuation of light
  - Shot noise
    - Fluctuation of number of photons to photodiode
  - Radiation pressure noise
    - Fluctuation of number of photons to mirror

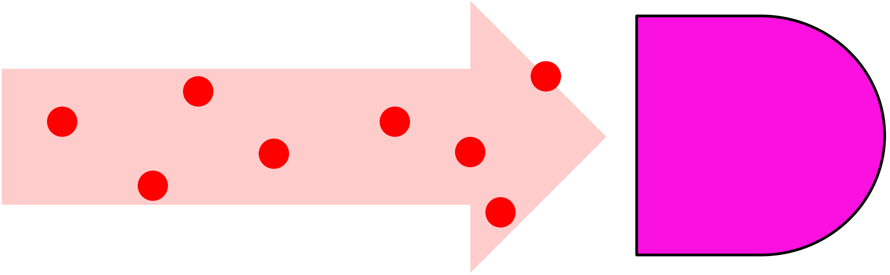
- Let’s calculate quantum noise limited sensitivity of gravitational wave detectors!

Shot Noise

- Number of photons to photodiodes fluctuates

$$N \pm \sqrt{N}$$





Photon energy

$$p_1 = \frac{hc}{\lambda}$$

- Quantum fluctuation of power

$$\delta P_{\mathrm{shot}} = \sqrt{\frac{2hcP_{\mathrm{PD}}}{\eta\lambda}}$$

Shot noise spectrum

Quantum efficiency

Number of photons

$$N = \frac{P_{\mathrm{PD}}}{p_1}$$

Shot Noise Limit of Michelson

- Power change $$\frac{\partial P_{\mathrm{PD}}}{\partial L_-} = \frac{2\pi P_0}{\lambda} \sin \frac{4\pi L_-}{\lambda}$$
- Shot noise

(recall Lecture 1)

$$\delta P_{\mathrm{shot}} = \sqrt{\frac{2hcP_{\mathrm{PD}}}{\eta\lambda}} = \sqrt{\frac{hcP_0}{\eta\lambda}\left(1 - \cos\frac{4\pi L_-}{\lambda}\right)}$$

- Shot noise limited sensitivity

$$\delta L_{\mathrm{shot}} = \delta P_{\mathrm{shot}} \left(\frac{\partial P_{\mathrm{PD}}}{\partial L_-}\right)^{-1} \to \frac{1}{2\pi}\sqrt{\frac{hc\lambda}{2\eta P_0}}$$

$$\frac{\sqrt{1-\cos\phi}}{\sin\phi} = \frac{\sqrt{2\sin^2\frac{\phi}{2}}}{2\sin\frac{\phi}{2}\cos\frac{\phi}{2}} = \frac{1}{\sqrt{2}\cos\frac{\phi}{2}}$$

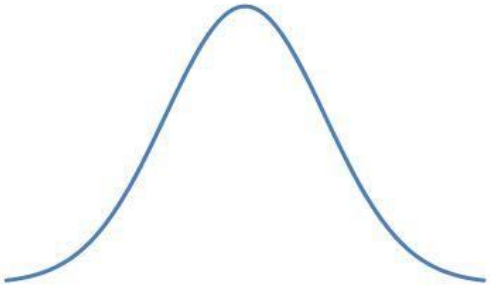
Better shot noise with higher input power

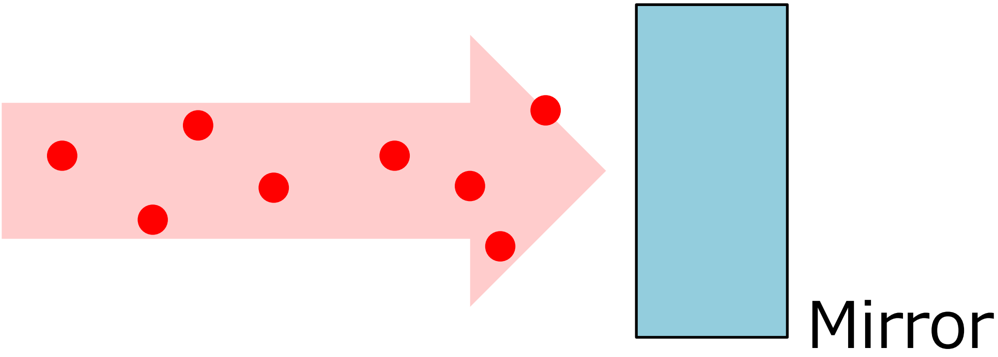
Best at dark fringe (where $P_{\mathrm{PD}}=0$)

Radiation Pressure Noise

- Number of photons to mirror fluctuates

$$N \pm \sqrt{N}$$





- Power fluctuation

$$\delta P_{\mathrm{rad}} = \sqrt{\frac{hcP_0}{\lambda}}$$

Assumed Michelson with input power P_0

- Mirror displacement

Force on mirror

Force to displacement

free-falling mirror

$$\delta L_{\mathrm{rad}} = \frac{2\delta P_{\mathrm{rad}}}{c} H(\omega) = \sqrt{\frac{4hP_0}{c\lambda}} \frac{1}{m\omega^2}$$

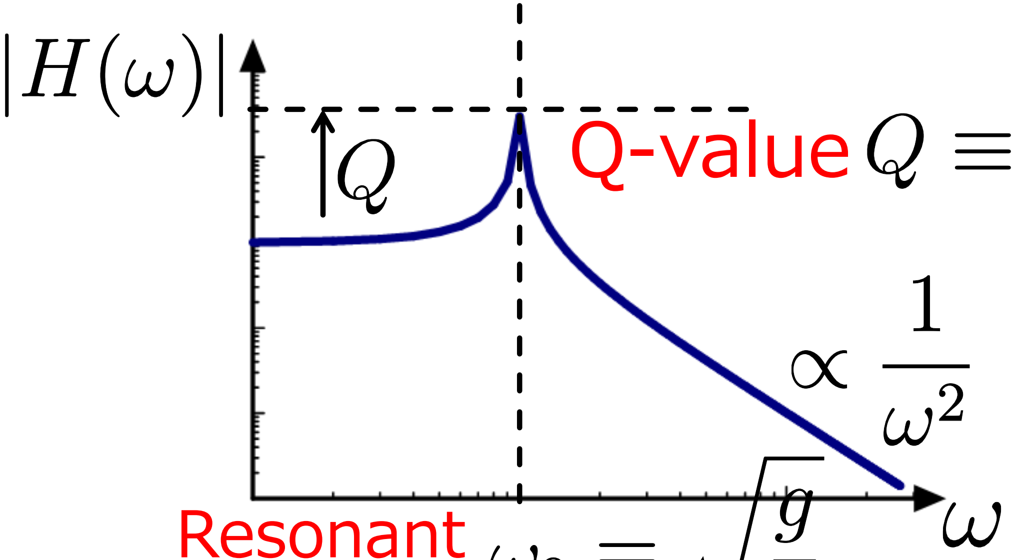
Susceptibility

- Equation of motion of a suspended mirror

$$m\ddot{x} = -\gamma'\dot{x} - \frac{mg}{l}x + f$$

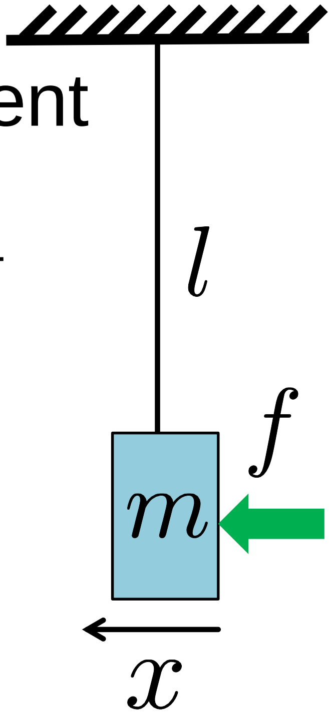
- Transfer function from force to displacement

$$H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{m}\frac{1}{-\omega^2 + \omega_0^2 + i\frac{\omega\omega_0}{Q}}$$



Q-value $Q \equiv \frac{\omega_0 m}{\gamma'}$

Resonant frequency $\omega_0 \equiv \sqrt{\frac{g}{l}}$



Standard Quantum Limit

- Shot noise is lower with higher power

$$h_{\rm shot} = \frac{1}{2\pi L}\sqrt{\frac{hc\lambda}{2P_0}}$$

Trade-off

- Radiation pressure noise is lower with lower power

$$h_{\rm rad} = \frac{1}{m\omega^2 L}\sqrt{\frac{8hP_0}{c\lambda}}$$

√2 for two arms
m is mirror mass
 (m/2 is reduced mass)
Assuming BS with infinite mass

- Standard Quantum Limit (SQL) for Michelson

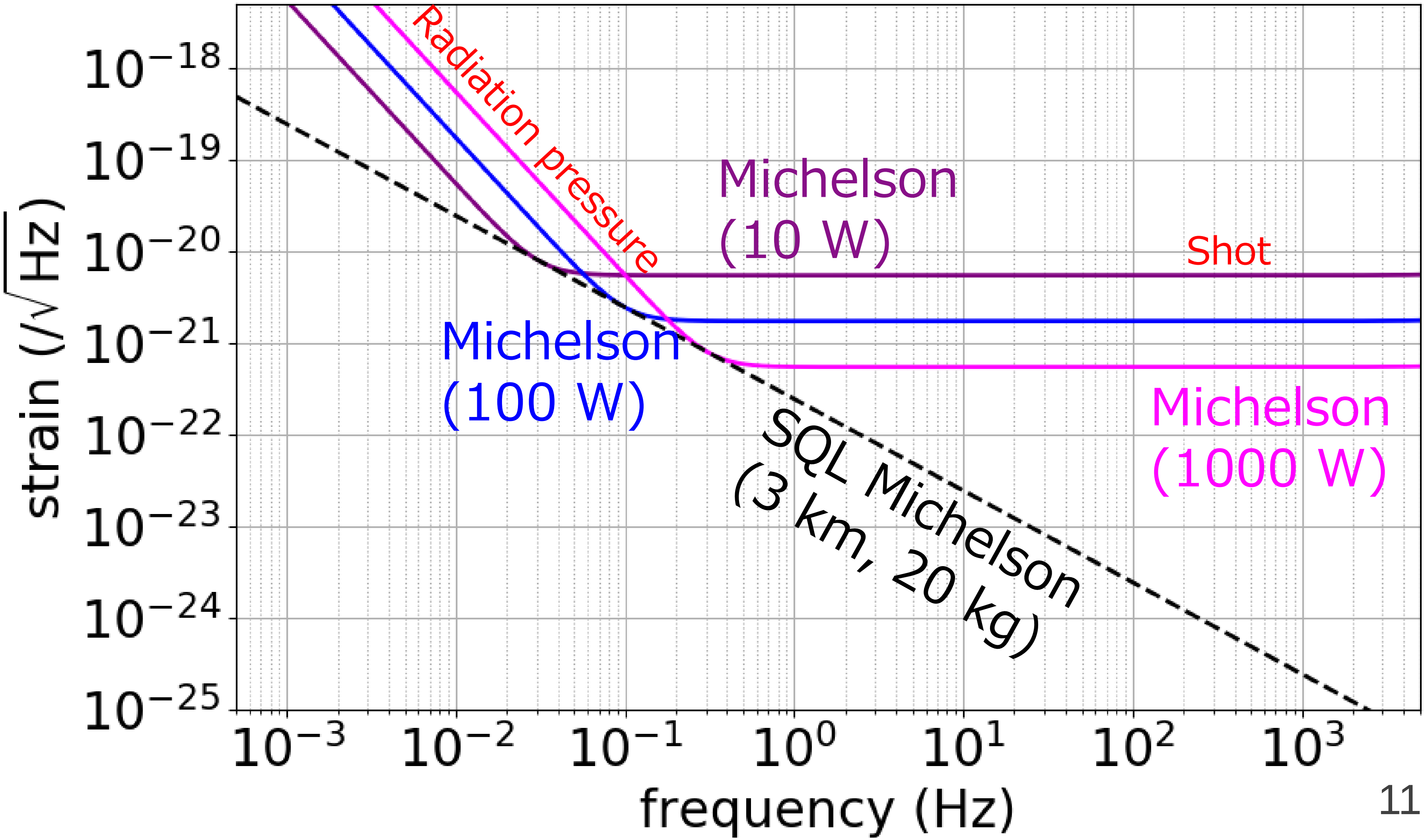
from Uncertainty principle

$\hbar \equiv h/(2\pi)$

$$h_{\rm SQL} = \sqrt{2h_{\rm shot}h_{\rm rad}} = \sqrt{\frac{4\hbar}{m\omega^2 L^2}}$$

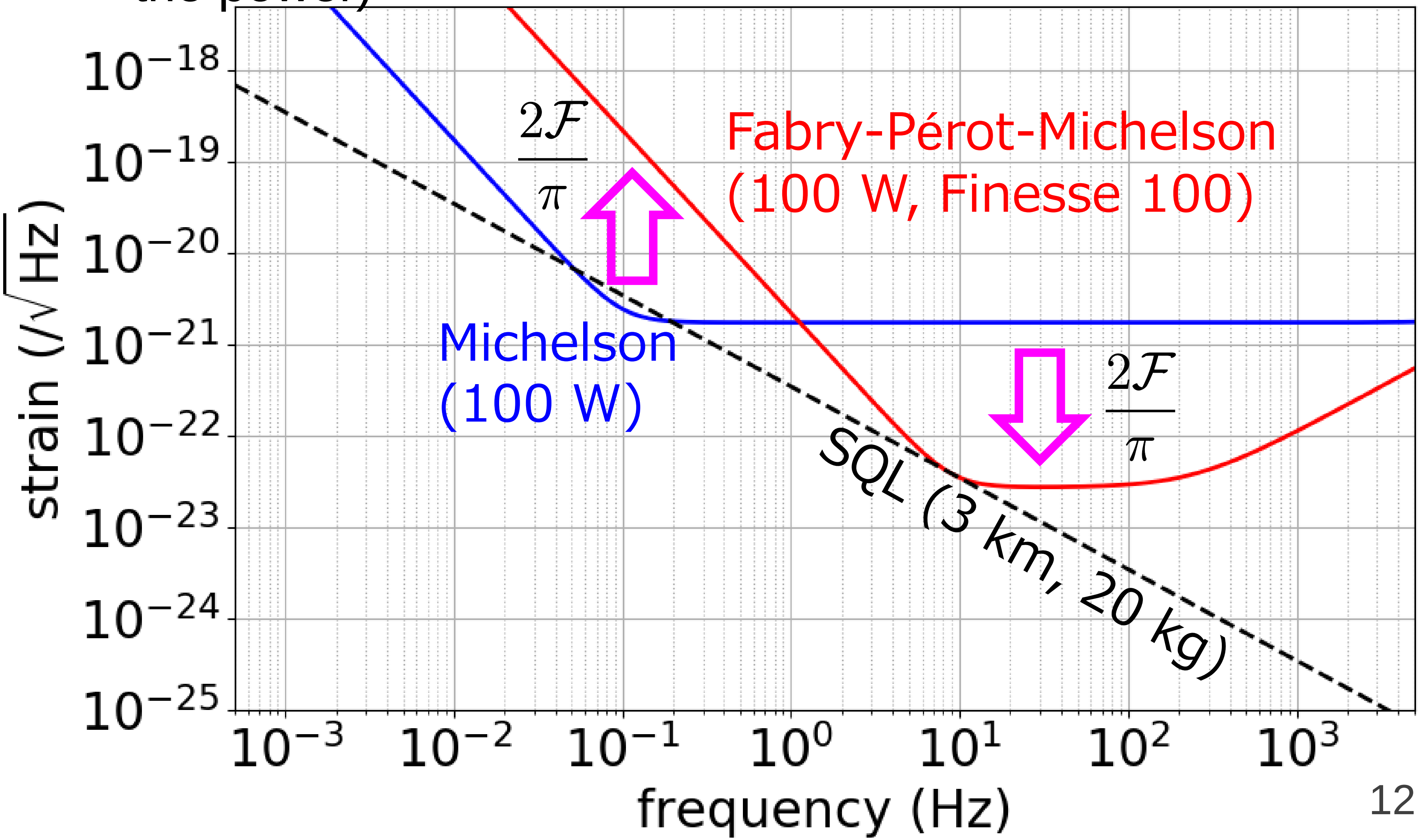
Input Power and Sensitivity

- SQL cannot be beaten by changing power



Use of Fabry-Pérot Cavities

- Still, SQL cannot be beaten (similar effect to increasing the power)

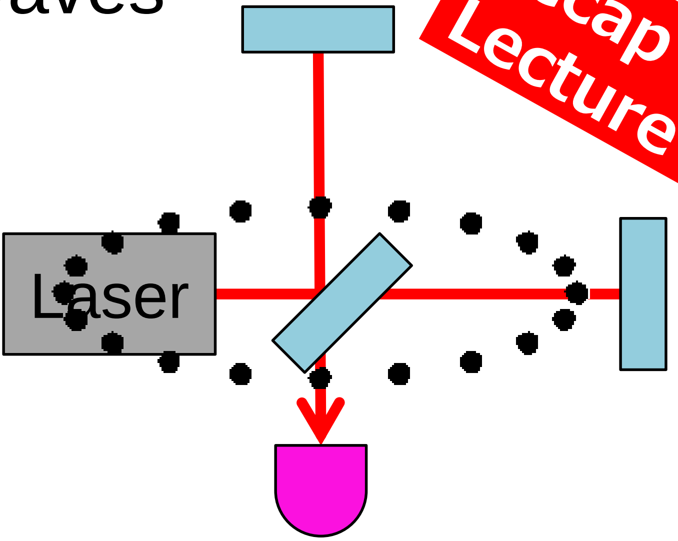


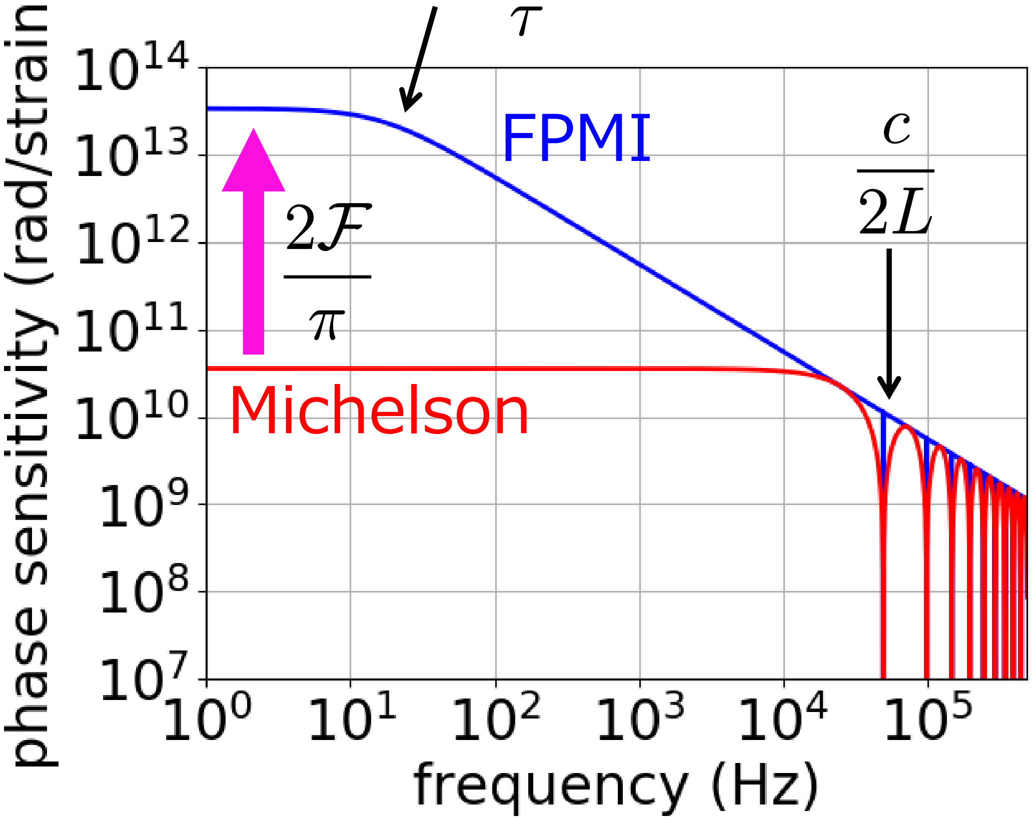
High-Frequency Response

- The effect of gravitational waves cancel at high frequencies

$$f_{\mathrm{c}} = \frac{1}{2\pi}\left(\underset{\tau}{\frac{2\mathcal{F}}{\pi}\frac{L}{c}}\right)^{-1} = \frac{c}{4L\mathcal{F}}$$

Recap of Lecture 1





For a given frequency, there is a limit where longer arm length and higher finesse won't help increasing the sensitivity

FPMI Quantum Noise

- Shot noise

$$h_{\mathrm{shot}} = \frac{1}{2\pi L}\sqrt{\frac{hc\lambda}{2P_0}}\frac{\pi}{2\mathcal{F}}\sqrt{1+(\omega\tau)^2} \quad \tau \equiv \frac{2L\mathcal{F}}{\pi c}$$

- Radiation pressure noise

$$h_{\mathrm{rad}} = \frac{2}{m\omega^2 L}\sqrt{\frac{8hP_0}{c\lambda}}\frac{2\mathcal{F}}{\pi}\frac{1}{\sqrt{1+(\omega\tau)^2}}$$

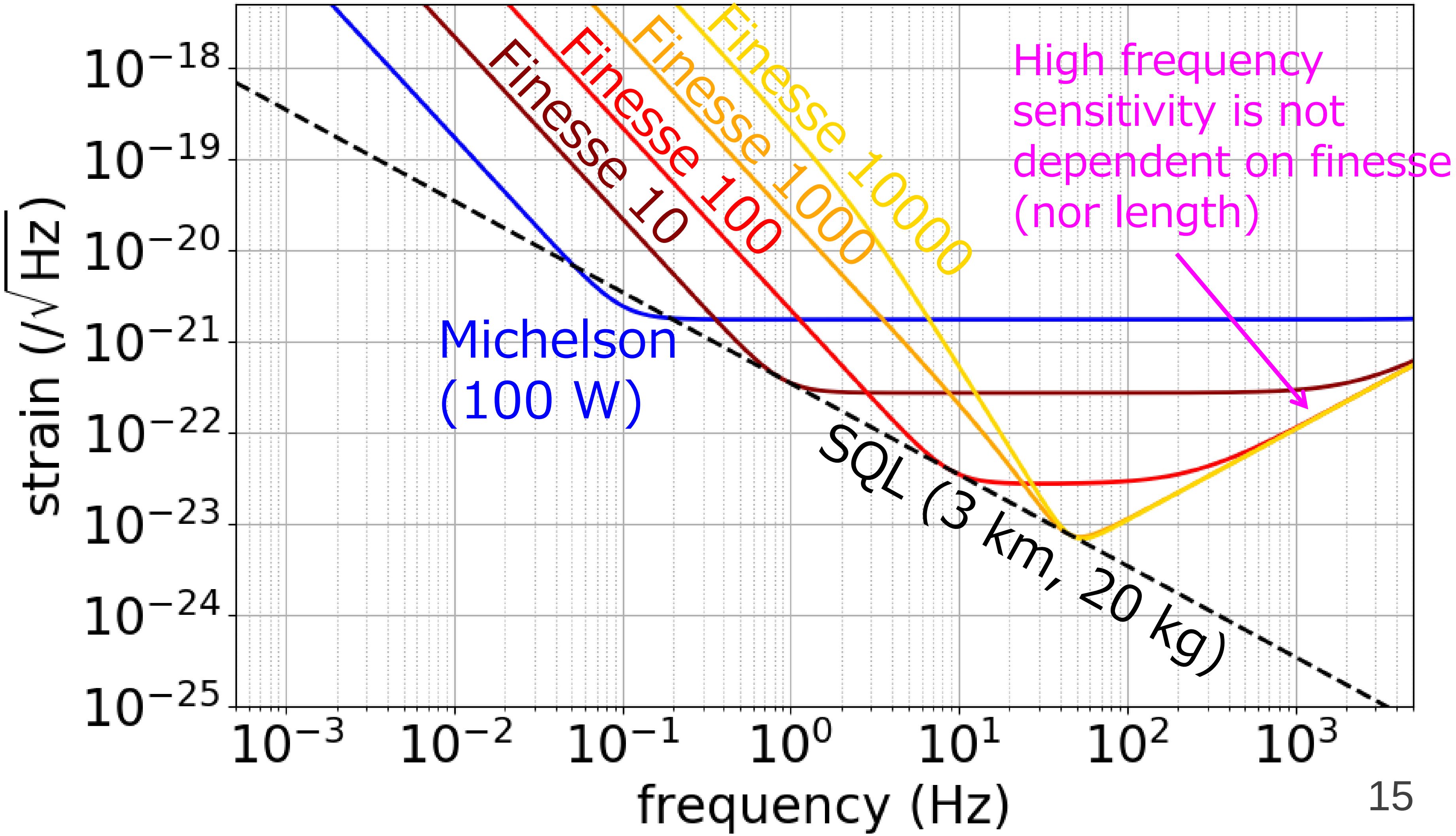
2 for two mirrors of a cavity
m is mirror mass (m/4 is reduced mass)

- Standard Quantum Limit (SQL) for FPMI

$$h_{\mathrm{SQL}} = \sqrt{2h_{\mathrm{shot}}h_{\mathrm{rad}}} = \sqrt{\frac{8\hbar}{m\omega^2 L^2}}$$

Finesse Dependence

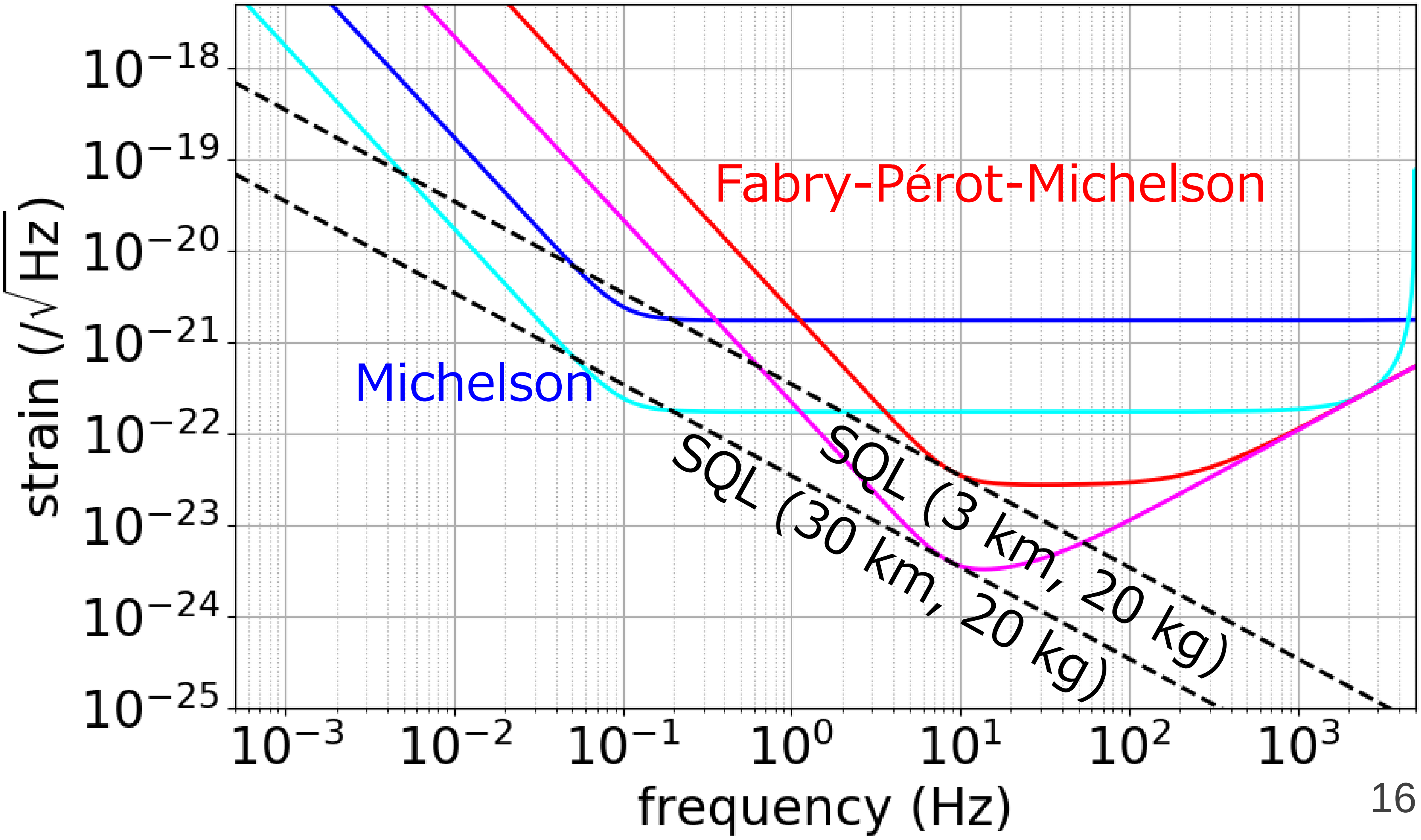
- Too high finesse narrows the detector bandwidth



High frequency sensitivity is not dependent on finesse (nor length)

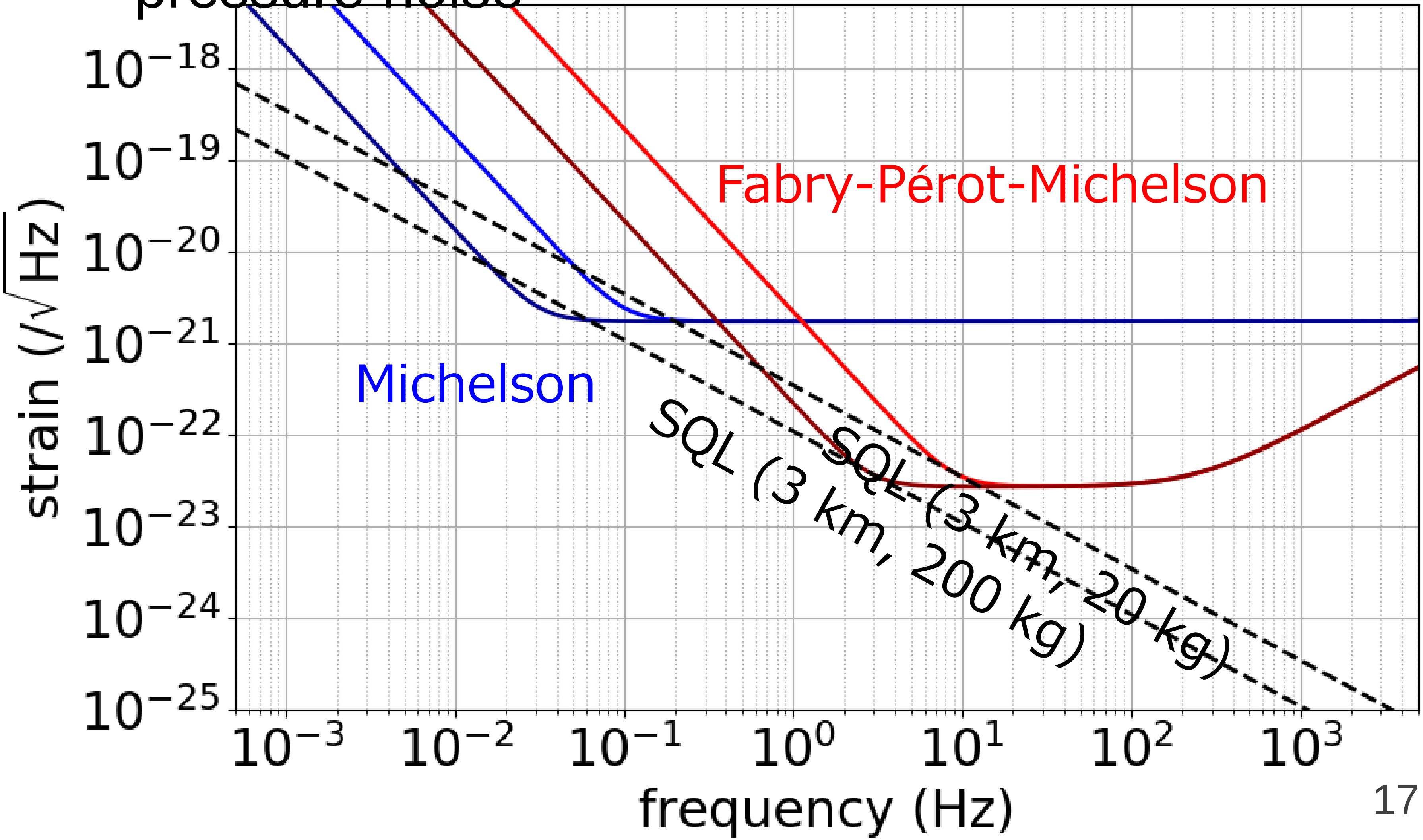
Arm Length Dependence

- Longer arm is better (but not at high frequencies)



Mirror Mass Dependence

- Heavier mass is better for reducing radiation pressure noise



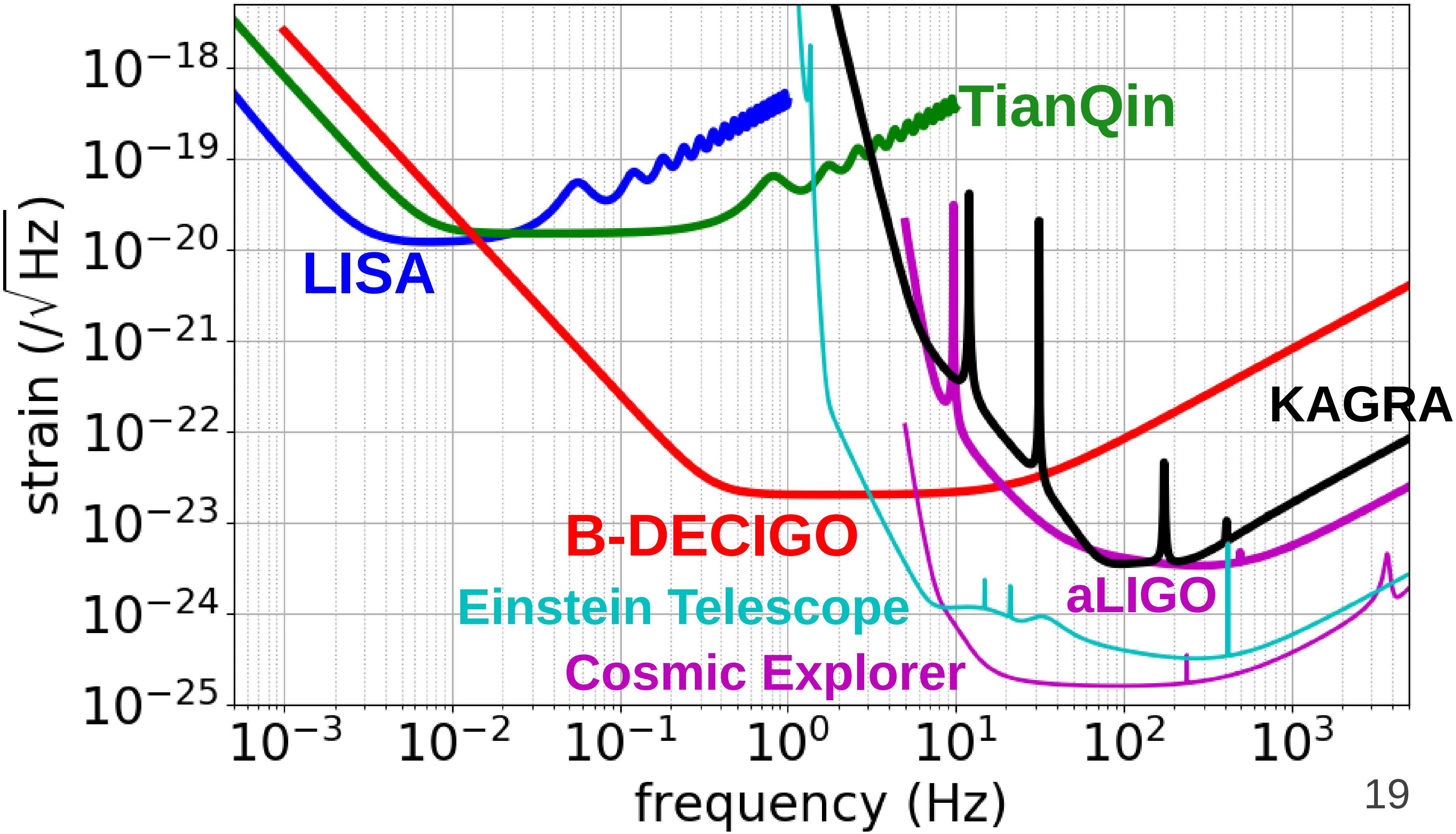
Sensitivity Curves of GW Detectors

- Now you know how to calculate quantum noise limited sensitivity
- Let’s look at the designed sensitivity curves of current and proposed GW detectors
- B-DECIGO
  Space-based Fabry-Pérot interferometer
- LISA, TianQin
  Space-based Optical transponder
  (similar to Michelson interferometer)
- Advanced LIGO, KAGRA
  Ground-based Fabry-Pérot-Michelson interferometer (with recycling cavities)

Sensitivity Curves of GW Detectors

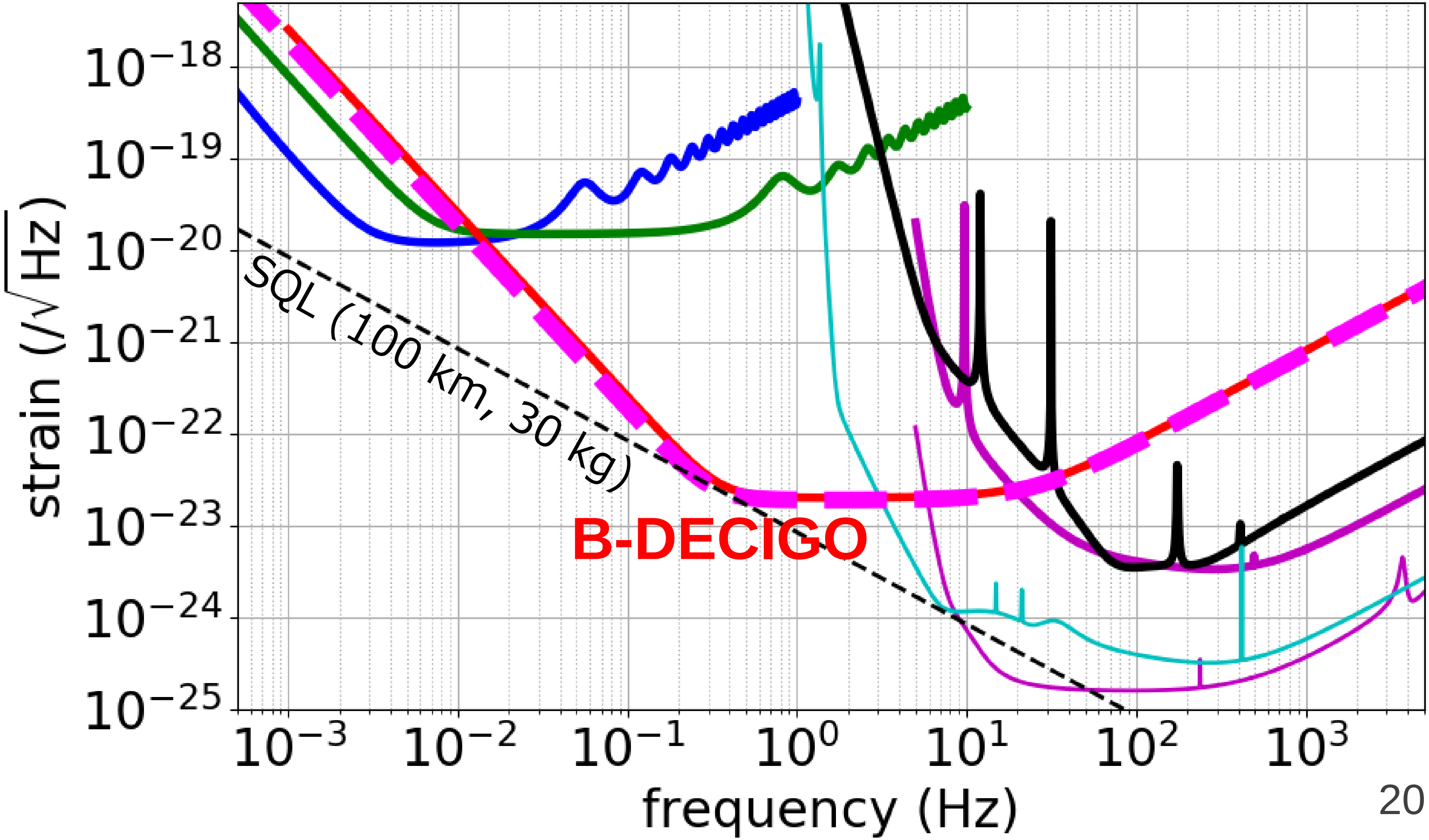
LISA: https://perf-lisa.in2p3.fr/
TianQin: arXiv:1902.04423 (from Yi-Ming Hu)
B-DECIGO: PTEP 2016, 093E01 (2016)

KAGRA: PRD 97, 122003 (2018)
aLIGO: LIGO-T1800044
ET: http://www.et-gw.eu/index.php/etdsdocument
CE: CQG 34, 044001 (2017)

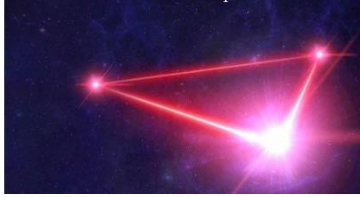


B-DECIGO

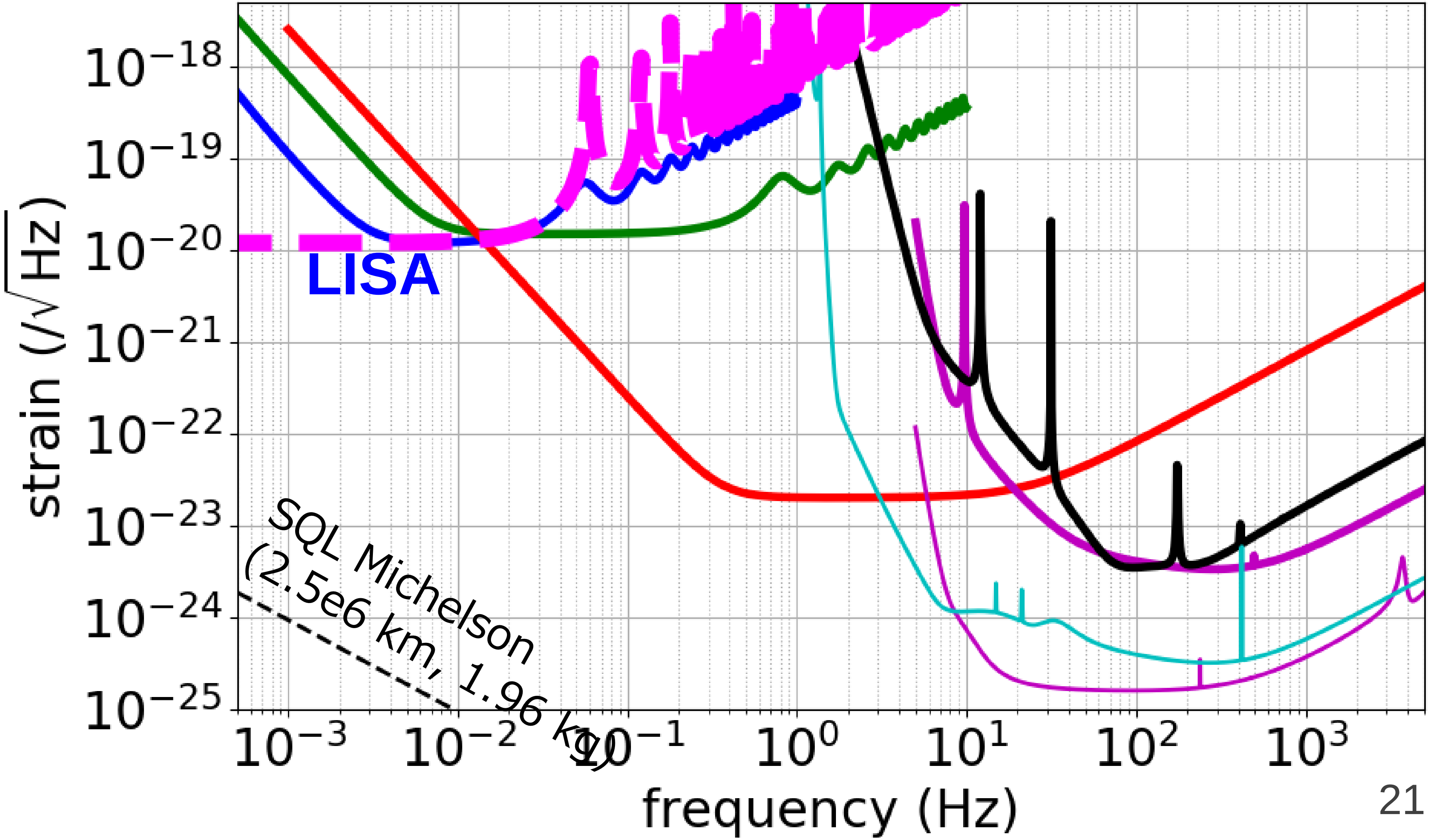
- Consistent with 100 km, 30 kg, 1 W, Finesse 30 (wavelength: 515 nm)



LISA

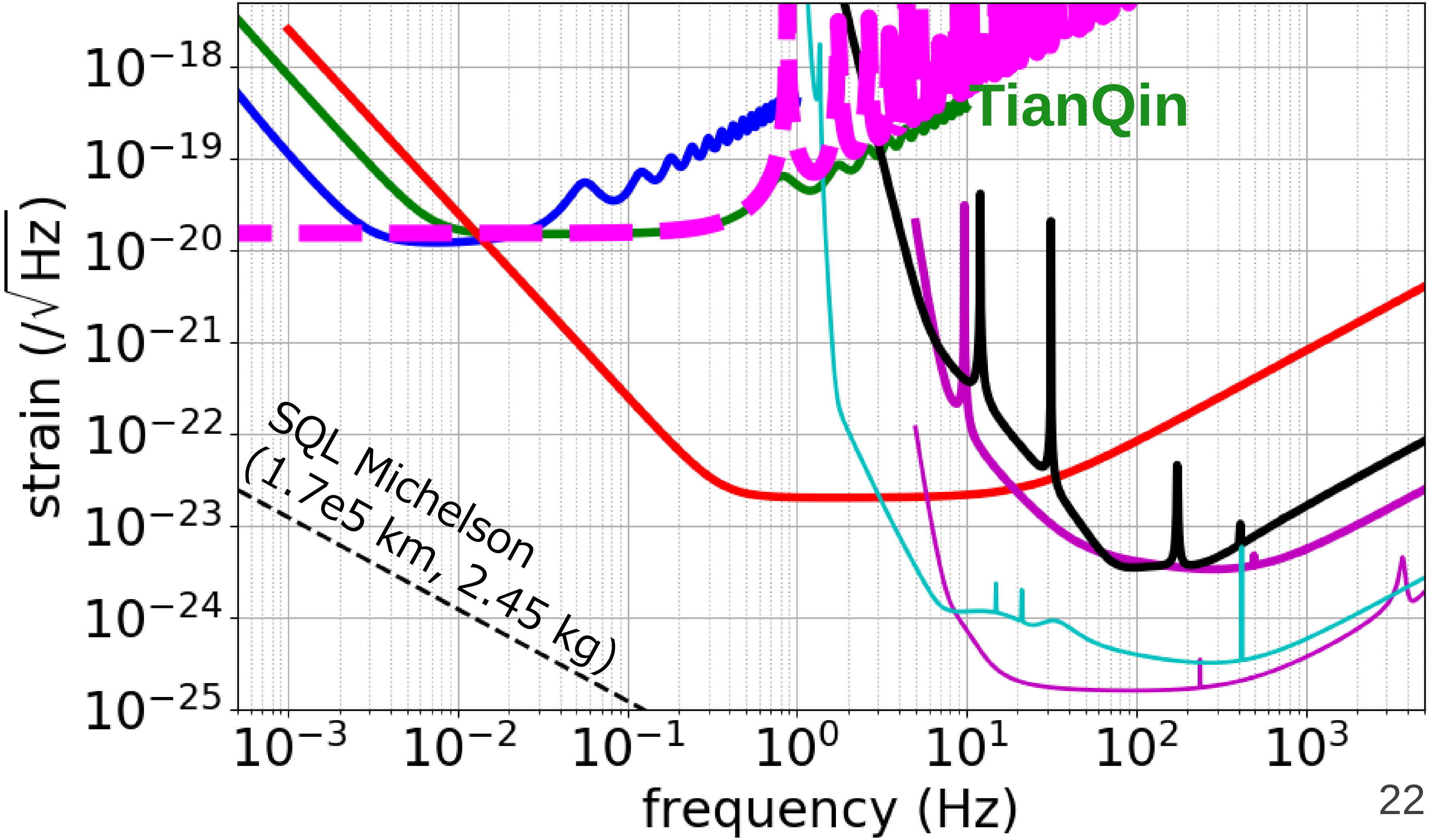


- Shot noise of 2.5e6 km, 3e-12 W Michelson (classical force noise at low frequencies)



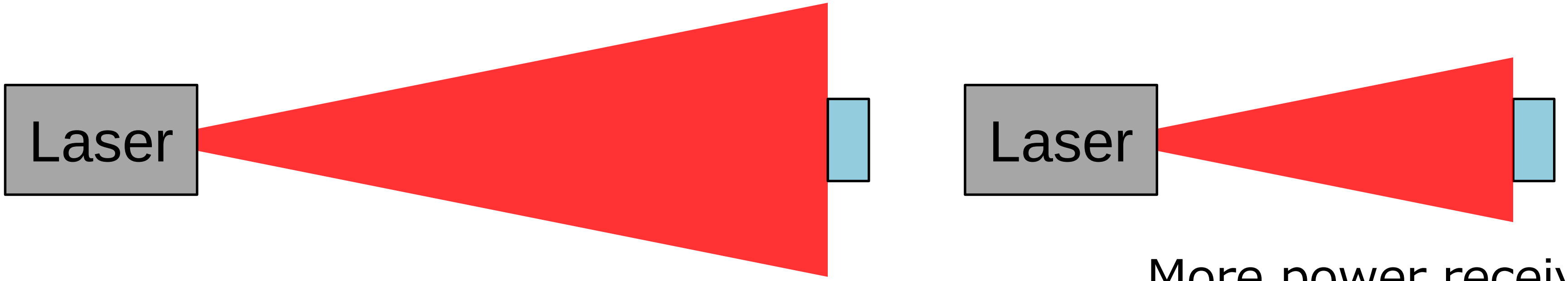
TianQin

- Shot noise of 1.7e5 km, 4e-10 W Michelson
  (classical force noise at low frequencies)



Optical Transponder

- LISA and TianQin uses small amount of light (1-100 pW) due to very long arm length
- Amount of light scales with $1/L^2$
  → shot noise floor stays the same
  (cut-off frequency shifts by $1/L$)

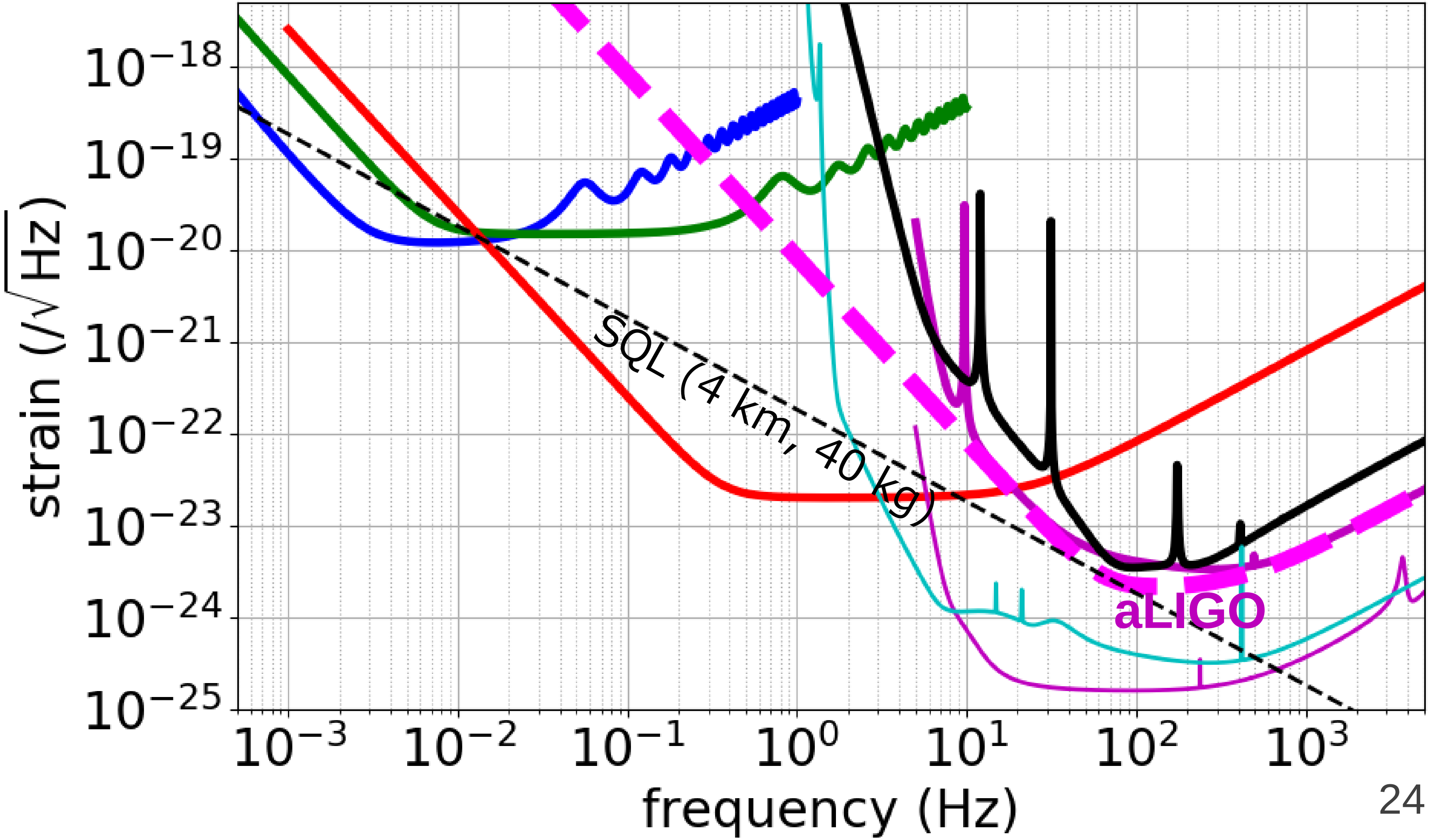


More power received with shorter arm

$$h_{\rm shot} = \frac{1}{2\pi L}\sqrt{\frac{hc\lambda}{2P_0}}$$

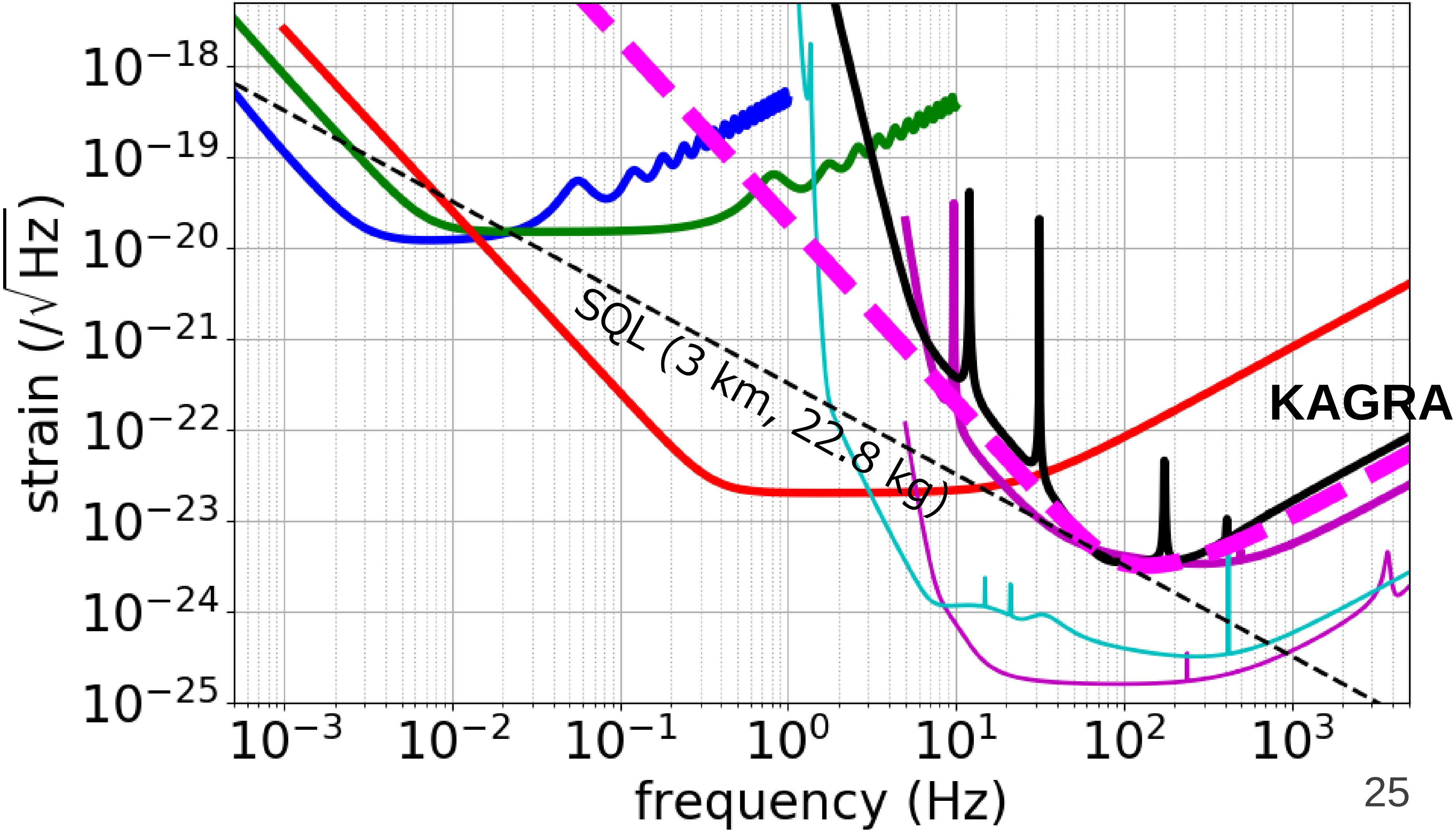
Advanced LIGO

- 4 km, 40 kg, 125x40/0.1 W, Finesse 450*0.1
(power recycling gain 40, signal recycling gain 0.1)



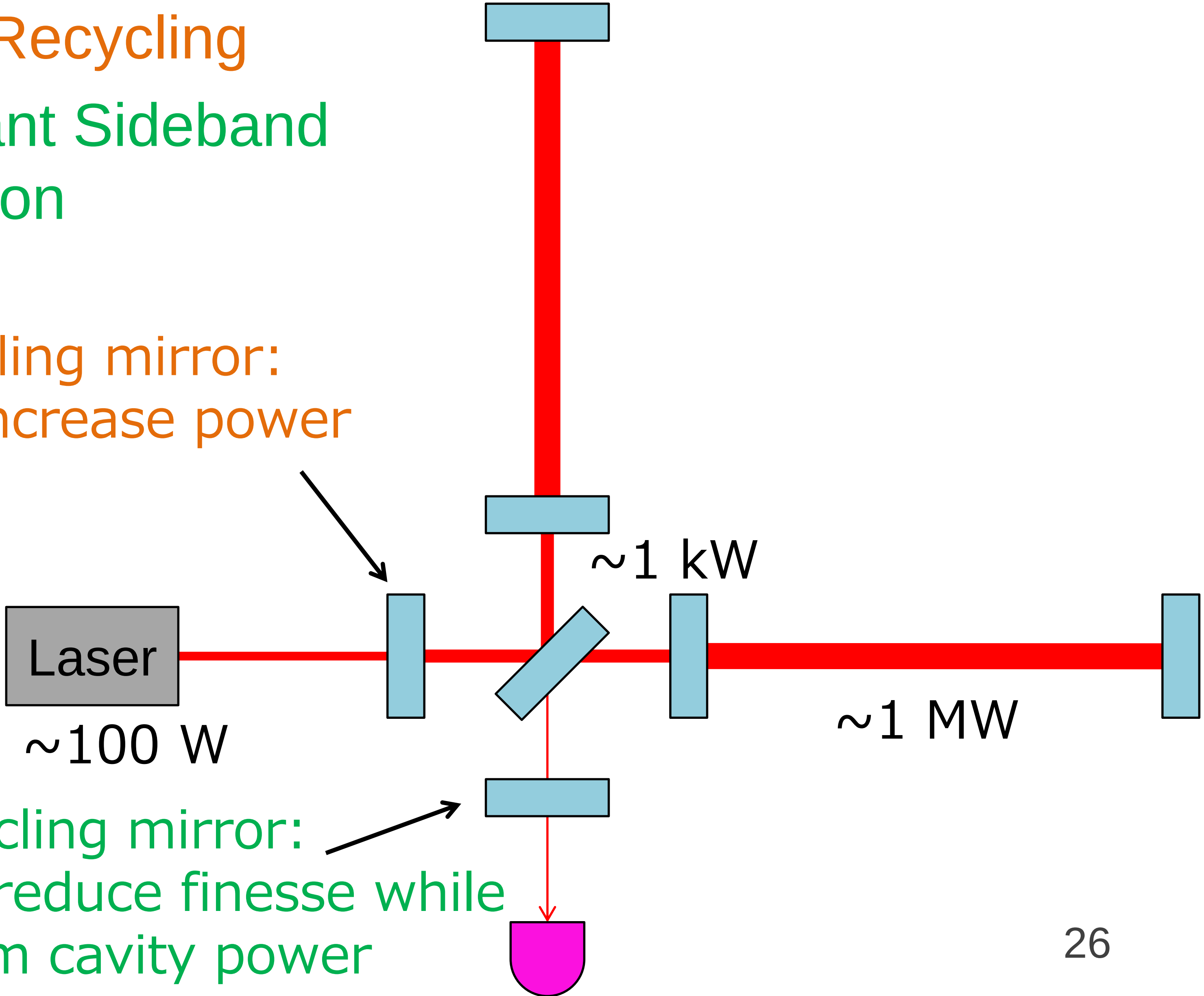
KAGRA

- 3 km, 22.8 kg, 67x10/0.07 W, Finesse 1530*0.07
  (power recycling gain 10, signal recycling gain 1/15=0.07)



Resonant Sideband Extraction

- Power Recycling
- Resonant Sideband Extraction



Power recycling mirror:
Effectively increase power

Signal recycling mirror:
Effectively reduce finesse while keeping arm cavity power

Some Details Neglected

- LISA, TianQin and DECIGO are triangular
- DECIGO is locked Fabry-Pérot interferometer (not Fabry-Pérot-Michelson interferometer)
- There are other sensing noises such as photodiode noise, laser frequency noise, oscillator phase noise etc...
- There are many other classical noises
  - Seismic noise
  - Gravity gradient noise (Newtonian noise)
  - Suspension thermal noise
  - Mirror coating thermal noise
  - Other force/displacement noises

→ To be addressed some in next Lecture

Summary

- Standard Quantum Limit (SQL) sets certain limit to the sensitivity of laser interferometers
- SQL can be reduced with larger mirror mass and longer arm length
- Higher power shifts the detector band to higher frequencies
- Higher finesse increases the sensitivity at the most sensitive band, but reduces the bandwidth
- LISA and TianQin use small fraction of power, and the detector band can be shifted by changing the arm length
- Resonant Sideband Extraction technique is used in ground-based detectors to effectively change finesse and power

Slides Available Online

1. Laser Interferometers (July 25 PM)
   https://tinyurl.com/YM20190725-1
2. Quantum Noise (July 25 PM)
   https://tinyurl.com/YM20190725-2
3. Sensitivity Design (July 26 AM)
   https://tinyurl.com/YM20190725-3
4. Status of KAGRA (July 26 AM)
   https://tinyurl.com/YM20190725-4