Laser Interferometry for Gravitational Wave Observations

1. Laser Interferometers

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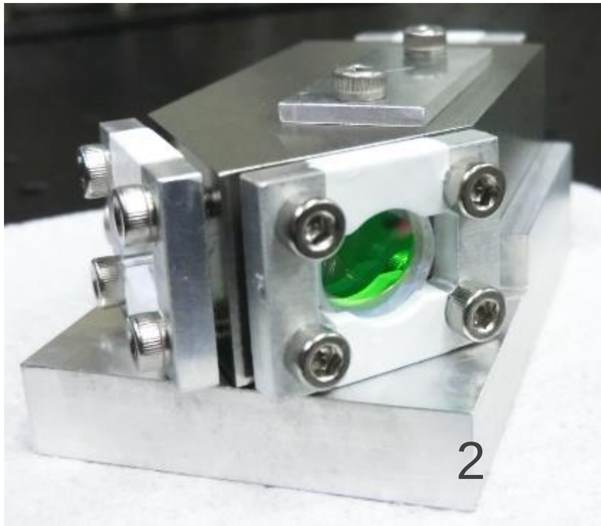
Self Introduction

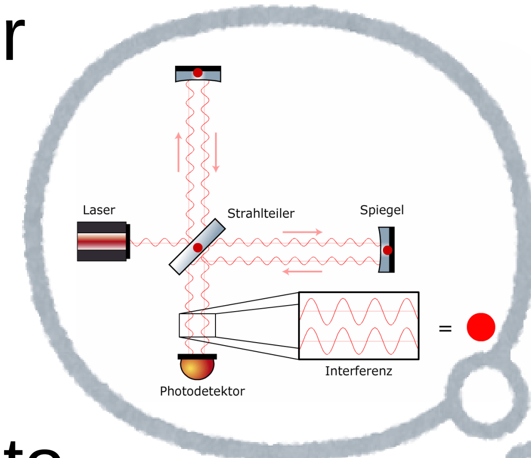
- Yuta Michimura (道村 唯太)
  Department of Physics, University of Tokyo
- Laser interferometric gravitational wave detectors
  - KAGRA
  - DECIGO
- Fundamental physics with laser interferometry
  - Lorentz invariance test
  - Macroscopic quantum mechanics
  - Axion search

  etc…



Aim of This Lecture

- Learn how laser interferometric gravitational wave detector works and learn how to calculate quantum noise of the detector



- You should be able to design your own interferometer after the lectures

Contents

1. Laser Interferometers (July 25 PM)
   - Michelson interferometer
   - Fabry-Pérot interferometer
2. Quantum Noise (July 25 PM)
   - Shot noise and radiation pressure noise
   - Standard quantum limit
3. Sensitivity Design (July 26 AM)
   - Force noise and displacement noise
   - Inspiral range and time to merger
   - Space interferometer design
4. Status of KAGRA (July 26 AM)
   - Status of KAGRA detector in Japan
   - Future prospects

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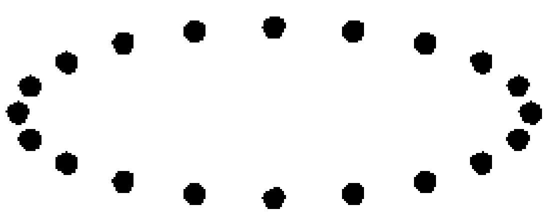
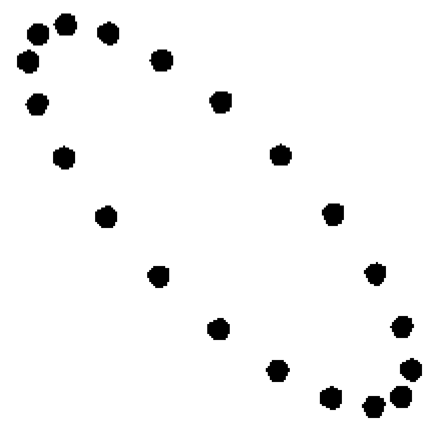
Gravitational Waves

- Ripples in space-time
- Stretches and squeezes length
- Amplitude: fraction of length change (strain)

$$h = \frac{\delta L}{L}$$

- Plus (+) and cross (x) polarizations





Detection of GWs

- Most common detector: laser interferometer
- Rai Weiss (MIT) proposed in 1960s

Internal report (1972)

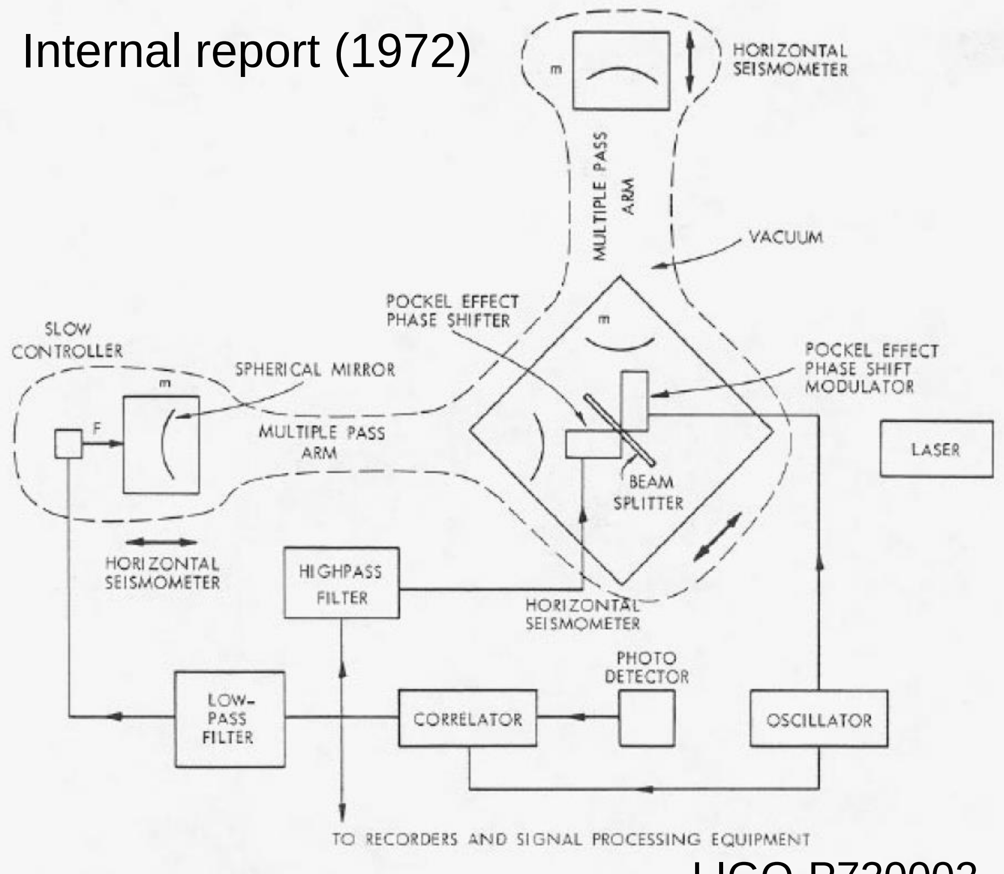
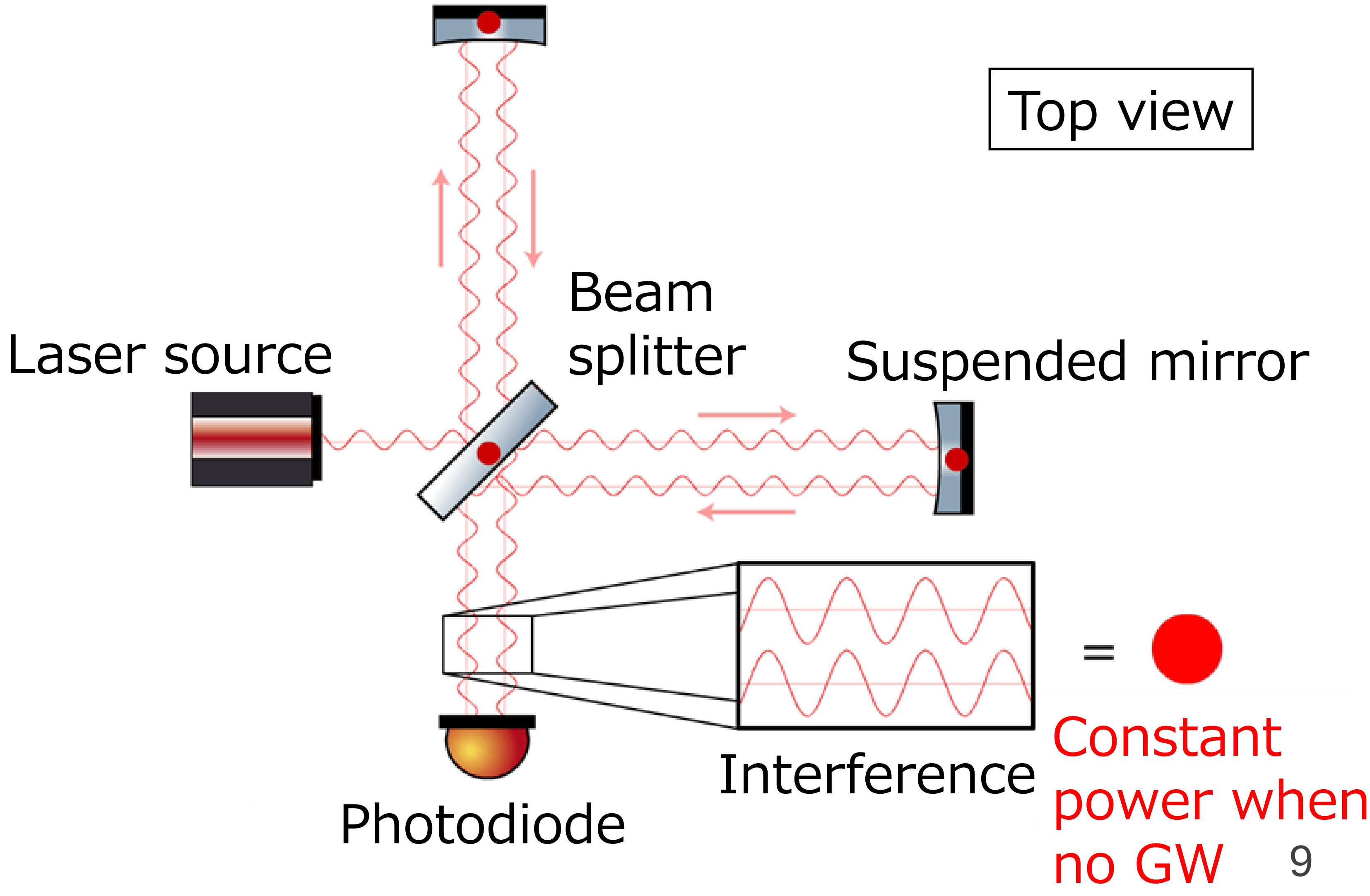


Fig. V-20. Proposed antenna.

LIGO-P720002

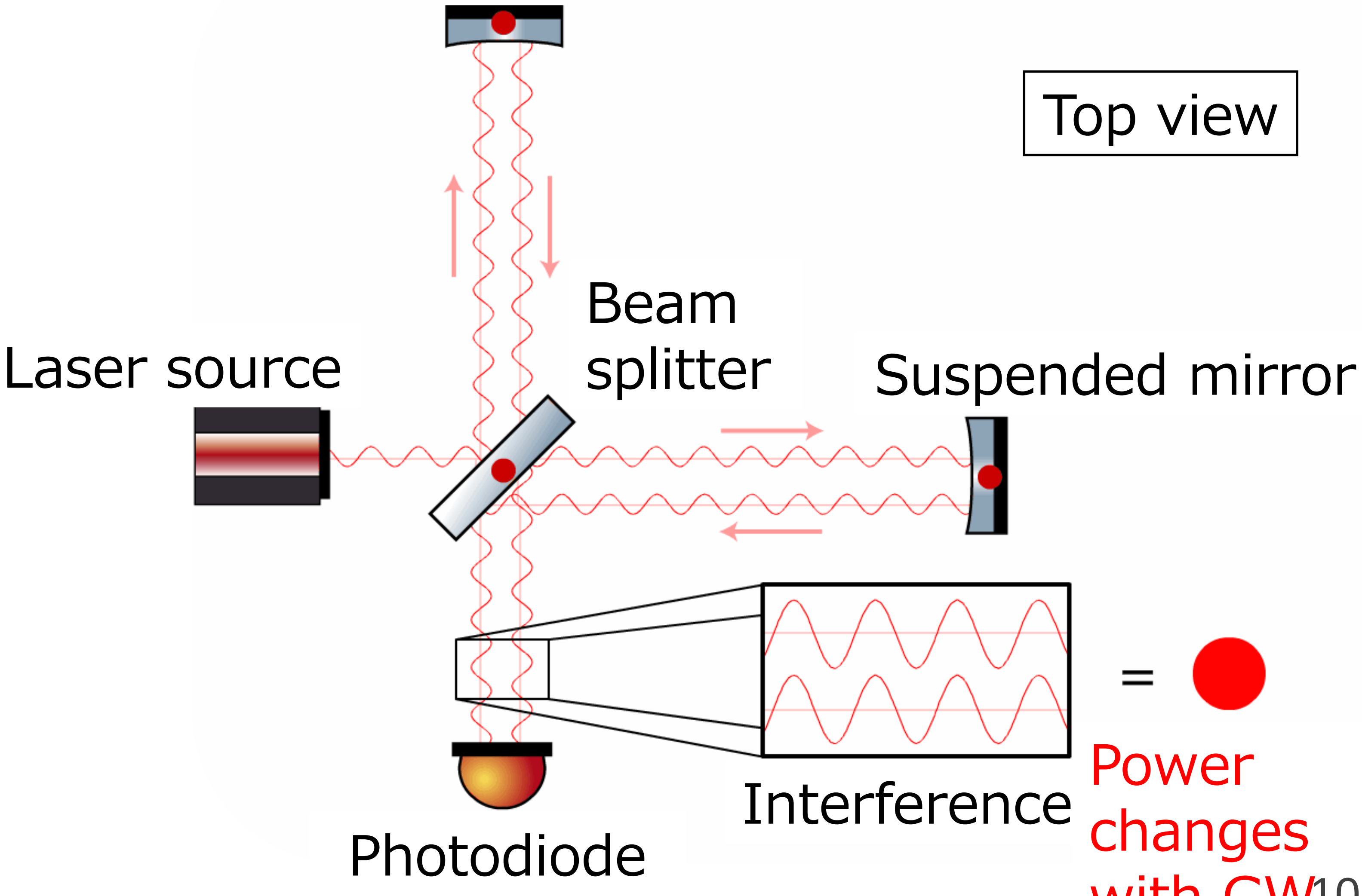
Laser Interferometric GW Detector

- measure differential arm length change



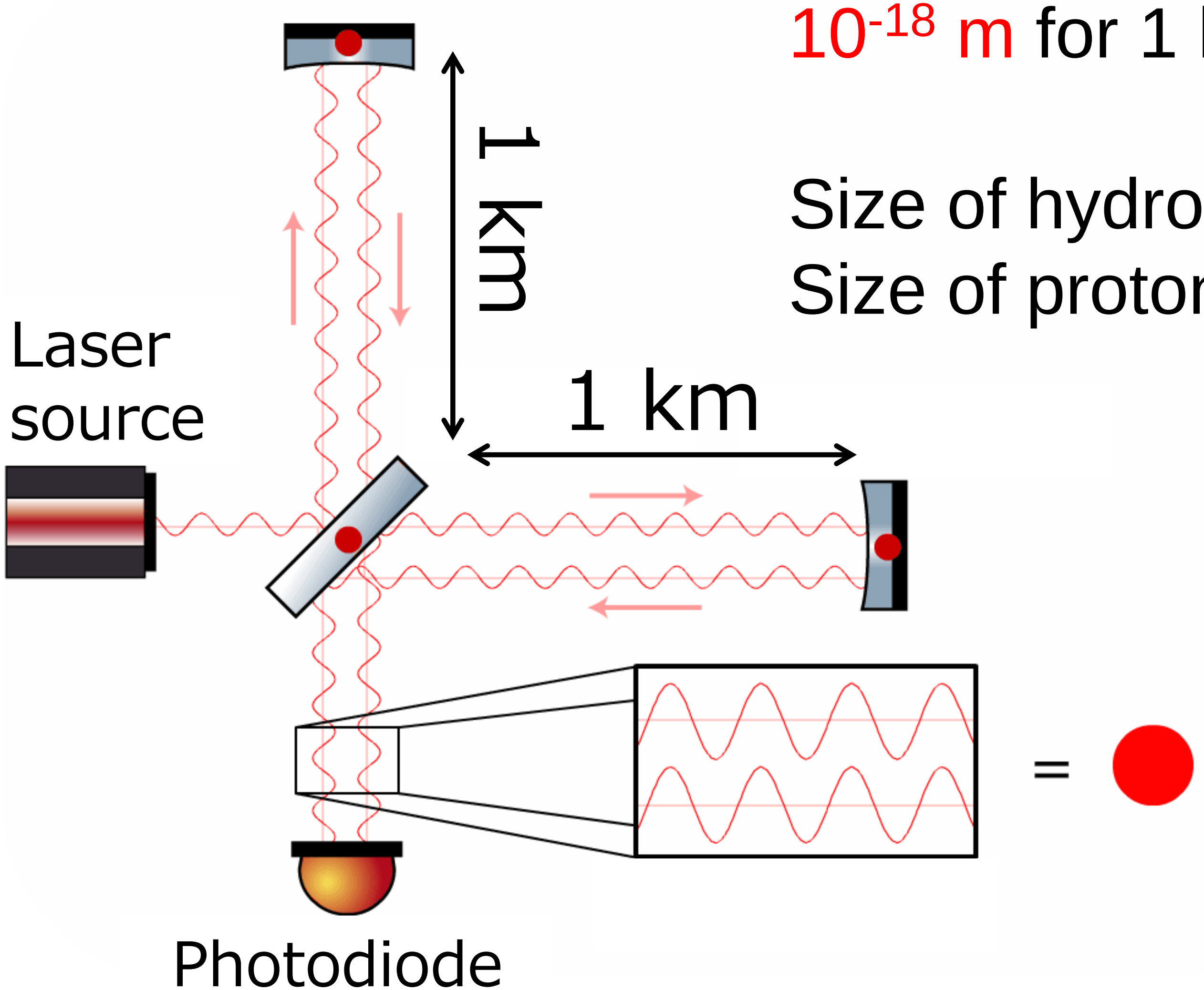
Laser Interferometric GW Detector

- measure differential arm length change



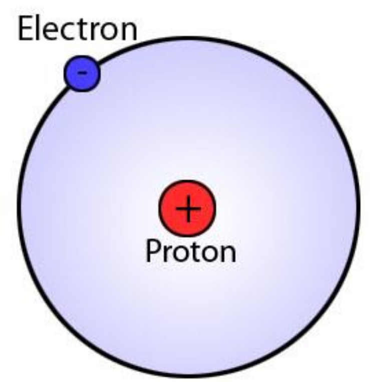
Amplitude of GW is Tiny

- For example, GW150914 had h ~ 10^{-21}



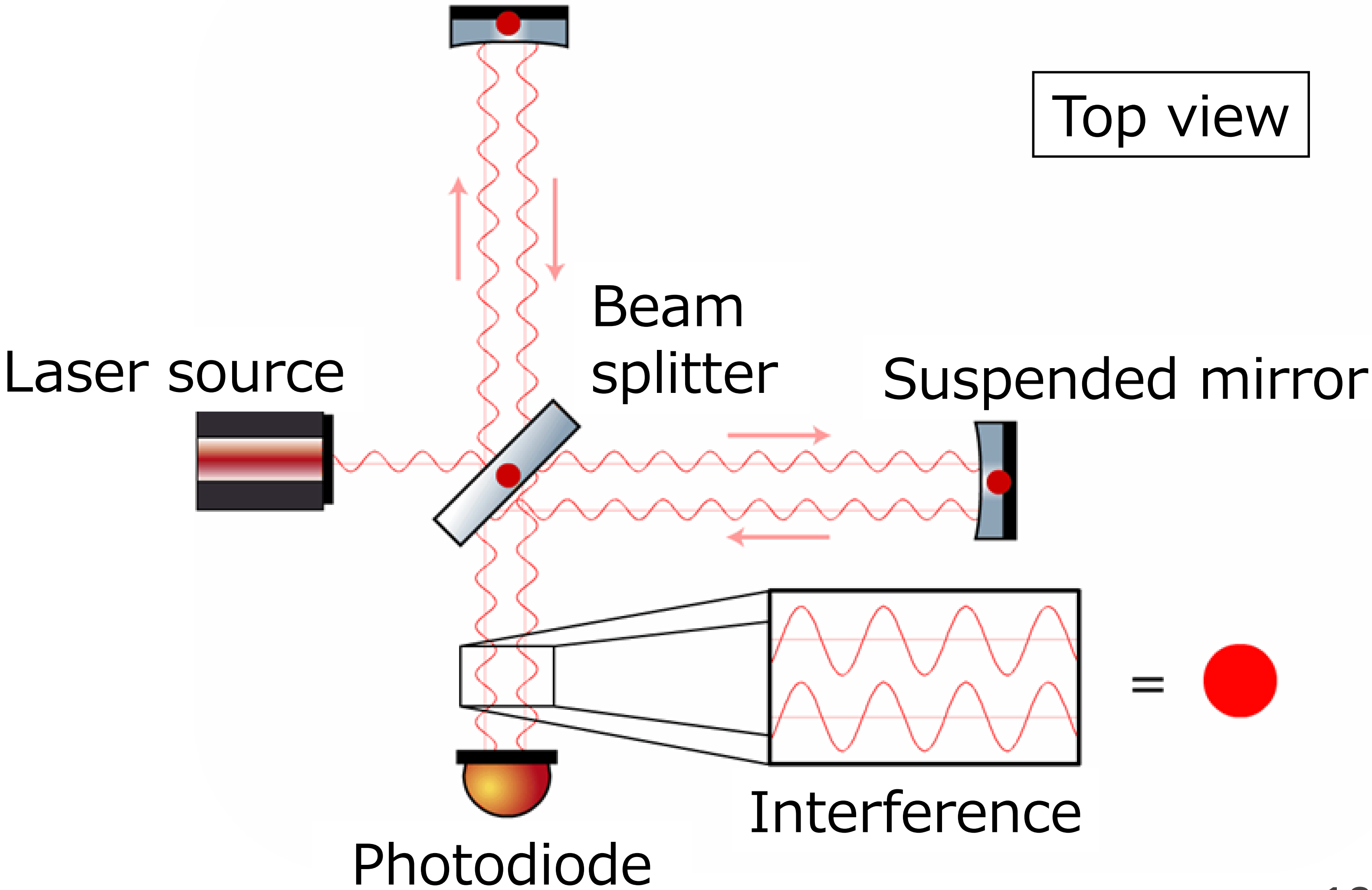
10^{-18} m for 1 km arm

Size of hydrogen atom: 10^{-11} m
Size of proton: 10^{-15} m



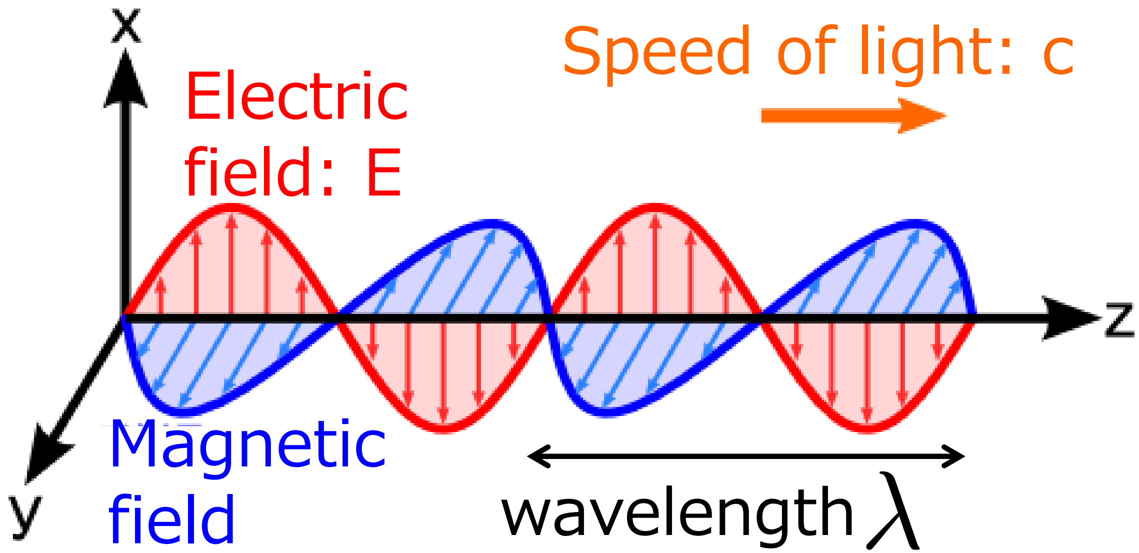
Michelson Interferometer

- Let's look into how Michelson interferometer works



Laser Beam

- Electro-magnetic waves



- Electric field can be written as

$$E = E_0 e^{i(\omega t - \phi)}$$

amplitude (E_0), angular frequency of laser (ω), phase (ϕ)

$$\omega = \frac{2\pi c}{\lambda}$$

$$\phi = \frac{2\pi L}{\lambda}$$

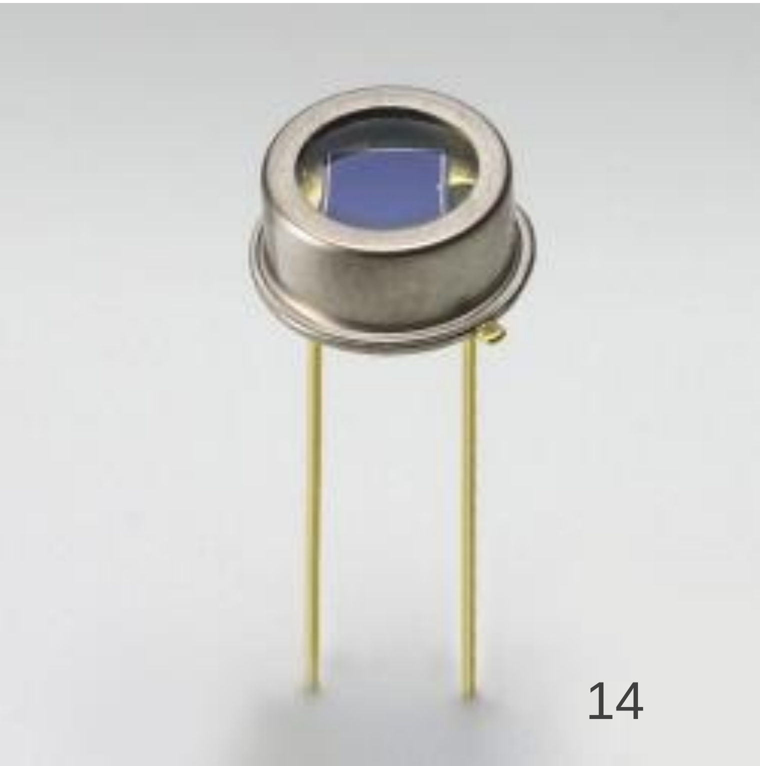
phase at distance L

Photodiodes

- Photodiodes (PDs)
  Convert photons into electrons
  Detects light power (square of amplitude)

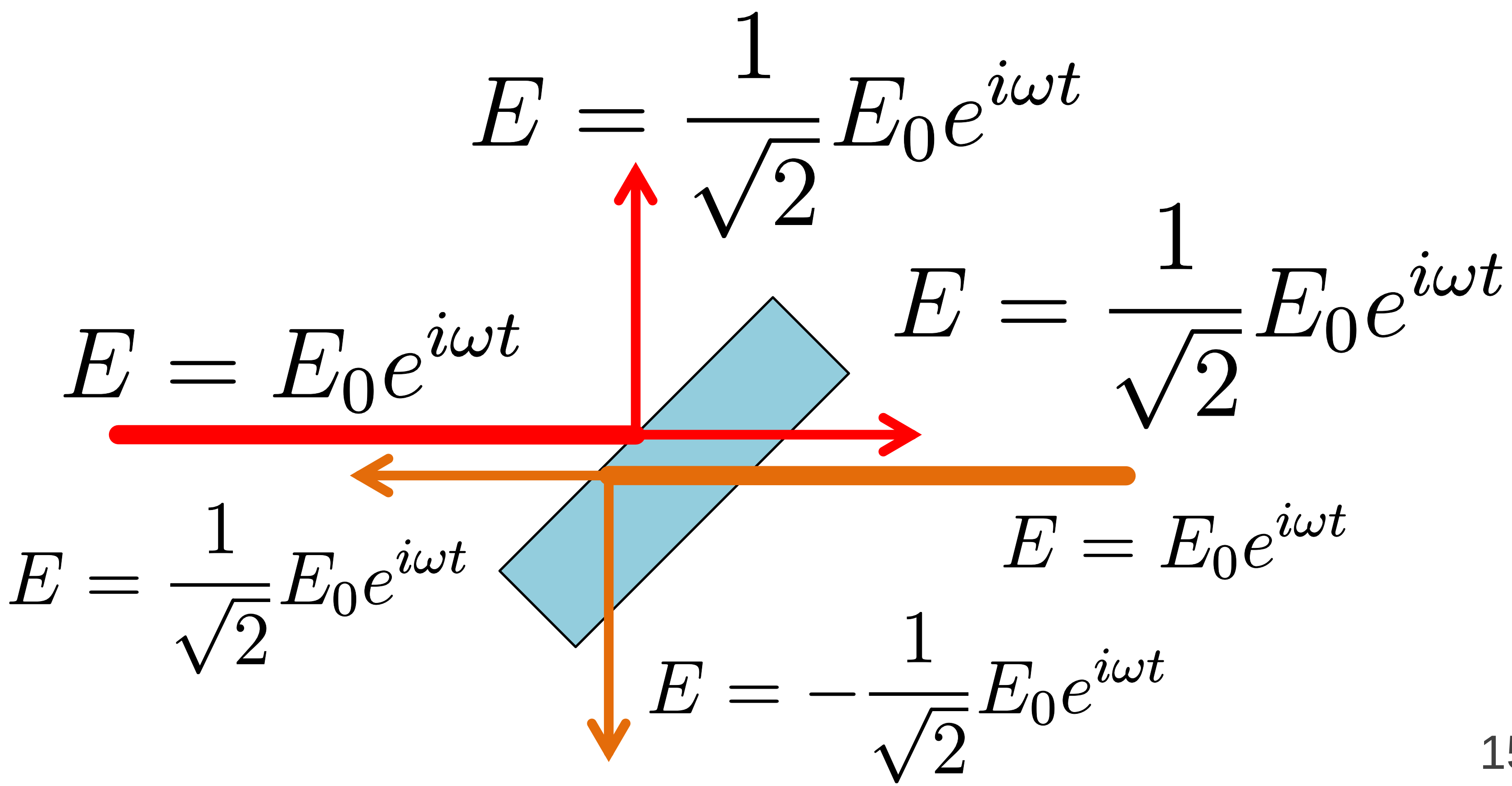
$$P \propto |E|^2 = E_0^2$$

We can only detect power change
Phase change cannot be detected directly



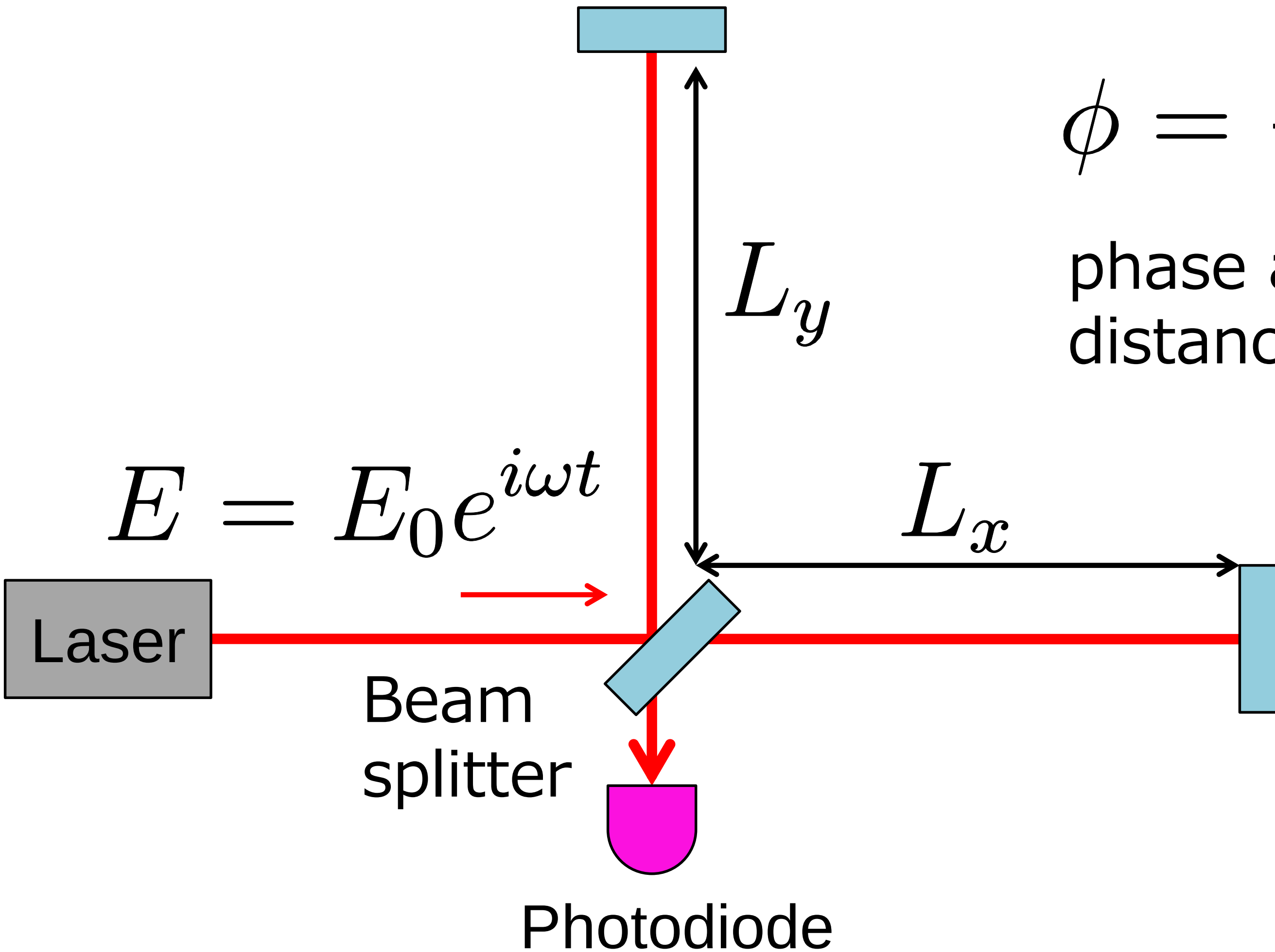
Beam Splitter

- Split beam in two
- Half in power, 1/√2 in amplitude
- Sign flip in back reflection



Output of Michelson Interferometer

- What is the power detected at the photodiode?



$$\phi = \frac{2\pi L}{\lambda}$$

phase at distance L

Output of Michelson Interferometer

- What is the power detected at the photodiode?

From Y-am From X-arm

$$P_{\mathrm{PD}} = \left| \frac{1}{2} E_0 e^{i(\omega t - \frac{4\pi L_y}{\lambda})} - \frac{1}{2} E_0 e^{i(\omega t - \frac{4\pi L_x}{\lambda})} \right|^2$$

$$= \frac{1}{4} |E_0|^2 \left| e^{-i\frac{4\pi L_y}{\lambda}} - e^{-i\frac{4\pi L_x}{\lambda}} \right|^2$$

$$= \frac{1}{2} P_0 \left(1 - \cos \frac{4\pi L_-}{\lambda} \right)$$

Input power

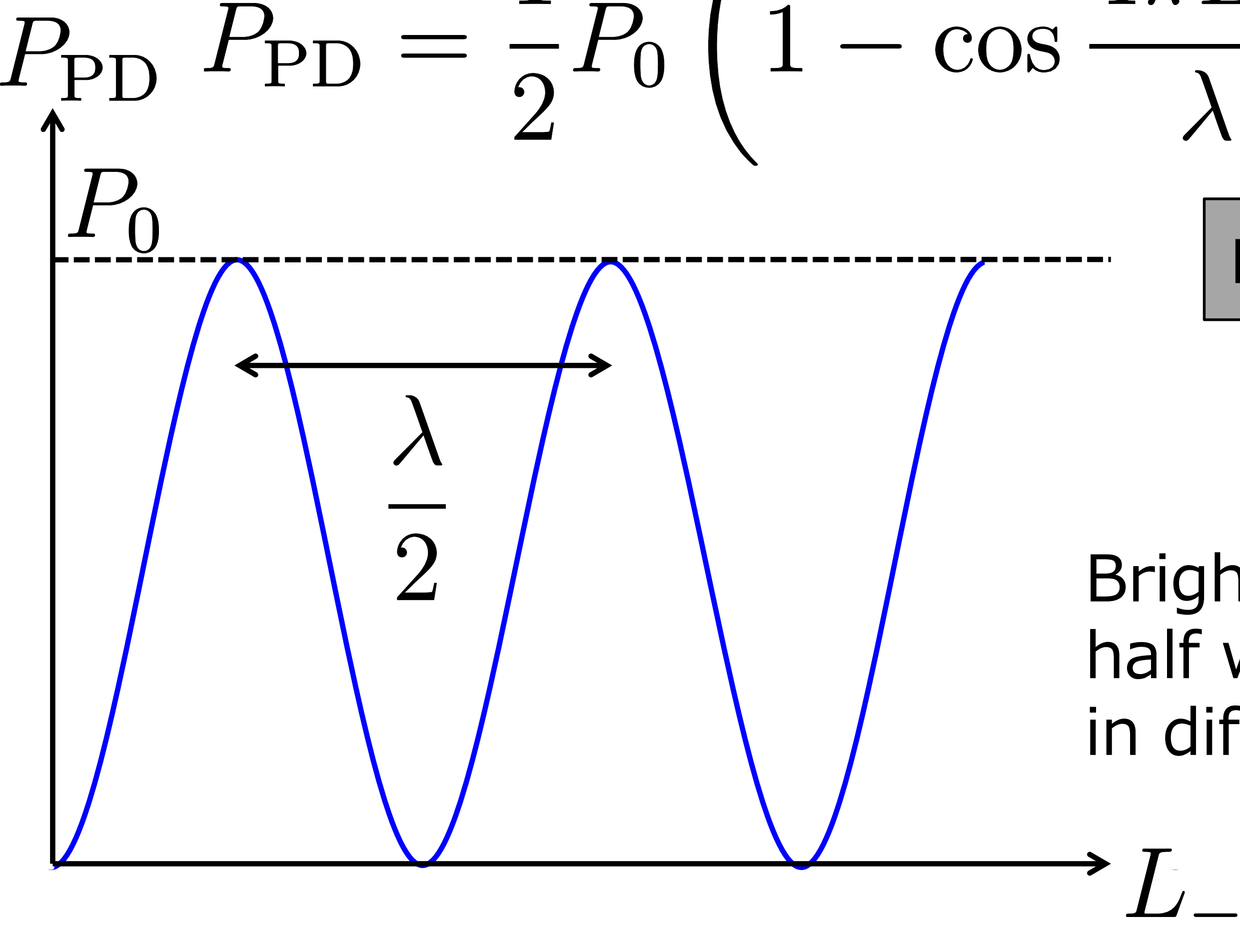
$$L_- = L_y - L_x$$

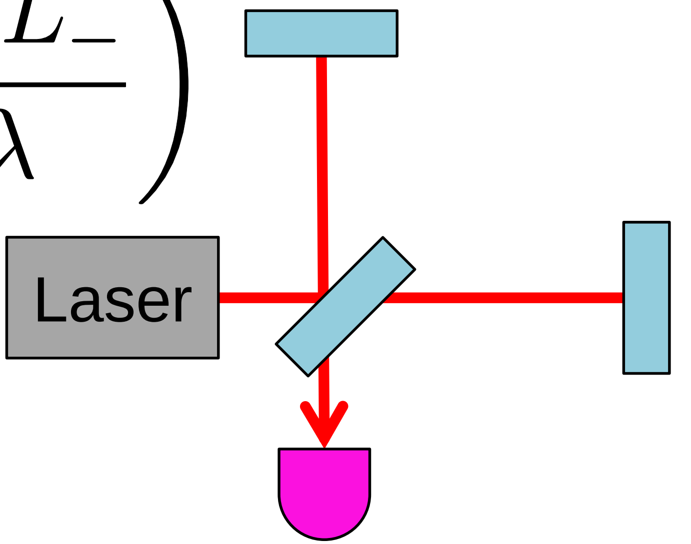
Differential arm length

Output of Michelson Interferometer

- Power changes with differential arm length change (interference)

$$P_{\mathrm{PD}} = \frac{1}{2} P_0 \left(1 - \cos \frac{4\pi L_-}{\lambda}\right)$$



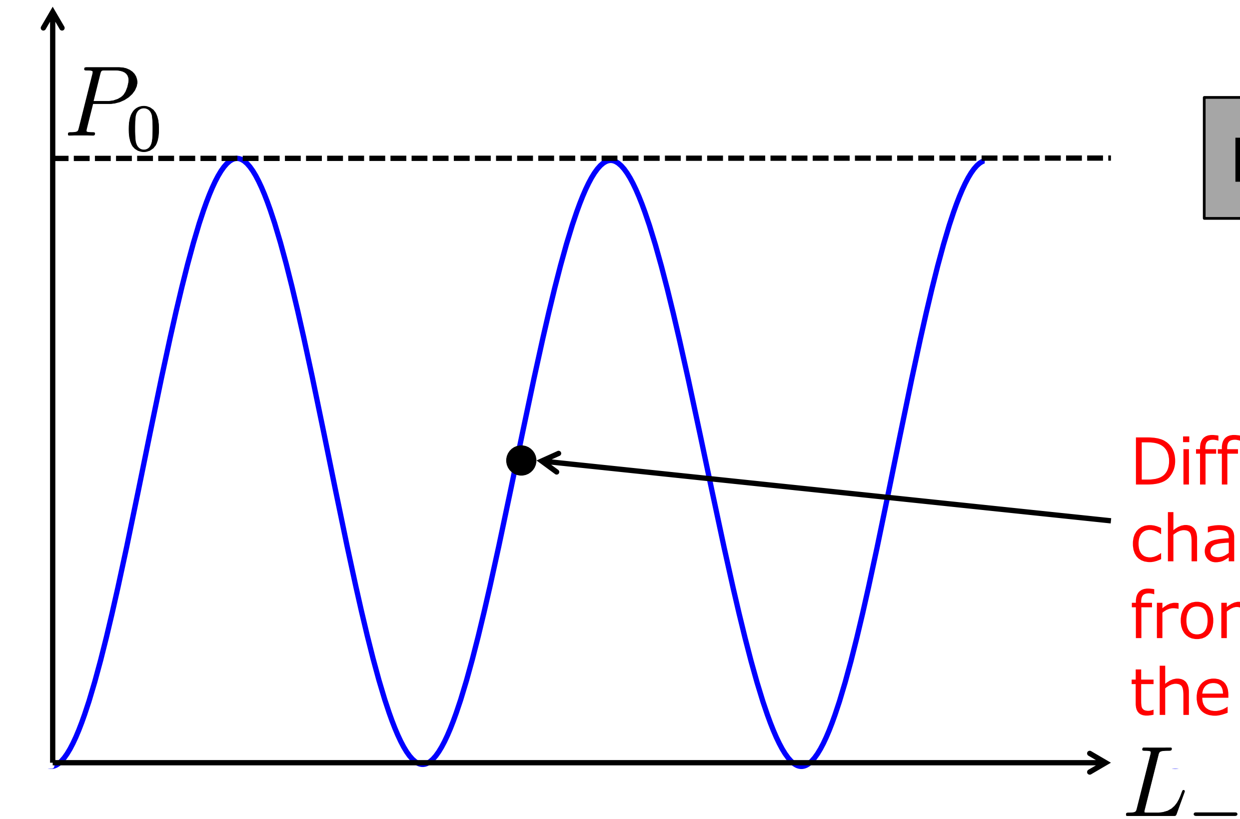


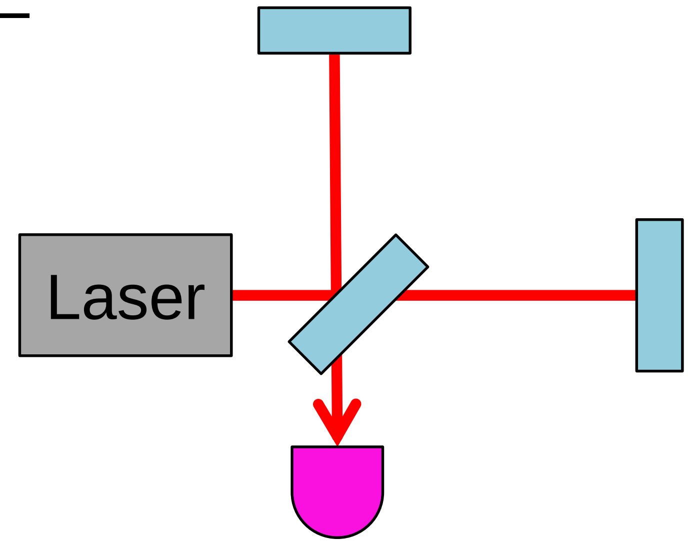
Bright fringe in every half wavelength change in differential arm length

Output of Michelson Interferometer

- Ratio between power change and length change

$$\frac{\partial P_{\mathrm{PD}}}{\partial L_-} = \frac{2\pi P_0}{\lambda} \sin \frac{4\pi L_-}{\lambda}$$





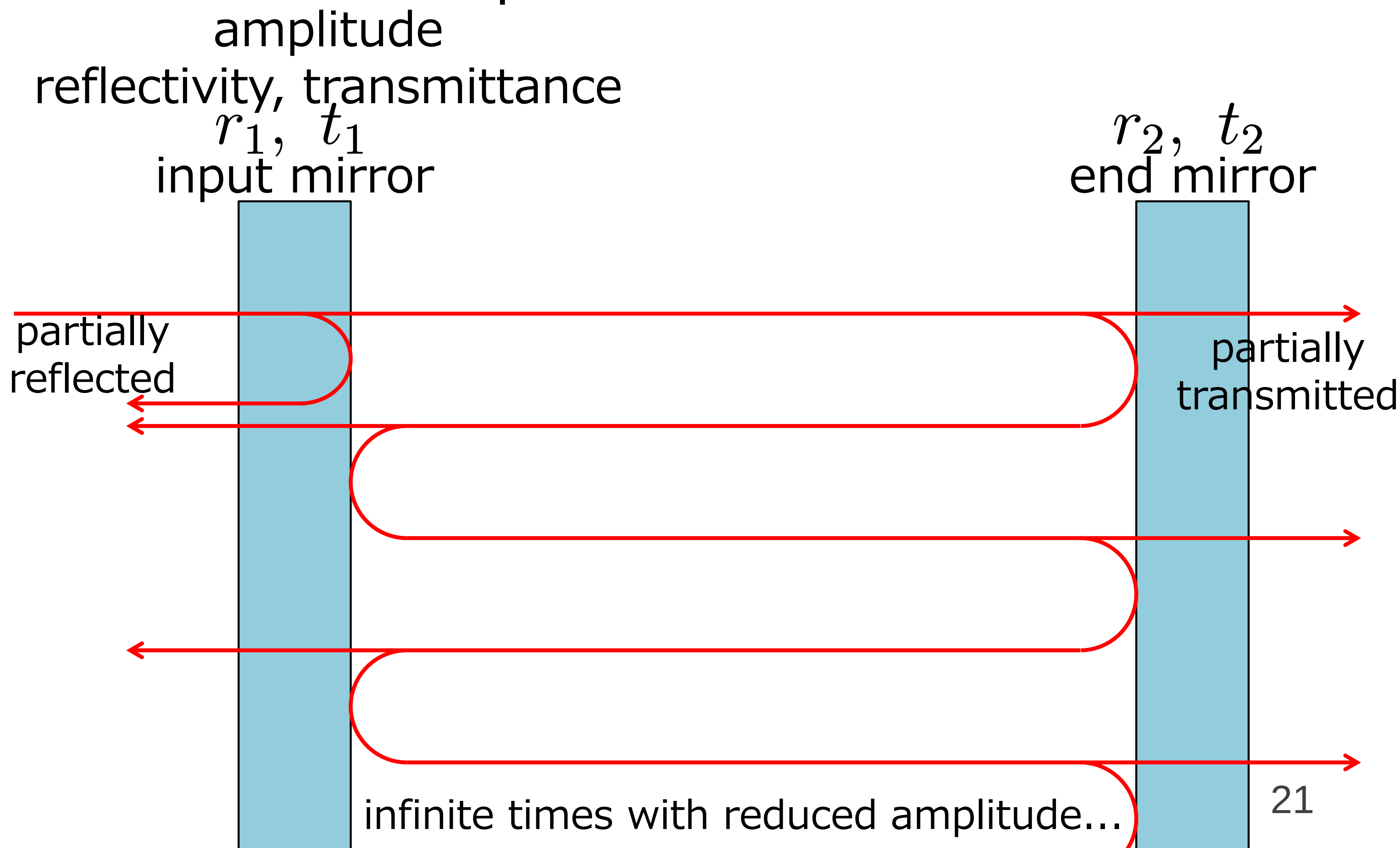
Differential arm length change can be detected from power change at the photodiode

How to Further Enhance the Signal

- Longer arms gives larger length change due to gravitational waves $\delta L = hL$
- But making arm length very long is tough (especially on Earth)
- Use Fabry-Pérot cavity
  laser light go back-and-forth many times to effectively enhance the arm length

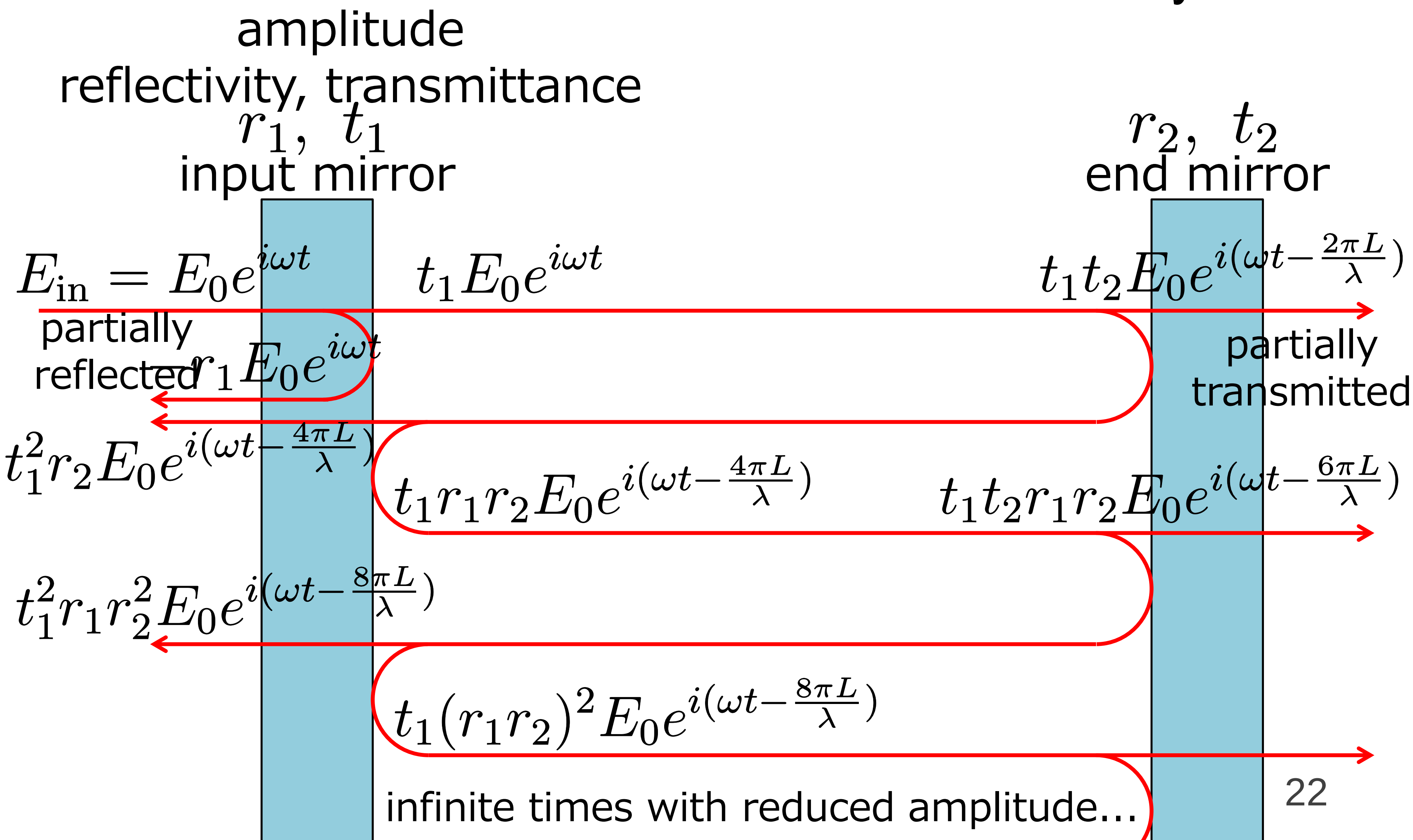
Fabry-Pérot Cavity

- Made from two parallel mirrors



Fabry-Pérot Cavity

- Let's calculate electric field inside the cavity



Intra-Cavity Field

- Intra-cavity field can be expressed as

$$E_{\text{cav}} = t_1 E_0 e^{i\omega t} + t_1 r_1 r_2 E_0 e^{i(\omega t - \frac{4\pi L}{\lambda})} + t_1 (r_1 r_2)^2 E_0 e^{i(\omega t - \frac{4\pi L}{\lambda})} + \ldots$$

$$= (t_1 + t_1 r_1 r_2 2e^{i\frac{4\pi L}{\lambda}} + t_1 (r_1 r_2)^2 2e^{i\frac{8\pi L}{\lambda}} + \ldots) E_0 e^{i\omega t}$$

infinite geometric series with a common ratio of $r_1 r_2 e^{i\frac{4\pi L}{\lambda}}$

input field

$$= \frac{t_1}{1 - r_1 r_2 e^{i\frac{4\pi L}{\lambda}}} E_{\text{in}}$$

Reflected Field

- Reflected field can be expressed as

$$E_{\text{refl}} = -r_1 E_0 e^{i\omega t} + t_1^2 r_2 E_0 e^{i(\omega t - \frac{4\pi L}{\lambda})} + t_1^2 r_1 r_2^2 E_0 e^{i(\omega t - \frac{4\pi L}{\lambda})} + \ldots$$

$$= (-r_1 + \underbrace{t_1^2 r_2 e^{i\frac{4\pi L}{\lambda}} + t_1^2 r_1 r_2^2 2 e^{i\frac{8\pi L}{\lambda}} + \ldots}) E_0 e^{i\omega t}$$

infinite geometric series with
a common ratio of $r_1 r_2 e^{i\frac{4\pi L}{\lambda}}$

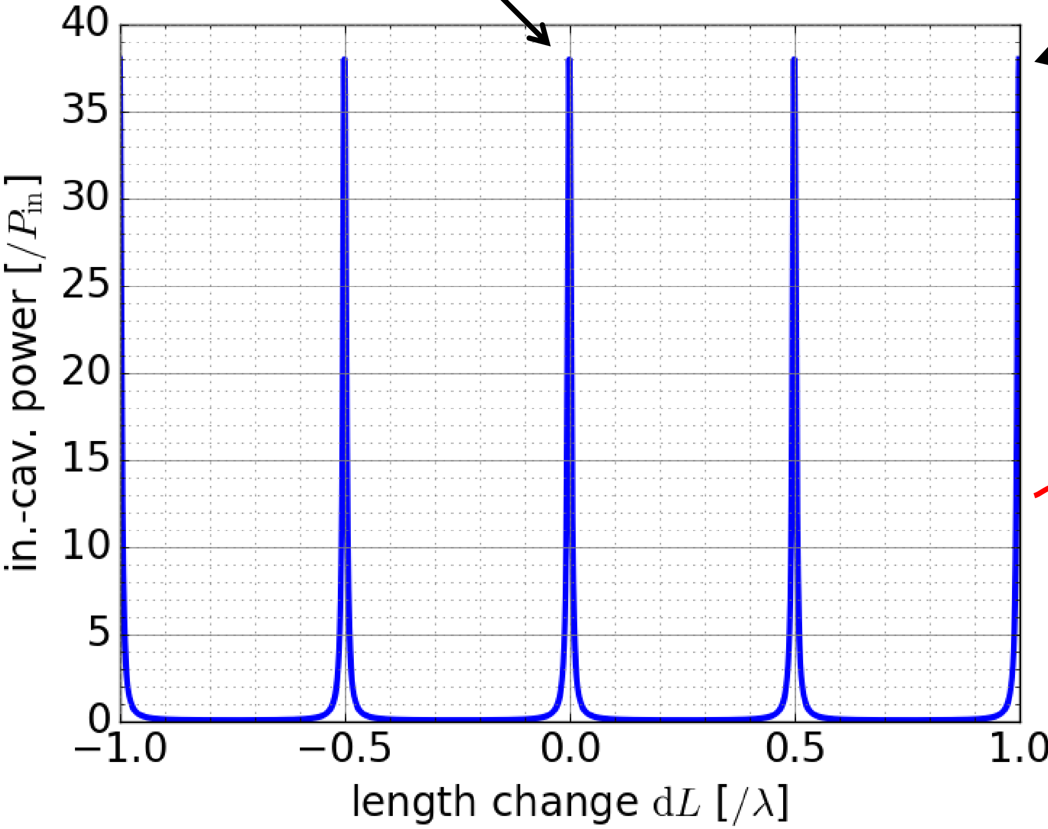
$$= \left(-r_1 + \frac{t_1^2 r_2 e^{i\frac{4\pi L}{\lambda}}}{1 - r_1 r_2 e^{i\frac{4\pi L}{\lambda}}} \right) E_{\text{in}}$$

Intra-Cavity Power

- Power inside the cavity

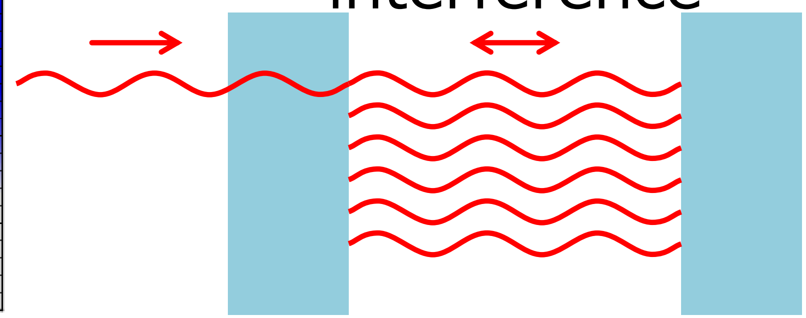
$$|E_{\rm cav}|^2 = \left| \frac{t_1}{1 - r_1 r_2 e^{i\frac{4\pi L}{\lambda}}} \right|^2 P_{\rm in}$$

resonance



Intra-cavity power can be much higher than input power on resonance

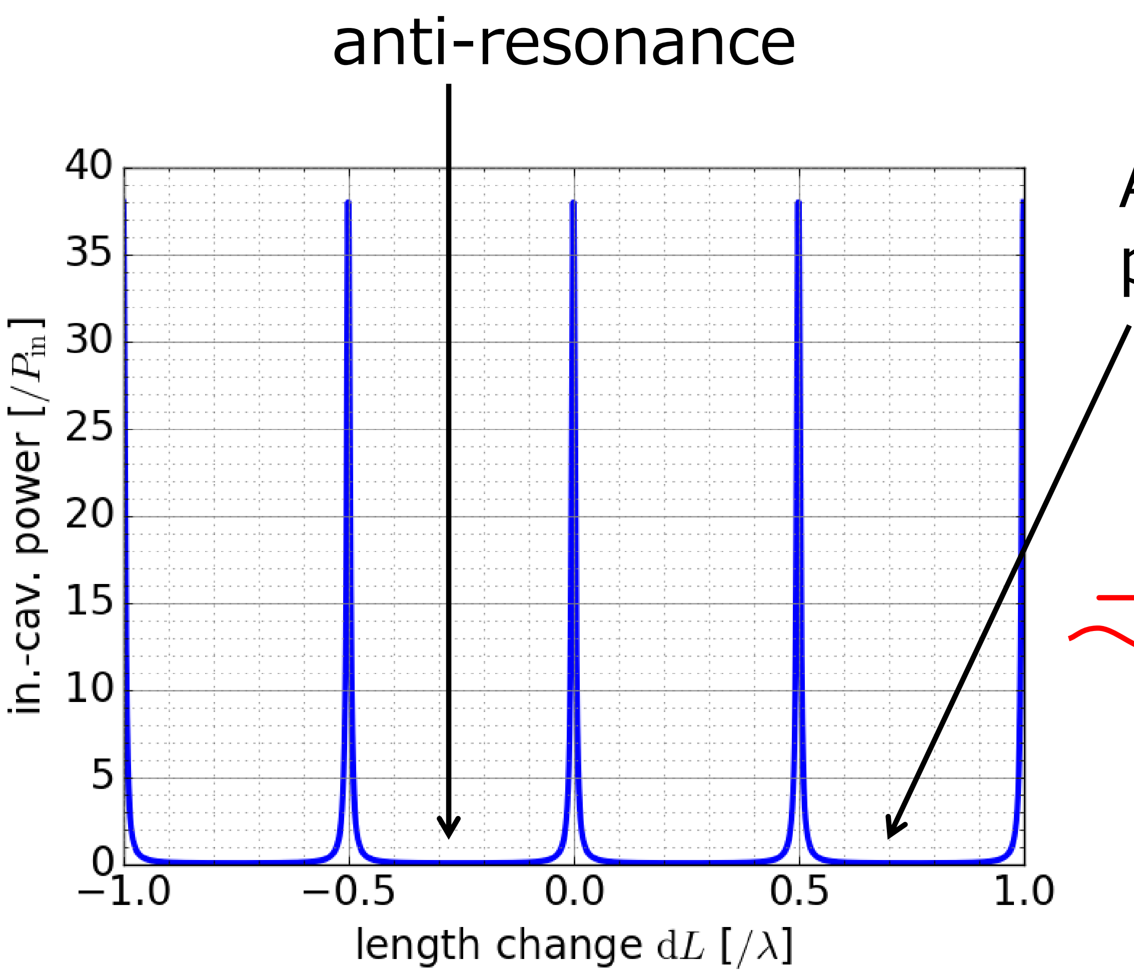
constructive interference

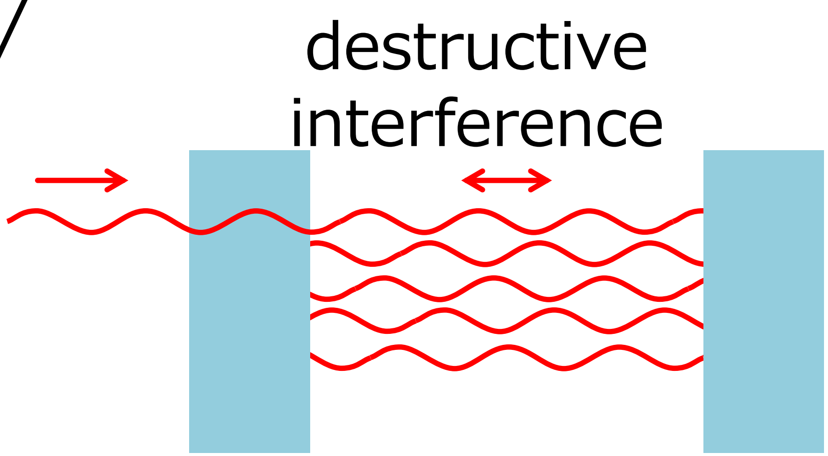


Intra-Cavity Power

- Power inside the cavity

$$|E_{\rm cav}|^2 = \left| \frac{t_1}{1 - r_1 r_2 e^{i\frac{4\pi L}{\lambda}}} \right|^2 P_{\rm in}$$

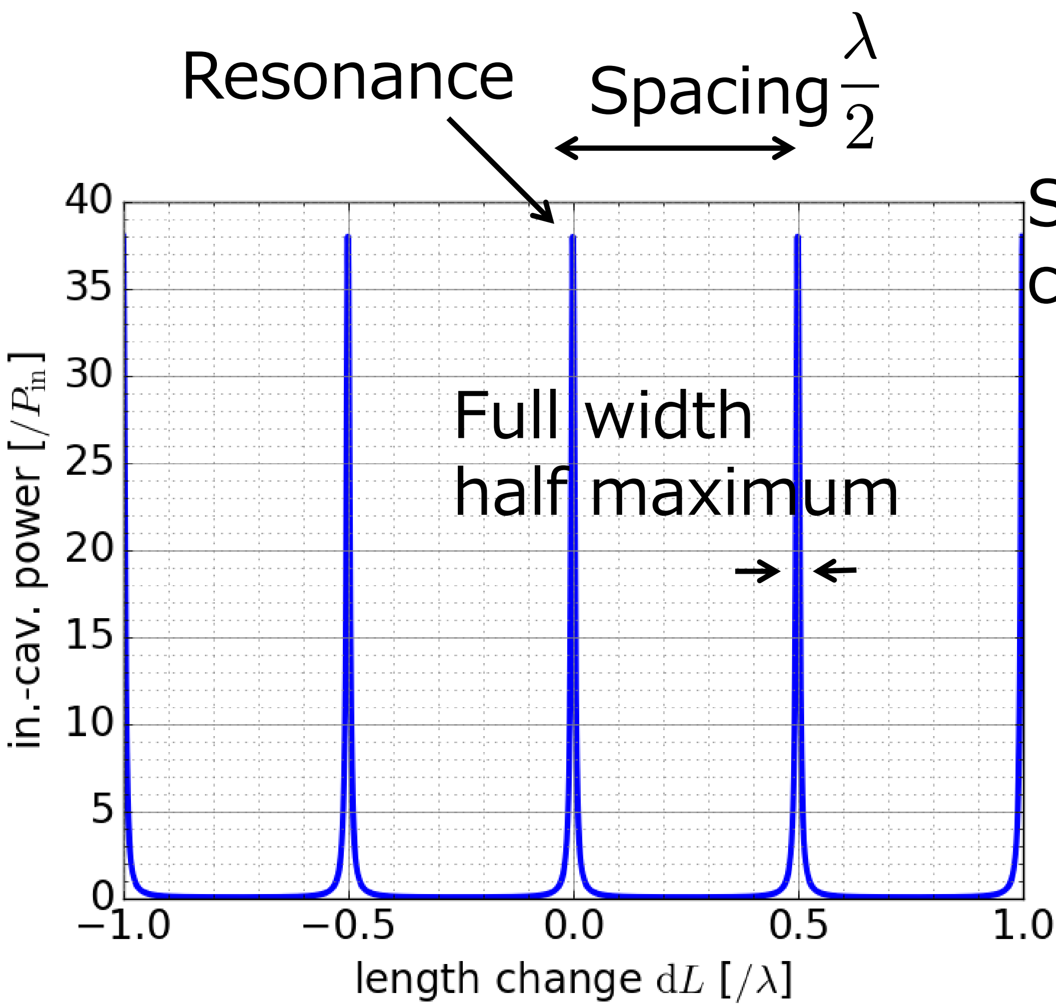




Finesse

- Power inside the cavity

$$|E_{\rm cav}|^2 = \left| \frac{t_1}{1 - r_1 r_2 e^{i\frac{4\pi L}{\lambda}}} \right|^2 P_{\rm in}$$



Sharpness of the resonance can be evaluated with

$$\frac{\text{Spacing}}{\text{FWHM}} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2} \equiv \mathcal{F}$$

Finesse

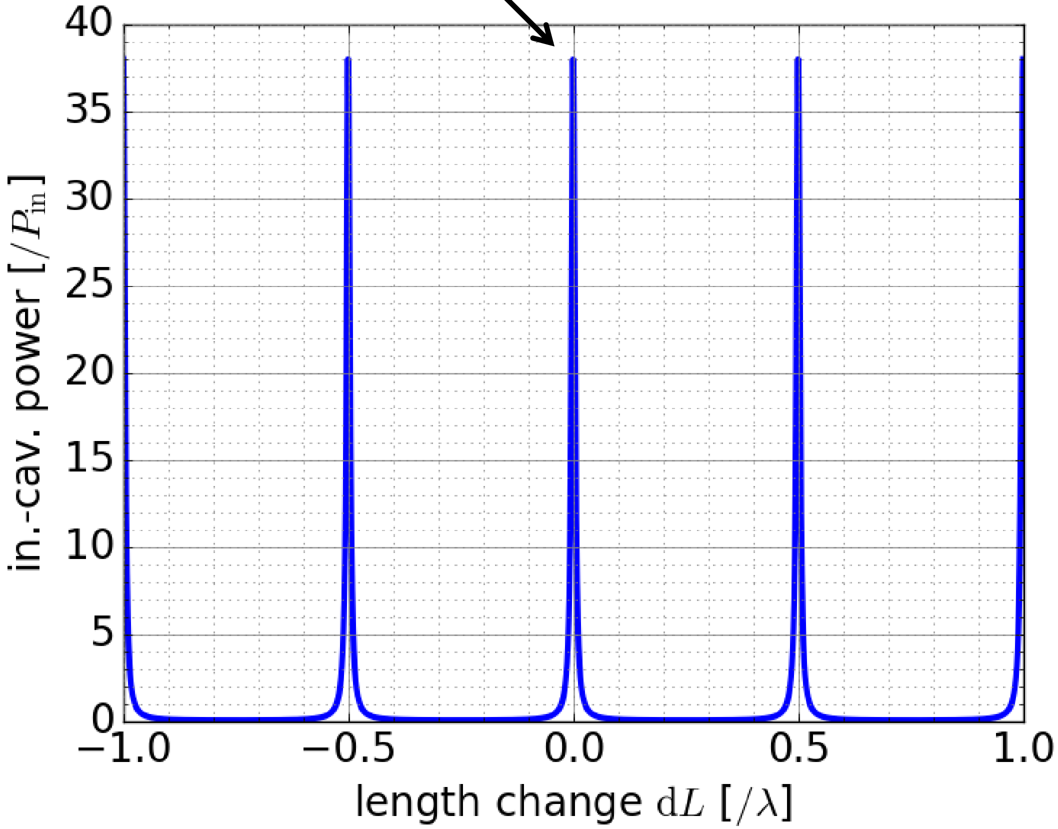
Higher finesse for higher reflectivity

Cavity Build-up

- Power inside the cavity

$$|E_{\mathrm{cav}}|^2 = \left| \frac{t_1}{1 - r_1 r_2 e^{i\frac{4\pi L}{\lambda}}} \right|^2 P_{\mathrm{in}}$$

Resonance



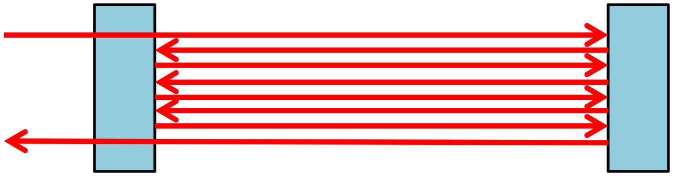
Intra-cavity power at resonance

$$|E_{\mathrm{cav}}|^2_{\mathrm{max}} = \left| \frac{t_1}{1 - r_1 r_2} \right|^2 P_{\mathrm{in}}$$

$$\simeq \boxed{\frac{2\mathcal{F}}{\pi}} P_{\mathrm{in}}$$

with $r_1 \sim 1$, $r_2 = 1$

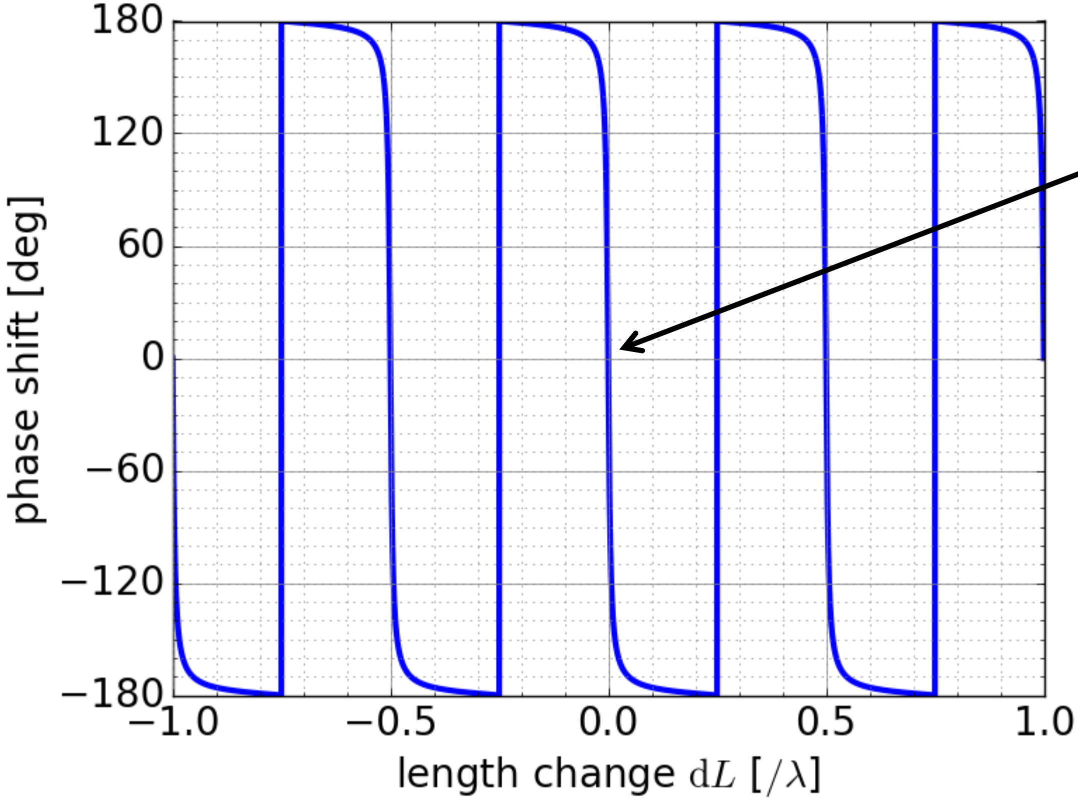
Cavity build-up



Phase of Reflected light

- Reflected field

$$E_{\mathrm{refl}} = \left(-r_1 + \frac{t_1^2 r_2 e^{i\frac{4\pi L}{\lambda}}}{1 - r_1 r_2 e^{i\frac{4\pi L}{\lambda}}} \right) E_{\mathrm{in}}$$



Phase of the reflected beam changes drastically at the resonance

$$\frac{\delta\phi}{\delta L} \simeq \frac{2\mathcal{F}}{\pi} \frac{4\pi}{\lambda}$$

Cavity build-up

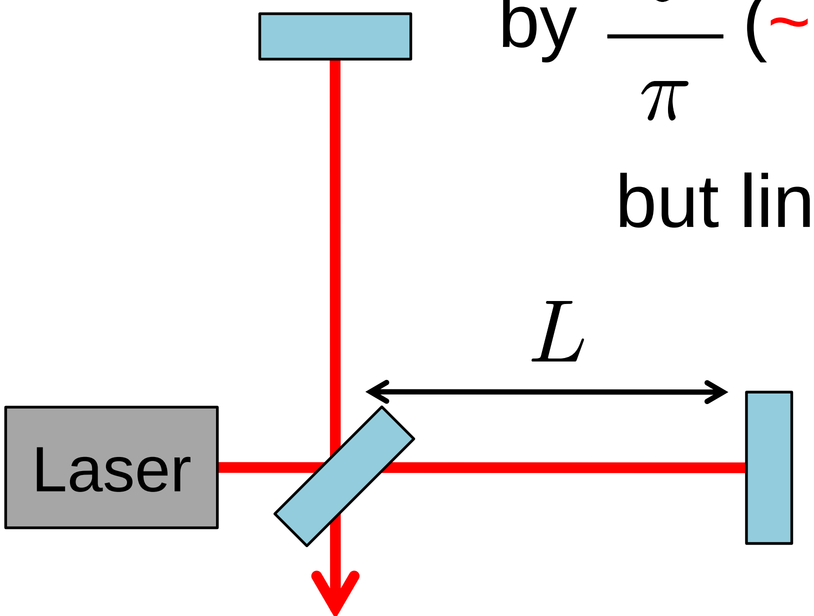
Michelson and Fabry-Pérot

- The phase of the reflected light is different by $\frac{2\mathcal{F}}{\pi}$

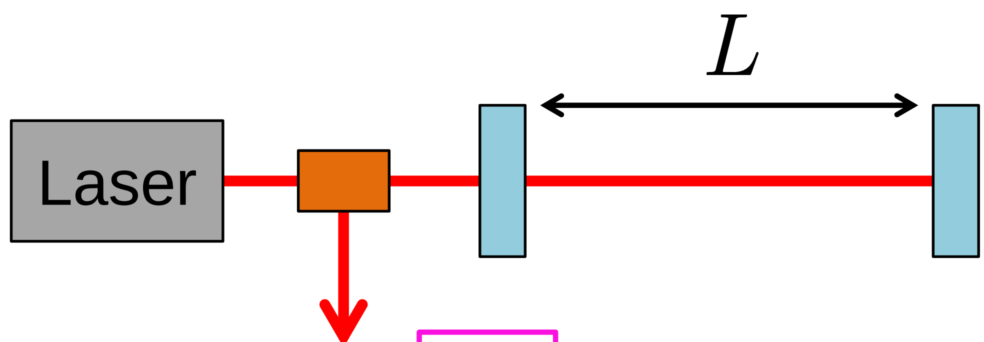
  $\rightarrow$ FP is more sensitive to mirror displacement

  by $\frac{2\mathcal{F}}{\pi}$ (~ finesse)

  but linear range is smaller



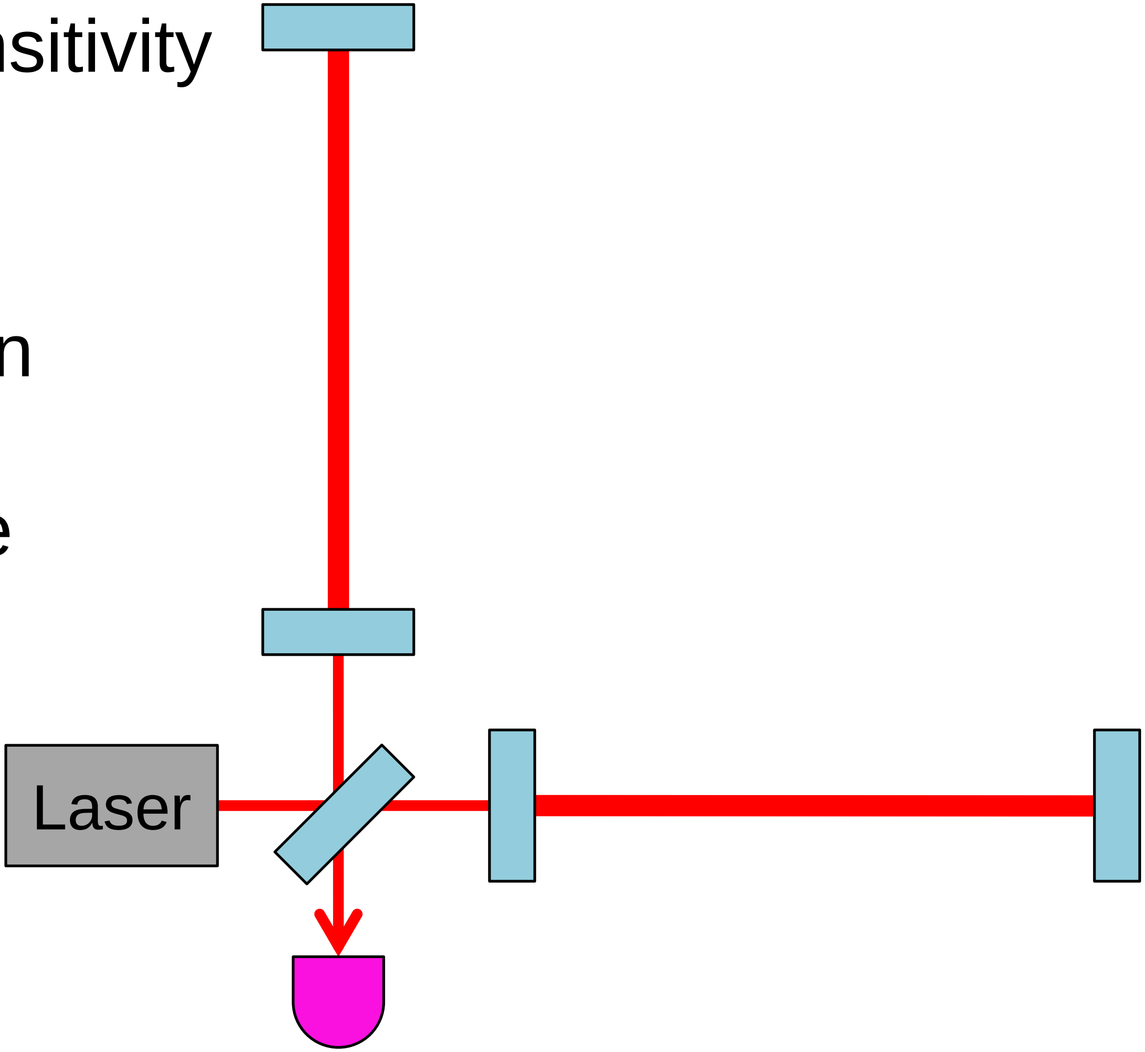
$$\phi = \frac{4\pi L}{\lambda}$$



$$\frac{\delta\phi}{\delta L} \simeq \boxed{\frac{2\mathcal{F}}{\pi}}\frac{4\pi}{\lambda}$$

Fabry-Pérot-Michelson Interferometer

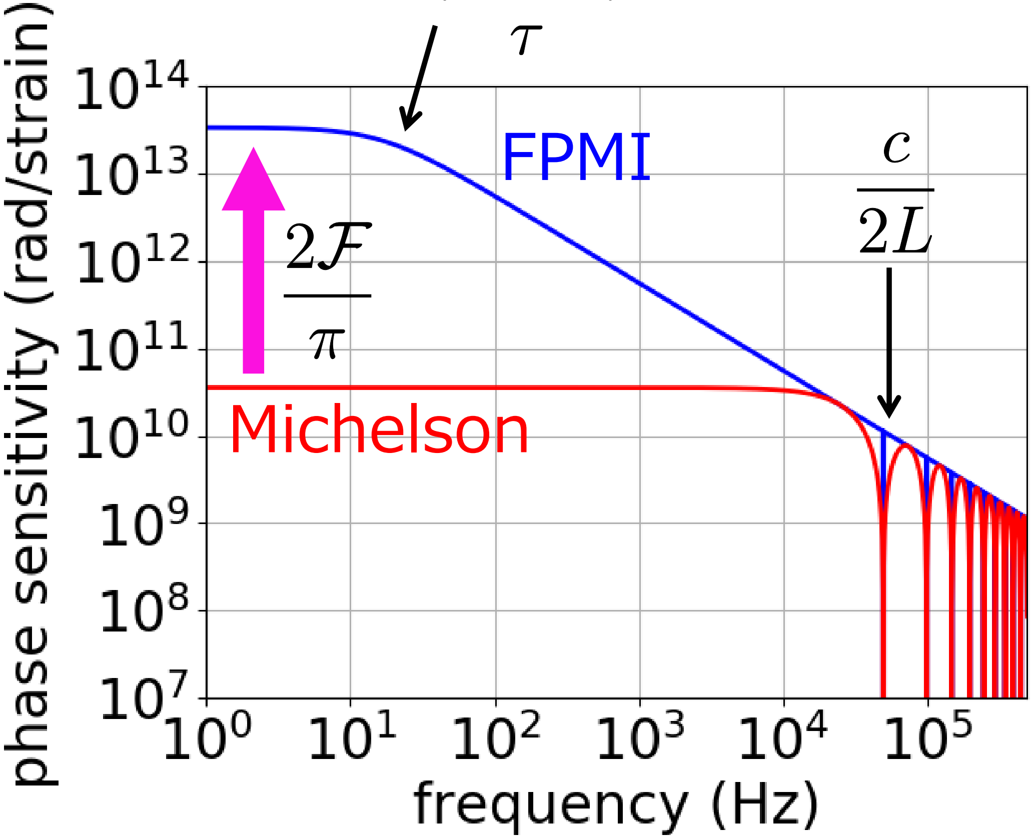
- Displacement sensitivity higher by $\frac{2\mathcal{F}}{\pi}$
- Commonly used in ground-based gravitational wave detectors

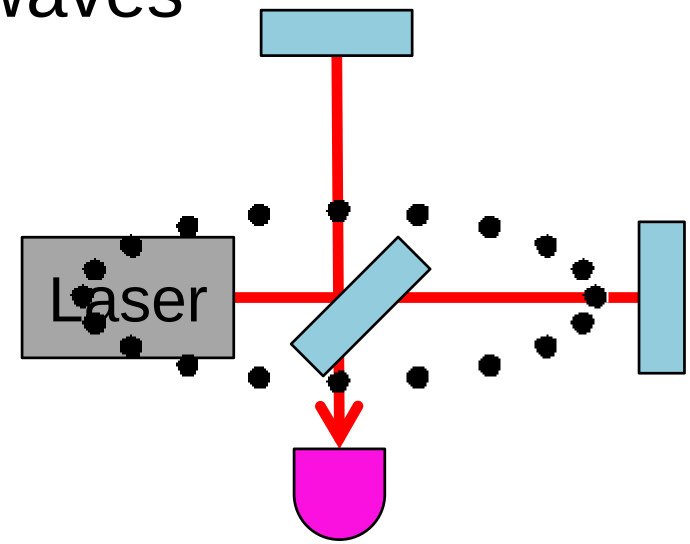


High-Frequency Response

- The effect of gravitational waves cancel at high frequencies

$$f_c = \frac{1}{2\pi}\left(\underbrace{\frac{2\mathcal{F}}{\pi}\frac{L}{c}}_{\tau}\right)^{-1} = \frac{c}{4L\mathcal{F}}$$





For a given frequency, there is a limit where longer arm length and higher finesse won't help increasing the sensitivity

Summary

- Gravitational waves create differential arm length change in Michelson interferometer
- Differential arm length change create power change at the output of the Michelson interferometer
- The signal can be enhanced by a factor of $\frac{2\mathcal{F}}{\pi}$ (Finesse: $\mathcal{F}$) by using Fabry-Pérot cavities
- The sensitivity at low frequencies can be increased with longer arm length and higher finesse

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