Displacement Noises in Laser Interferometric Gravitational Wave Detectors

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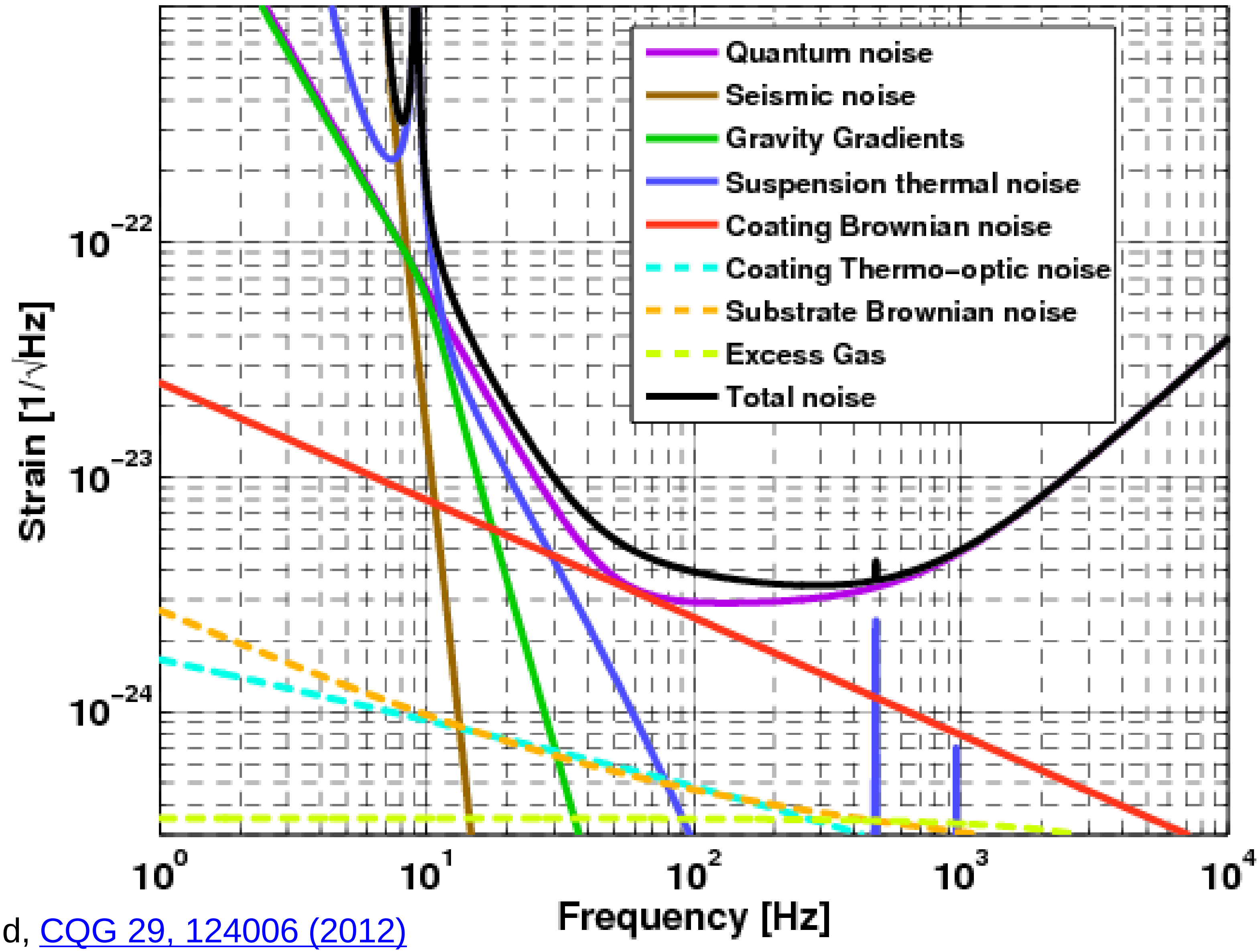
Slides

http://granite.phys.s.u-tokyo.ac.jp/michimura/lectures/GWPhysics20171212.pdf

or

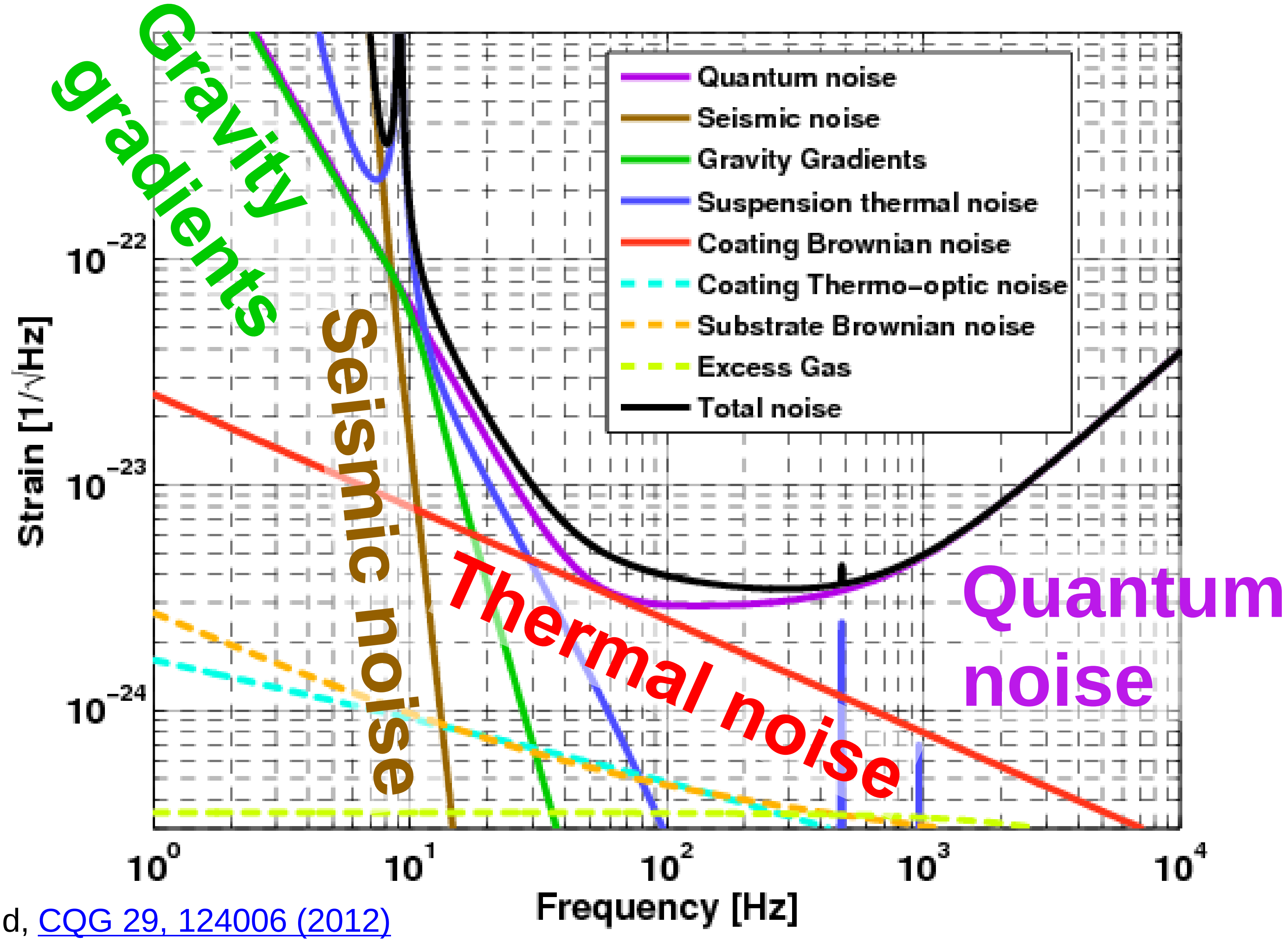
https://tinyurl.com/GWPhysics20171212

Advanced LIGO Design Sensitivity



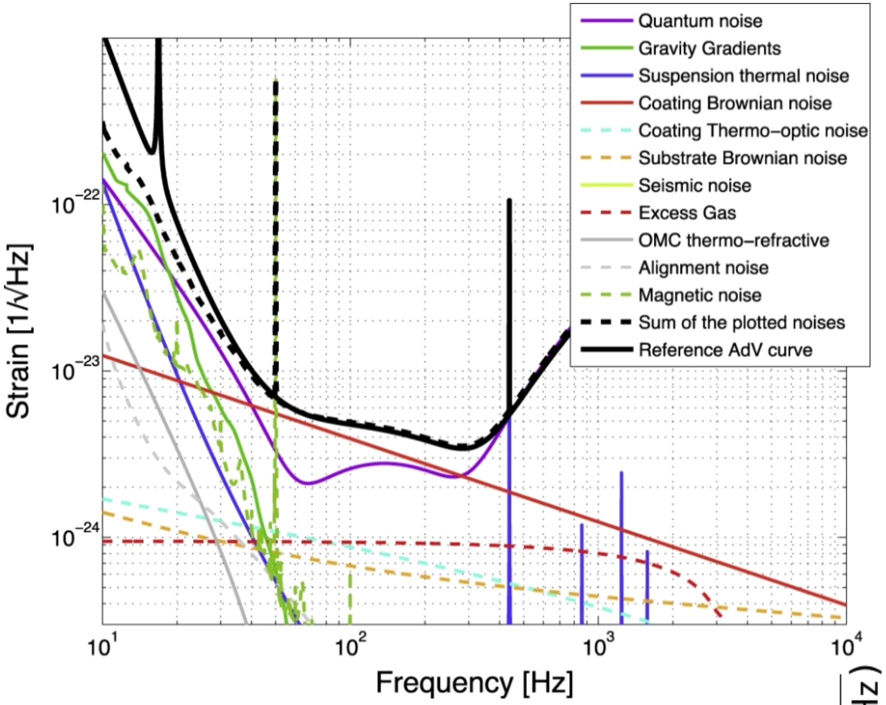
S. Hild, CQG 29, 124006 (2012)

Advanced LIGO Design Sensitivity

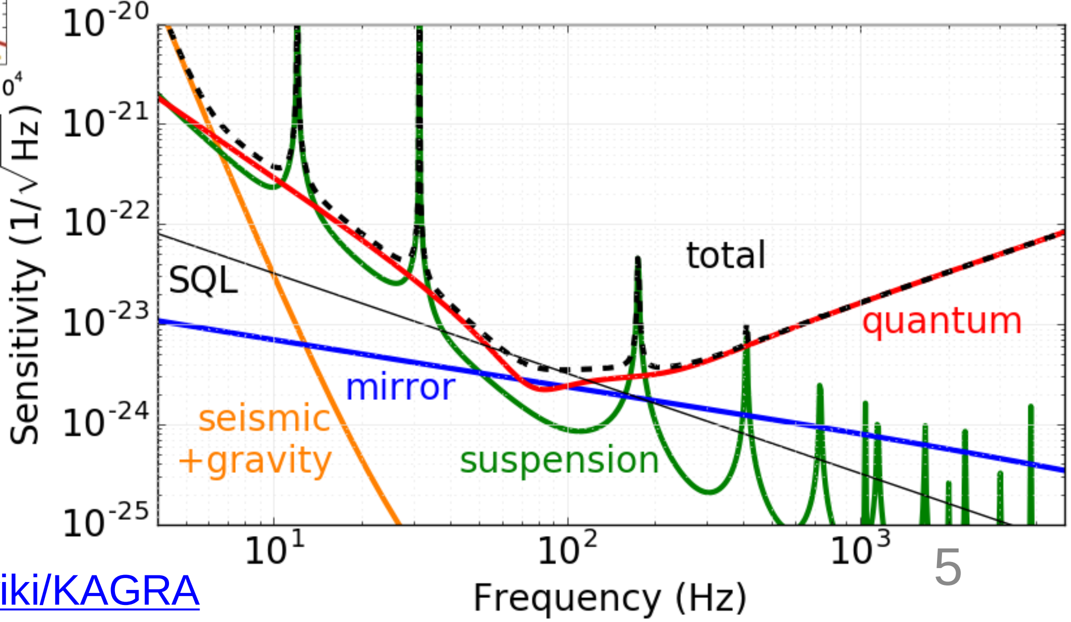


S. Hild, CQG 29, 124006 (2012)

Advanced Virgo and KAGRA



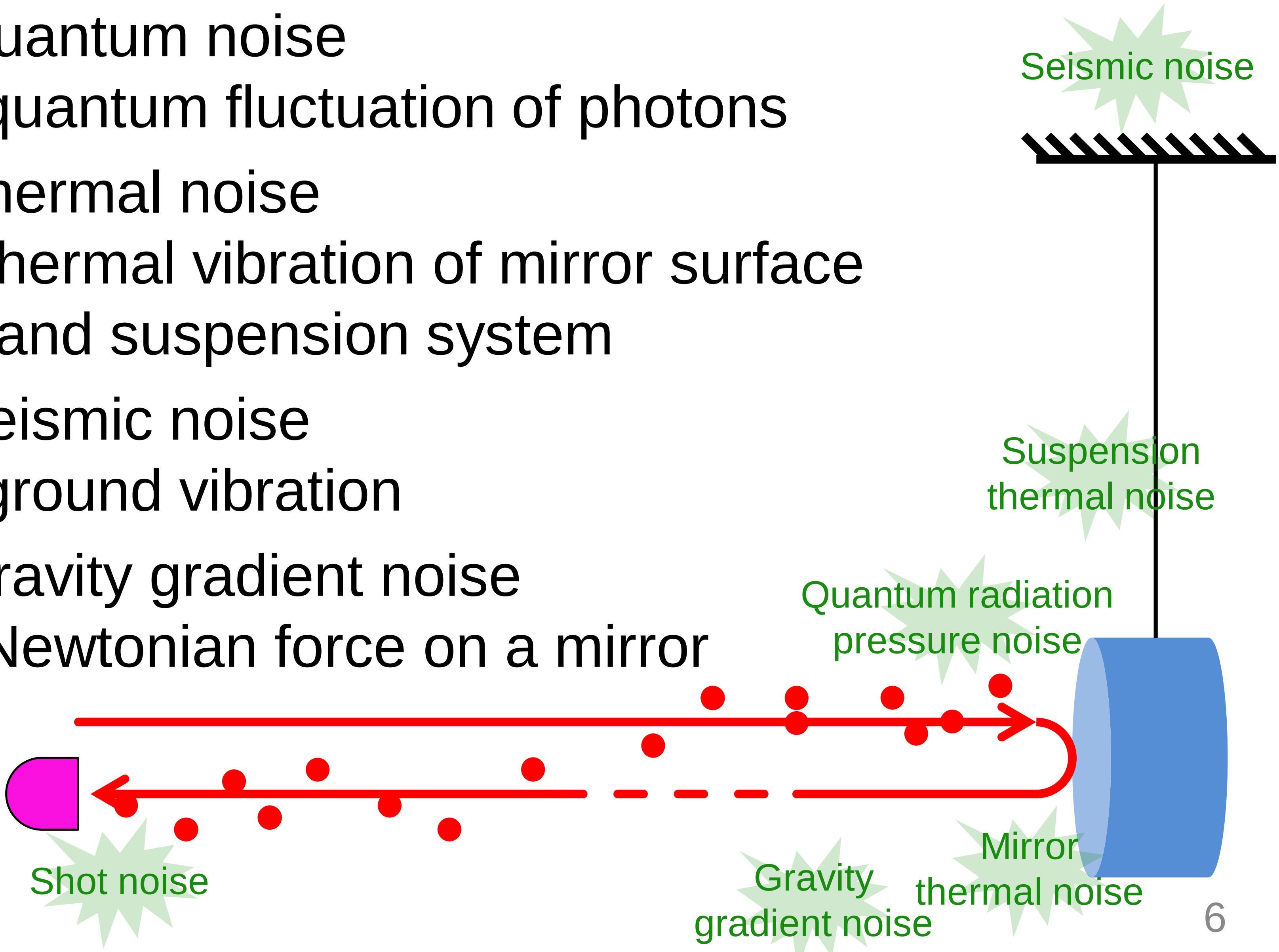
F. Acernese+, CQG 32, 024001 (2015)



http://gwwiki.icrr.u-tokyo.ac.jp/JGWwiki/KAGRA

Fundamental Noises

- Quantum noise
  quantum fluctuation of photons
- Thermal noise
  thermal vibration of mirror surface
  and suspension system
- Seismic noise
  ground vibration
- Gravity gradient noise
  Newtonian force on a mirror



Contents

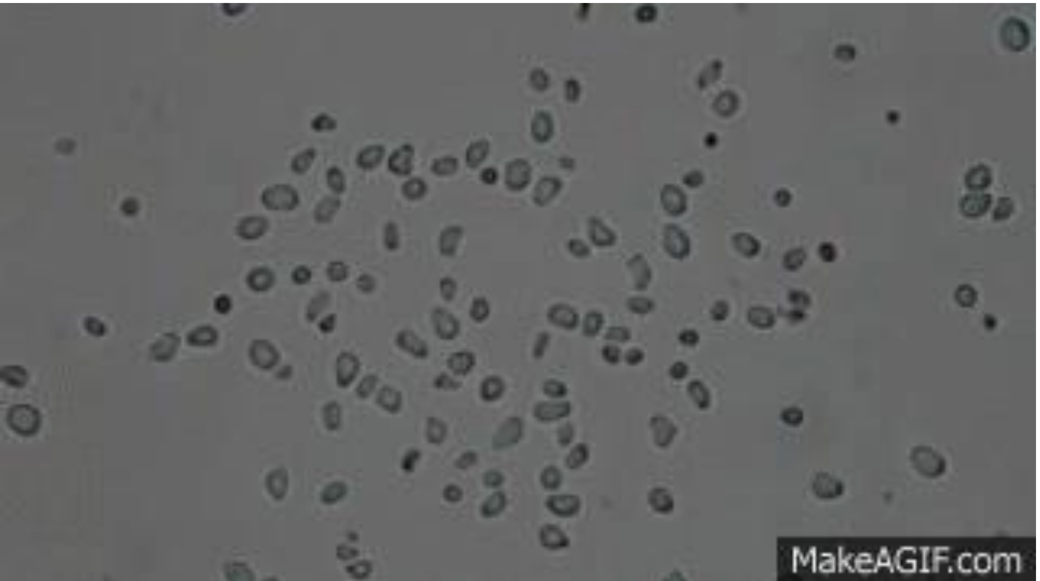
- Thermal noise
  - Brownian motion, Johnson-Nyquist noise
  - Fluctuation-dissipation theorem
  - Suspension thermal noise
  - Mirror thermal noise
  - Coating thermal noise

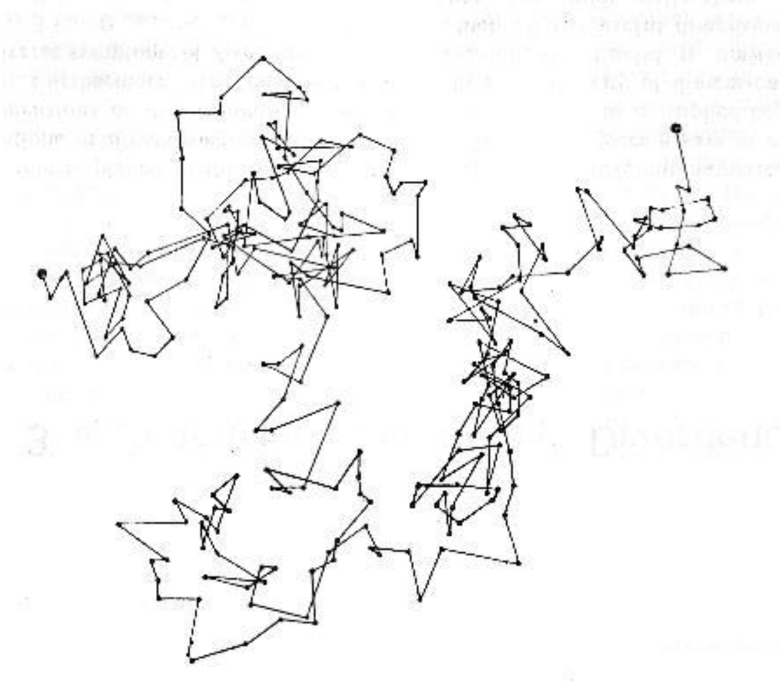
- Seismic noise
  - Seismic vibration
  - Vibration isolation
  - Gravity gradient noise
  - Underground site

Thermal Noise

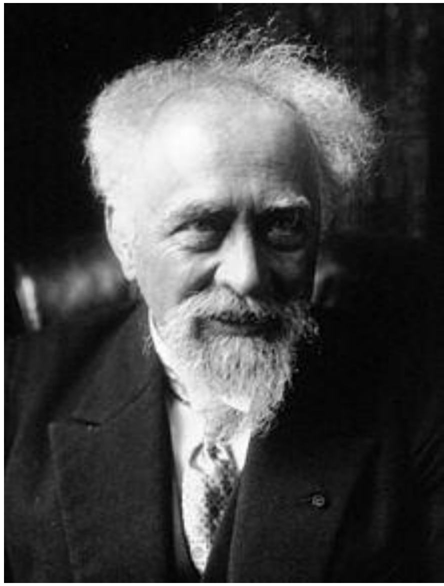
Brownian Motion

- 1827 Robert Brown
  Discovered random motion of pollen grains in water
- Also proved that this motion exists in non-organic matter
  → not related to life





Molecular-Kinetic Theory



- 1905 Albert Einstein
  Molecular-Kinetic Theory
  → Quantitative calculations on
  the movement of small particles in
  a liquid (Ph. D. Thesis)
- 1908 Jean B. Perrin
  Experimental verification using
  colloid liquid

→ Existence of atoms
Statistical physics

Johnson-Nyquist Noise

- 1926 John B. Johnson observed voltage fluctuation across a resister
- 1928 Harry Nyquist

$$V^2(f) = 4k_\mathrm{B}TR$$

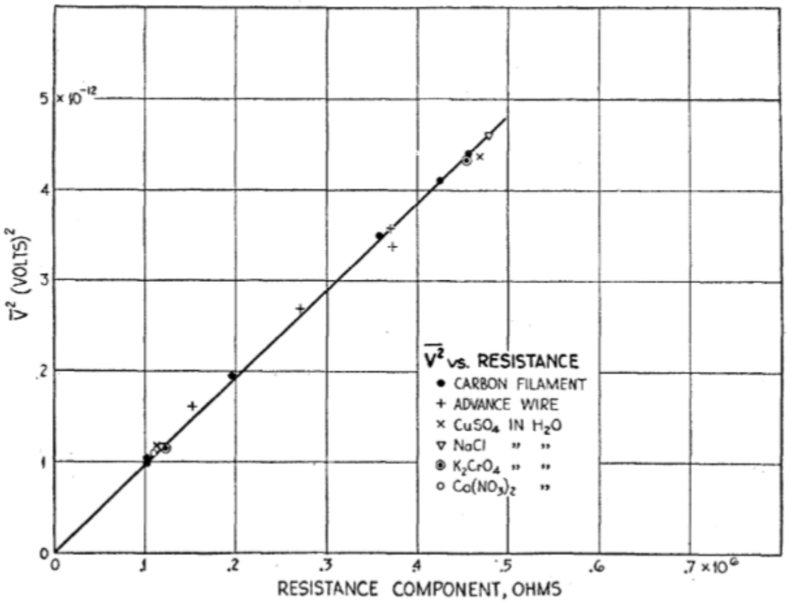
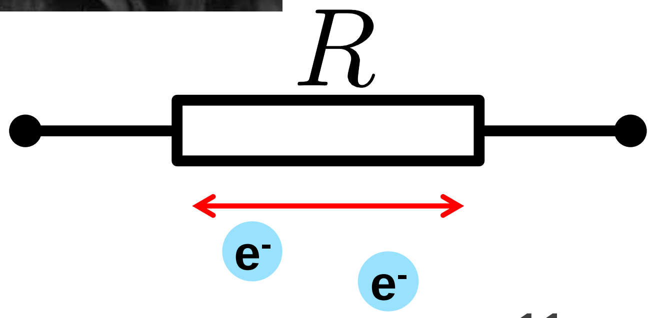


Fig. 4. Voltage-squared vs. resistance component for various kinds of conductors.

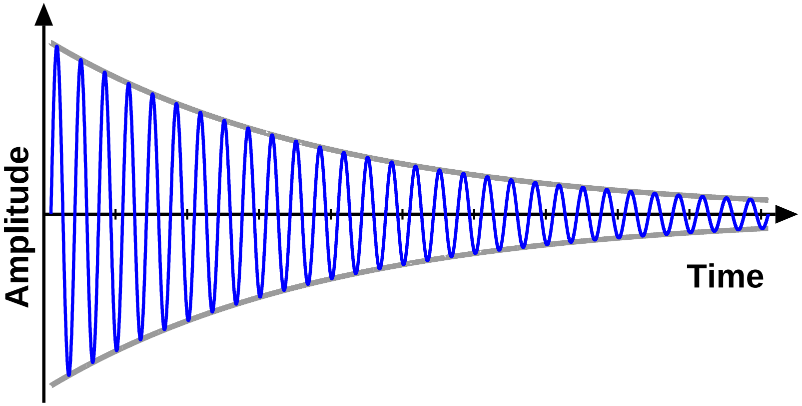


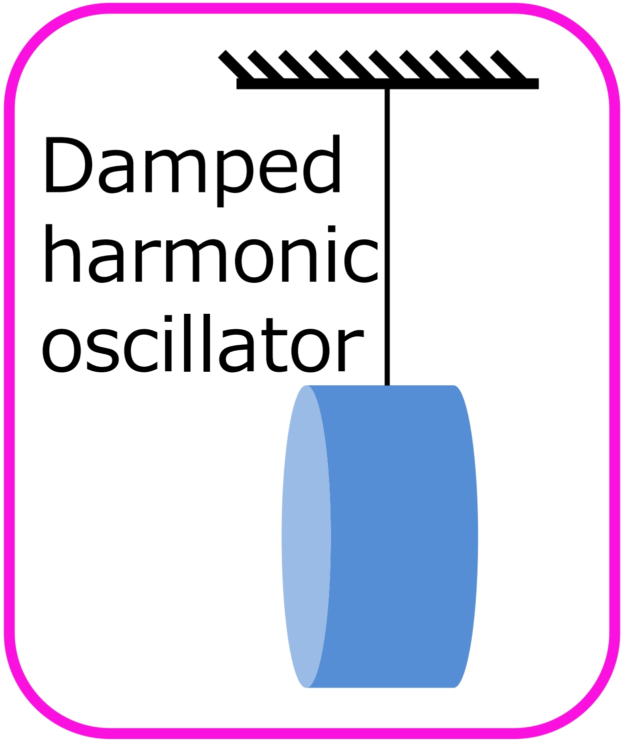
J. B. Johnson, Phys. Rev. 32, 97 (1928)

Generalization of Brownian Motion

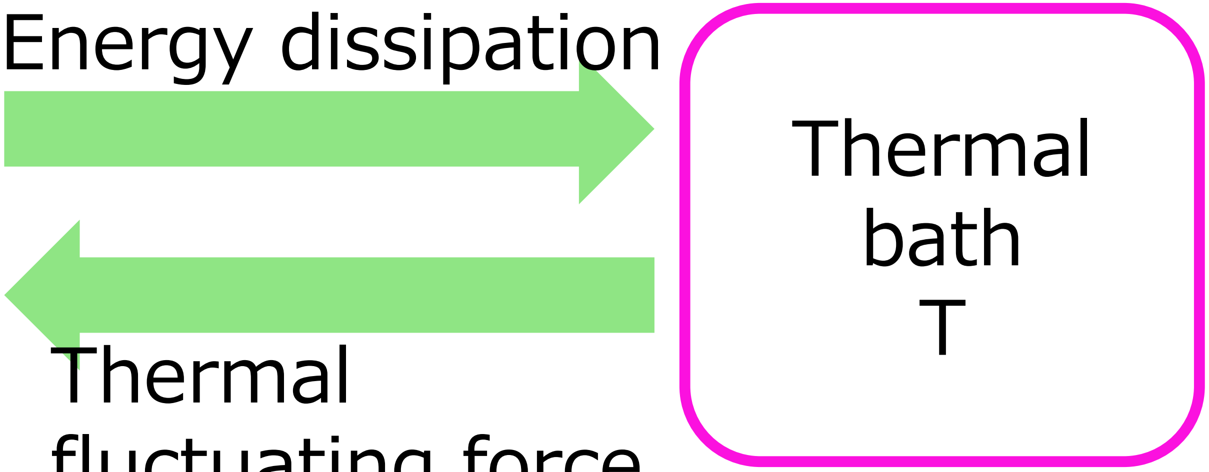
- 1951-52 Herbert Callen et al.

  Fluctuation-Dissipation Theorem





Damped harmonic oscillator



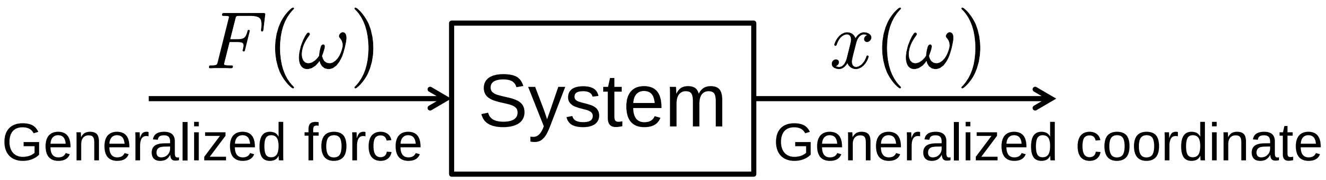
Energy dissipation

Thermal fluctuating force

Thermal bath
T

Fluctuation-Dissipation Theorem

- Linear System



- System impedance

$$Z(\omega) = \frac{F(\omega)}{v(\omega)} = \frac{F(\omega)}{i\omega x(\omega)}$$

- Thermal fluctuating force for x is

$$F_{\mathrm{th}}^2(\omega) = 4k_{\mathrm{B}}T\mathrm{Re}(Z(\omega))$$

- Analogy with Johnson-Nyquist noise
  Voltage V ↔ Force F
  Current I ↔ Velocity v
  Resistor R ↔ Real part of impedance Z

Fluctuation-Dissipation Theorem

- Thermal fluctuating force for x is

$$F_{\mathrm{th}}^2(\omega) = 4k_{\mathrm{B}}T\mathrm{Re}(Z(\omega))$$

- Thermal fluctuation of x will be

$$x_{\mathrm{th}}^2(\omega) = \frac{F_{\mathrm{th}}^2(\omega)}{\omega^2|Z(\omega)|^2}$$
$$= \frac{4k_{\mathrm{B}}T}{\omega^2}\mathrm{Re}(Y(\omega))$$

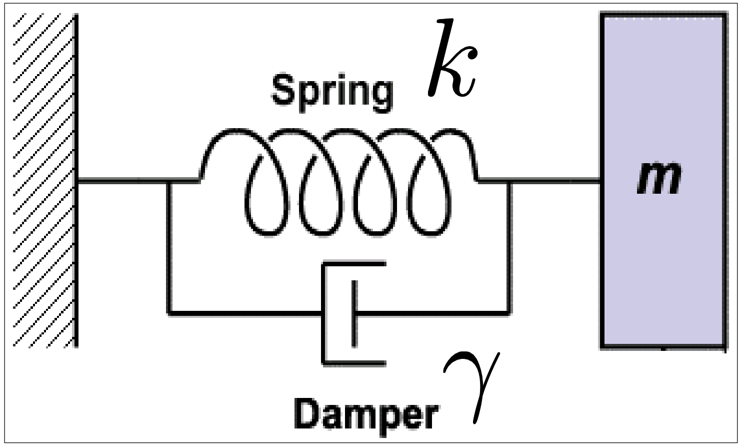
admittance

$$Y(\omega) \equiv Z^{-1}(\omega)$$

Harmonic Oscillator

- Damped harmonic oscillator (viscous damping)

$$m\ddot{x} + \gamma\dot{x} + kx = F$$



- Admittance

$$Y(\omega) = \frac{i\omega x(\omega)}{F(\omega)}$$

$$= \frac{i\omega}{-\omega^2 m + i\omega\gamma + k}$$

$$= \frac{\gamma + i(k/\omega - \omega m)}{\gamma^2 + (k/\omega - \omega m)^2}$$

- Thermal noise

$$x_{\mathrm{th}}^2(\omega) = \frac{4k_{\mathrm{B}}T}{\omega^2}\mathrm{Re}(Y(\omega)) = \frac{4k_{\mathrm{B}}T\gamma}{\gamma^2\omega^2 + (k - m\omega^2)^2}$$

Harmonic Oscillator

- Resonant frequency and Q value

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad\qquad Q = \frac{m\omega_0}{\gamma}$$

- Thermal noise

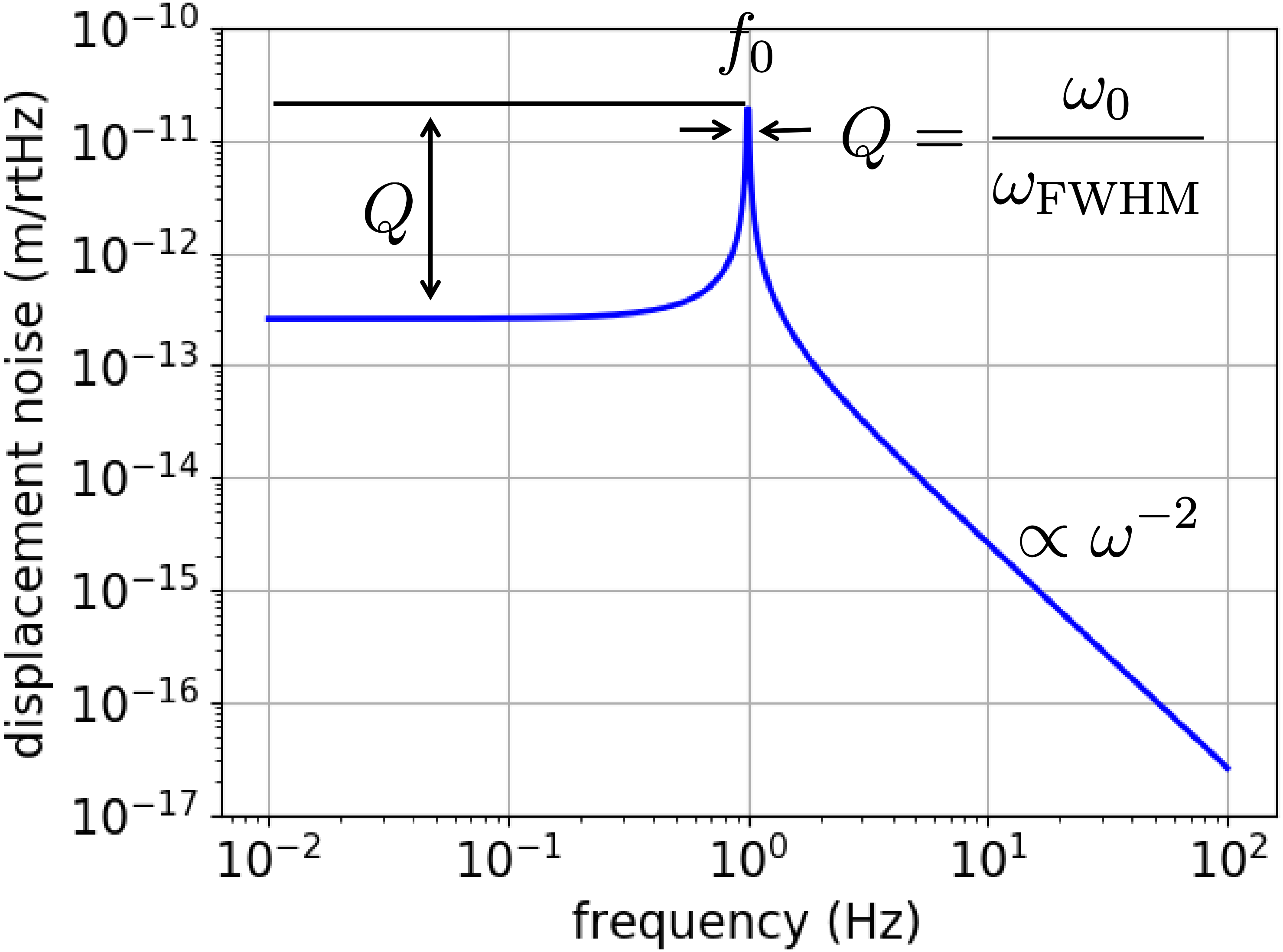
$$\begin{aligned} x_{\mathrm{th}}^2(\omega) &= \frac{4k_{\mathrm{B}}T\gamma}{\gamma^2\omega^2 + (k - m\omega^2)^2} \\ &= \frac{4k_{\mathrm{B}}T\omega_0}{mQ} \frac{1}{(\omega^2 - \omega_0^2)^2 + \frac{\omega_0^2\omega^2}{Q^2}} \end{aligned}$$

$$\omega \ll \omega_0 \qquad \omega = \omega_0 \qquad \omega \gg \omega_0$$

$$\frac{4k_{\mathrm{B}}T}{m\omega_0^3}\frac{1}{Q} \qquad \frac{4k_{\mathrm{B}}T}{m\omega_0^3}Q \qquad \frac{4k_{\mathrm{B}}T\omega_0}{m\omega^4}\frac{1}{Q}$$

Harmonic Oscillator

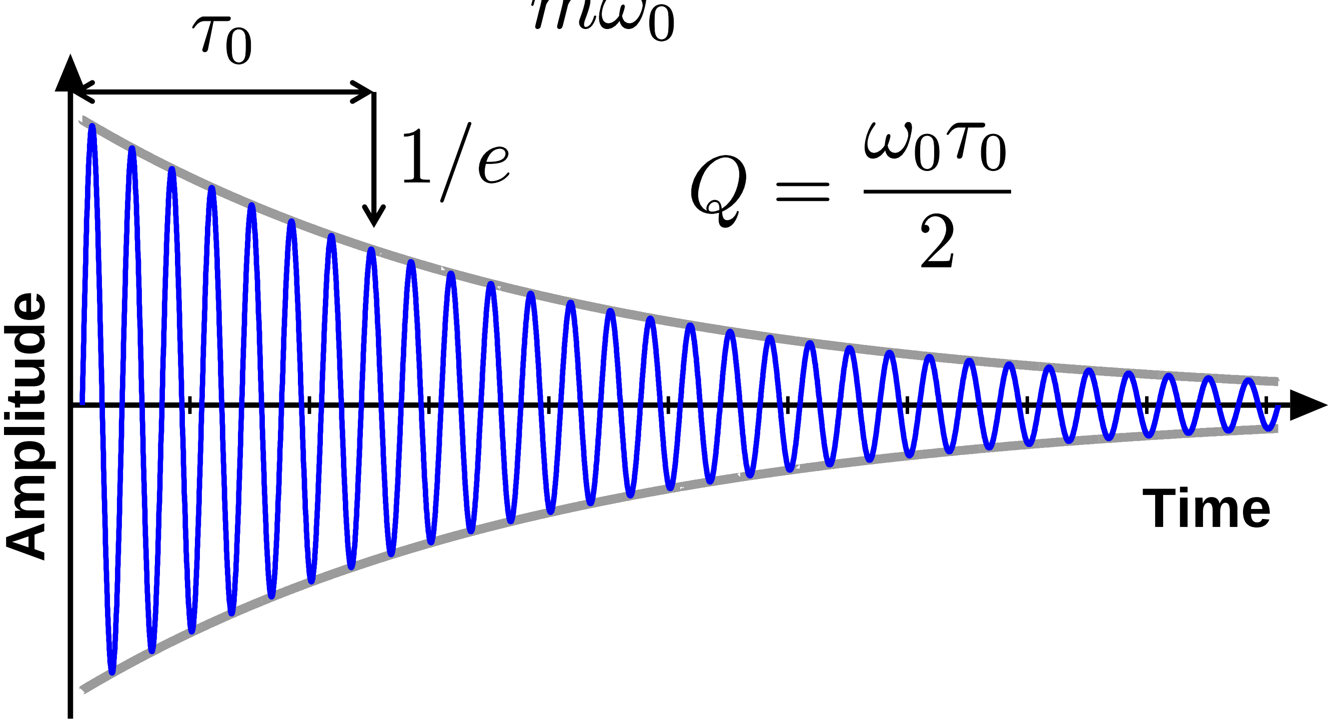
- Thermal noise spectrum



Harmonic Oscillator

- Resonant frequency and Q-value can be measured from the impulse response

$$x(t) = \frac{1}{m\omega_0} e^{-\frac{\omega_0 t}{2Q}} \sin \omega_0 t$$



$$Q = \frac{\omega_0 \tau_0}{2}$$

tuning fork

Structural Damping

- Viscous damping
  (e.g. gas damping, eddy-current damping)

$$m\ddot{x} + \gamma\dot{x} + kx = F$$

- Structural damping
  (e.g. internal friction of materials)
  - elastic material

$$F_{\text{spring}} = -kx$$ (Hooke’s Law)

  - anelastic material

loss angle

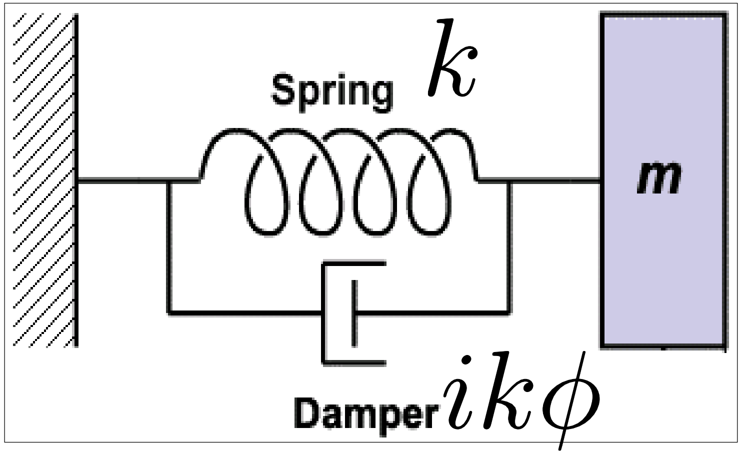
$$F_{\text{spring}} = -k(1 + i\phi(\omega))x$$

Delay between displacement and force
→ energy dissipation

Structural Damping

- Damped harmonic oscillator (structural damping)

$$m\ddot{x} + k(1 + i\phi)x = F$$



- Admittance

$$Y(\omega) = \frac{k\phi\omega + i(k\omega - m\omega^3)}{k^2\phi^2 - (k - m\omega^2)^2}$$

- Thermal noise

$$x_{\mathrm{th}}^2(\omega) = \frac{1}{\omega}\frac{4k_{\mathrm{B}}Tk\phi}{k^2\phi^2 + (k - m\omega^2)^2}$$

Structural Damping

- Resonant frequency and Q value

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad\qquad Q = \frac{1}{\phi(\omega)}$$

- Thermal noise

$$x_{\mathrm{th}}^2(\omega) = \frac{1}{\omega}\frac{4k_{\mathrm{B}}Tk\phi}{k^2\phi^2 + (k - m\omega^2)^2}$$

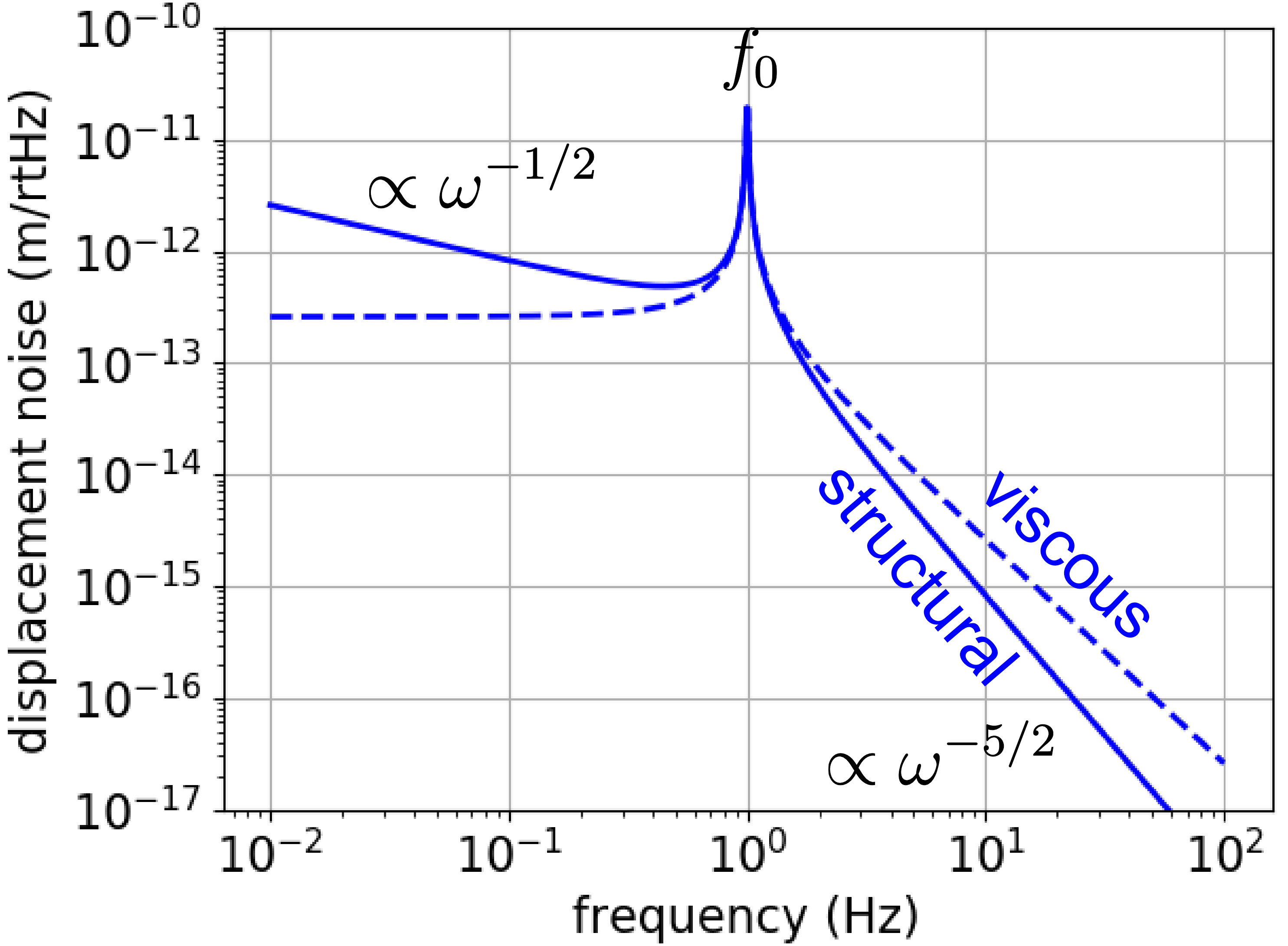
$$= \frac{4k_{\mathrm{B}}T\omega_0^2\phi}{m\omega}\frac{1}{\omega_0^4\phi^2 + (\omega^2 - \omega^2)^2}$$

$$\omega \ll \omega_0 \qquad\qquad \omega = \omega_0 \qquad\qquad \omega \gg \omega_0$$

$$\frac{4k_{\mathrm{B}}T}{m\omega_0^2}\frac{\phi}{\omega} \qquad\qquad \frac{4k_{\mathrm{B}}T}{m\omega_0^3}\frac{1}{\phi} \qquad\qquad \frac{4k_{\mathrm{B}}T\omega_0}{m\omega^4}\frac{\phi}{\omega}$$

Structural Damping

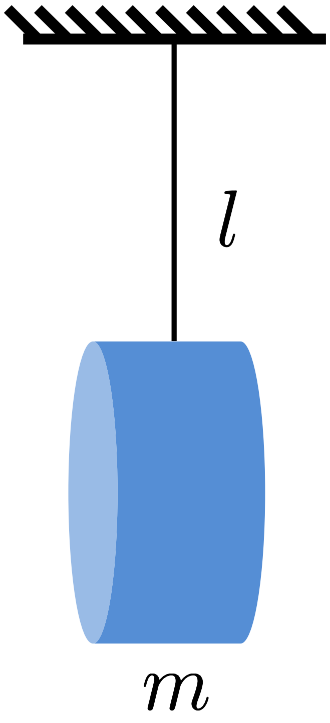
- Thermal noise spectrum when $\phi(\omega) \sim \mathrm{const.}$



Pendulum Thermal Noise

- Restoring force

$$F_{\mathrm{spring}} = -(mg/l + k_{\mathrm{el}}(1+i\phi))x$$
$$= -(mg/l + k_{\mathrm{el}})(1+i\phi_{\mathrm{p}})x$$



- Gravitational dilution

$$\phi_{\mathrm{p}} \equiv \underbrace{\frac{k_{\mathrm{el}}}{mg/l + k_{\mathrm{el}}}}\phi \simeq \frac{\phi}{2l}\sqrt{\frac{EI}{mg}}$$

loss angle of pendulum
10^{-5}~10^{-7}

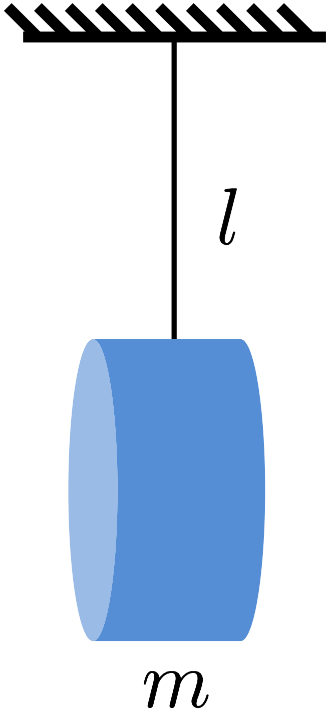
dilution factor
10^{-3}~10^{-2}

intrinsic loss angle of wire 10^{-4}~10^{-3}

$$k_{\mathrm{el}} = \frac{\sqrt{TEI}}{2l^2} = \frac{\sqrt{mgEI}}{2l^2}$$

T: wire tension
E: wire Young's modulus
I: wire moment of inertia

Pendulum Thermal Noise



- Suspension thermal noise ($\omega \gg \omega_0$)

$$x_{\mathrm{susp}}^2(\omega) = \frac{4k_{\mathrm{B}}T\omega_0}{m\omega^4}\frac{\phi_{\mathrm{p}}}{\omega}$$

$$= \frac{4k_{\mathrm{B}}T}{m\omega^5}\sqrt{\frac{g}{l}}\frac{\phi}{2l\omega}\sqrt{\frac{EI}{mg}}$$

- To lower the thermal noise
  - lower temperature $x_{\mathrm{susp}}(\omega) \propto \sqrt{T\phi}$
  - lower loss angle
  - longer suspension $x_{\mathrm{susp}}(\omega) \propto l^{-3/2}$
  - heavier mirror

$x_{\mathrm{susp}}(\omega) \propto m^{-1/4}$ assuming $I \propto A^2$

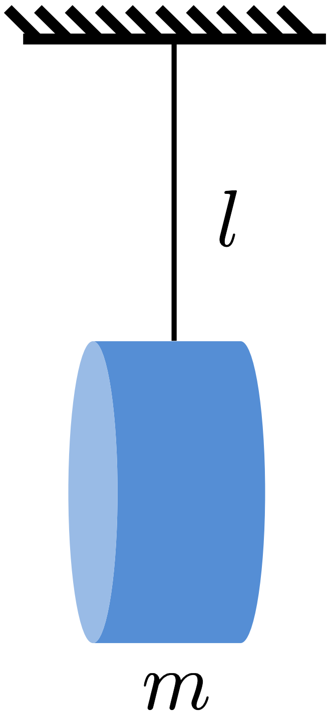
A: wire cross section $m \propto A$

Pendulum Thermal Noise

- Suspension thermal noise ($\omega \gg \omega_0$)

$$x^2_{\mathrm{susp}}(\omega) = \frac{4k_{\mathrm{B}}T\omega_0}{m\omega^4}\frac{\phi_{\mathrm{p}}}{\omega}$$

- Calculate suspension thermal noise for m=10 kg, f_0= 1Hz, T = 300 K, φ_p = 10^{-6} at 100 Hz. $k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J/K}$



Pendulum Thermal Noise

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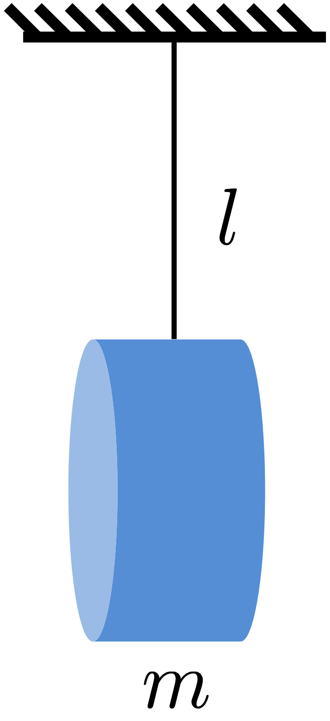
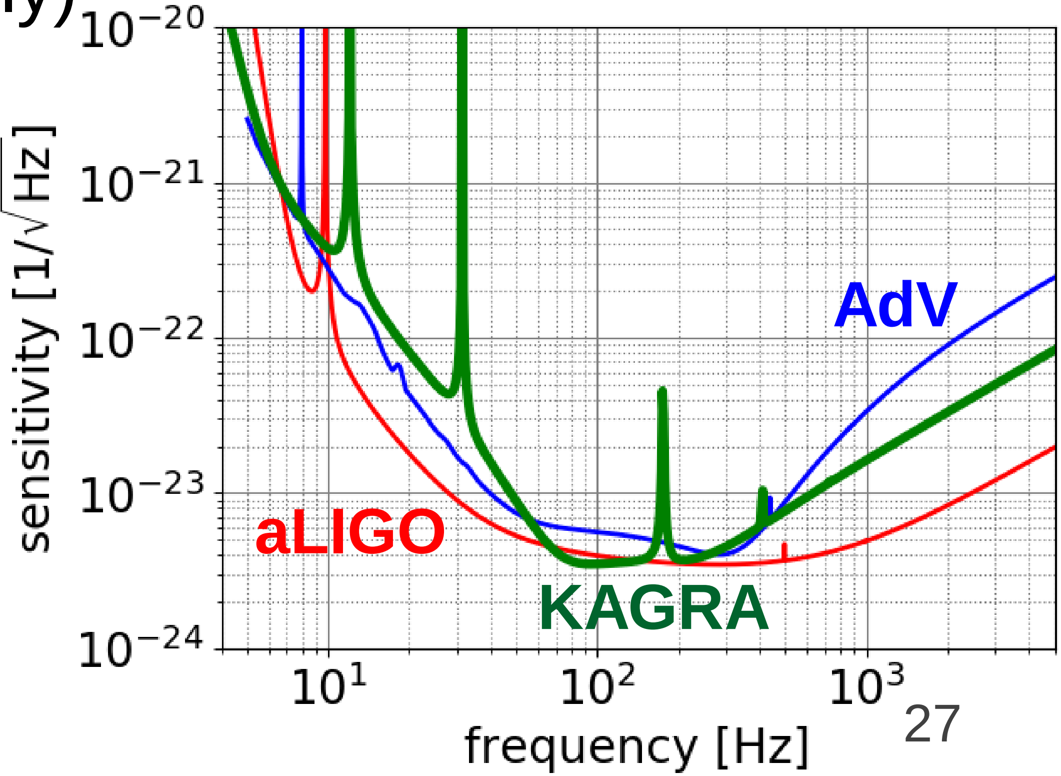


$$x_{\mathrm{susp}}(\omega) = 1.0 \times 10^{-20}\left(\frac{100\,\mathrm{Hz}}{f}\right)^{5/2}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$$

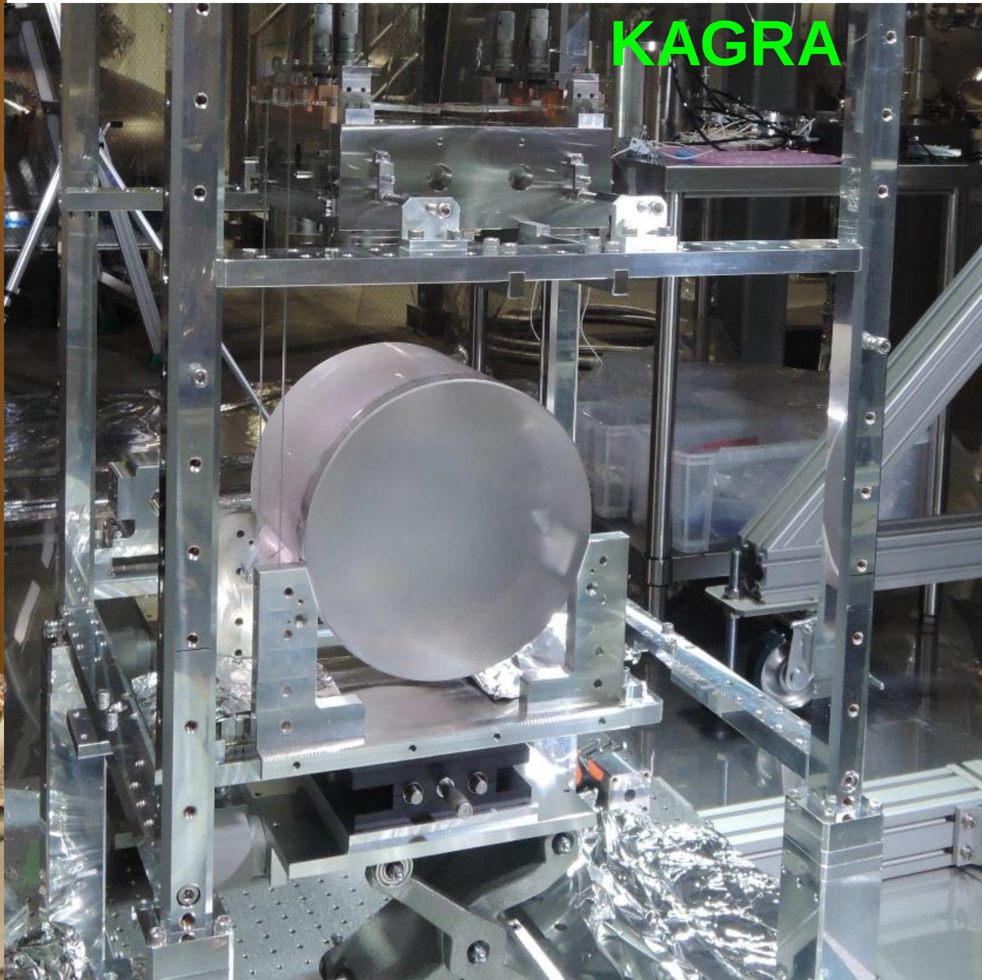
Sufficient for GW detection!

Suspension Thermal Noise in 2G

- Advanced LIGO (US)
  40 kg, 60 cm,
  fused silica,
  295 K, φw=~1e-7
- Advanced Virgo (Italy)
  42 kg, 70 cm,
  fused silica,
  295 K, φw=~1e-7
- KAGRA (Japan)
  23 kg, 23 cm,
  sapphire,
  22 K, φw=2e-7



Suspensions



Challenges in Cryogenics

- KAGRA is the first km-scale GW detector to use cryogenics to reduce thermal noise
- Fused silica has large mechanical loss at cryogenic temperatures

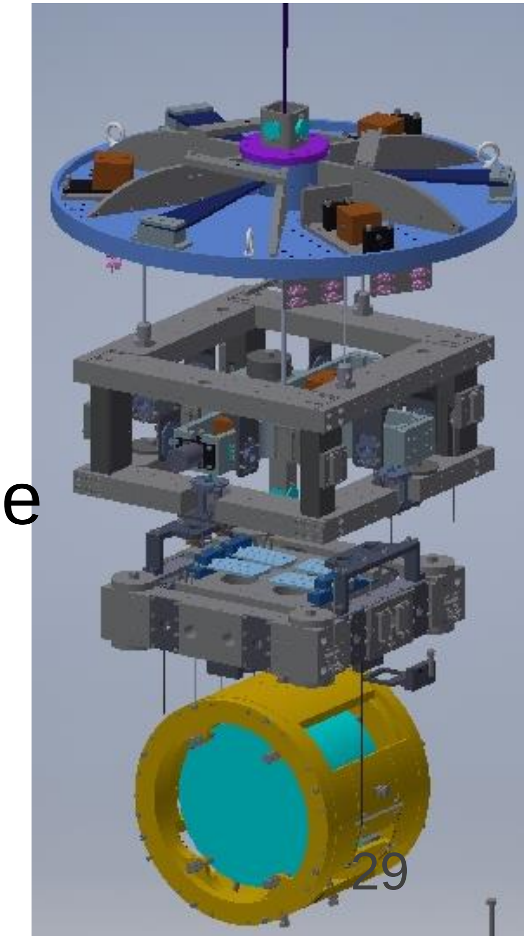
  → sapphire
- Limitation in mirror size: 23 kg

  (small compared with aLIGO/AdV)

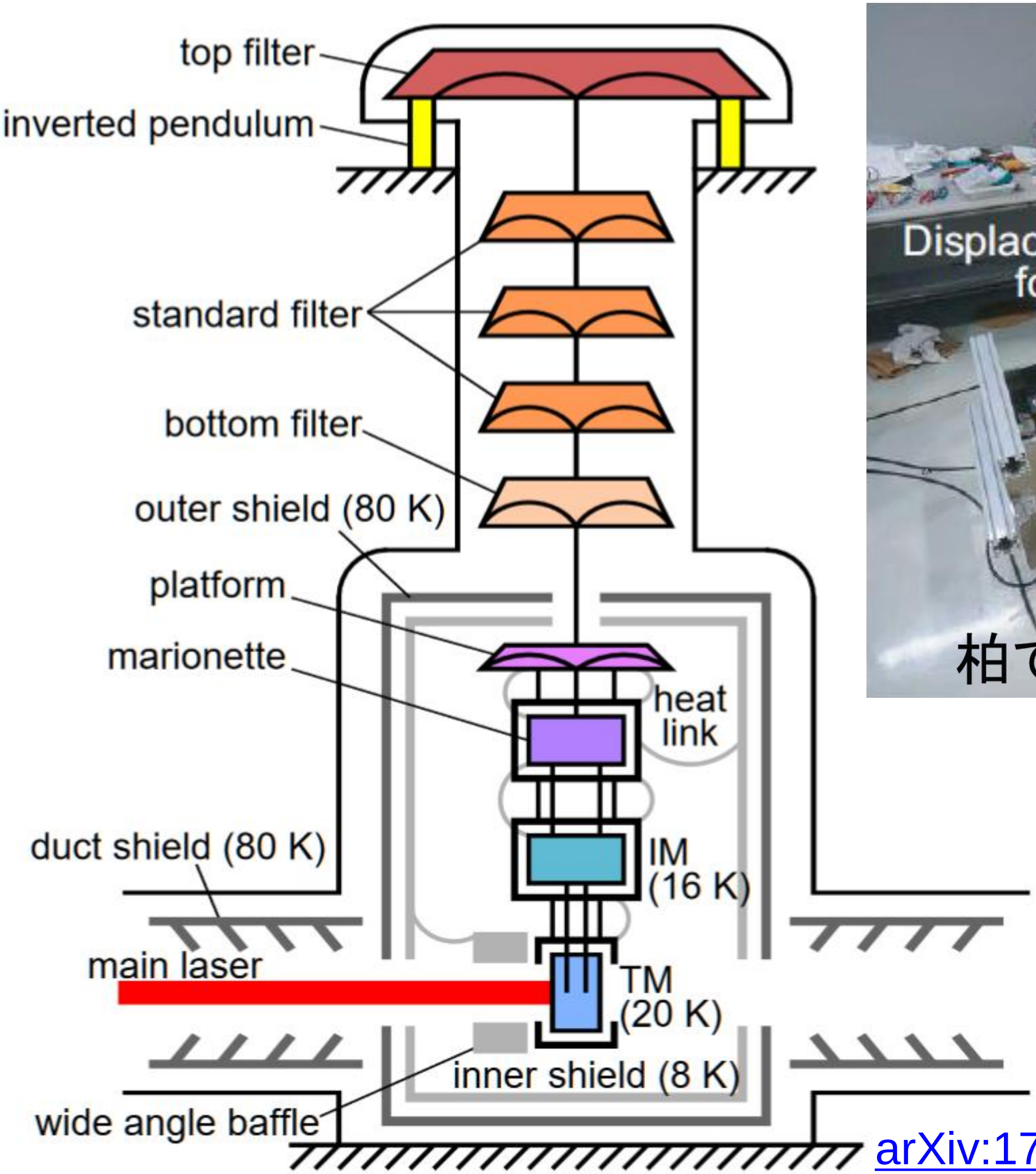
  → smaller beam size

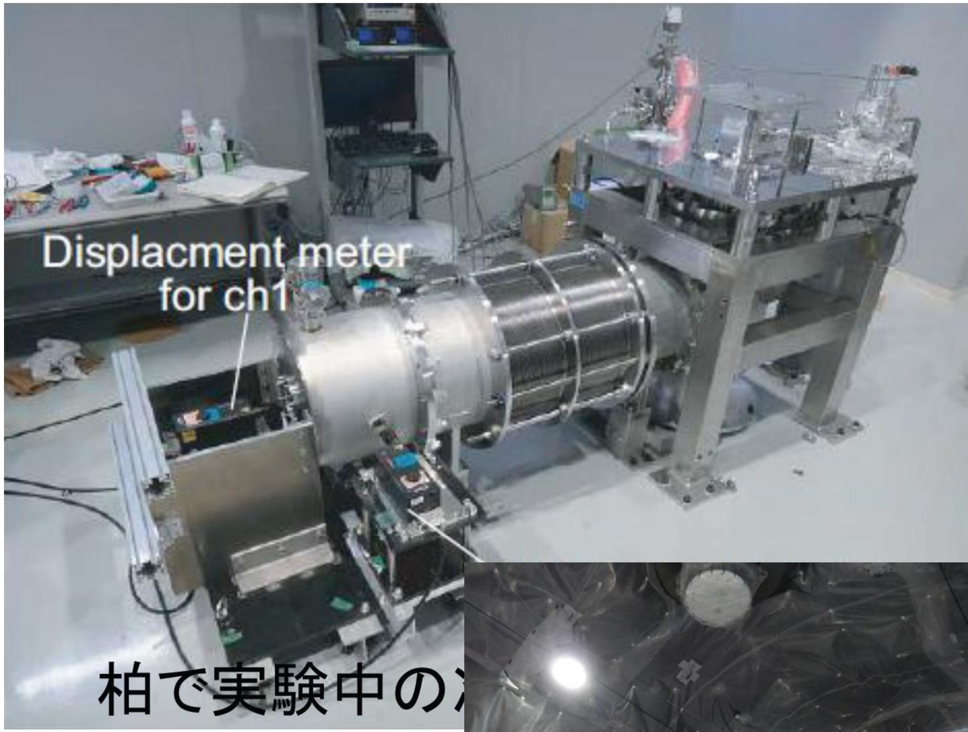
  worse quantum and thermal noise
- Thicker wire for heat extraction

  → larger suspension thermal noise
- Vibration of cryocoolers



KAGRA Cryogenics

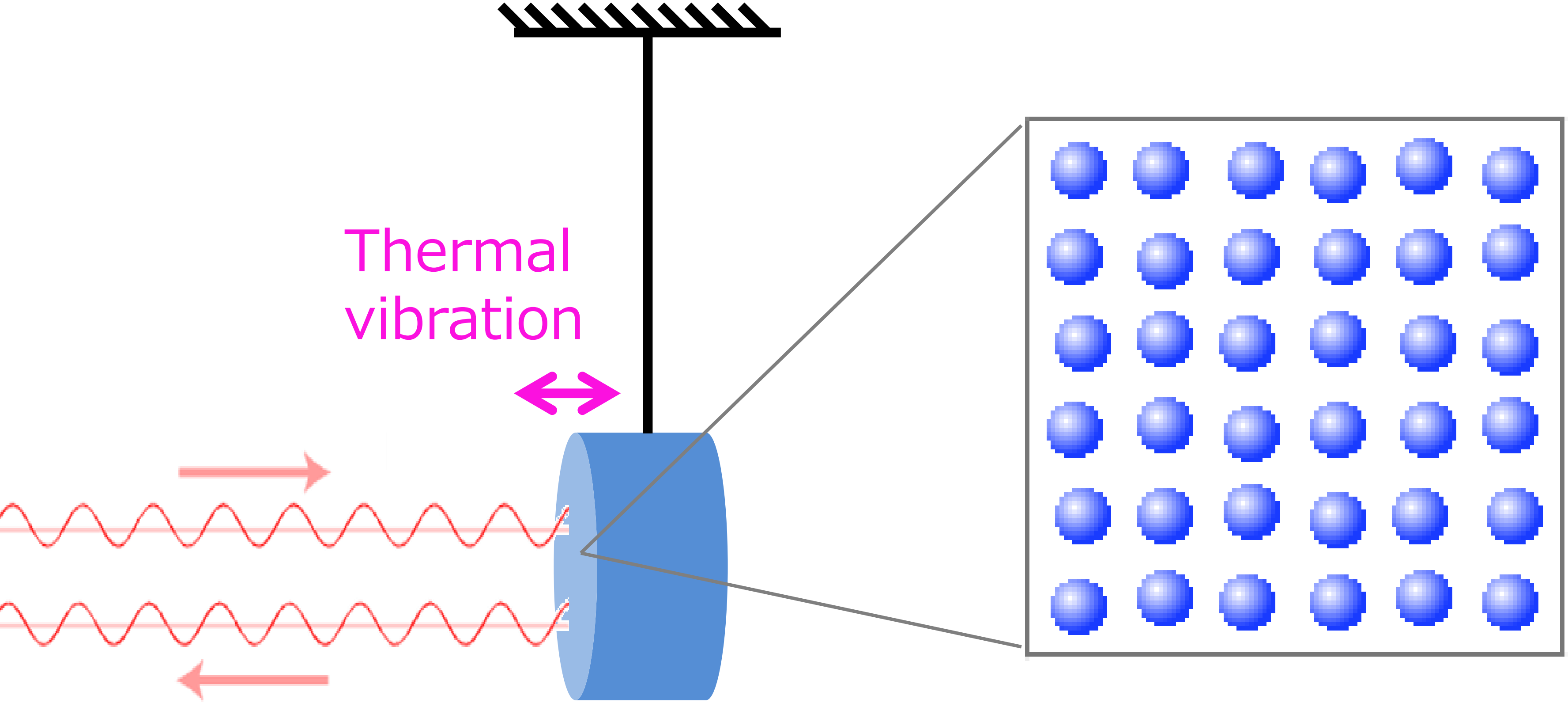




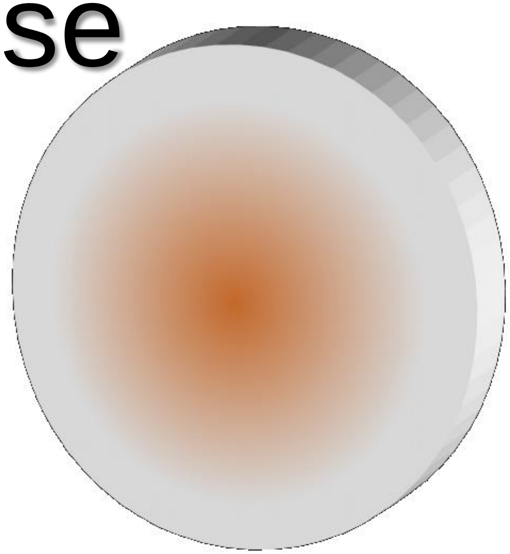
arXiv:1710.04823

Mirror Thermal Noise

- Thermal vibration of mirror surface



Mirror Thermal Noise



- Thermal vibration of mirror surface
- Structural damping, $\omega \ll \omega_0$

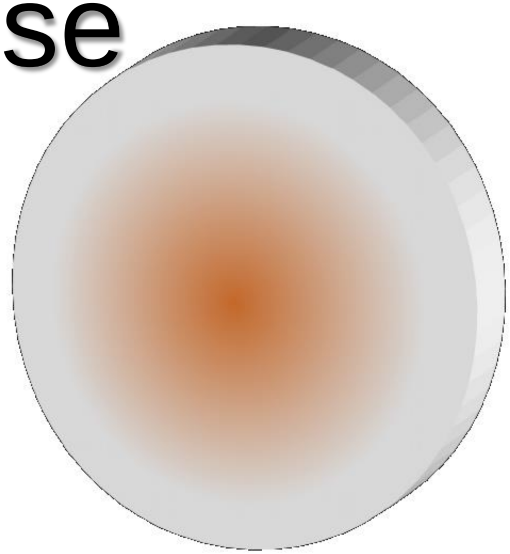
$$x_{\rm mir}^2(\omega) = \frac{4k_{\rm B}T}{m_{\rm eff}\omega_0^2}\frac{\phi}{\omega}$$

- $m_{\rm eff}$ is reduced mass
  function of mirror aspect ratio, mirror radius and beam radius ratio, Poisson's ratio
- ω_0 is resonant frequency of vibration mode

$$\omega_0 \propto \sqrt{E}$$

- You also have to sum up all the vibration modes

Mirror Thermal Noise



- Simple formula

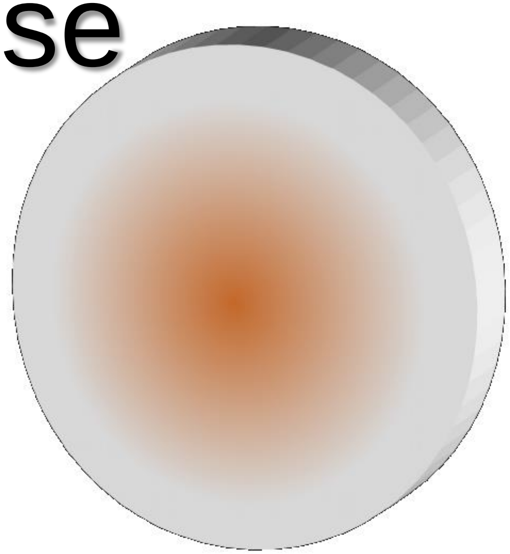
$$x_{\mathrm{mir}}^2(\omega) = \frac{4k_{\mathrm{B}}T\phi}{\omega}\frac{1-\sigma}{\sqrt{\pi}Ew}$$

- To lower the thermal noise
  - lower temperature
  - lower loss angle
  - larger beam size (needs larger mirror)

$$x_{\mathrm{mir}}(\omega) \propto \sqrt{T\phi}$$

$$x_{\mathrm{mir}}(\omega) \propto 1/\sqrt{w}$$

Mirror Thermal Noise



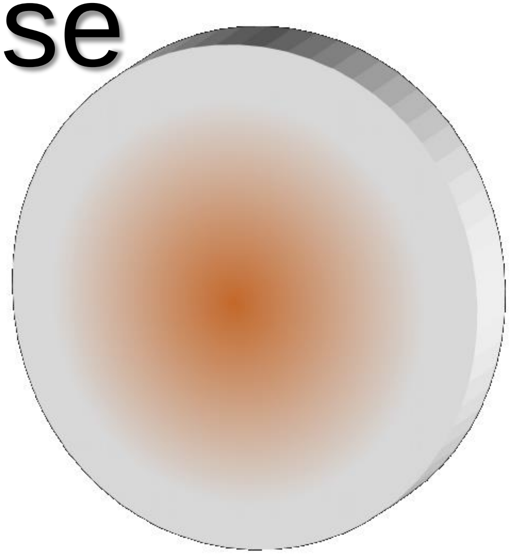
- Simple formula

$$x_{\mathrm{mir}}^2(\omega) = \frac{4k_{\mathrm{B}}T\phi}{\omega}\frac{1-\sigma}{\sqrt{\pi}Ew}$$

- Calculate mirror thermal noise for fused silica (E = 73 GPa, σ = 0.17, φ=1e-6) w = 5 cm, T = 300 K, at 100 Hz

$$k_{\mathrm{B}} = 1.38\times10^{-23}\,\mathrm{J/K}$$

Mirror Thermal Noise



- Simple formula

$$x^2_{\mathrm{mir}}(\omega) = \frac{4k_{\mathrm{B}}T\phi}{\omega}\frac{1-\sigma}{\sqrt{\pi}Ew}$$

- Calculate mirror thermal noise for fused silica (E = 73 GPa, σ = 0.17, φ=1e-6) w = 5 cm, T = 300 K, at 100 Hz

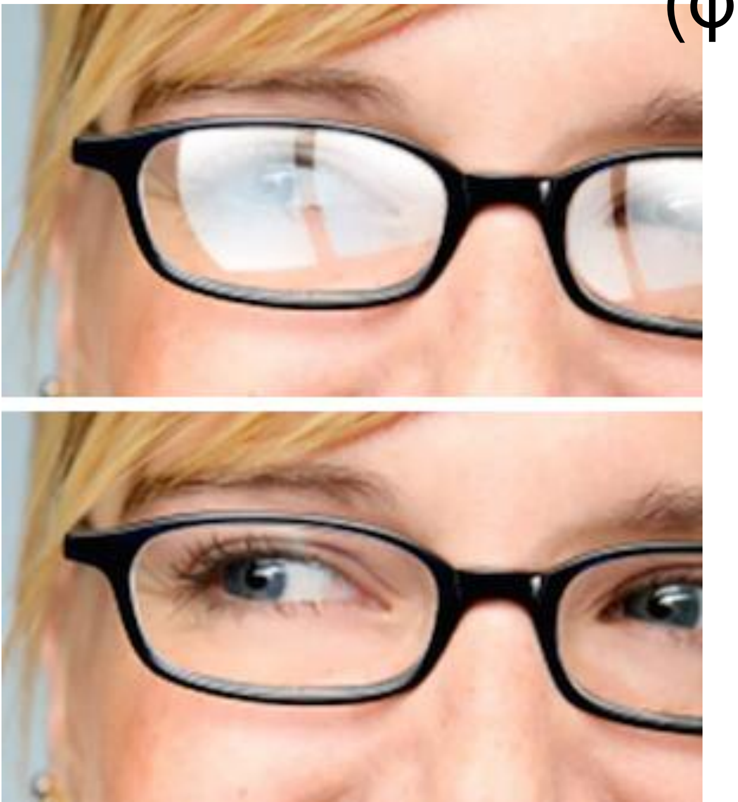
$$k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J/K}$$

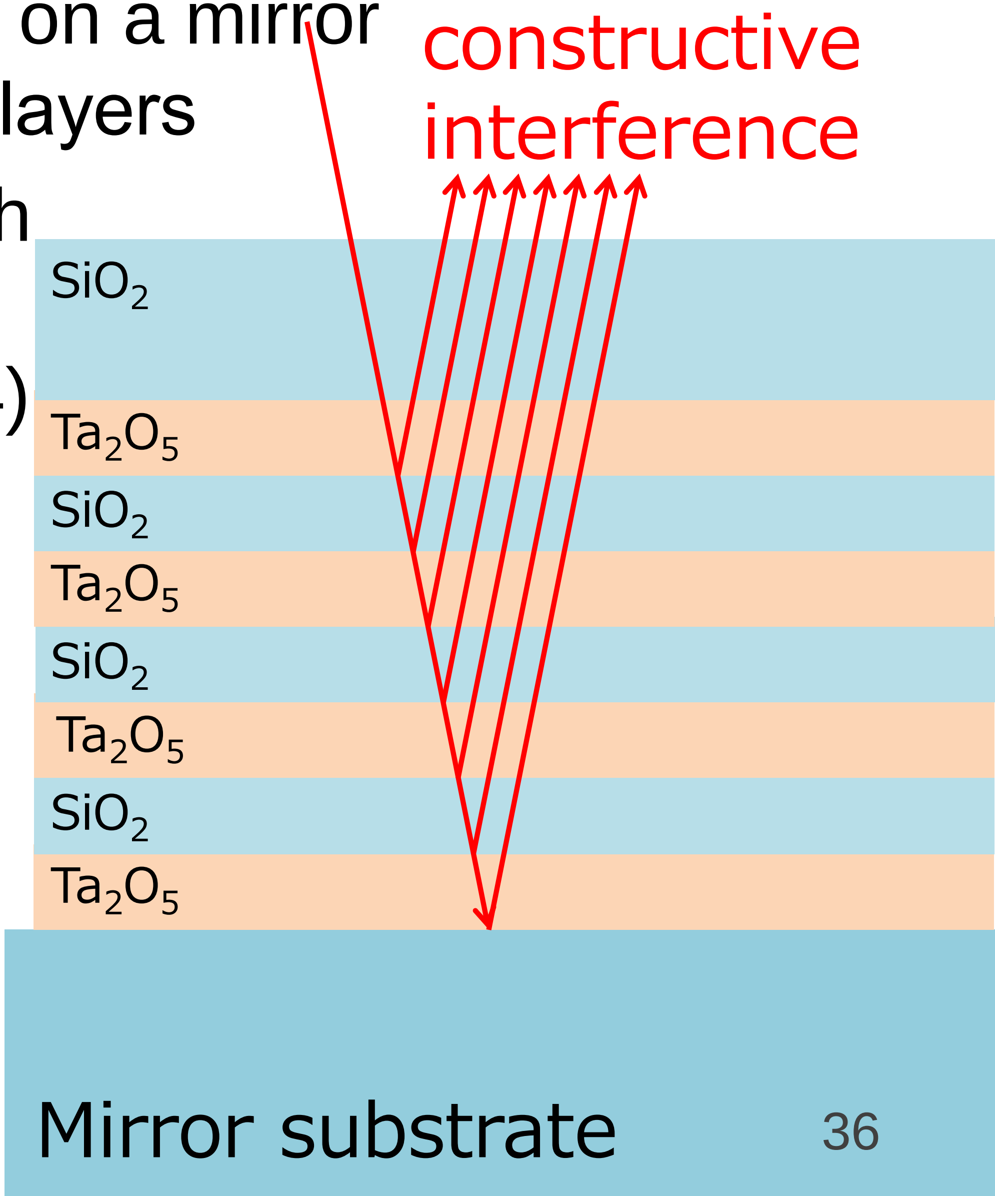
$$x_{\mathrm{mir}}(\omega) = 5.8 \times 10^{-20} \left(\frac{100\ \mathrm{Hz}}{f}\right)^{1/2}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

Sufficient for GW detection!

Coating Thermal Noise

- High reflective coating on a mirror
  λ/4 thick alternative layers
- Active area of research due to low loss angle (φ~1e-4)





Coating Thermal Noise

- Formula (N. Nakagawa+, PRD 65, 102001 (2002))

$$x^2_{\rm coat}(\omega) = \frac{4k_{\rm B}T\phi}{\omega}\frac{2d_{\rm co}(1+\sigma_{\rm co})(1-2\sigma_{\rm co})}{\pi E w^2}$$

- To lower the thermal noise
  - lower temperature
  - lower loss angle
  - larger beam radius (needs larger mirror)

$$x_{\rm coat}(\omega) \propto \sqrt{T\phi}$$

$$x_{\rm coat}(\omega) \propto 1/w$$

Coating Thermal Noise

- Formula (N. Nakagawa+, PRD 65, 102001 (2002))

$$x^2_{\mathrm{coat}}(\omega) = \frac{4k_{\mathrm{B}}T\phi}{\omega}\frac{2d_{\mathrm{co}}(1+\sigma_{\mathrm{co}})(1-2\sigma_{\mathrm{co}})}{\pi E w^2}$$

- Calculate coating thermal noise for silica/tantala coating (E = 72 Gpa, σ = 0.17, φ = 4e-4) d = 10 um, w = 5 cm, T = 300 K, at 100 Hz

$$k_{\mathrm{B}} = 1.38 \times 10^{-23}\,\mathrm{J/K}$$

Coating Thermal Noise

- Formula (N. Nakagawa+, PRD 65, 102001 (2002))

$$x_{\rm coat}^2(\omega) = \frac{4k_{\rm B}T\phi}{\omega}\frac{2d_{\rm co}(1+\sigma_{\rm co})(1-2\sigma_{\rm co})}{\pi E w^2}$$

- Calculate coating thermal noise for silica/tantala coating (E = 72 Gpa, σ = 0.17, φ = 4e-4) d = 10 um, w = 5 cm, T = 300 K, at 100 Hz

$$k_{\rm B} = 1.38 \times 10^{-23}\ {\rm J/K}$$

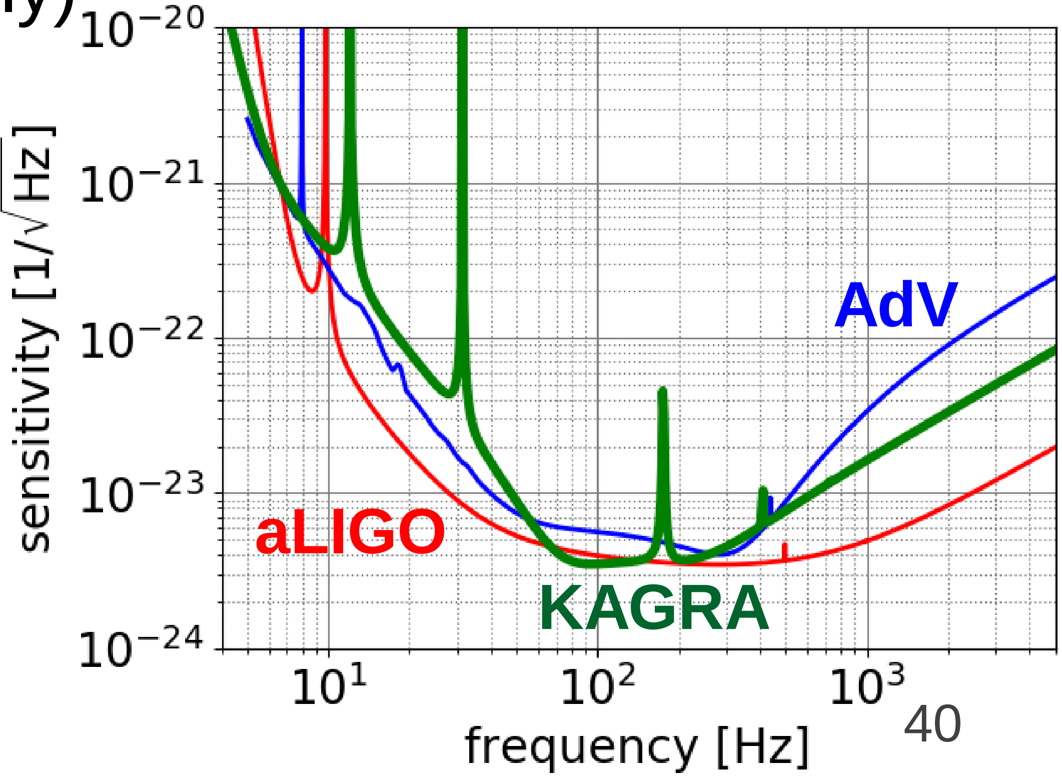
$$x_{\rm coat}(\omega) = 1.7 \times 10^{-20} \left(\frac{100\ {\rm Hz}}{f}\right)^{1/2}\ {\rm m}/\sqrt{\rm Hz}$$

Sufficient for GW detection!

Mirror/Coating Thermal Noise in 2G

- Advanced LIGO (US)
  w= 5.5 / 6.2 cm,
  fused silica,
  295 K, φ=~1e-6
- Advanced Virgo (Italy)
  w= 4.9 / 5.8 cm,
  fused silica,
  295 K, φ=~1e-6
- KAGRA (Japan)
  w= 3.5 / 3.5 cm,
  sapphire,
  22 K, φ=1e-8

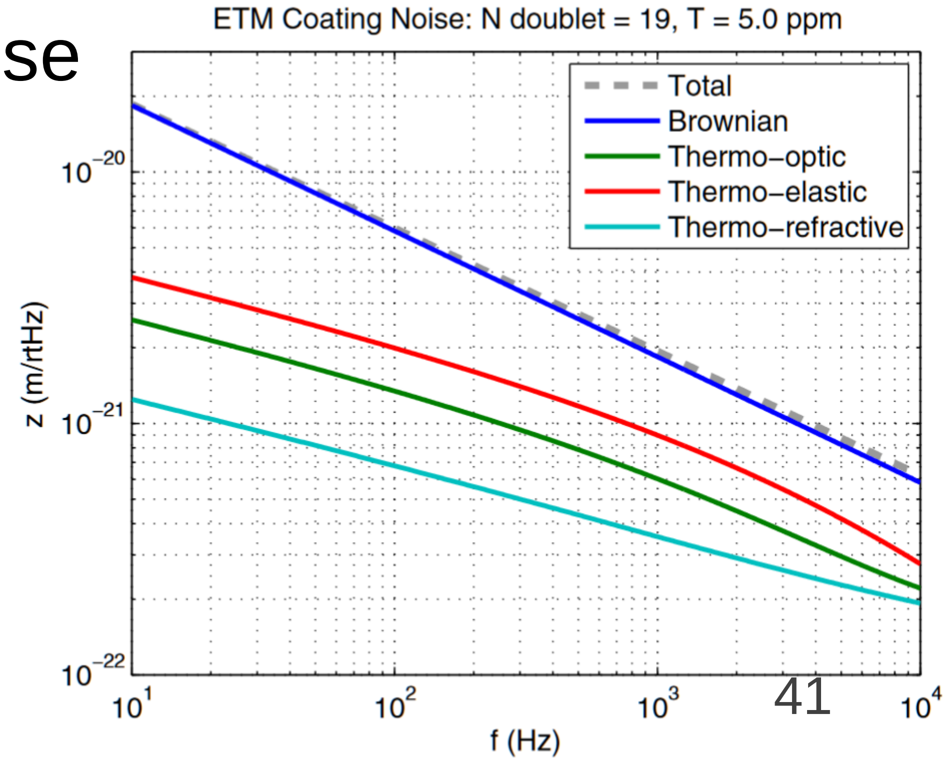
Mirror thermal noise is not significant, but coating thermal noise is troublesome in 2G



Other Coating Thermal Noises

- From thermal dissipation in the coating leads to temperature fluctuation in the coating, which causes:
  - Thermo-elastic noise
    thermal expansion
  - Thermo-refractive noise
    refractive index change

- These two can cancel



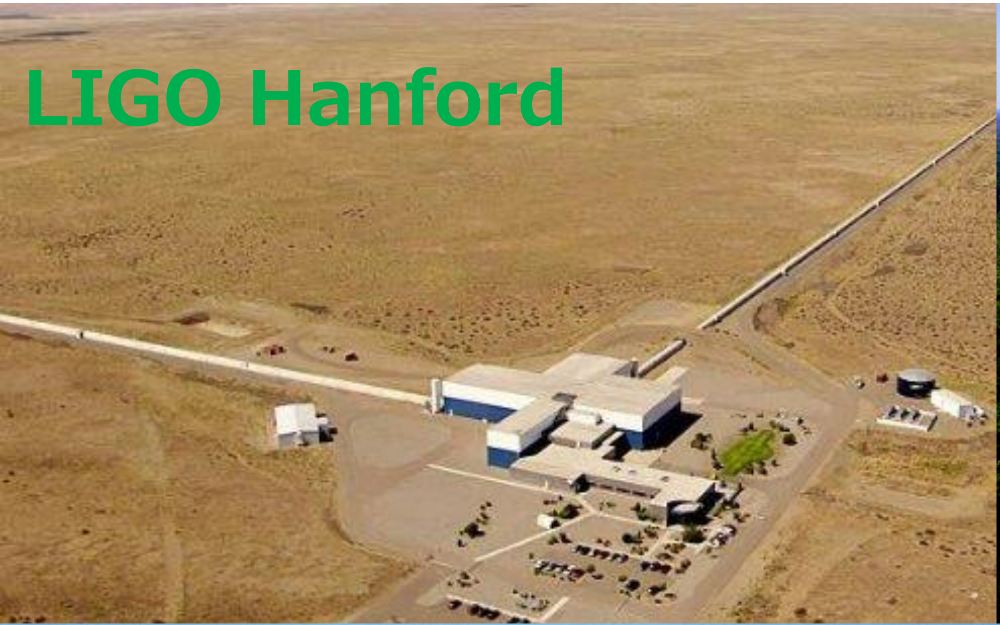
M. Evans+, PRD 78, 102003 (2009)

Seismic Noise

Seismic Vibration

- Ground vibrates even if there is no earthquakes
- Vibration level differs by
  - day and night
  - stormy day and calm day
  - urban place and countryside
  - surface and underground
- Site selection is very important for GW detectors

GW Detector Sites


LIGO Hanford


LIGO Livingston


Virgo

KAGRA

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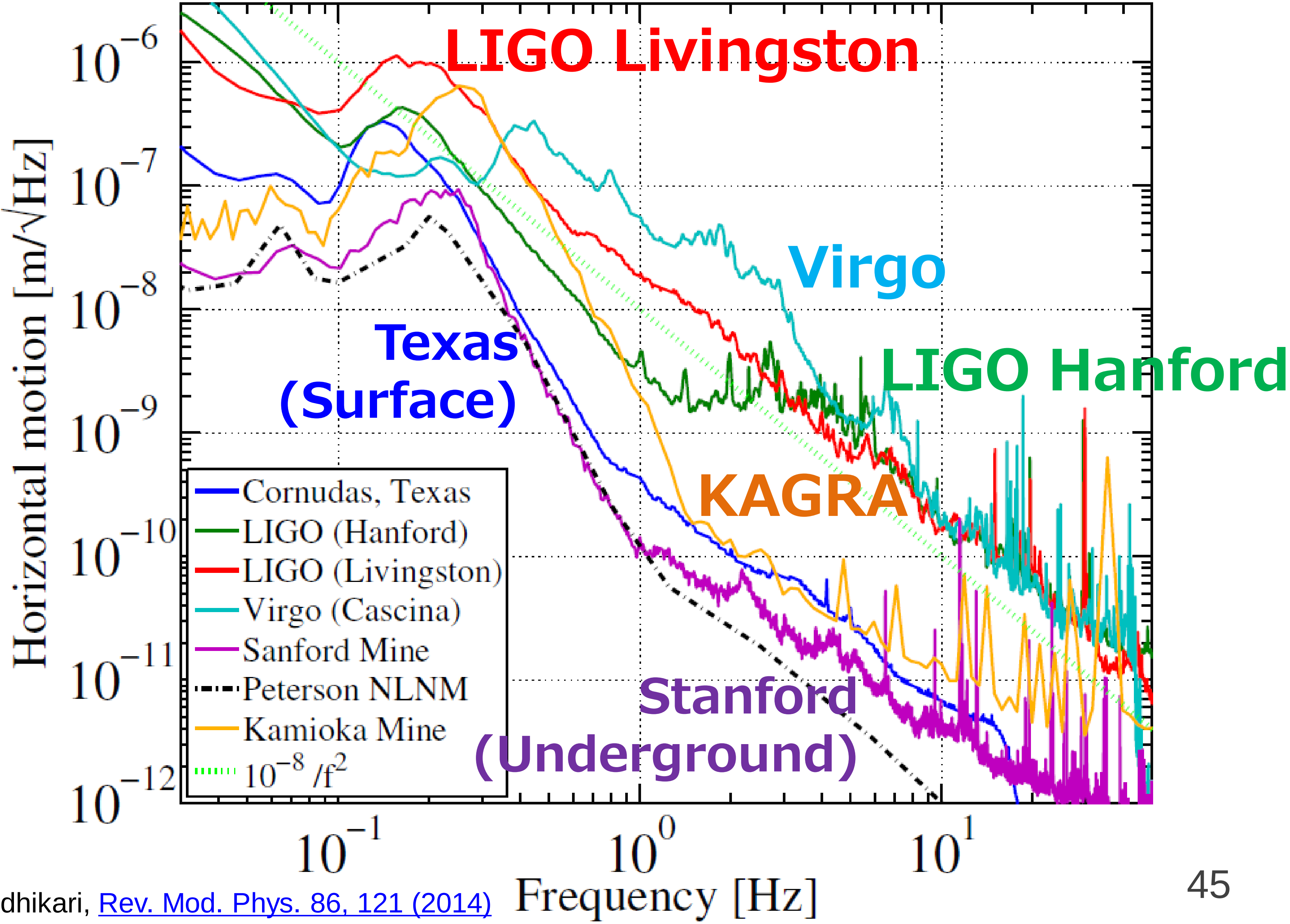
Seismic Vibration Between Sites



R. X. Adhikari, Rev. Mod. Phys. 86, 121 (2014)

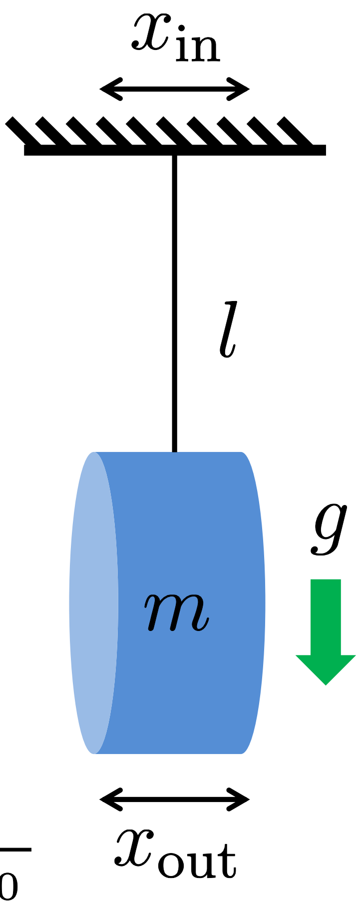
Vibration Isolation

- Typical seismic noise (according to Saulson)

$$x_{\mathrm{seis}}(\omega) = \begin{cases} 10^{-9}\,\mathrm{m}/\sqrt{\mathrm{Hz}} & \text{1-10 Hz} \\ 10^{-9}\,(10\,\mathrm{Hz}/f)^2\ \mathrm{m}/\sqrt{\mathrm{Hz}} & \text{>10 Hz} \end{cases}$$

- Needs vibration attenuation by ~9 orders of magnitude
- Use pendulums to attenuate the ground vibration

Simple Pendulum



- Equation of motion

$$m\ddot{x}_{\text{out}} = -\frac{mg}{l}(x_{\text{out}} - x_{\text{in}}) - \gamma\dot{x}_{\text{out}}$$

- Transfer function from x_{in} to x_{out}

$$H(\omega) = \frac{x_{\text{out}}(\omega)}{x_{\text{in}}(\omega)} = \frac{\frac{g}{l}}{-\omega^2 + \frac{g}{l} + i\frac{\gamma}{m}\omega}$$

$$= \frac{\omega_0^2}{-\omega^2 + \omega_0^2 + i\frac{\omega\omega_0}{Q}}$$

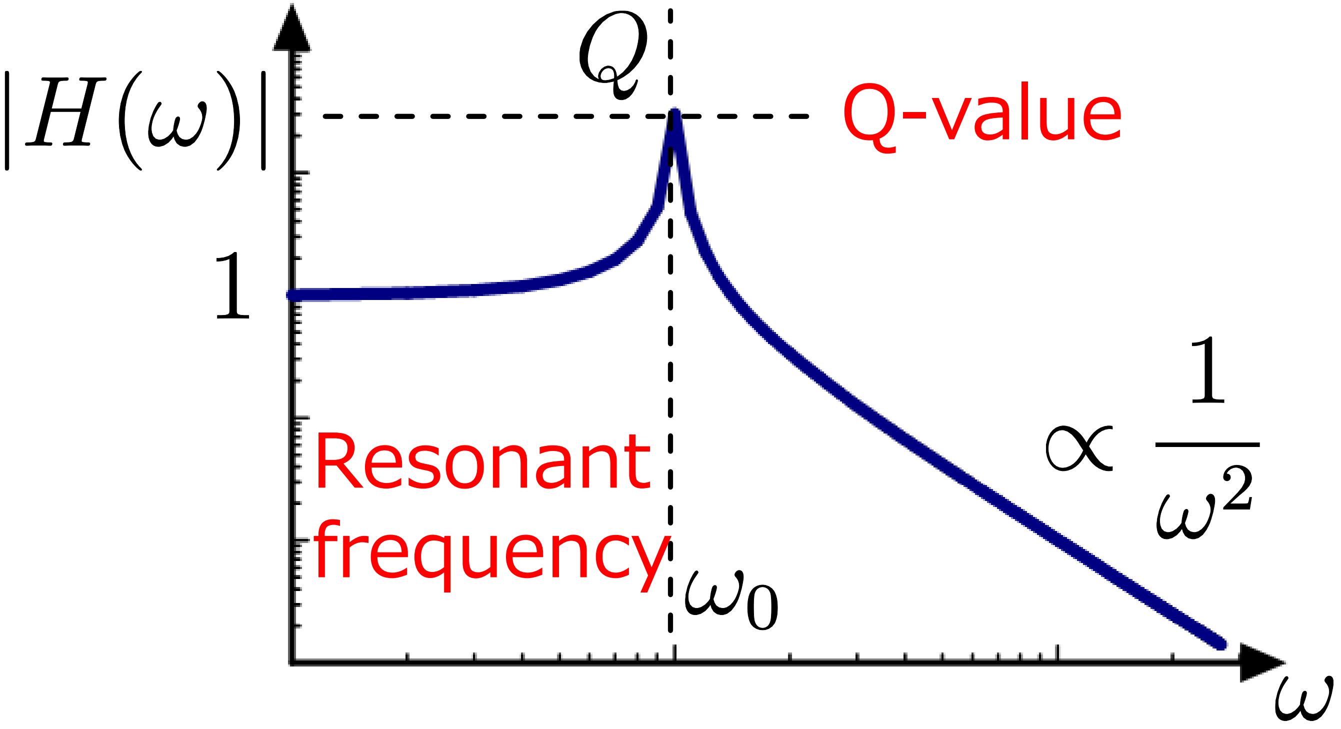
where

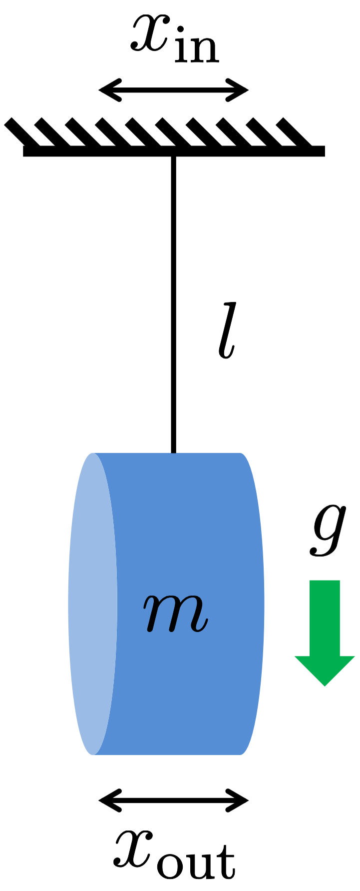
$$\omega_0 = \sqrt{\frac{g}{l}} \qquad Q = \frac{m\omega_0}{\gamma}$$

Simple Pendulum

- Seismic attenuation ratio

$$H(\omega) = \frac{\omega_0^2}{-\omega^2 + \omega_0^2 + i\frac{\omega\omega_0}{Q}}$$

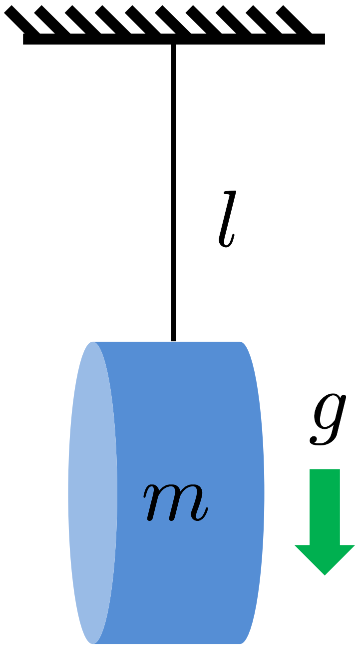


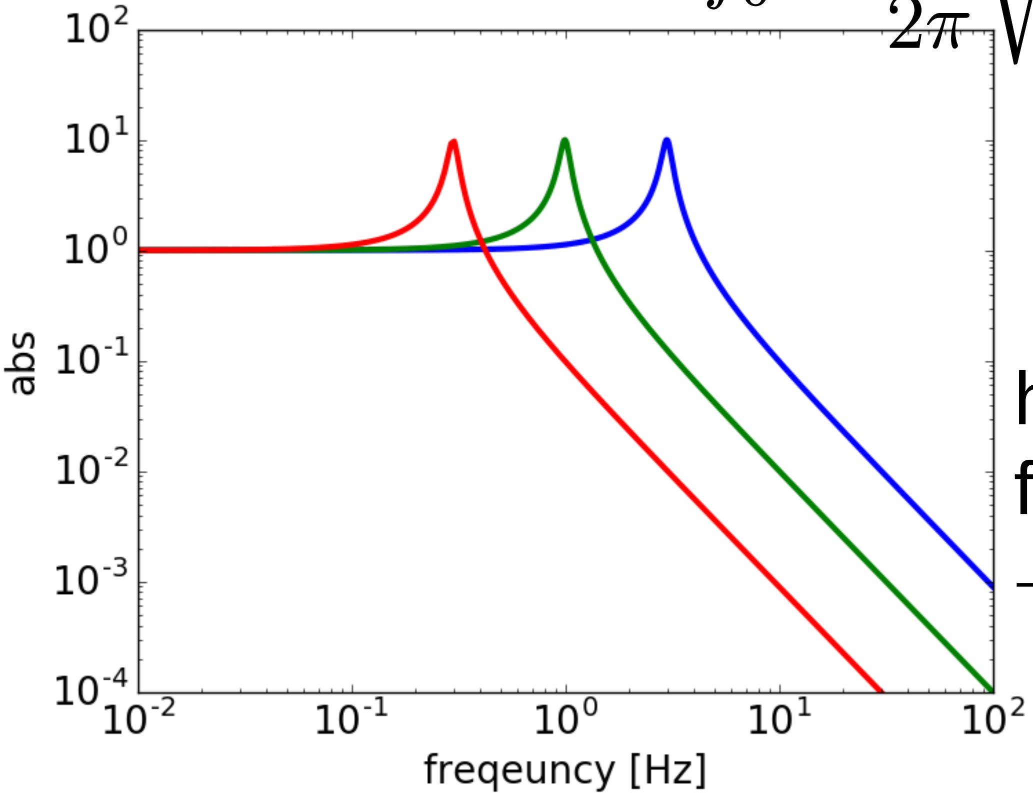


Resonant Frequency

- Longer suspension gives lower resonant frequency, and thus better attenuation ratio at high frequencies
- $l = 1\ \mathrm{m}$ gives

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \simeq 0.5\ \mathrm{Hz}$$

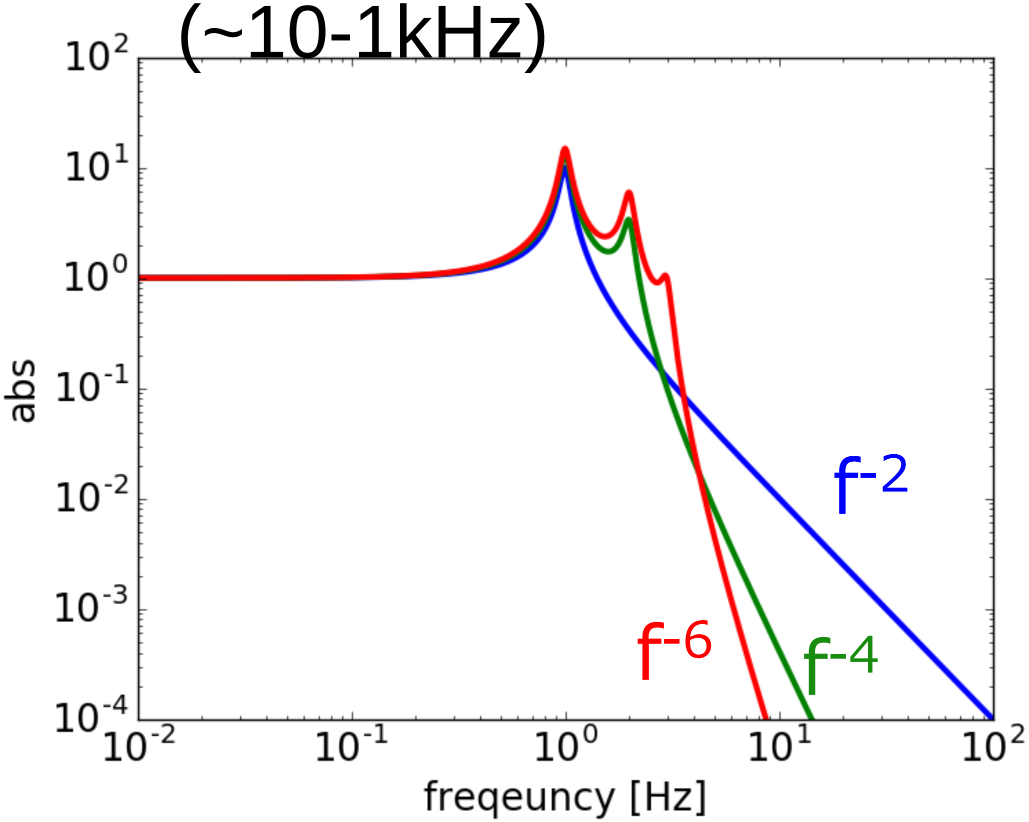


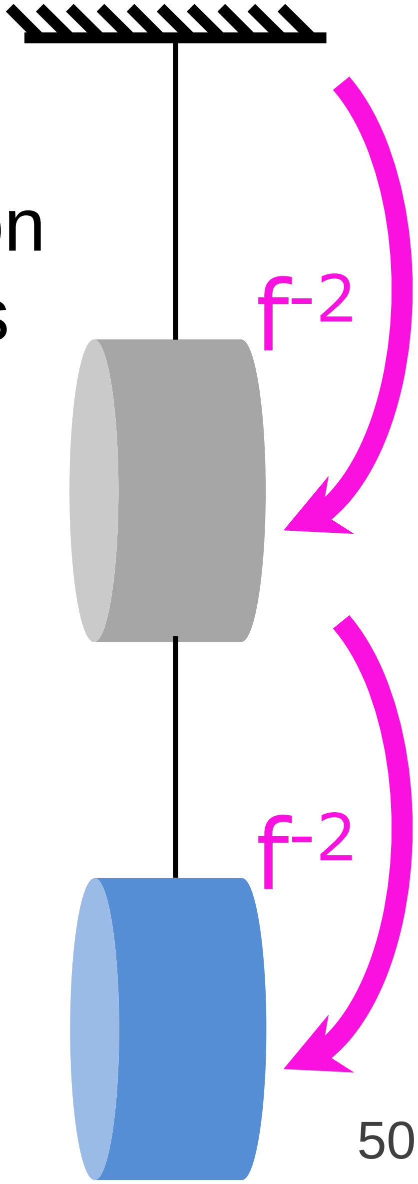


hard to get low resonant frequency on Earth
→ space GW detectors

Multiple-Stage Pendulum

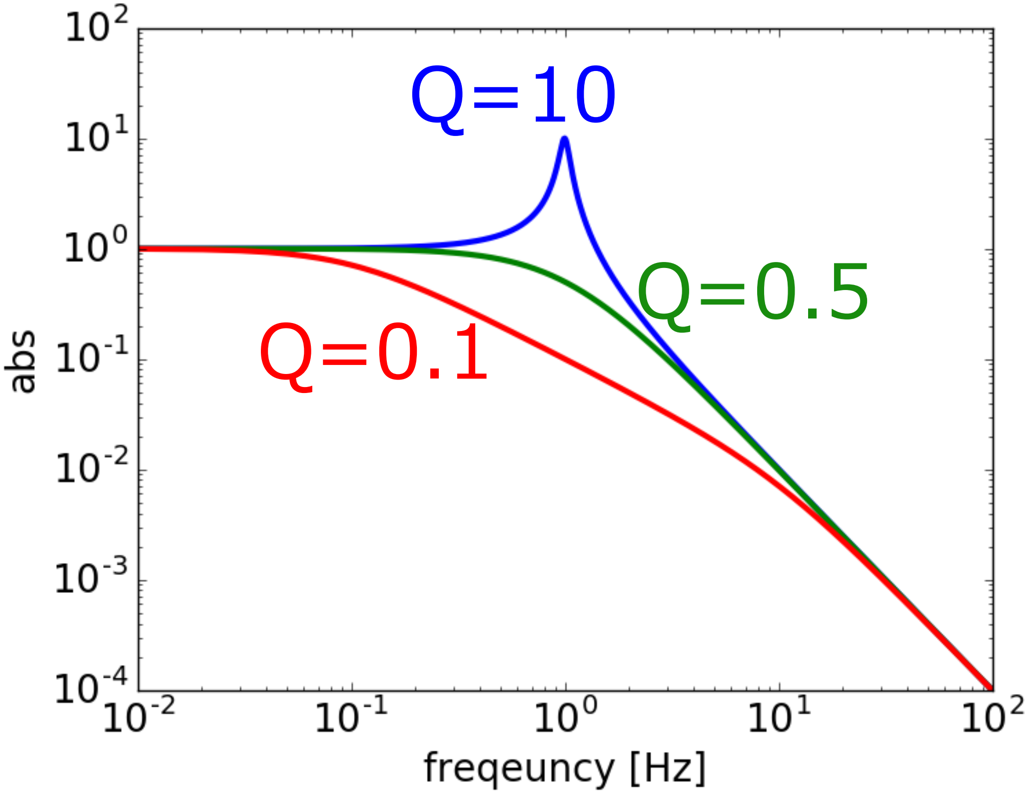
- n-stage pendulum has f^{-2n} attenuation at high frequencies
- Gives enough isolation at observation band of ground-based GW detectors (~10-1kHz)





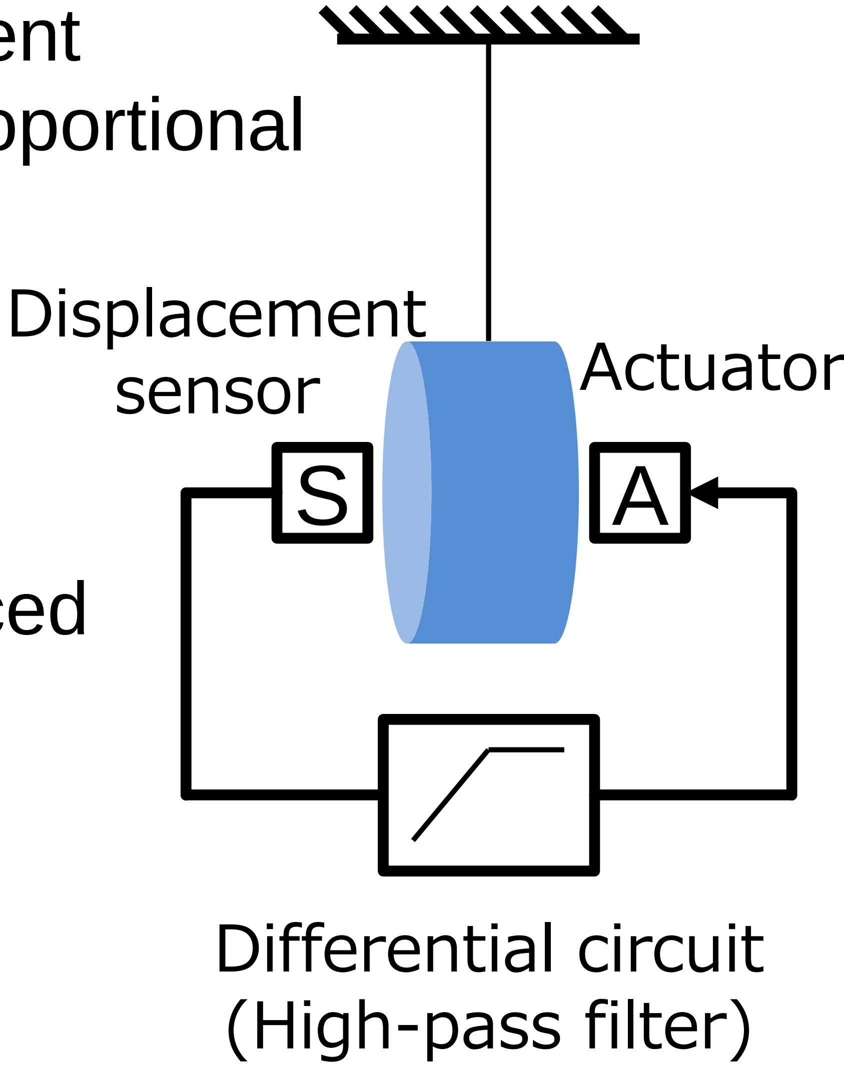
Pendulum Q-value

- Higher Q-value gives better thermal noise, but needs longer time it takes to calm down
- Very low Q-value is bad for seismic attenuation
  (Q < 0.5 is over damping)



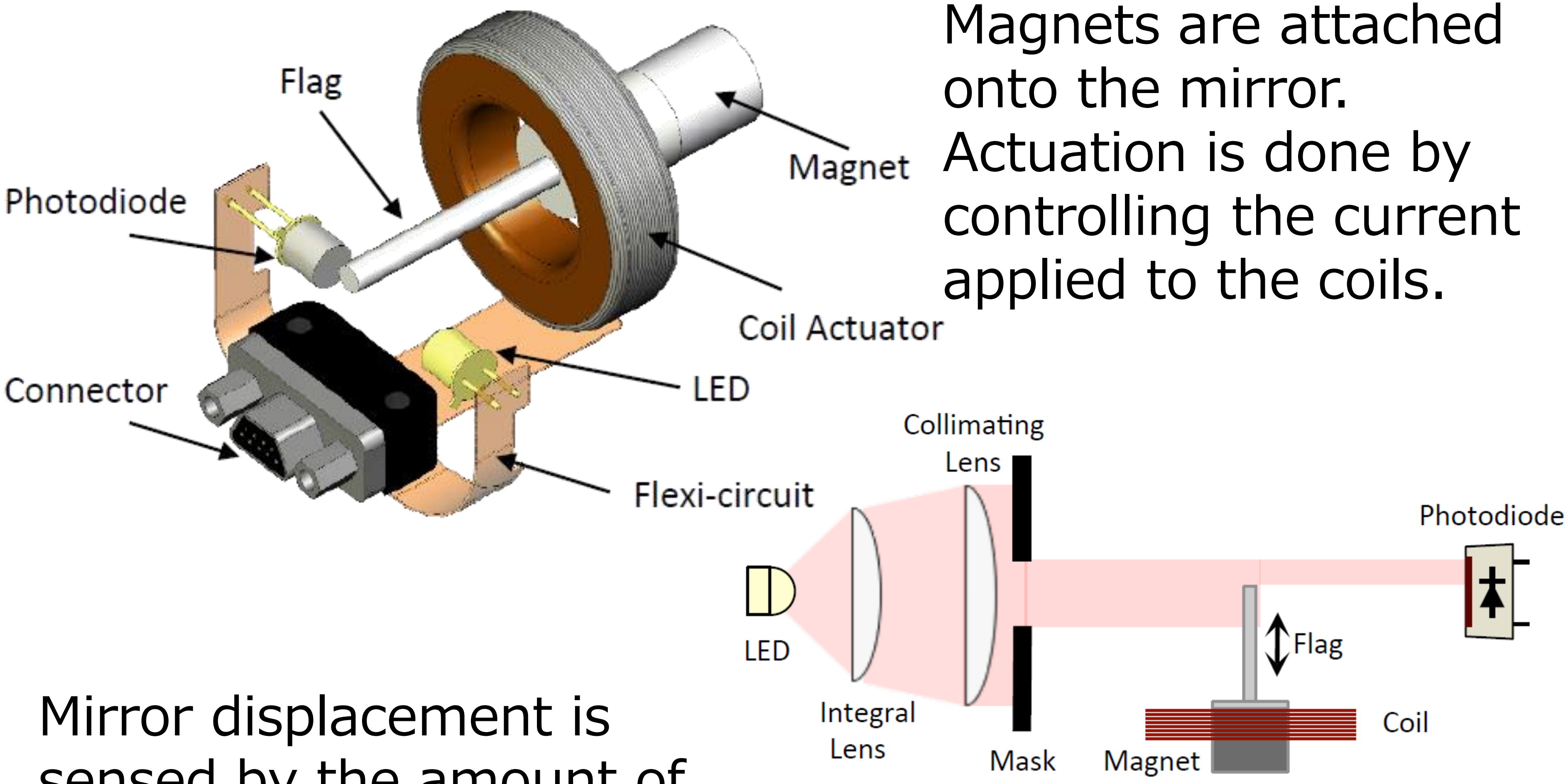
Active Damping

- Measure mirror displacement and give damping force proportional to mirror velocity using differential circuit
- No degradation in thermal noise
- Control noises are introduced
  circuit noise,
  magnetic noise,
  sensor noise,
  actuator noise……



OSEM

- Optical Sensor ElectroMagnet

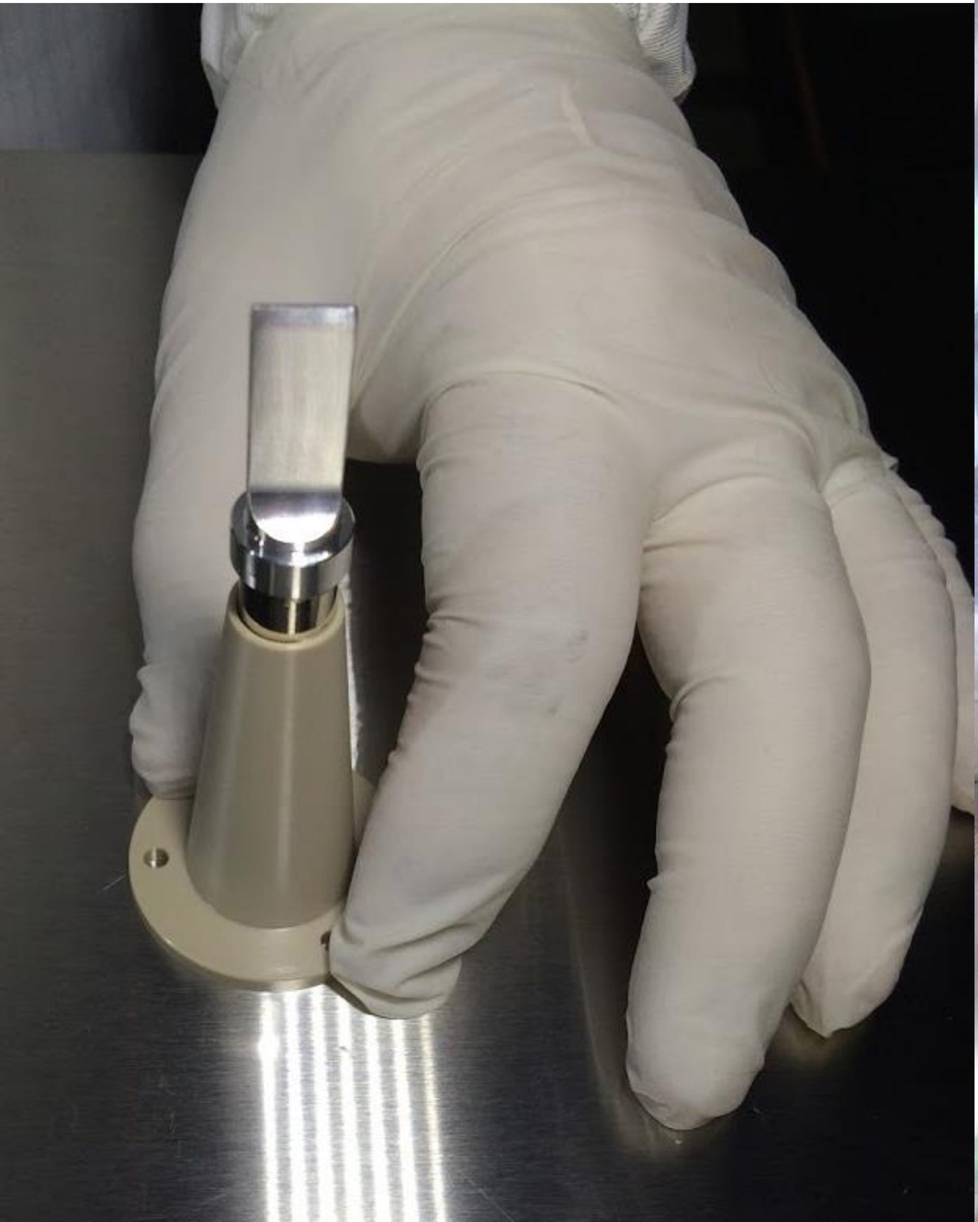


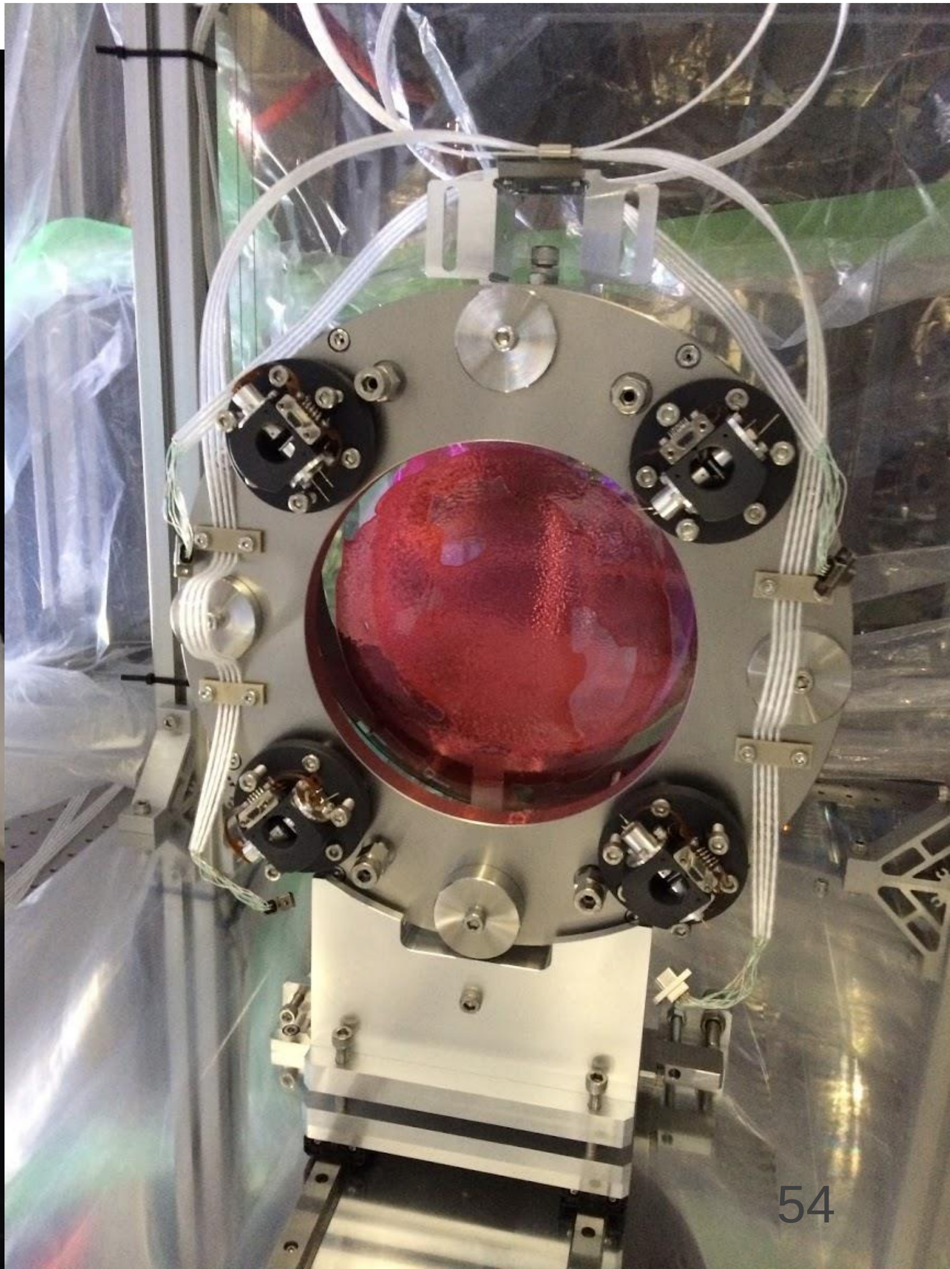
Magnets are attached onto the mirror. Actuation is done by controlling the current applied to the coils.

Mirror displacement is sensed by the amount of light shaded by the flags.

Class. Quantum Grav. 29, 115005 (2012)

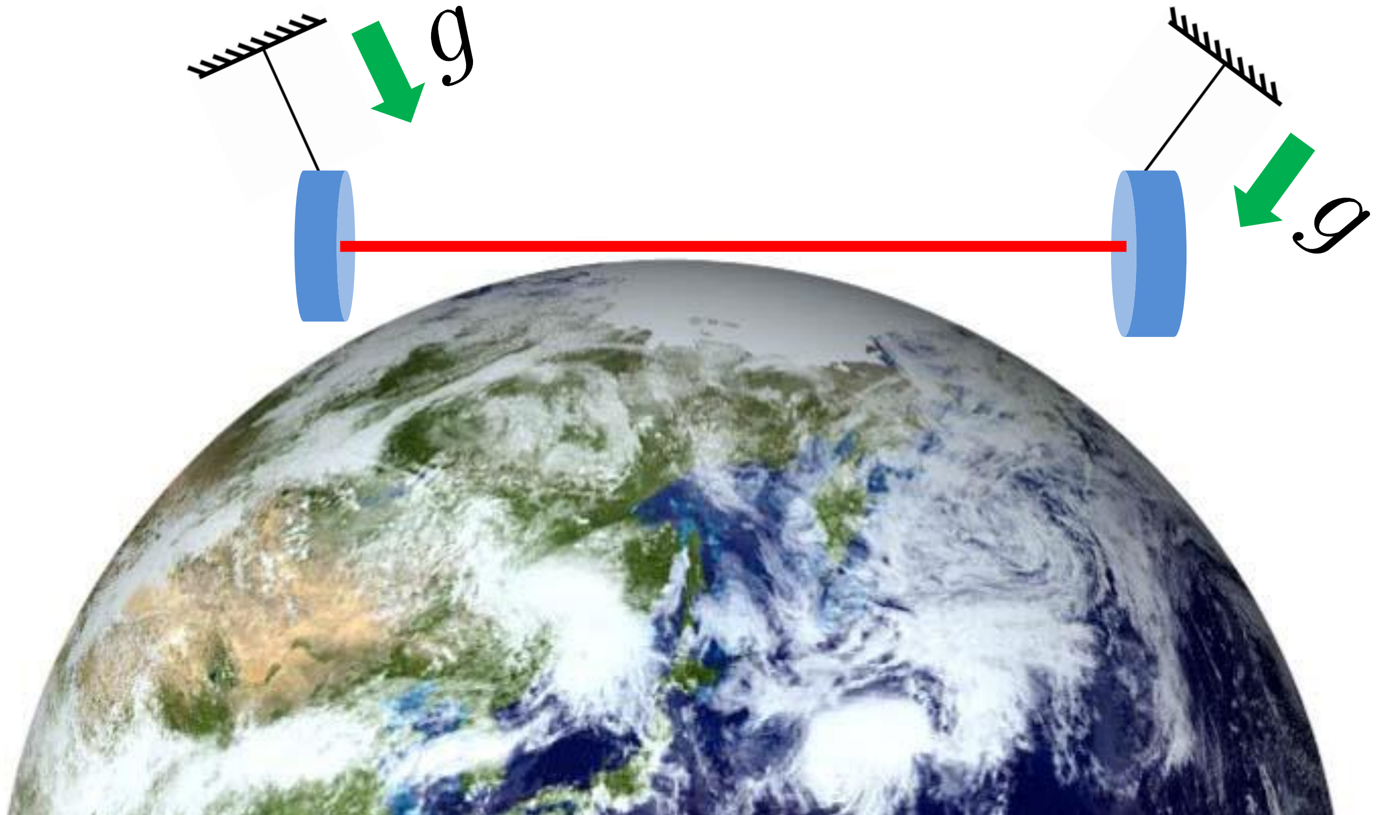
KAGRA OSEM

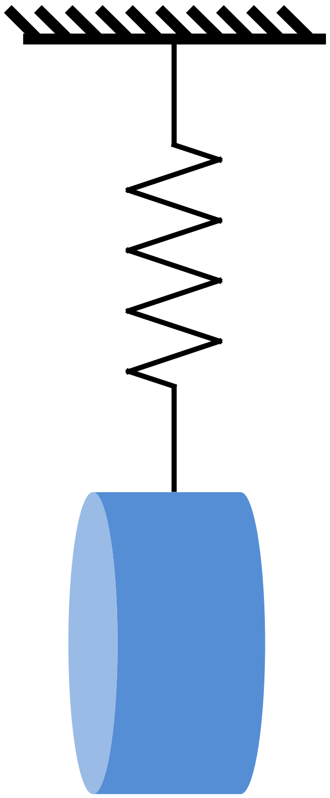




Vertical Vibration Isolation

- Vertical vibration isolation is also necessary since the beam axis is not perfectly horizontal





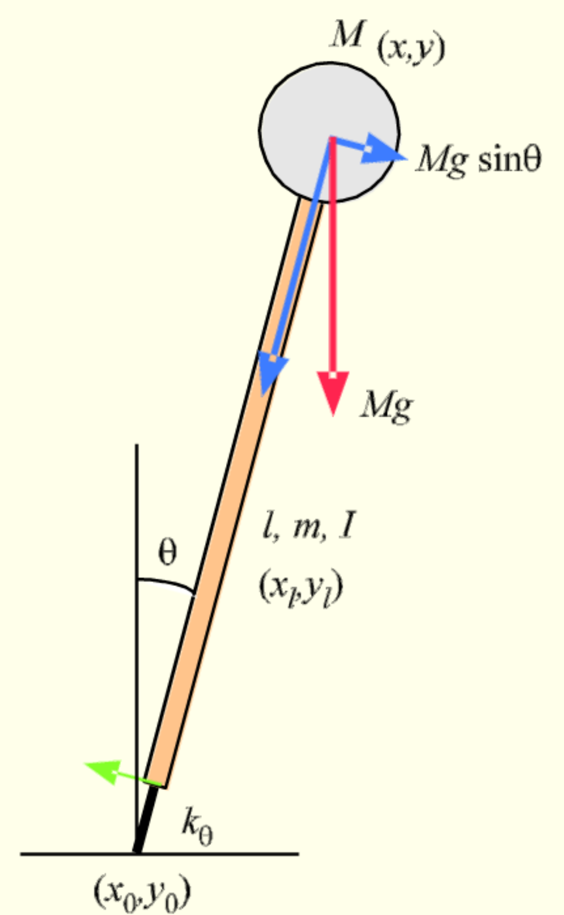
Low Frequency Vibration Isolation

- Usual pendula are not practical for making resonant frequency low

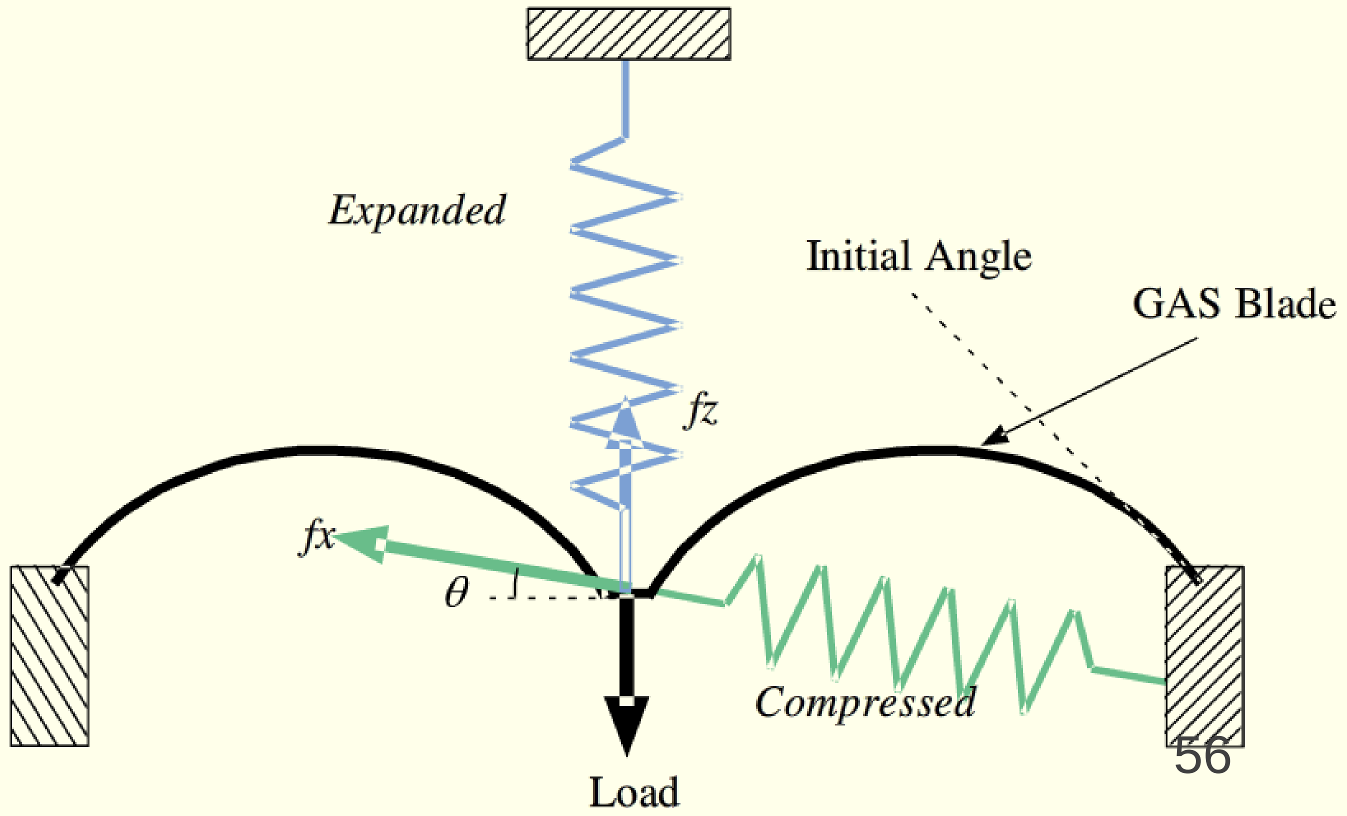
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

0.5 Hz for 1 m-long pendulum

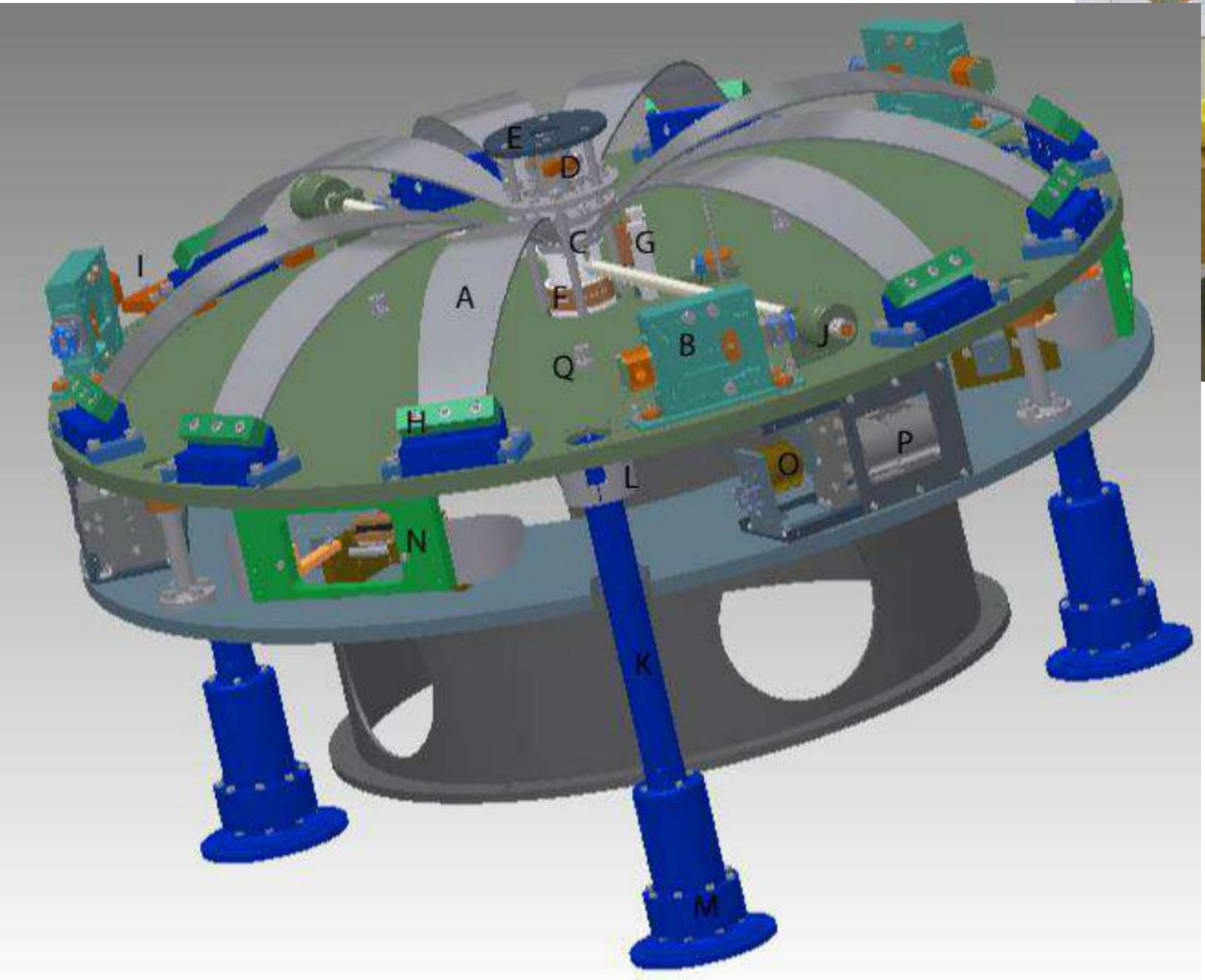
Inverted pendulum

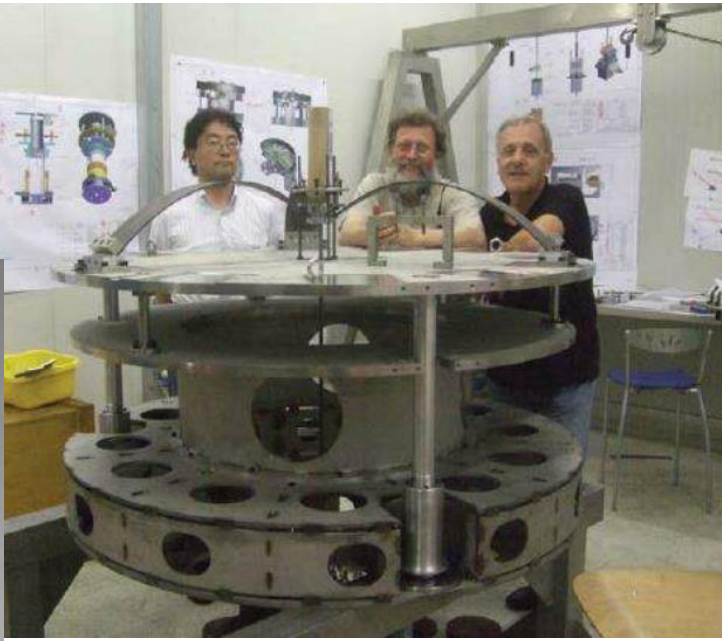


Geometric-Anti-Spring filter



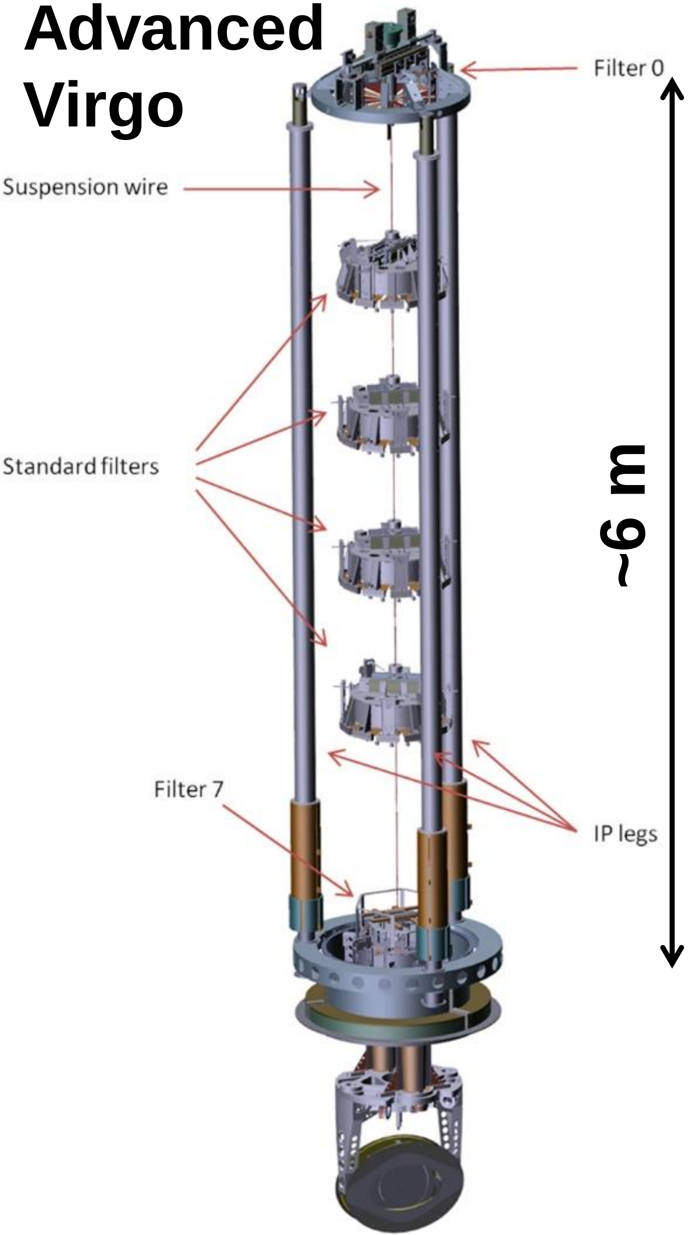
Low Frequency Vibration Isolation





Vibration Isolation System in 2G

Advanced Virgo



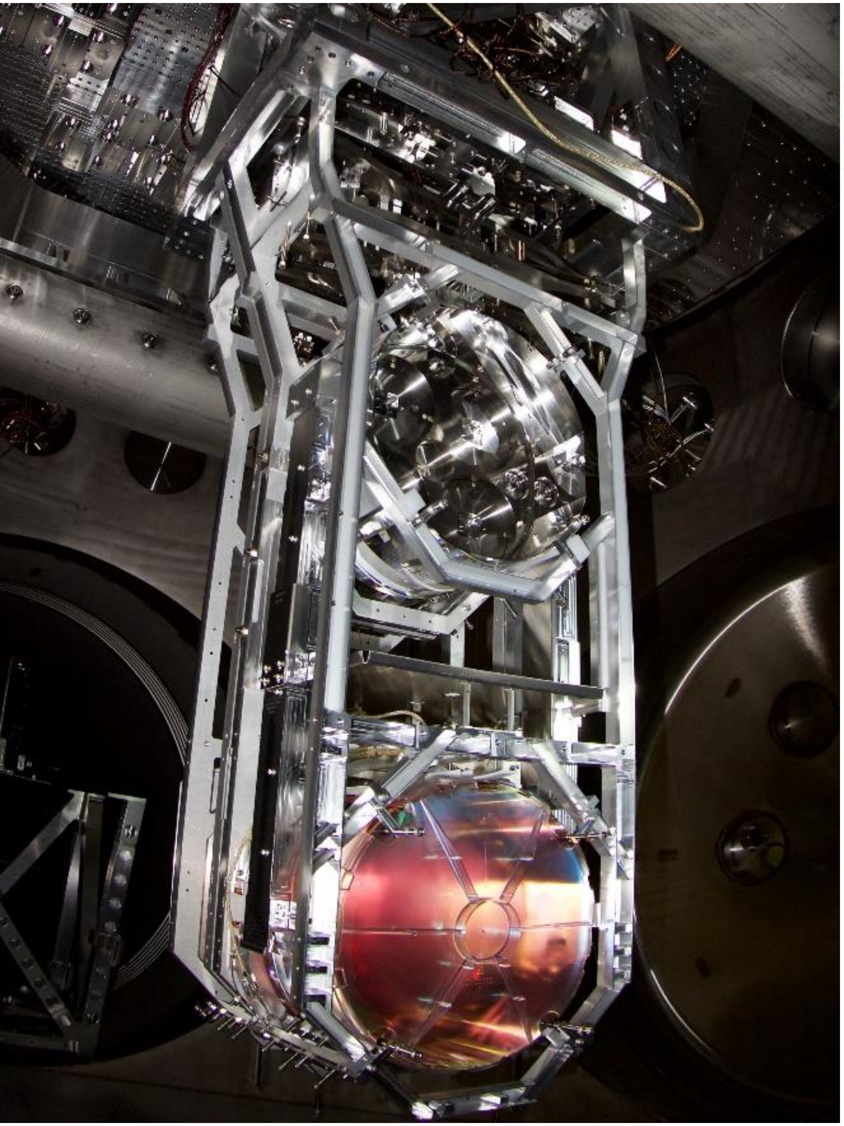
~6 m

KAGRA (BS)

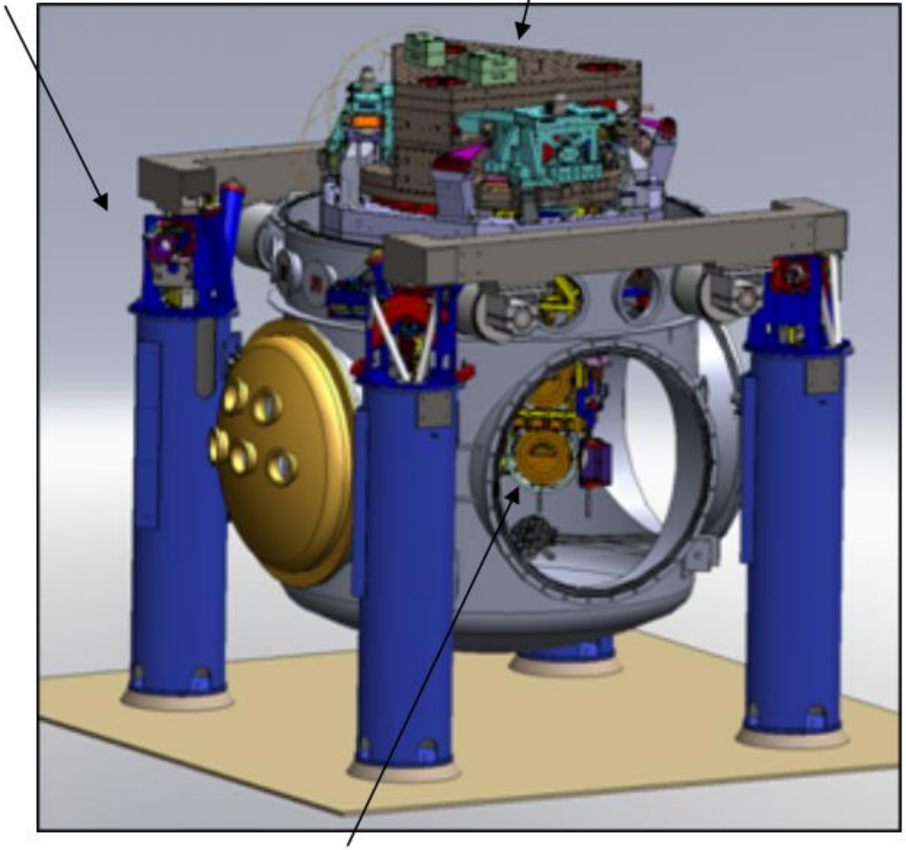
~14 m

KAGRA

Vibration Isolation System in 2G



Advanced LIGO



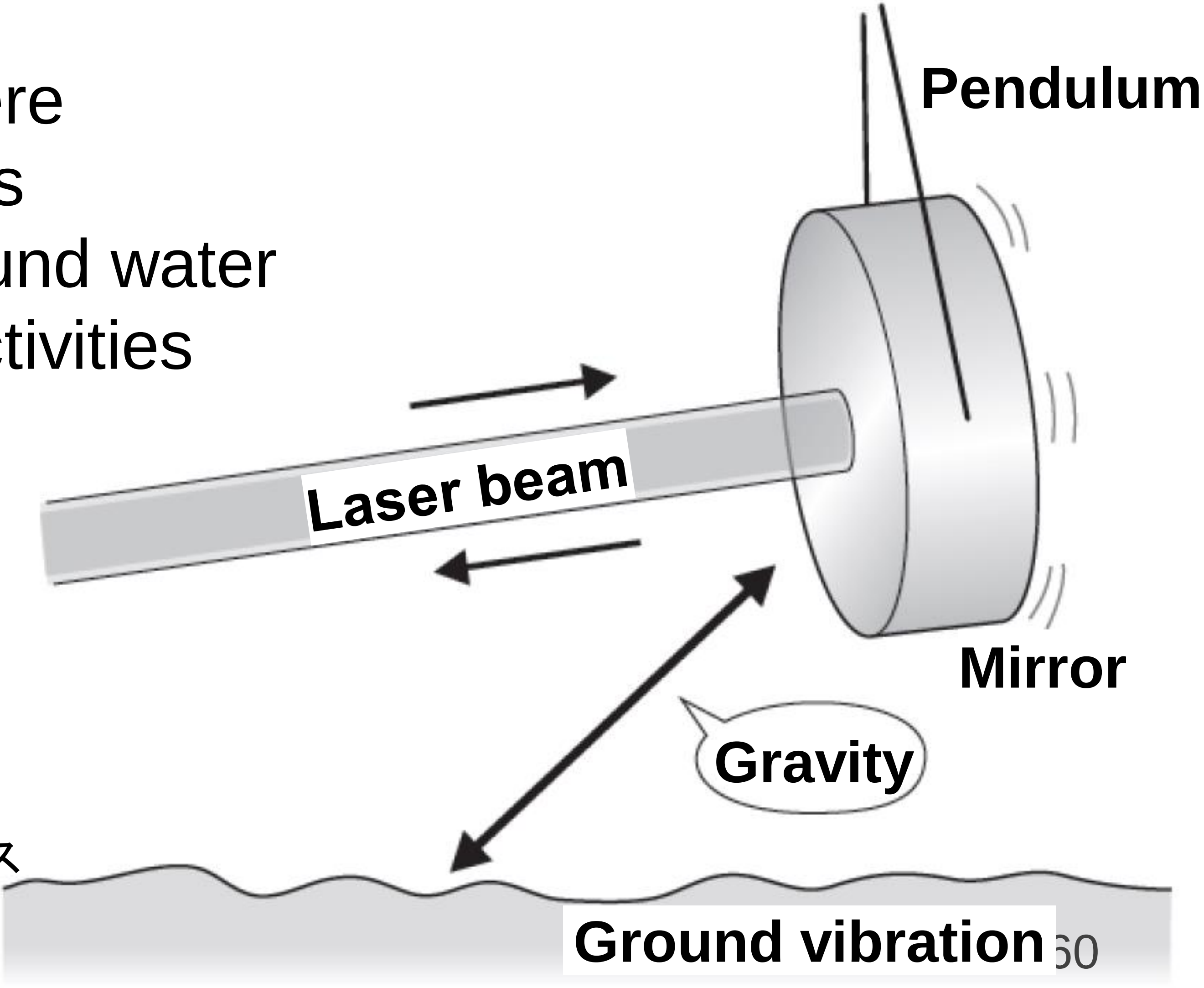
hydraulic external pre-isolator (HEPI) (one stage of isolation)

active isolation platform (2 stages of isolation)

quadruple pendulum (four stages of isolation) with monolithic silica final stage

Gravity Gradient Noise

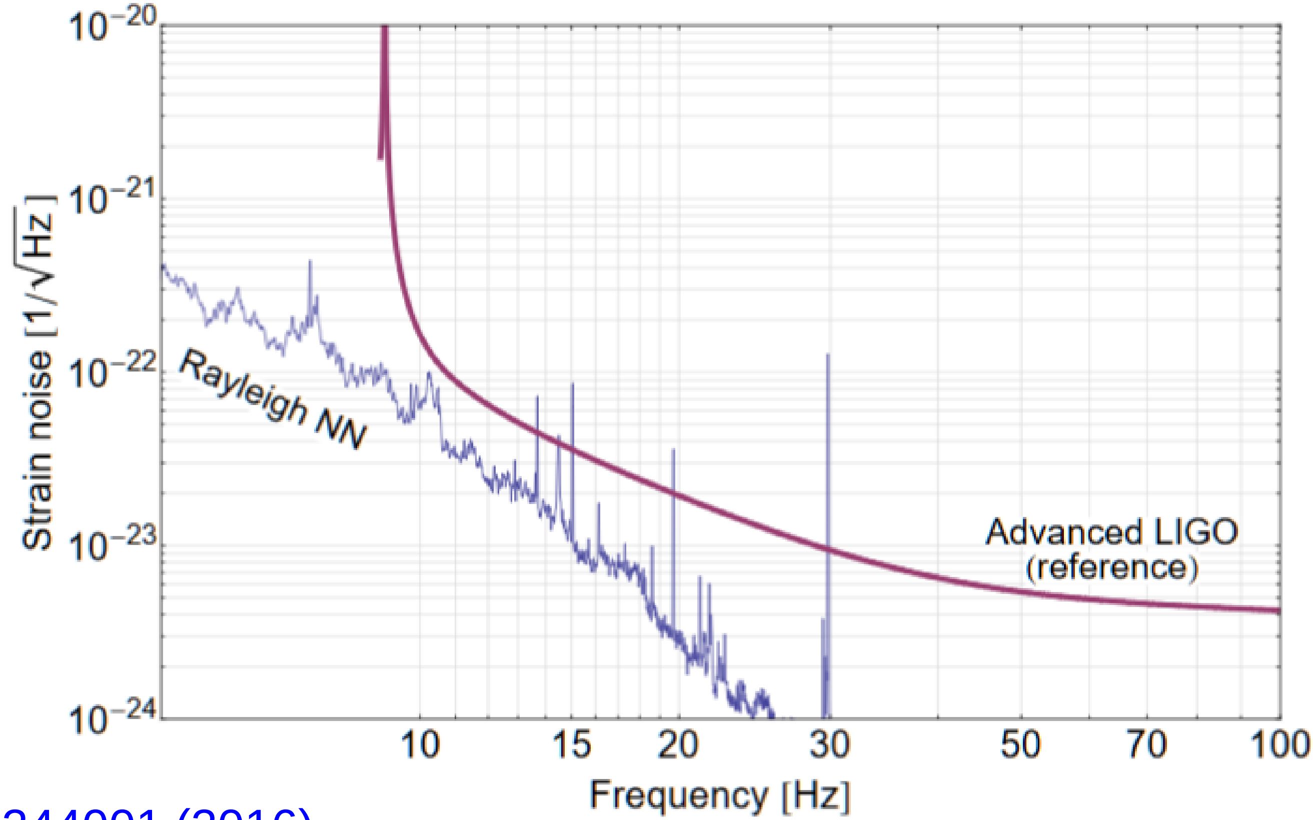
- Newtonian gravitational force acting on a mirror
  - - ground
  - - atmosphere
  - - sea waves
  - - underground water
  - - human activities



Modified from
講談社ブルーバックス
『重力波とはなにか』
安東正樹著

Gravity Gradient Noise

- Newtonian gravitational force acting on a mirror
- Could be an issue for 2G



M. Coughlin+, [CQG 33 244001 (2016)]

FIG. 1: Predicted average Newtonian noise in Advanced LIGO.

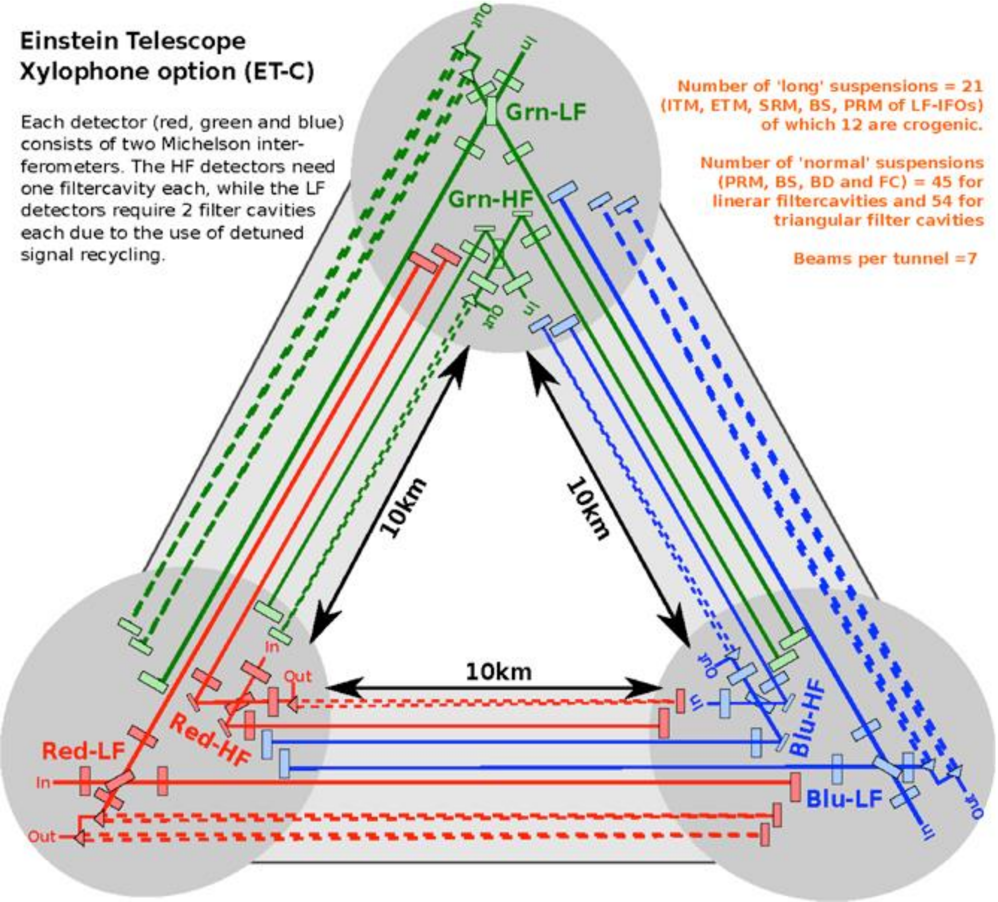
Underground GW Detector

- Underground site gives lower seismic noise (and lower gravity gradient noise)
- KAGRA is the first km-scale underground detector
- Future GW project Einstein Telescope also considers using underground site

KAGRA site (March 2014)

Einstein Telescope

- Triangular, 10 km, underground
- Xylophone configuration
  LF detector with cryogenic and low power
  HF detector with room temperature and high power



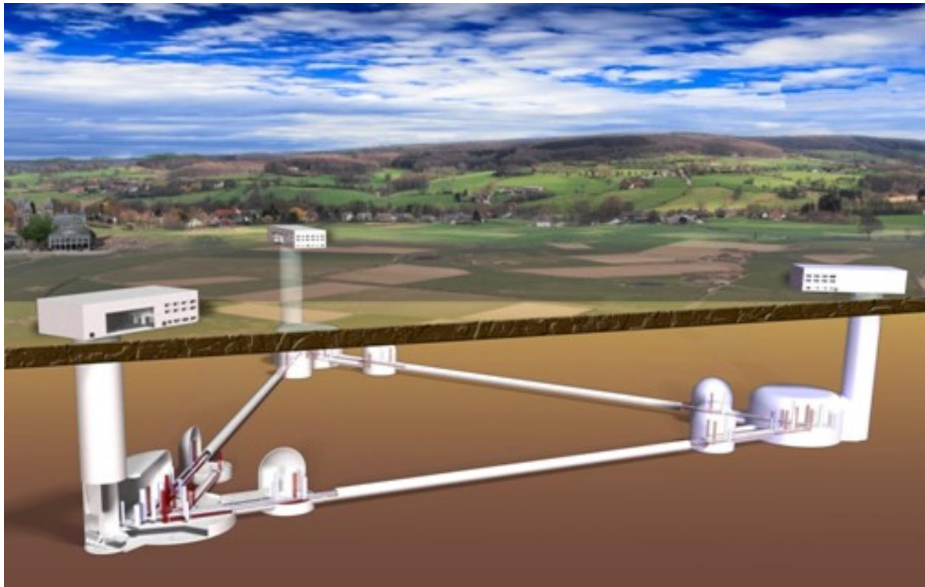
Einstein Telescope
Xylophone option (ET-C)

Each detector (red, green and blue) consists of two Michelson interferometers. The HF detectors need one filtercavity each, while the LF detectors require 2 filter cavities each due to the use of detuned signal recycling.

Number of 'long' suspensions = 21 (ITM, ETM, SRM, BS, PRM of LF-IFOs) of which 12 are crogenic.

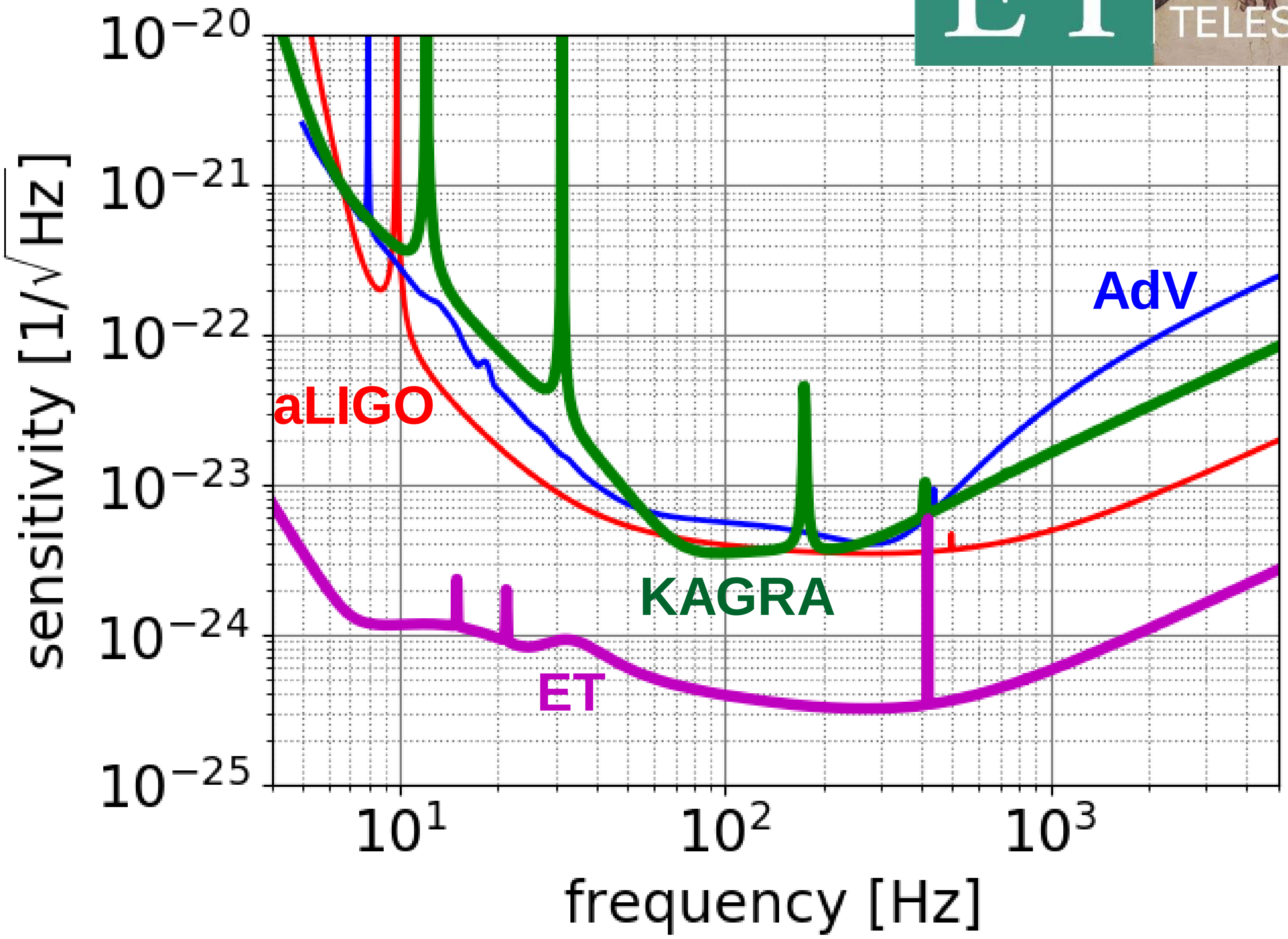
Number of 'normal' suspensions (PRM, BS, BD and FC) = 45 for linear filtercavities and 54 for triangular filter cavities

Beams per tunnel =7



Einstein Telescope

- Roughly x10 the 2G sensitivity



Assignment from Raffaele Flaminio (by next week)

Assignment n.1

Consider a Fabry-Perot cavity made of two mirrors M1 and M2. M1 is the input mirror and it has amplitude reflectivity r. M2 is the end mirror and it is perfect reflector. The cavity wavelength is L.

E_i is the electric field amplitude of the laser beam entering into the cavity. λ is its wavelength. The cavity is kept in resonance with the laser field.

A gravitational wave produces a small variation of the cavity length x(t)<<λ with linear spectral density $\tilde{x}(f)$.

1. Calculate the variation of the electric field $\tilde{E}_c(f)$ inside the cavity.

2. Calculate the variation of the electric field $\tilde{E}_c(f)$ reflected by the cavity.

Assignment n.2

Consider a double pendulum made of two pendulums in cascade.

The double pendulum temperature is T=300 K.

The first pendulum has a length l_1=1 m and a mass m_1=100 kg. the second one hangs form the first mass and has a length l_2=1 m and a mass of m_2=50 kg.

The first pendulum is affected by a mechanical loss of $\phi_1=10^{-4}$ i.e. the gravitational restoring force has an imaginary part equal to 10^{-4} of its real part.

The second pendulum is affected by a mechanical loss of $\phi_2=10^{-6}$ i.e. the gravitational restoring force has an imaginary part equal to 10^{-6} of its real part.

Using the fluctuation dissipation theorem, calculate the spectrum of the second mass displacement due to thermal noise.

Show that at high frequency the effect of the losses in the upper pendulum can be neglected.

Give an interpretation of this result.

Summary

- Thermal noise
  - suspension / mirror / coating thermal noise
  - cryogenics, low loss angle, longer suspension, and larger mirror reduces thermal noise
  - cryogenics is attractive but challenging
- Seismic noise
  - site selection is very important
  - multiple-stage, low resonant frequency vibration isolation techniques are used
  - underground site is preferred for reducing seismic noise and gravity gradient noise
- KAGRA is currently the only km-scale cryogenic underground GW detector in the world