# ABCD matrices for flipped mirror

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## Glossary

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|---|---|
| $\theta$ | angle of incidence on AR surface |
| $\phi$ | angle of incidence on HR surface |
| $n$ | mirror refractive index |
| $d$ | mirror thickness |

## 1 Introduction

ABDC martices for reflection from flipped mirror are shown.

## 2 Basic ABCD matrices

### 2.1 Free space

Refractive index $n$, length $d$.

$$\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix} \tag{2.1}$$

### 2.2 Curved mirror

Radius of curvature $R$, angle of incidence $\phi$.

#### 2.2.1 Tangential plane

$$\begin{pmatrix} 1 & 0 \\ -\frac{2}{R\cos\phi} & 1 \end{pmatrix} \tag{2.2}$$

*July 10, 2015: Added some notes on refractive index.

### 2.2.2 Sagittal plane

$$\begin{pmatrix} 1 & 0 \\ -\dfrac{2\cos\phi}{R} & 1 \end{pmatrix} \tag{2.3}$$

## 2.3 Interface

Initial refractive index $n_1$, final refractive index $n_2$, incident angle $\theta_1$, refraction angle $\theta_2$. From Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$. *Note that Siegmann uses "reduced slopes".* He says "this actual slope is multiplied by the local index of refraction at the ray position" in p.583.

### 2.3.1 Tangential plane

$$\begin{pmatrix} \dfrac{\cos\theta_2}{\cos\theta_1} & 0 \\ 0 & \dfrac{n_1\cos\theta_1}{n_2\cos\theta_2} \end{pmatrix} \tag{2.4}$$

### 2.3.2 Sagittal plane

$$\begin{pmatrix} 1 & 0 \\ 0 & \dfrac{n_1}{n_2} \end{pmatrix} \tag{2.5}$$

# 3 ABCD matrices for a flipped mirror

## 3.1 Tangential plane

$$M_t = \begin{pmatrix} \dfrac{\cos\theta}{\cos\phi} & 0 \\ 0 & \dfrac{n\cos\phi}{cos\theta} \end{pmatrix} \begin{pmatrix} 1 & \dfrac{d}{n} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\dfrac{2}{R\cos\phi} & 1 \end{pmatrix} \begin{pmatrix} 1 & \dfrac{d}{n} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \dfrac{\cos\phi}{\cos\theta} & 0 \\ 0 & \dfrac{\cos\theta}{n\cos\phi} \end{pmatrix} \tag{3.1}$$

$$= \begin{pmatrix} 1-\dfrac{2d}{nR\cos\phi} & -\dfrac{2d\cos^2\theta}{n^2\cos^2\phi}\left(1-\dfrac{d}{nR\cos\phi}\right) \\ -\dfrac{2n\cos\phi}{R\cos^2\theta} & 1-\dfrac{2d}{nR\cos\phi} \end{pmatrix} \tag{3.2}$$

When mirror thickness is small $d \to 0$,

$$M_t = \begin{pmatrix} 1 & 0 \\ -\dfrac{2n\cos\phi}{R\cos^2\theta} & 1 \end{pmatrix} \tag{3.3}$$

So, effective curvature is multiplied by $\dfrac{\cos^2\theta}{n\cos^2\phi}$ compared with un-flipped case. ABCD matrix for flipped and un-flipped are different by $-1/n$ when incident angle is 0.

### 3.2 Sagittal plane

$$M_s = \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix} \begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2\cos\phi}{R} & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{n} \end{pmatrix} \tag{3.4}$$

$$= \begin{pmatrix} 1 - \frac{2d\cos\phi}{nR} & \frac{2d}{n^2}\left(1 - \frac{d\cos\phi}{nR}\right) \\ -\frac{2n\cos\phi}{R} & 1 - \frac{2d\cos\phi}{nR} \end{pmatrix} \tag{3.5}$$

When mirror thickness is small $d \to 0$,

$$M_s = \begin{pmatrix} 1 & 0 \\ -\frac{2n\cos\phi}{R} & 1 \end{pmatrix} \tag{3.6}$$

So, effective curvature is multiplied by $-1/n$ compared with un-flipped case.