# Testing Spontaneous Wave-Function Collapse Models on Classical Mechanical Oscillator

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# 参考文献

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# 量子力学

シュレディンガー方程式

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)\right]\underset{\text{波動関数}}{\underline{\Psi(x,t)}}$$

- ➢ コペンハーゲン解釈

波動関数を「確率振幅」とする（ボルンの確率解釈）。観測の瞬間波束は収縮する。

↕ 古典量子境界なし

- ➢ 多世界解釈

確率解釈も古典的測定系も必要としない、あらゆる状態が重ね合わされている。

# 巨視的実在論

- ➢ 巨視的物体の観測量は、観測の有無に関わらず決まった値をもつ。⟺ 量子論

Einstein「月は私が見ている時だけ存在すると思うか？」　古典量子境界あり

- ➢ 提唱されているモデル

Spontaneous collapse models

✓ Continuous Spontaneous Localization (CSL)
✓ Diosi-Penrose (DP)

# Spontaneous Collapse Models

✓ CSL model (the most advanced model)

シュレディンガー方程式に非線形(non-linear)、確率(stochastic)項を導入。局在化距離、陽子局在化レートの2つがfree parameter

✓ DP model (gravity-induced)

重力が波束を収縮させる。重力の効果が大きくなると、非線形、確率項が大きくなり、局在化レートも高くなる。

自発的な波束の収縮 ⇨ 観測にはCat Stateが必要？

# マスター方程式

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}_{qm}|\Psi\rangle \quad + \quad \text{確率項}$$

密度行列 $\hat{\rho} = \sum_k p_k|\Psi_k\rangle\langle\Psi_k|$ の言葉で書いたのが

$$\boxed{\underline{\frac{\partial\hat{\rho}}{\partial t} = -\frac{i}{\hbar}[\hat{H}_{qm},\hat{\rho}]} - \frac{D_{sp}}{\hbar^2}[\hat{x},[\hat{x},\hat{\rho}]]}$$

マスター
(Lindblad)
方程式

フォン•ノイマン方程式

⇩ 古典化

$$\boxed{\frac{\partial\rho}{\partial t} = \{H,\rho\} + \underline{D_{sp}\frac{\partial^2}{\partial p^2}\rho}}$$

リウヴィル方程式

Cat State 不要

ρ : 密度関数

ランダムに生じる運動量拡散

# 他のデコヒーレンス要因

$$\frac{\partial \rho}{\partial t} = \{H, \rho\} + \underline{D_{sp} \frac{\partial^2}{\partial p^2} \rho}$$ 波束の収縮に伴う揺動力

$$\underline{+2\gamma \frac{\partial}{\partial p} p\rho + D_{th} \frac{\partial^2}{\partial p^2} \rho}$$ ランジュバン方程式

$$+\underline{D_m \frac{\partial^2}{\partial p^2} \rho}$$ 測定に伴う反作用

新たなForce Spectrum $\underline{D_m + D_{th} + D_{sp}}$

(未知の)Heating効果と見なせる：$D_{th}'$

# Heating Effect

$$T' = \left(1 + \frac{D_{sp}}{D_{th}}\right) T \equiv T + \underline{\Delta T_{sp}}$$

Decoherence の効果をSN比で論じられる

$$D_{sp} = 2k_B \Delta T_{sp} \gamma m$$

✓ DP model

$$\boxed{D_{DP} = \frac{\hbar m G \rho}{12\sqrt{\pi}} \left(\frac{a}{\sigma_{DP}}\right)^3}$$

格子定数
ここでは
5e-10 m

空間分解能(<<a)。DP model でのfree parameter

$$\Delta T_{DP} = \frac{\hbar G \rho}{12\sqrt{\pi} k_B} \left(\frac{a}{\sigma_{DP}}\right)^3 \tau \simeq 4 \times 10^{-5} \tau \ [s]$$

# Heating：DP model

$$\Delta T_{DP} = \frac{\hbar G \rho}{12\sqrt{\pi} k_B} \left( \frac{a}{\sigma_{DP}} \right)^3 \tau \simeq 4 \times 10^{-5} \tau \ [s]$$

$$\tau = \frac{1}{2\gamma}$$ ：relaxation time

およそ原子核サイズ1e-14 mを仮定

上昇温度は…

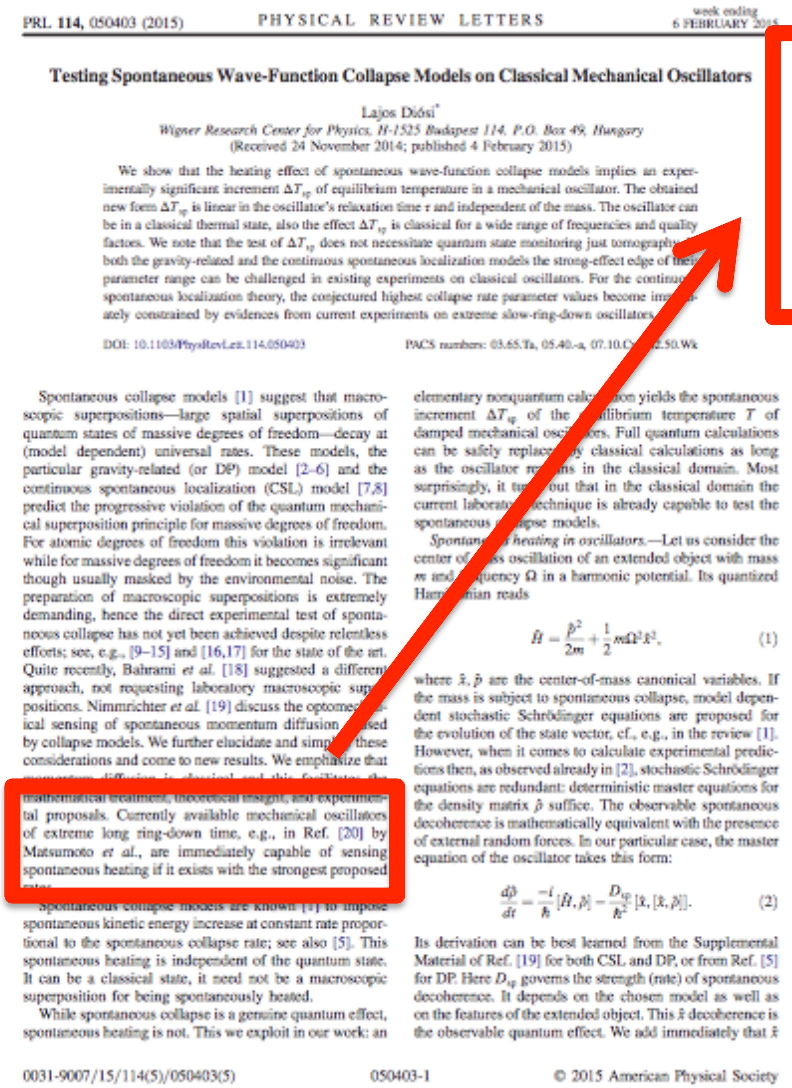
✧ 質量依存性なし

✧ DP modelの場合free parameterは1つ

✧ 緩和時間に比例。散逸が小さいほど大

オプトメカではこれまでf*Qが指標となってきたが、このモデルではQ/fが重要

# 検証可能性の高い実験



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## Testing Spontaneous Wave-Function Collapse Models on Classical Mechanical Oscillators

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We show that the heating effect of spontaneous wave-function collapse models implies an experimentally significant increment $\Delta T_{sp}$ of equilibrium temperature in a mechanical oscillator. The obtained new form $\Delta T_{sp}$ is linear in the oscillator's relaxation time $\tau$ and independent of the mass. The oscillator can be in a classical thermal state, also the effect $\Delta T_{sp}$ is classical for a wide range of frequencies and quality factors. We note that the test of $\Delta T_{sp}$ does not necessitate quantum state monitoring just tomography; in both the gravity-related and the continuous spontaneous localization models the strong-effect edge of their parameter range can be challenged in existing experiments on classical oscillators. For the continuous spontaneous localization theory, the conjectured highest collapse rate parameter values become immediately constrained by evidences from current experiments on extreme slow-ring-down oscillators.

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Spontaneous collapse models [1] suggest that macroscopic superpositions—large spatial superpositions of quantum states of massive degrees of freedom—decay at (model dependent) universal rates. These models, the particular gravity-related (or DP) model [2–6] and the continuous spontaneous localization (CSL) model [7,8] predict the progressive violation of the quantum mechanical superposition principle for massive degrees of freedom. For atomic degrees of freedom this violation is irrelevant while for massive degrees of freedom it becomes significant though usually masked by the environmental noise. The preparation of macroscopic superpositions is extremely demanding, hence the direct experimental test of spontaneous collapse has not yet been achieved despite relentless efforts; see, e.g., [9–15] and [16,17] for the state of the art. Quite recently, Bahrami *et al.* [18] suggested a different approach, not requesting laboratory macroscopic superpositions. Nimmrichter *et al.* [19] discuss the optomechanical sensing of spontaneous momentum diffusion caused by collapse models. We further elucidate and simplify these considerations and come to new results. We emphasize that momentum diffusion is classical and this facilitates the mathematical treatment, theoretical insight, and experimental proposals. Currently available mechanical oscillators of extreme long ring-down time, e.g., in Ref. [20] by Matsumoto *et al.*, are immediately capable of sensing spontaneous heating if it exists with the strongest proposed rates.

Spontaneous collapse models are known [1] to impose spontaneous kinetic energy increase at constant rate proportional to the spontaneous collapse rate; see also [5]. This spontaneous heating is independent of the quantum state. It can be a classical state, it need not be a macroscopic superposition for being spontaneously heated.

While spontaneous collapse is a genuine quantum effect, spontaneous heating is not. This we exploit in our work: an elementary nonquantum calculation yields the spontaneous increment $\Delta T_{sp}$ of the equilibrium temperature $T$ of damped mechanical oscillators. Full quantum calculations can be safely replaced by classical calculations as long as the oscillator remains in the classical domain. Most surprisingly, it turns out that in the classical domain the current laboratory technique is already capable to test the spontaneous collapse models.

*Spontaneous heating in oscillators.*—Let us consider the center of mass oscillation of an extended object with mass $m$ and frequency $\Omega$ in a harmonic potential. Its quantized Hamiltonian reads

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\Omega^2\hat{x}^2, \tag{1}$$

where $\hat{x}, \hat{p}$ are the center-of-mass canonical variables. If the mass is subject to spontaneous collapse, model dependent stochastic Schrödinger equations are proposed for the evolution of the state vector, cf., e.g., in the review [1]. However, when it comes to calculate experimental predictions then, as observed already in [2], stochastic Schrödinger equations are redundant: deterministic master equations for the density matrix $\hat{\rho}$ suffice. The observable spontaneous decoherence is mathematically equivalent with the presence of external random forces. In our particular case, the master equation of the oscillator takes this form:

$$\frac{d\hat{\rho}}{dt} = \frac{-i}{\hbar}[\hat{H}, \hat{\rho}] - \frac{D_{sp}}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}]]. \tag{2}$$

Its derivation can be best learned from the Supplemental Material of Ref. [19] for both CSL and DP, or from Ref. [5] for DP. Here $D_{sp}$ governs the strength (rate) of spontaneous decoherence. It depends on the chosen model as well as on the features of the extended object. This $\hat{x}$ decoherence is the observable quantum effect. We add immediately that $\hat{x}$

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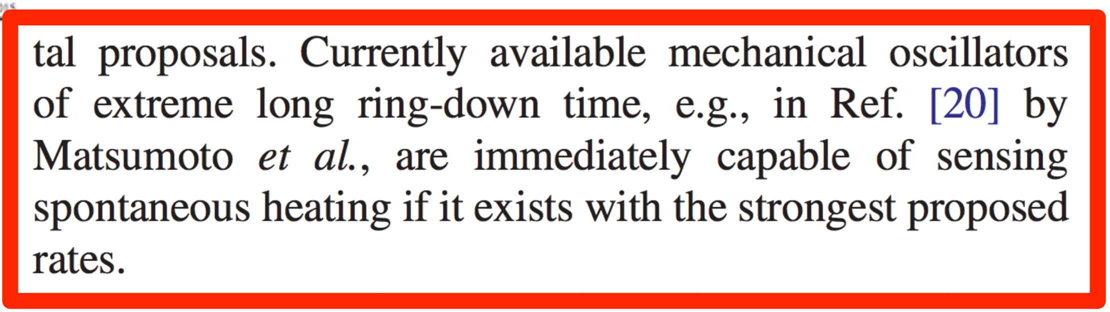

tal proposals. Currently available mechanical oscillators of extreme long ring-down time, e.g., in Ref. [20] by Matsumoto *et al.*, are immediately capable of sensing spontaneous heating if it exists with the strongest proposed rates.

Diosiの論文では(Hammerの論文でも)共振周波数が0.5Hzになっているが、Q値50万で

$$\Delta T_{DP} = 6.4K$$

# Heating : CSL model

$$D_{CSL} = \frac{4\pi\hbar^2\rho m}{m_0^2 d}\lambda_{CSL}\sigma_{CSL}^2$$

局在化距離。
1e-7mとされる。

標準原子単位

運動量拡散の起こる平面
の厚み

$$\boxed{2.2\times10^{-17}\ Hz \le \lambda_{CSL} \le 2.2\times10^{-8\pm2}\ Hz}$$

局在化レート

$$\Delta T_{CSL} = \frac{4\pi\hbar^2\rho}{k_B m_0^2 d}\lambda_{CSL}\sigma_{CSL}^2\tau$$

$$\simeq \frac{\tau[s]}{d[m]}6.4\times10^{-8\pm2}K$$

$$5.1\times10^{1\pm2}K$$

# まとめ

- 自発的に波束が収縮するというモデルの検証にはCat Stateは不必要。古典的な運動量拡散という形で現れる。
- 運動量拡散由来の信号をHeating Effectと見なせば、加熱温度は測定対象の質量に依存せず、緩和時間に比例する。
- 緩和時間の非常に長い振り子という機械振動子は、パラメータの上限値を大幅に更新するポテンシャルを秘める。