The LISA concept

KOHEI YAMAMOTO
D1 IN KAJITA LAB

Who am I ?

Name: Kohei Yamamoto

2013/4 – 2017/3 : *Faculty of engineering, Kyoto University*

Bachelor's Thesis: Evolution of Stress Tensor of Electron and Nuclei based on the Primary Rigged QED

2017/4 – 2019/3 : *Department of Physics, the University of Tokyo (Kajita lab)*

Master's Thesis: Laser modulation system for interferometer sensing and control for Detuned RSE in KAGRA

2019/4 – 2019/6 (7?) : *Department of Physics, the University of Tokyo (Kajita lab)*

Thanks to Izumi-san, now staying in ISAS/JAXA in Sagamihara as a visitor

2019/6 - : *Department of Physics, Leibniz University Hannover*

Max Planck Institute for Gravitational Physics, Hannover (Gerhard Heinzel Lab)

Supposed to belong to the **LISA experimental group** (Specifically, Optical Bench subgroup)

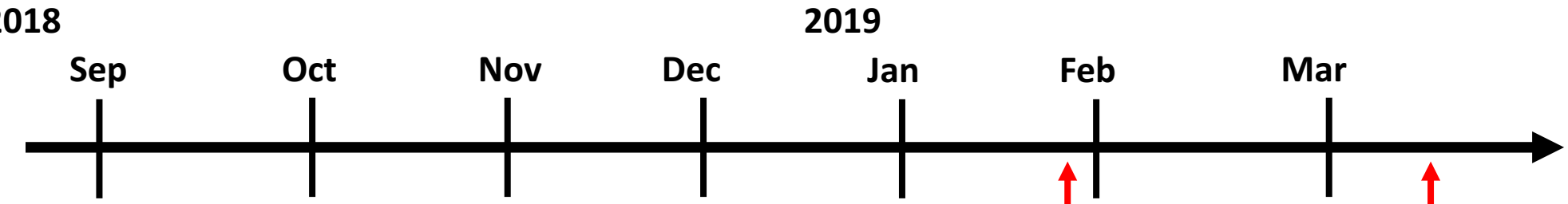
Thank you for inviting me today!

Here is two requests I received when invited.

Takano, “Please tell us how you got admission from AEI”

Enomoto-san, “Tell us how LISA detects GWs !!!”

Until I got a new position



Sent the first email
to Prof. Gerhard Heinzel

Take TOEFL👎

Skype Conversation:
They have almost 10 people

Good opportunity to let them know me

keep in touch on email

nervous waiting period

Complete applications
- CV
- Letters
- GPA
- Essay

AEI sets no deadline of applications

- Interview by Zoom
- Got admission on next day👍

Thank you for inviting me today!

Here is two requests I received when invited.

Takano, “Please tell us how you got admission from AEI”

Enomoto-san, “Tell us how LISA detects GWs !!!”

Today's talk

- Now, I study LISA with Izumi-san based on the document, called "Yellow Book".

- Today's talk basically focuses on the generic part of LISA, like the science target, experimental concept, anticipated noises and whatever..

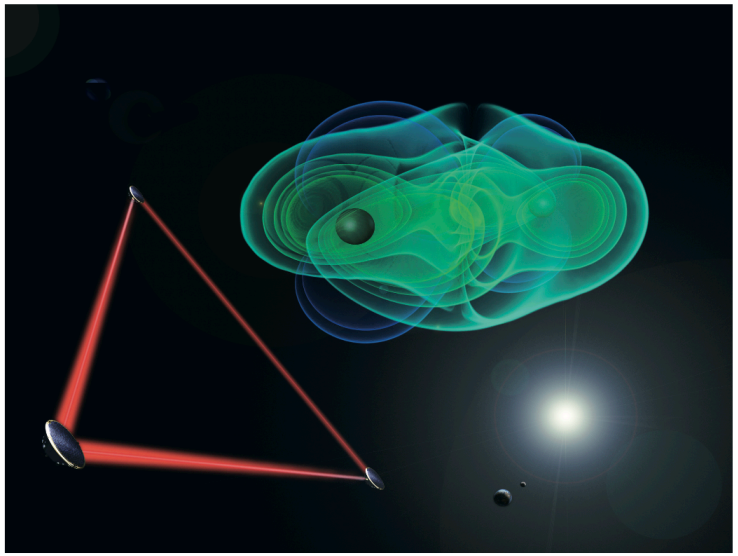
※ Some parts of the contents in this slide might be out of fashion now…

esa

ESA/SRE(2011)3
February 2011

LISA

Unveiling a hidden Universe



Assessment Study Report

Contents

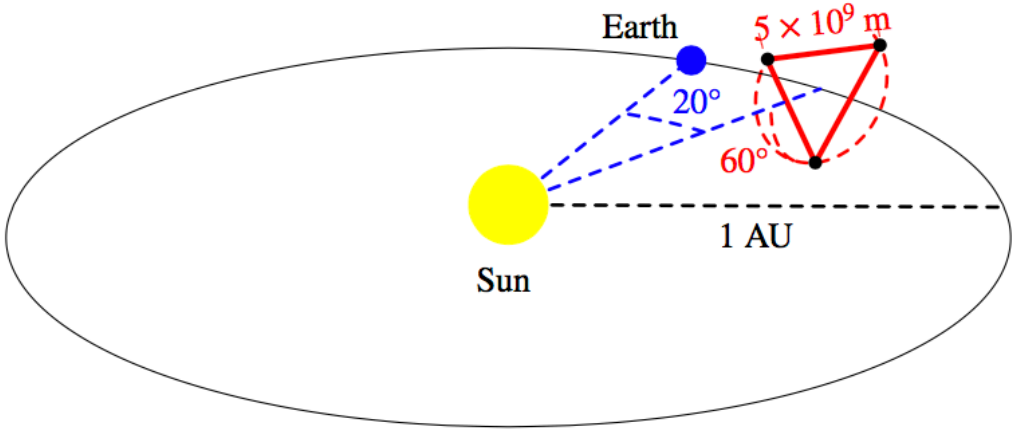
- **The LISA concept**

- Science Requirements Overview
  - Science Objectives
  - Instrumental Noise model and Noise allocation

- Towards the detection of GW
  - Time-Delay Interferometry
  - Payload

The LISA concept

First of all, anyway, I list **three key concepts** of LISA.
(I'll explain the payload of LISA more in detail later)



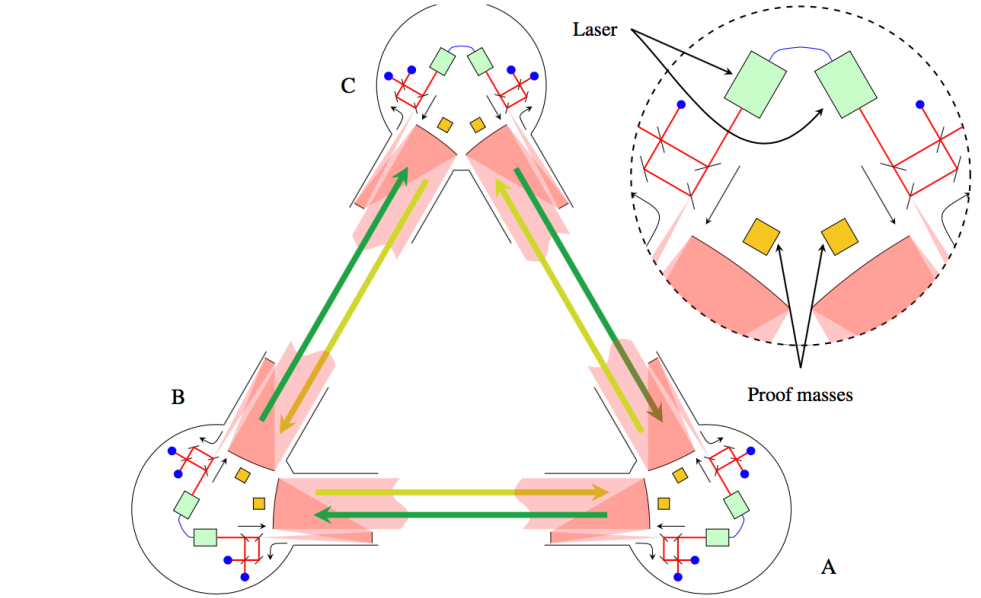
- **Break up the measurement to three parts**
- **Drag-free operation**
  - Surround the test mass by S/C and control the relative position of TM and S/C
  - To prevent the test mass from the time-varying disturbance
- **Continuous laser ranging scheme, called "transponder" mode**
  - One S/C returns the high power beam which is phase-locked to the received beam.

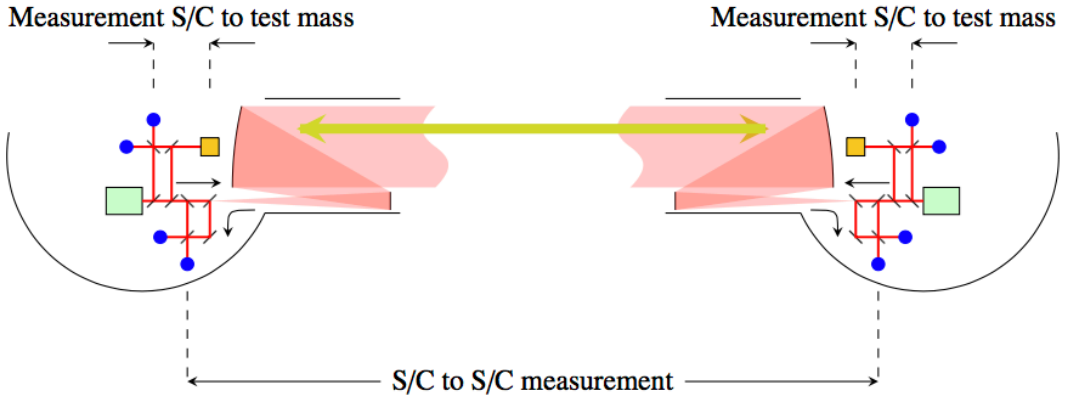
The LISA concept/Break up the measurment to 3 parts

- What we want to measure, the path length b/w test masses, is split into 3
  - One: S/C to S/C
  - Two: Test Mass to S/C

- This is necessary for the total phase measurement from mass to mass.

Explain the detail later…





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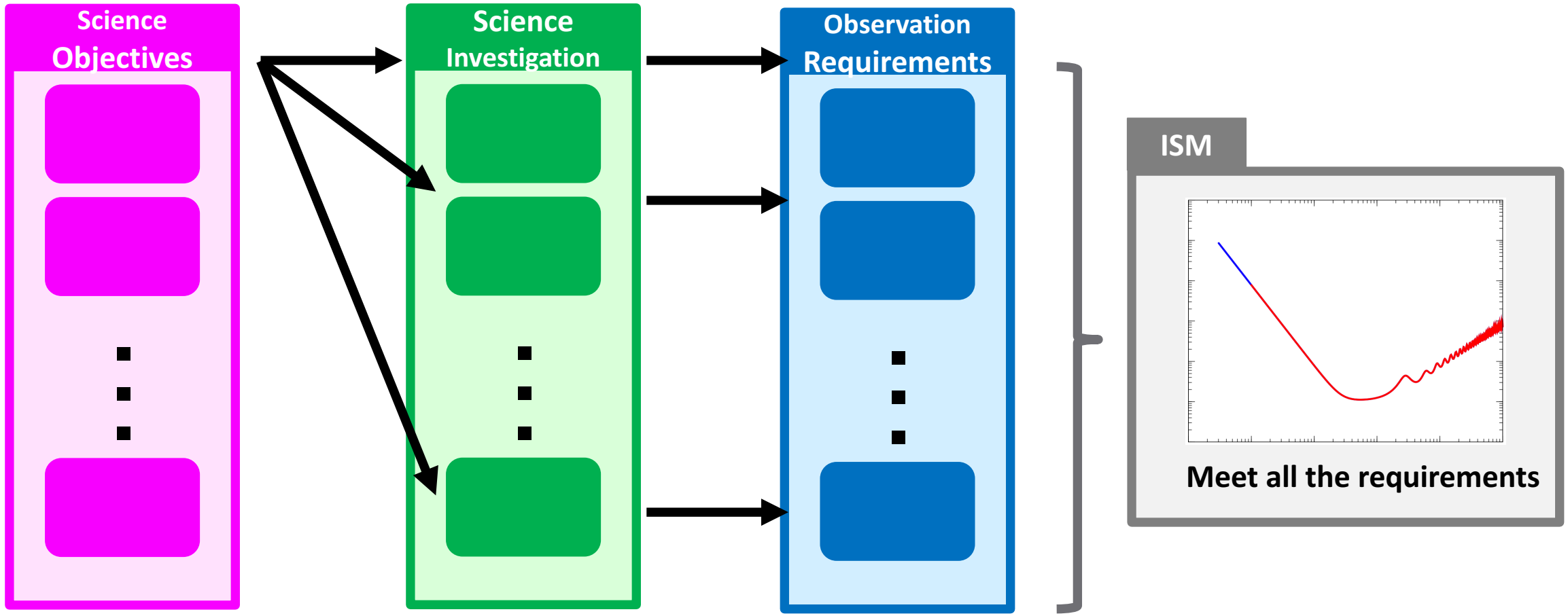
From Science Objectives to Instrumental Noise Model (ISM)

Here, I show how to set the instrumental noise model of LISA.
In this part, I'll briefly explain each in the schematic, especially science objectives and ISM.

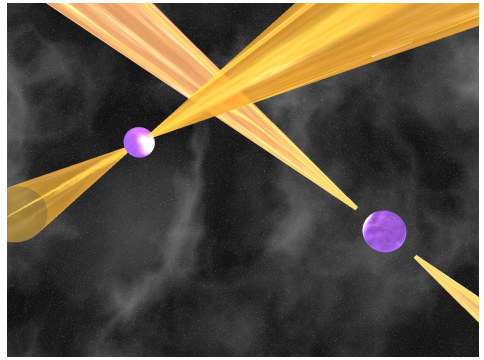


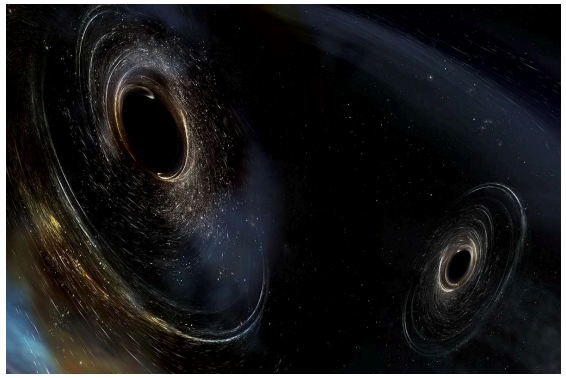
Science Objectives

Here, I list up the science objectives in LISA band

- **Test of General Relativity**
  MBHs merger and EMRI
- **Determine the history of BHs to generate the MBHs in galactic nuclei today**
  map the mass of BHs with redshift
- **Survey thousands of new compact stellar-mass binaries**
  Formation history of them and Structure of Galaxy
- **Precision Cosmology**
- **New physics**
  Cosmological background, Cosmic string and unforeseen sources

Science Objectives/Test of General Relativity





So far, we found no evidence of the violation of GR

- Test in solar system
- Binary pulsars
  Period derivative
- GWs detected with the ground-based detector

LISA tests GR in the much stronger gravitational field with the SNR as high as several thousands.

There are two sources good for this purpose

- **Merger of MBHs**
- **Extreme Mass Ratio Inspirals (EMRI)**

Science Objectives/Test of General Relativity/MBHs

- $3 \times 10^5\,\mathrm{M}_\odot < M(1+z) < 3 \times 10^7\,\mathrm{M}_\odot$
  ※Roughly comparable mass

- **Inspiral:** The last $10^4 - 10^5$ cycles in inspiral phase can be detected
  - Mass and spins can be determined to high accuracy
  - Leads to good prediction of the burst waveform

- **Merger:** The predicted waveform based on GR can be compared to the observed burst signals. -> Ideal test of GR
  Amplitude of the burst waveform is so large that it is visible by eye in the data stream (see Fig.1).

- **Ringdown:** We can derive the mass and spin of each BH from the ringdown parameter: Frequency and Damping time.

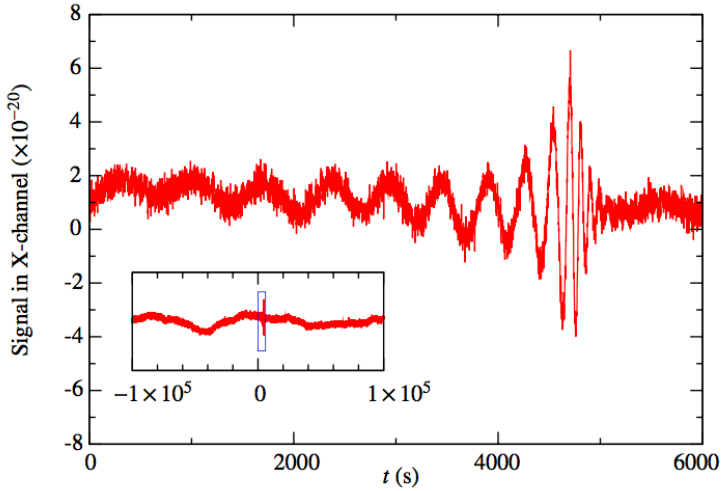


Fig.1

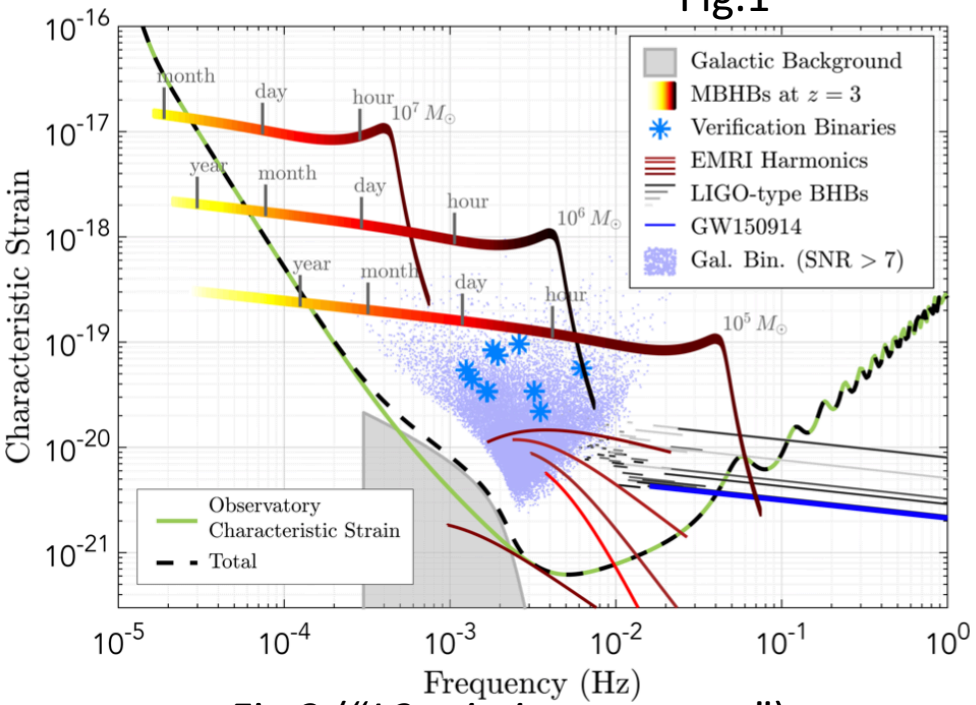
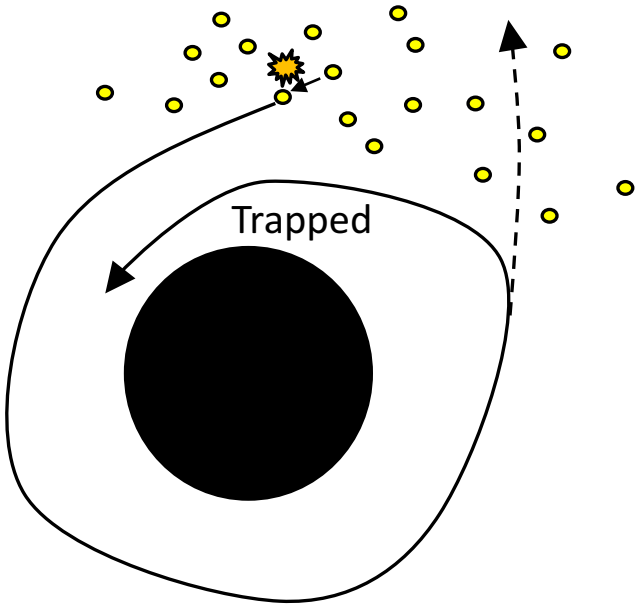


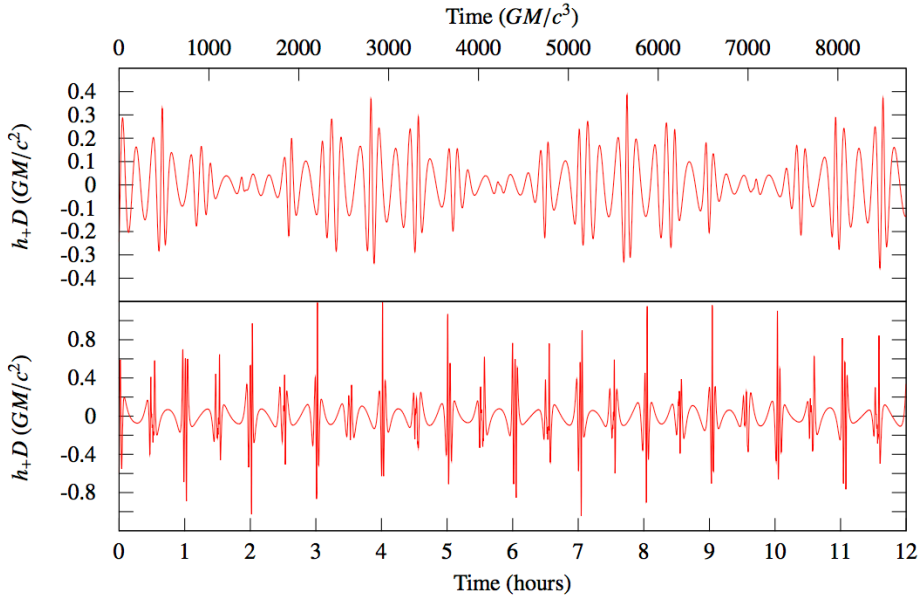
Fig.2 (“*L3 mission concepts*”)

Science Objectives/Test of General Relativity/EMRI

- Mass ratio: $10^{-4} - 10^{-7}$
- Non-zero eccentricity
- Duration time near ISCO: few decades
  ->For LISA's entire observation time, the orbits are highly relativistic
- A fit of the waveform is very sensitive
  -> Computationally cost, but extremely accurate determination of parameters, *i.e.* determine mass and spin to accuracy of $10^{-5} - 10^{-3}$

This leads to the precise observational test of the "Kerr-ness" of the central MBH.





Generic EMRI waveforms

Science Investigation and Observation Requirement

Here is the science investigations and observation requirements of “Test of GR”

SI: research needed to fulfill scientific objectives
OR: observable quantities necessary for the SI and the precision with which they must be measured

Science Investigations	Observation Requirement
SR 4.1 Detect gravitational waves directly and measure their properties precisely.	**OR 4.1.1** LISA shall have capability to detect and study three or more optically observable verification binaries between 1 mHz and 10 mHz with SNR > 20 in two years of mission lifetime. **OR 4.1.2** LISA shall be capable of observing the gravitational waves from at least 50 % of all $z \sim 2$ coalescing binary systems consisting of compact objects with masses between $10^5\,M_\odot$ and $10^6\,M_\odot$ and mass ratios between 1 : 1 and 1 : 3. LISA shall detect these systems with SNR ≥ 5 in each of five equal logarithmic frequency bands between 0.03 mHz (or the lowest observed frequency) and the highest inspiral frequency.
SR 4.2 Test whether the central massive objects in galactic nuclei are the black holes of GR.	**OR 4.2** LISA shall have the capability to detect gravitational waves emitted during the last year of inspiral for a $10\,M_\odot$ black hole orbiting a $3 \times 10^5\,M_\odot$ to $3 \times 10^6\,M_\odot$ black hole at 1 Gpc with SNR > 30. LISA shall have a science mission duration with adequate observation time for EMRIs to sweep over a range of r/M to map space-time, and to provide a good sample of events.
SR 4.3 Perform precision tests of dynamical strong-field gravity.	**OR 4.3** Observe the merger and ring-down radiation from all $1 \times 10^5\,M_\odot$ to $1 \times 10^6\,M_\odot$ black holes formed from approximately equal mass, $M_1 < 3M_2$, mergers to $z \leq 8$, measuring the mass and spin parameters, M and a, of the final black hole to $0.1M$. This will include essentially all systems with these masses as the rates are expected to be vanishingly small for higher redshifts (a $z < 5$ requirement, for example, would also include almost all likely events.).

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Instrument Sensitivity Model (ISM)

ISM : The noise permitted so that the sensitivity fulfill the science requirements

※This combines an instrumental noise mode with the antenna's transfer functions

$$\tilde{h}(f) = \underbrace{\sqrt{5}}\,\underbrace{\frac{2}{\sqrt{3}}}\,\underbrace{T(f)}\,\underbrace{\frac{\sqrt{S_{\delta x,\mathrm{IMS}}(f) + S_{\delta x,\mathrm{DRS}}(f)}}{L}}$$

Average over the full sky

$= 1/\sin(60°)$: Projection effect

Antenna's TF

Noise model (budget=requirement)

- IMS: Displacement noise from the measuring system
- DRS: Displacement noise from Spurious accelerations on TM

ISM/Antenna's TF

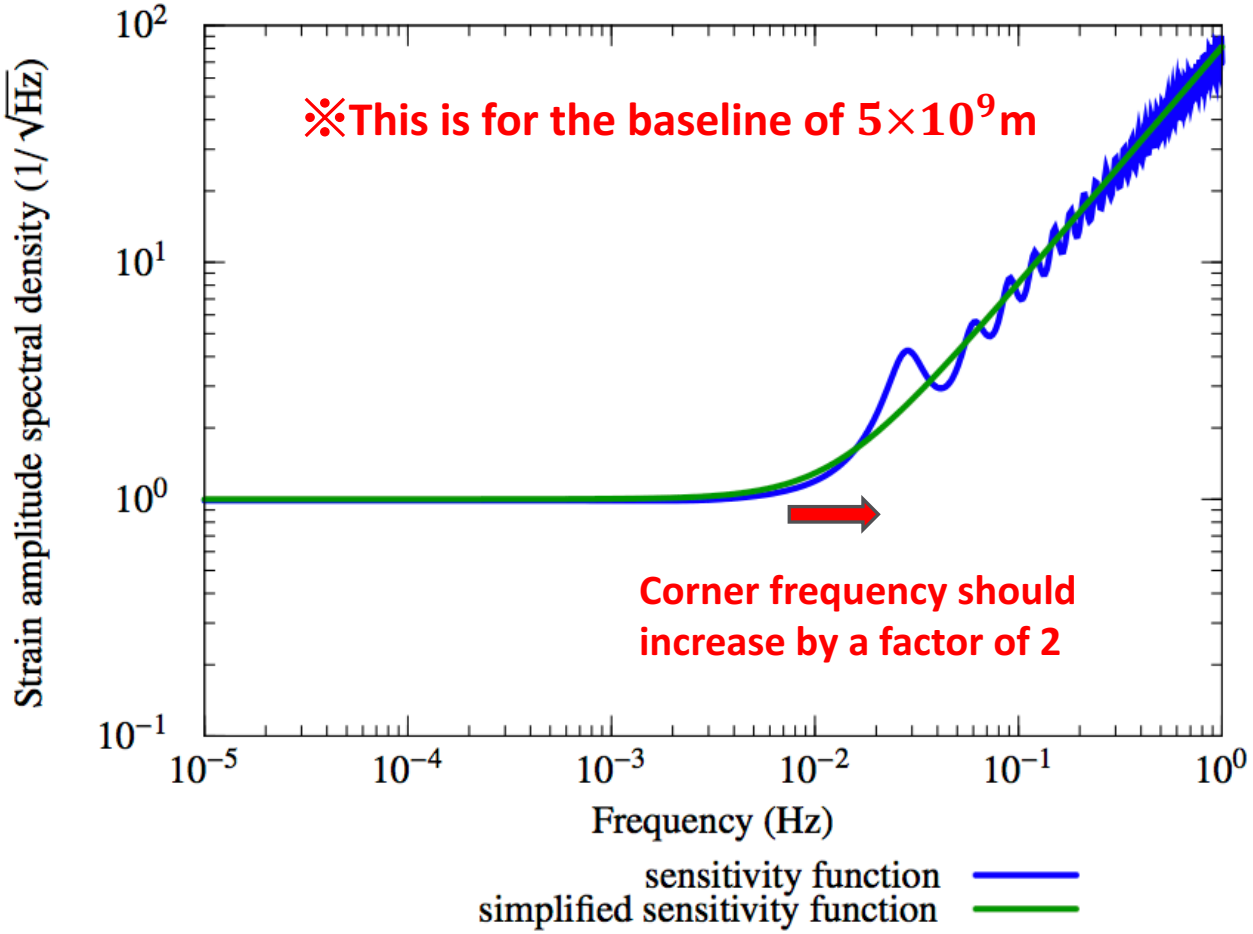
This is what we usually call the frequency response of the detector to GWs

For the approximation of T(f), the following equation can be used,

$$T(f) \approx \sqrt{1+\left(\frac{f}{af_0}\right)^2}$$

f_0: The frequency at which the cancellation of the GW signal starts, $= \frac{c}{2L} = 30\text{mHz}$

a: 0.41



ISM/Noise model

Dominant at low frequency

- $\delta x_{DRS}(f)$: acceleration noise, converted to displacement, of the Disturbance Reduction System (DRS)

$$\widetilde{\delta a}_{\mathrm{DRS}}(f) = \Delta a_0 \times 10^{-15}\,\mathrm{m/s^2/\sqrt{Hz}} \times \sqrt{1+\left(\frac{f}{f_H}\right)^4}\sqrt{1+\frac{f_L}{f}} \quad \text{for} \quad 3\times10^{-5}\,\mathrm{Hz} \le f \le 1\,\mathrm{Hz}$$

$\Delta a_0 = 3,\ f_H = 8\mathrm{mHz},\ f_L = 0.1\mathrm{mHz}$

Dominant at high frequency

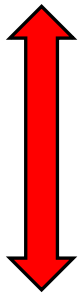
- $\delta x_{IMS}(f)$: displacement noise from the Interferometric Measuring System (IMS)

$$\widetilde{\delta x}_{\mathrm{IMS}}(f) = \Delta x_0 \times 10^{-12}\,\mathrm{m/\sqrt{Hz}} \times \sqrt{1+\left(\frac{f_0}{f}\right)^4} \quad \text{for} \quad 3\times10^{-5}\,\mathrm{Hz} \le f \le 1\,\mathrm{Hz}$$

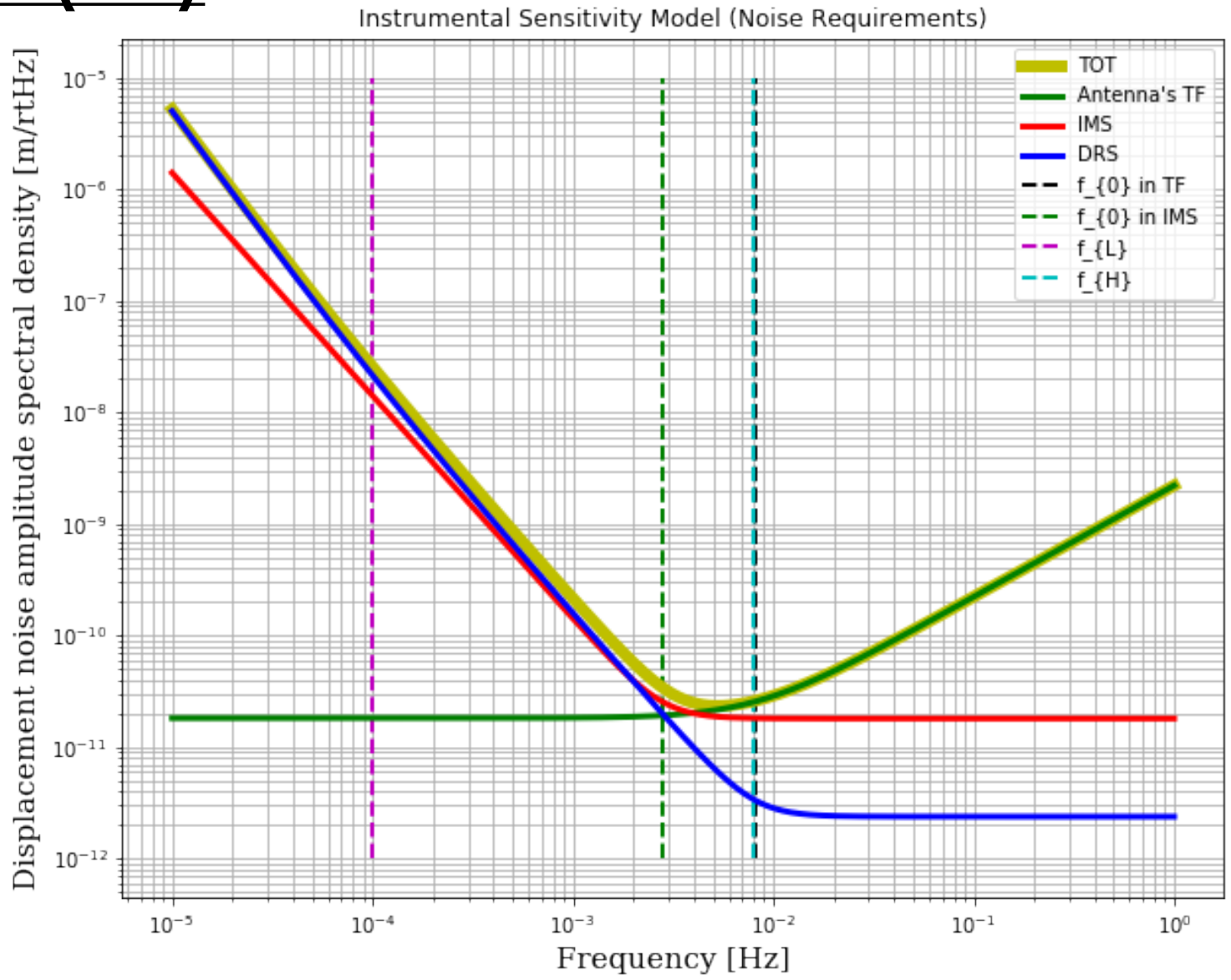
$\Delta x_0 = 18,\ f_H = 2.8\mathrm{mHz}$

Instrument Sensitivity Model (ISM)

Here is the displacement noise amplitude spectral density from the ISM.



We need to foresee the amount of noise of a certain subsystem. This is called allocation



Noise allocation

In amplitude spectral density, the allocation is 65% of the budget, *i.e.* requirement.
This margin is for contingency in science performance.

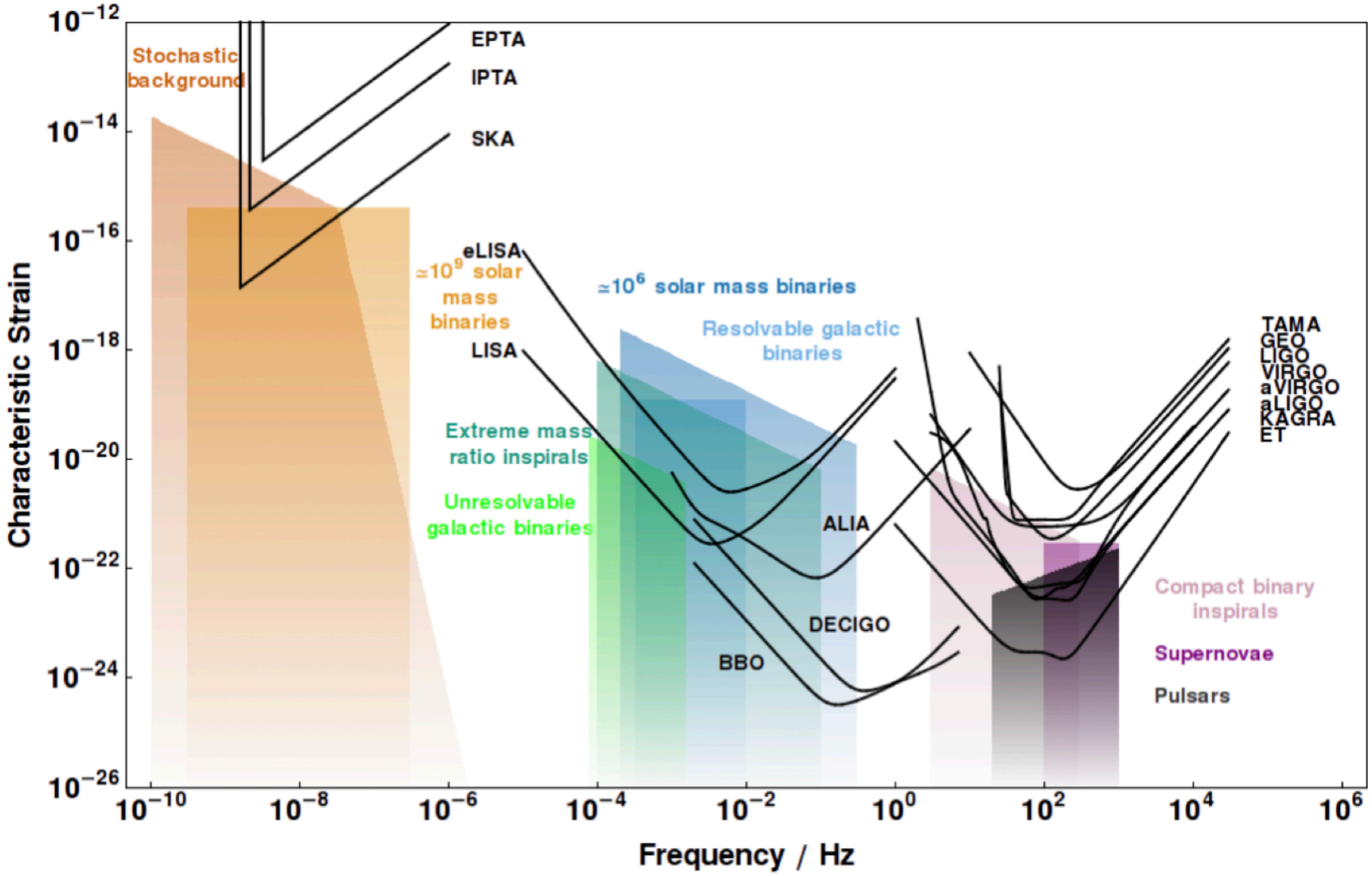
Table 3.3.: *Summary of IMS Subsystem Noise Allocations*

Effect	Total per group $\times 10^{-12}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$	Sub-allocation $\sqrt{1+(2.8\,\mathrm{mHz}/f)^4}$	Comments
Total IMS error/noise budget	18		
Total of subsystem allocations	11.7		RSS of subsystems
Subsystem allocations			Dominant
Shot noise	7.7		100 pW received power
Pathlength noise	7		
Pointing errors		5.3	
Telescope pathlength stability		1	
Optical bench pathlength stability		4.5	
Measurement noise	5.4		RSS
Photoreceiver errors		3	
Residual laser frequency noise		2	
Residual clock frequency noise		2	
Phasemeter noise		1	
Intensity noise		1	
Phase reconstruction		1	
Stray light		2	

Table 3.4.: *Summary of DRS Subsystem Noise Allocations*

Effect	Total per group $\times 10^{-16}\ \mathrm{m/s^2}/\sqrt{\mathrm{Hz}}$	Sub-allocation $\sqrt{1+(f/8\,\mathrm{mHz})^4}\sqrt{1+0.1\,\mathrm{mHz}/f}$	Comments
Total acceleration noise budget	30		
Total of subsystem allocations	19.5		RSS of subsystems
Electrostatics		12.0	
Brownian		9.1	
S/C magnetic		7.0	
S/C coupling		6.0	
S/C cross-coupling		4.5	
Thermal		4.0	
Interplanetary magnetic		4.0	
Misc small effects		4.0	

LISA band



Ref: C. J. Moore et al, "*Gravitational-wave sensitivity curves", Classical and Quantum Gravity*

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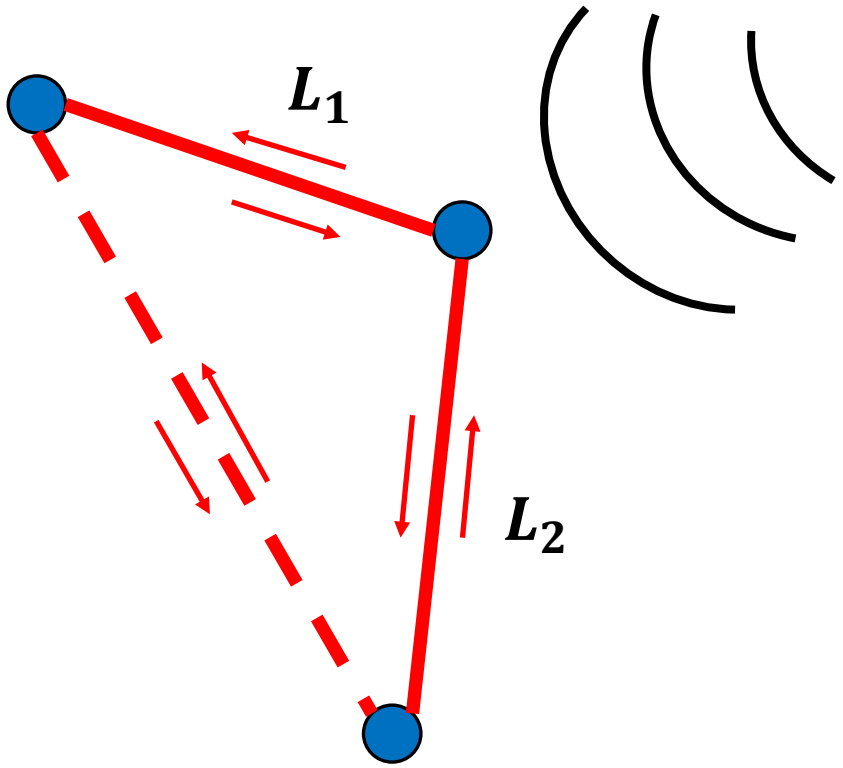
- **Towards the detection of GW**
  - **Phase measurement**
  - **Frequency stabilization**

Towards the detection of GWs



- The basic concept is “Michelson interferometer” of the two path
  ※ Third links contributes to the improvement by a factor of $\sqrt{2}$

- If we configure the standard MI, the frequency noise is too large to detect the GW **because $\Delta L = |L_1 - L_2|$ can be up to 10^8m**

- LISA measures L_1 and L_2 independently and combines the data series in post processing, called Time-delay Interferometry

Two main topics here

Now we confront two questions

How does LISA measure the path length?

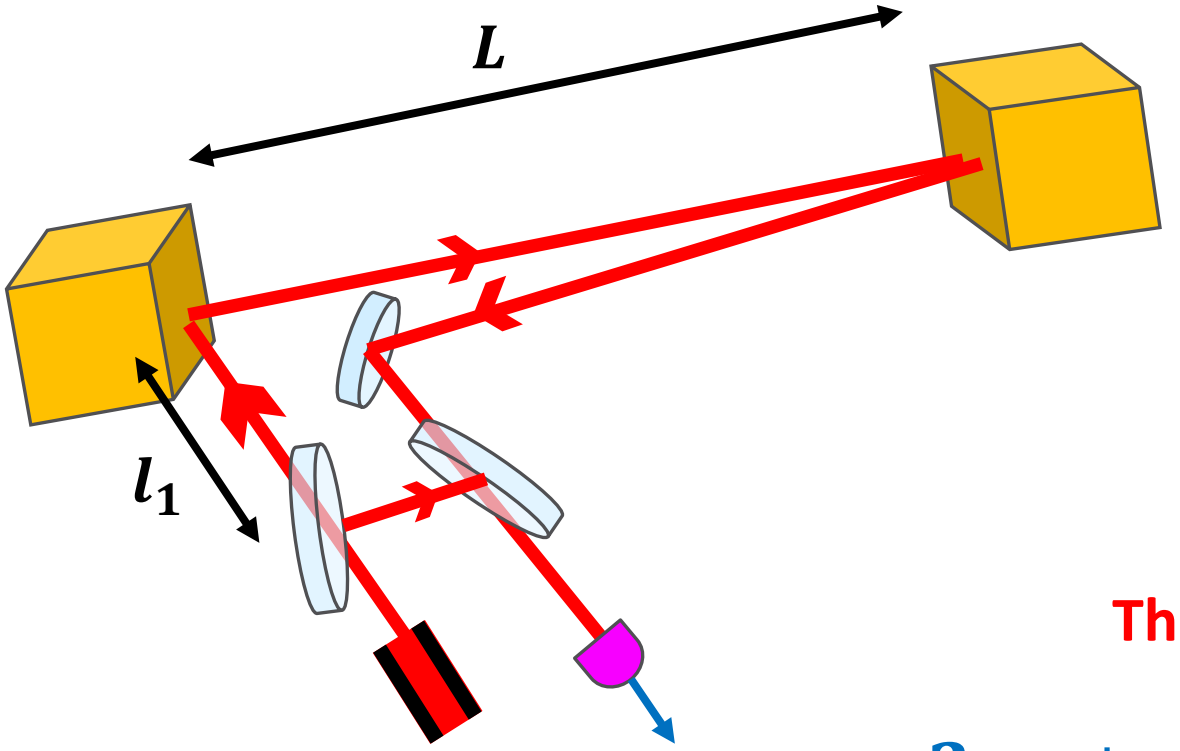
- Mass Interferometer
- Science Interferometer

How does LISA stabilize the laser frequency?

- Pre-stabilization
- Arm locking (Transponder)
- Time-Delay Interferometry

Phase measurement

Can we directly measure the distance from test mass to test mass?

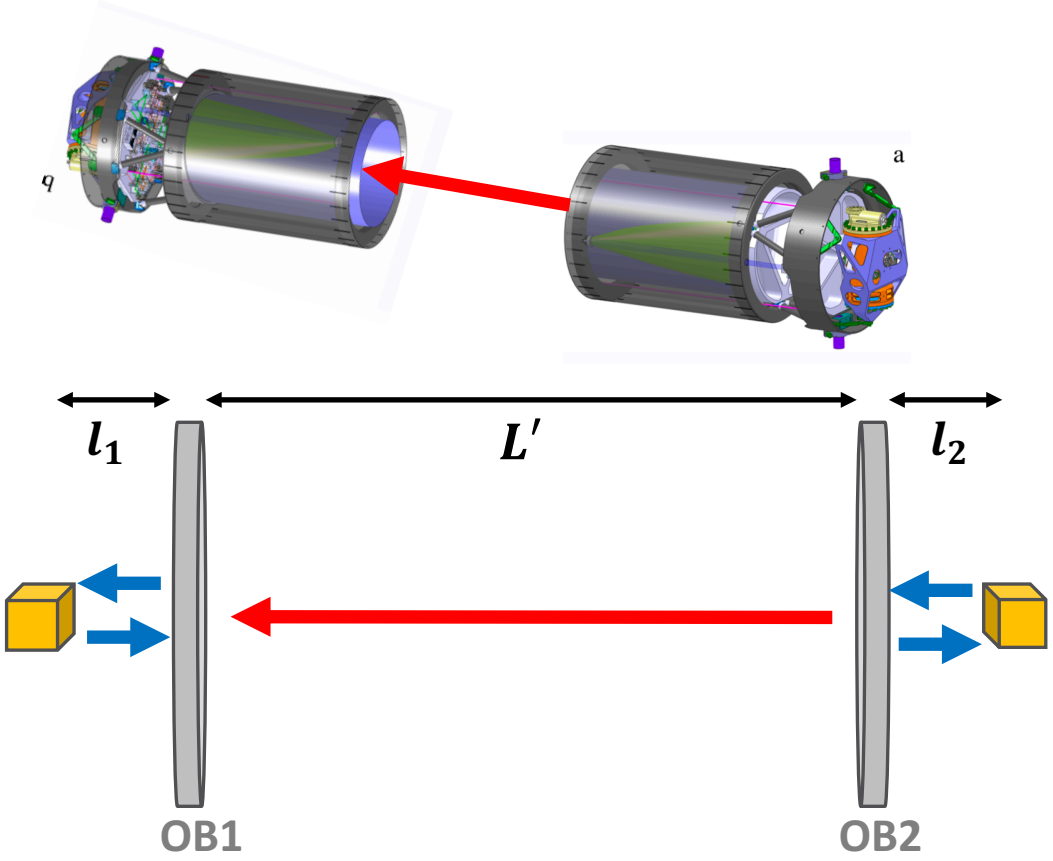


With only one phase measurement, it's difficult to extract the $2\varphi_L$ from the measured phase difference, φ_{sig}.

The phase measurement is broken up to 3 parts

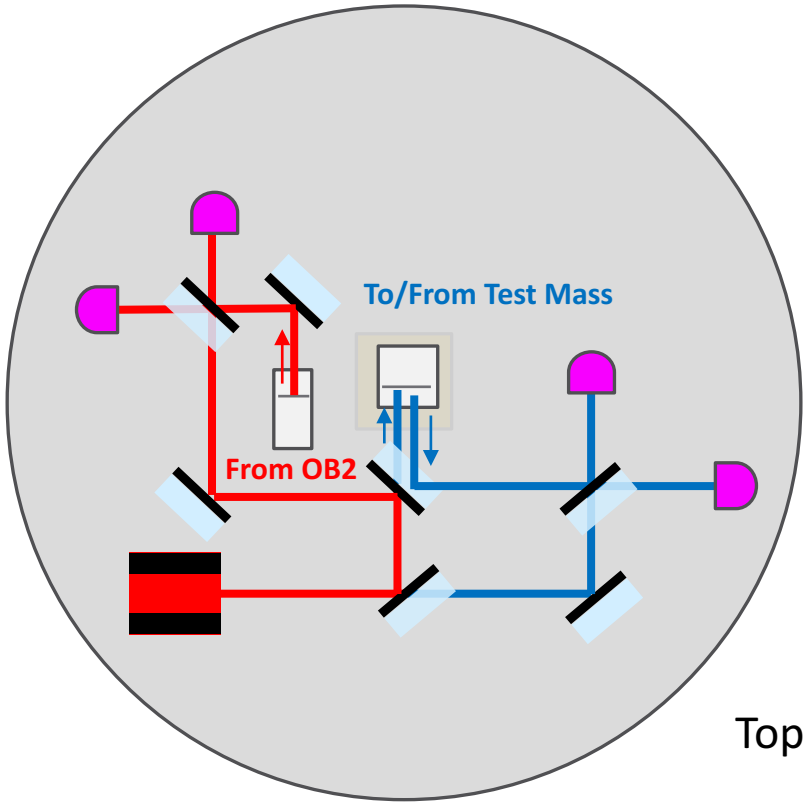
$$\varphi_{sig} = 2\varphi_L + \varphi_{l1}$$

Phase measurement/3parts



- Conceptually, one way path is enough for the measurement of $L = L' + l_1 + l_2$
- **Science Interferometer:** measure $\varphi_{L'}$
- **Mass Interferometer:** measure φ_{l1} (and φ_{l2} on OB2)

However, the GW signal is obviously under the frequency noise level $\widetilde{\delta\nu}_{free} = 3\text{MHz/rtHz}$, while $\widetilde{\delta\nu}_{req} = 1.2\times10^{-6}\text{Hz/rtHz}$



Top view

Frequency Stabilization

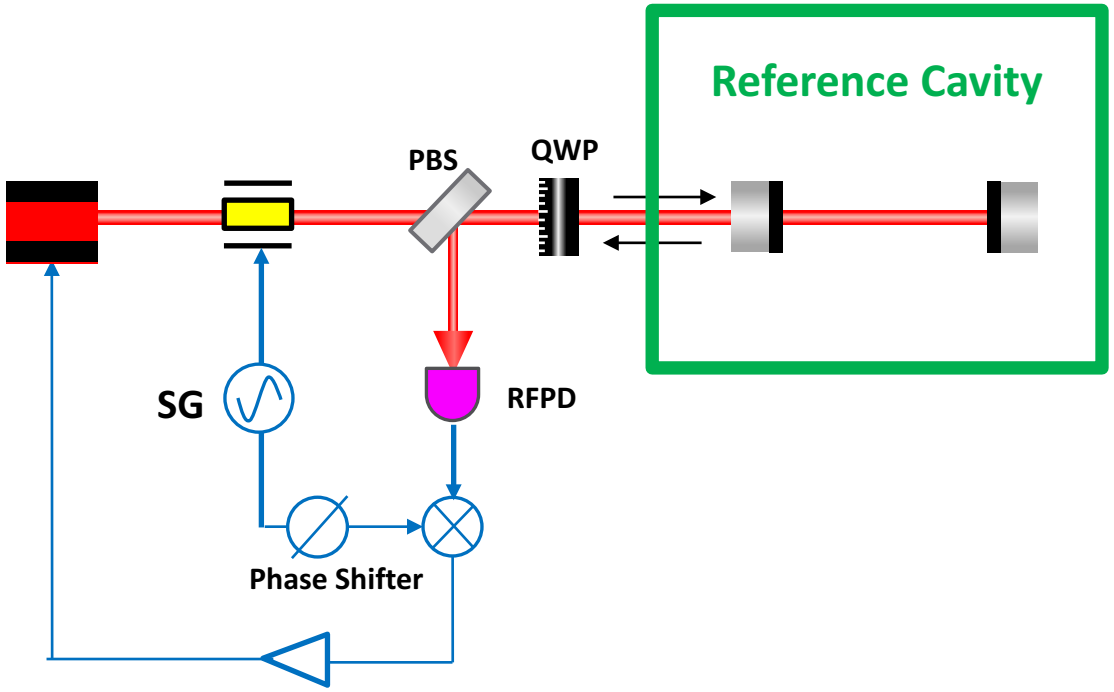
Here is the noise requirement of frequency stabilization

Stabilisation stage	Performance after stabilisation in Hz/ $\sqrt{\text{Hz}}$
Free running	$\frac{10^4}{f}$
Pre-stabilisation	$\frac{280}{f} \times \sqrt{1+\left(\frac{2.8\,\text{mHz}}{f}\right)^4}$
Arm locking	$0.3 \times \sqrt{1+\left(\frac{2.8\,\text{mHz}}{f}\right)^4}$
TDI	$4 \times 10^{-7} \times \sqrt{1+\left(\frac{2.8\,\text{mHz}}{f}\right)^4}$

- The reduction of frequency noise by about **12 orders of magnitude** is needed

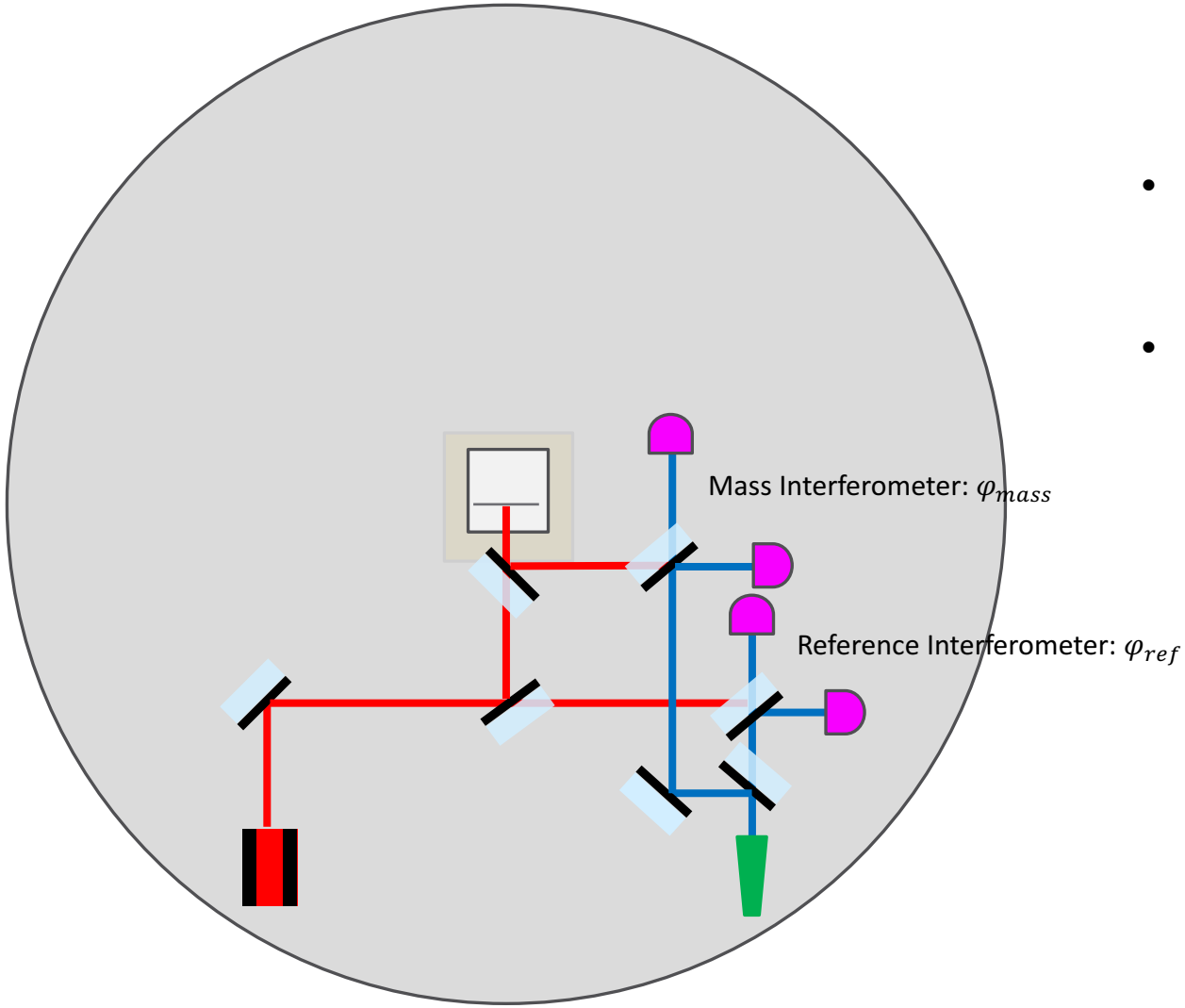
- **Pre-stabilization:** the basic scheme with the reference cavity
- **Arm locking:** PLL between S/Cs (transponder)
- **TDI:** Linear combination of data series in post processing

Frequency Stabilization/Pre-stabilization



- The usual scheme with a reference cavity is implemented
- One of the laser in S/C is stabilized by this PDH technique
- The other in the same S/C is phase-locked to this stabilized laser by so-called, **reference interferometer**

Frequency Stabilization/Pre-stabilization



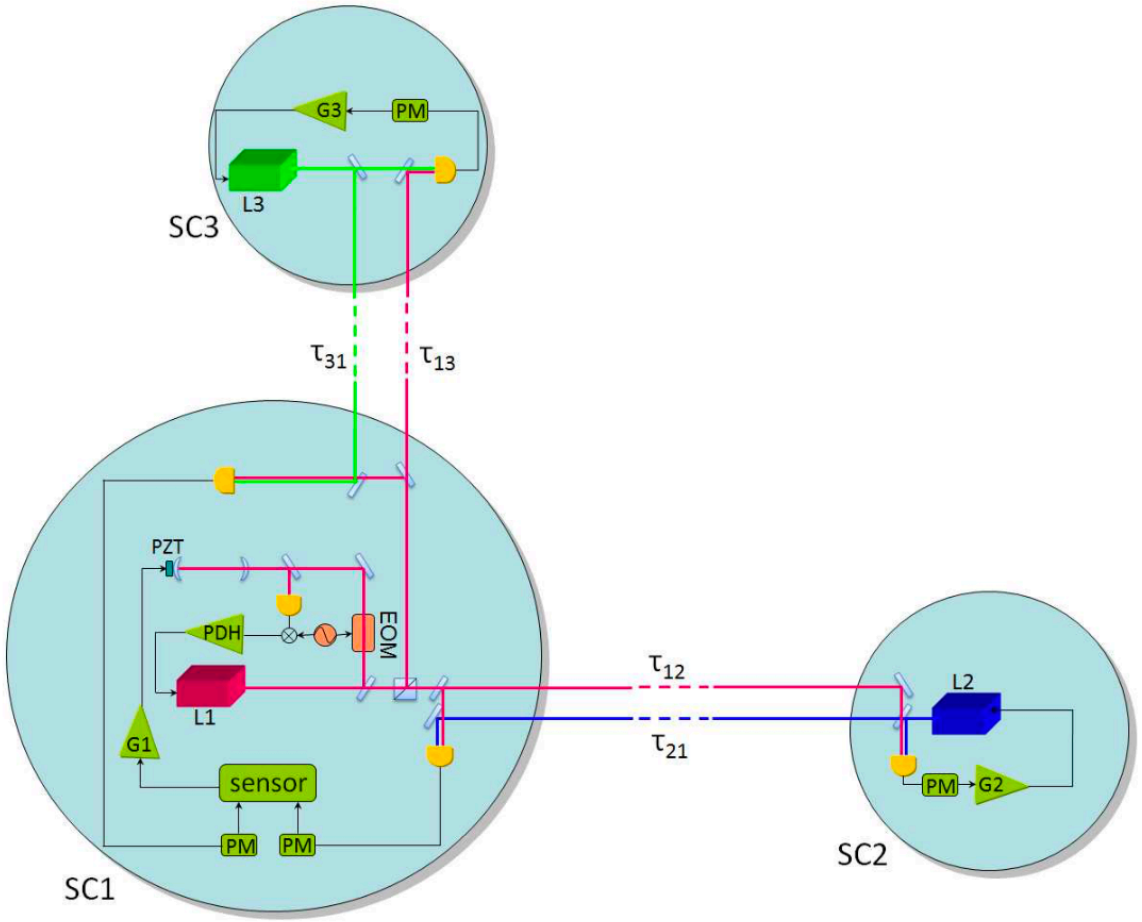
- Reference interferometer is used for the phase-locking b/w the primary laser and secondary one
- By using the secondary laser also for the mass interferometer, we can further reduce the phase(frequency) noise.
  - $\varphi_{ref}(t) = \varphi_1(t) - \varphi_2(t) + n_f(t)$
  - $\varphi_{mass}(t) = \varphi_1(t) - \varphi_2(t) + k\Delta L_{mass} + n_f(t)$

    $\varphi_{1,2}$: Phase of primary and secondary laser

    n_f: noise in back-link fiber

    ΔL_{mass}: relative position of test mass against OB

Frequency Stabilization/Arm locking (transponder)

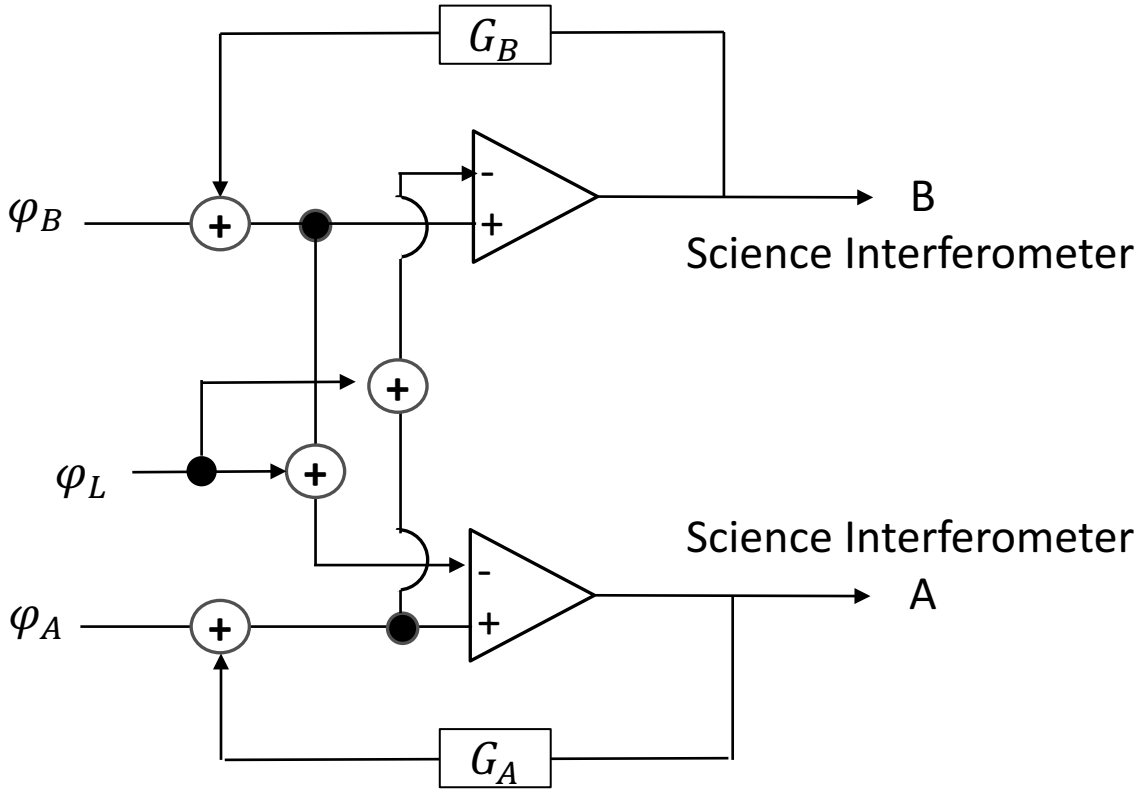


- In transponder mode, the transmitted beam is phase-locked to the received beam
- This is for generating the high power replica
- At the same time, this act as the second frequency stabilization

Ref: Yian Yu, doctral thesis, UF

Frequency Stabilization/Arm locking (transponder)

We can see how the frequency noise is cancelled by the phase-locking b/w transmitted and received laser



Block Diagram of the transponder mode

Without the phase-locking, the signal is highly affected by frequency noise

$$A = \varphi_A - \varphi_B + \varphi_L$$

$$\begin{cases} -G_B B + \varphi_B - (\varphi_L - G_A A + \varphi_A) = B & (1) \\ -G_A A + \varphi_A - (\varphi_L - G_B B + \varphi_B) = A & (2) \end{cases}$$

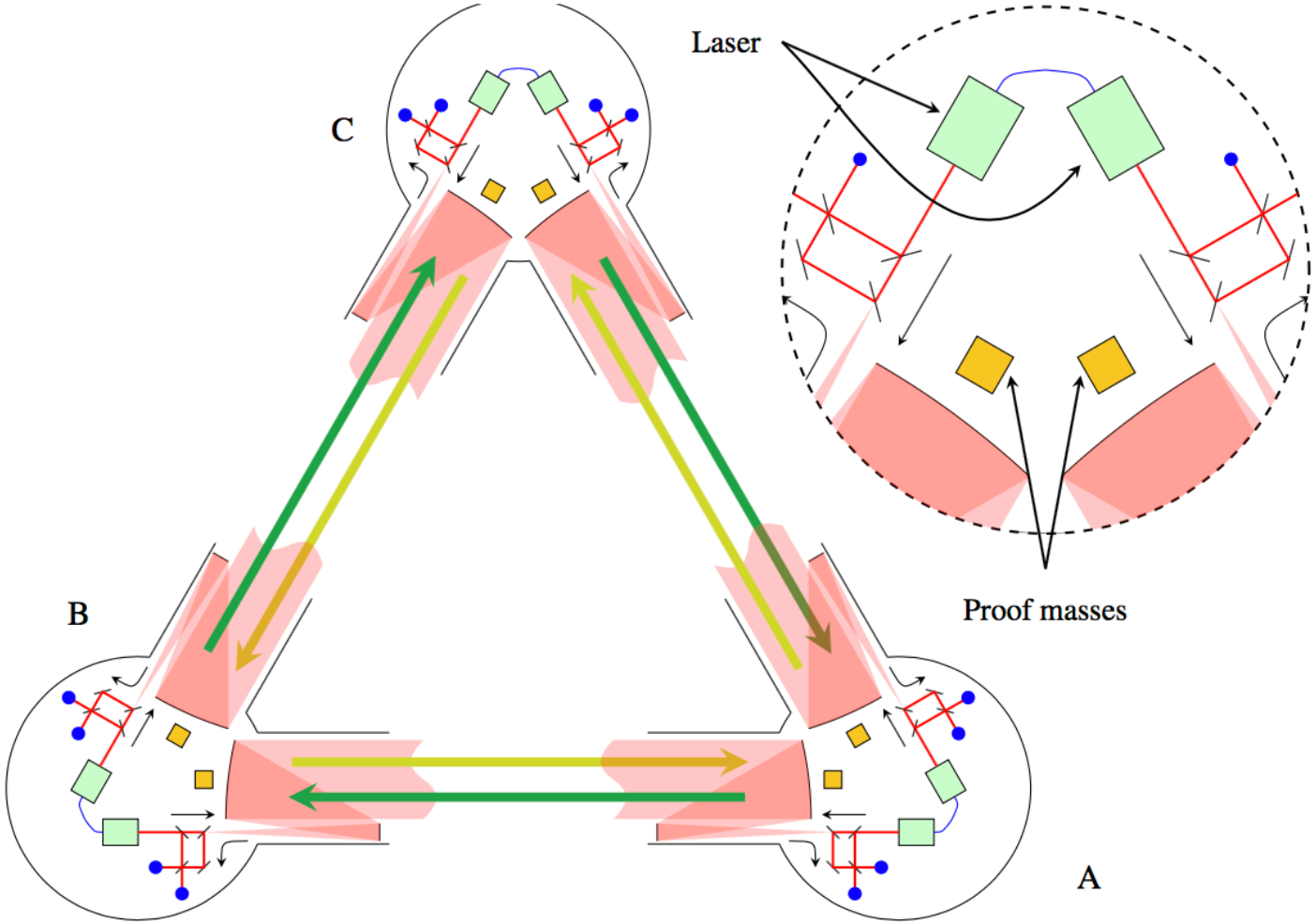
From (1) and (2), the signal of the science interferometer is derived:

$$\begin{cases} A = -\dfrac{1+2G_B}{1+G}\varphi_L + \dfrac{\varphi_A}{1+G} - \dfrac{\varphi_B}{1+G} & (3) \\ B = -\dfrac{1+2G_A}{1+G}\varphi_L + \dfrac{\varphi_B}{1+G} - \dfrac{\varphi_A}{1+G} & (4) \end{cases}$$

When $G_A, G_B \gg 1$

φ_L $\quad$ 0

Frequency Stabilization

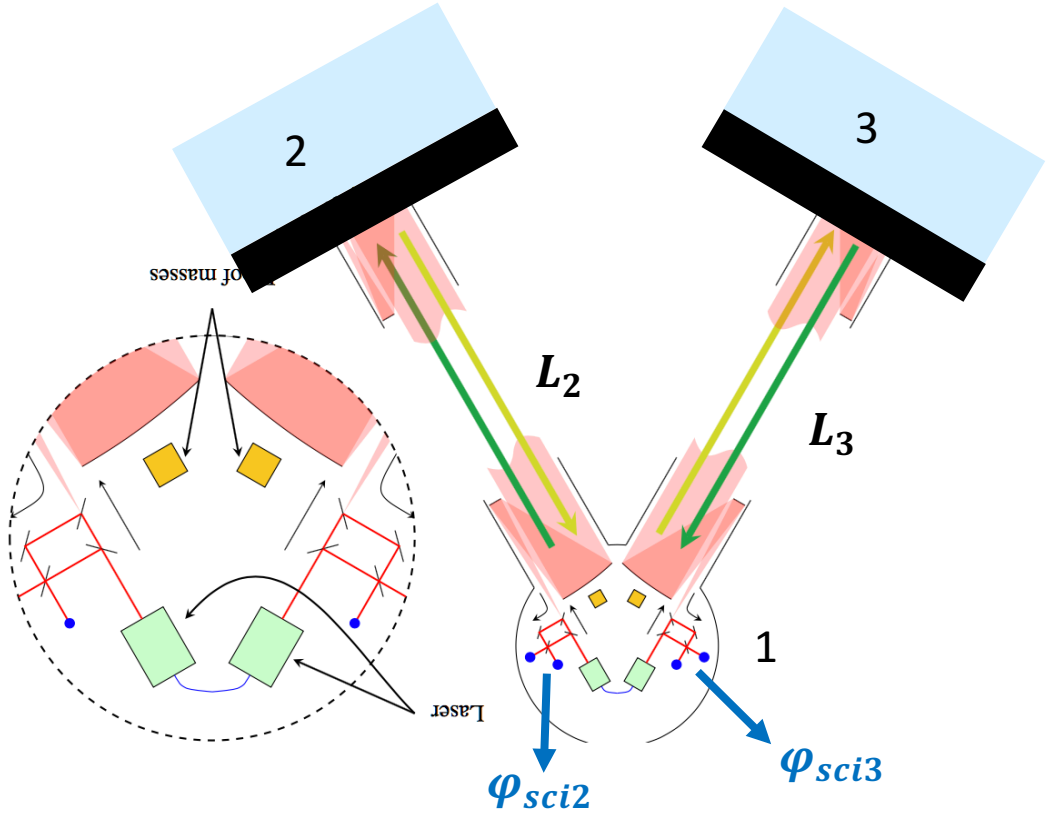


- Only one master laser has the cavity to be pre-stabilized.
- Other lasers are stabilized by the reference interferometer or arm locking
- This leads to the frequency noise with the same order as the ground-based detector
- But the large difference in the path length prevent the residual noise from being cancelled
- LISA utilize the technique, called TDI, during **post-processing** to solve this problem

Frequency Stabilization/TDI

Assumption:

1. Lasers in the one S/C 1 can be considered as “identical” ones -> BS in the standard MI
2. S/C 2 & 3 can be considered as “mirrors” because of transponder
3. Arm length is fixed



With these assumptions, the signal of the science interferometer can be derived like the following.

$$\varphi_{sci2}(t) = \varphi_1(t) - \varphi_1\left(t - \frac{2L_2}{c}\right) + h_2(t) \quad (1)$$

$$\varphi_{sci3}(t) = \varphi_1(t) - \varphi_1\left(t - \frac{2L_3}{c}\right) + h_3(t) \quad (2)$$

From these data series, we can calculate the linear combination such that the phase noise is cancelled.

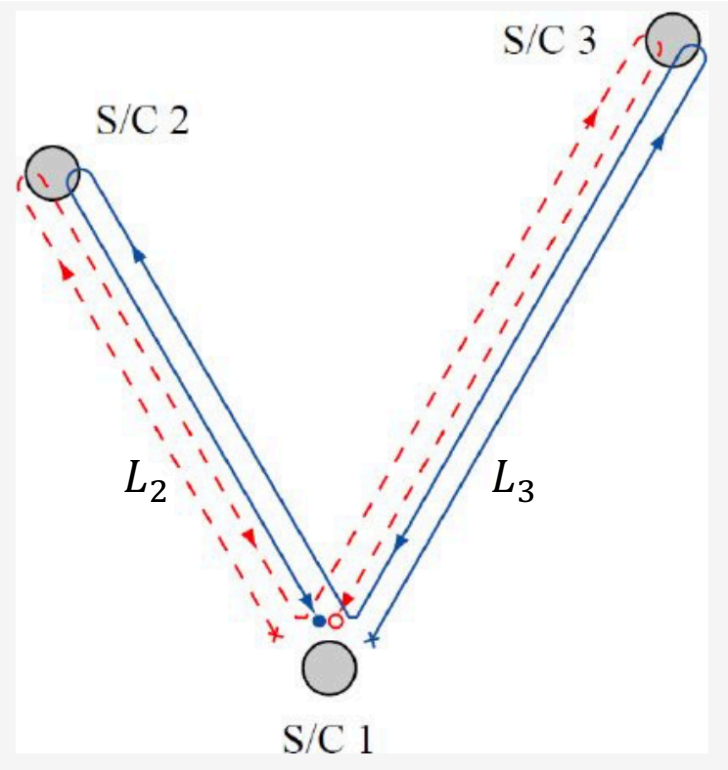
$$X \equiv \varphi_{sci2}(t) - \varphi_{sci3}(t) - \varphi_{sci2}\left(t - \frac{2L_3}{c}\right) + \varphi_{sci3}\left(t - \frac{2L_2}{c}\right)$$

$$= h_2(t) - h_3(t) - h_2\left(t - \frac{2L_3}{c}\right) + h_3\left(t - \frac{2L_2}{c}\right)$$

This is called **Michelson X-combination**

Frequency Stabilization/TDI

This combination can be interpreted as the **reconstruction of the equal-armed Michelson interferometer** and cancel the common phase noise like a standard MI does.



$$X = [\varphi_{sci2}(t) + \varphi_{sci3}\left(t - \frac{2L_2}{c}\right)] - [\varphi_{sci3}(t) + \varphi_{sci2}\left(t - \frac{2L_3}{c}\right)]$$

These two terms are considered as lasers which have propagates in the same distance, $L_2 + L_3$, and are combined at time of t at S/C 1.

We can naturally understand
this combination cancel the phase noise

Ref: Tinto “Time-Delay Interferometry”

Frequency Stabilization/TDI

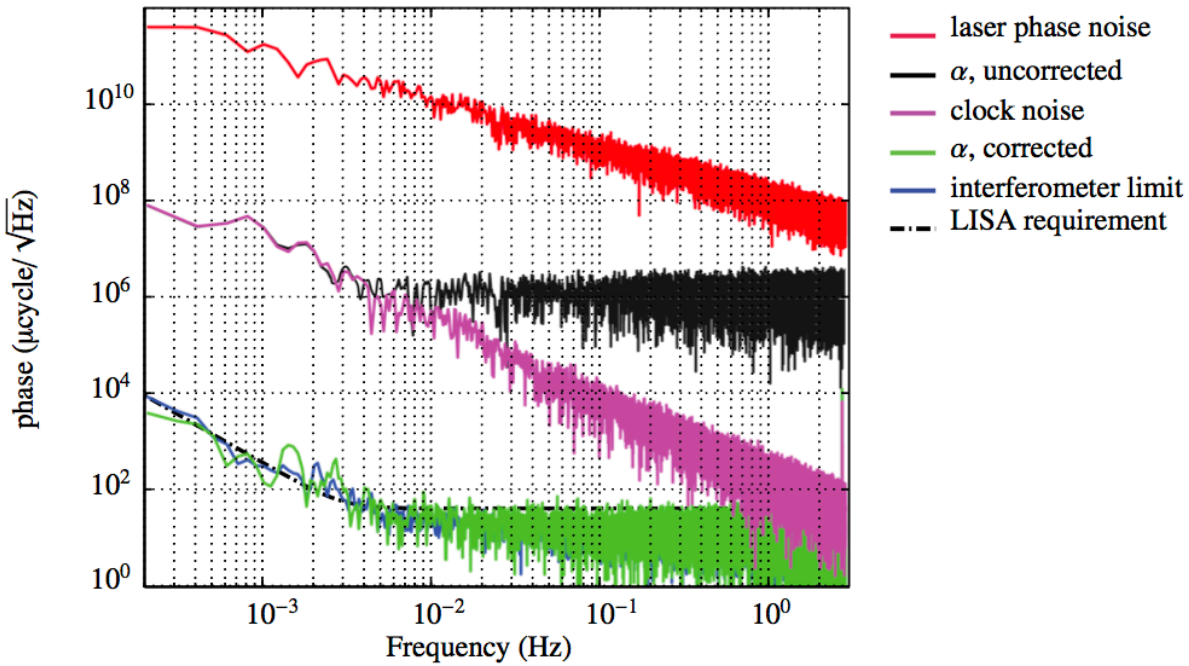


Figure 4.7.: *Displacement measurements of the LISA interferometry test bed, showing injected laser phase noise (red); interpolated and clock noise corrected Sagnac TDI variable α (green), demonstrating phase noise cancellation by about 9 orders-of-magnitude, down to the interferometer noise floor (blue). Note:* 1 μcycle ≃ 1 pm *displacement equivalent. Adapted from de Vine et al. (2010).*

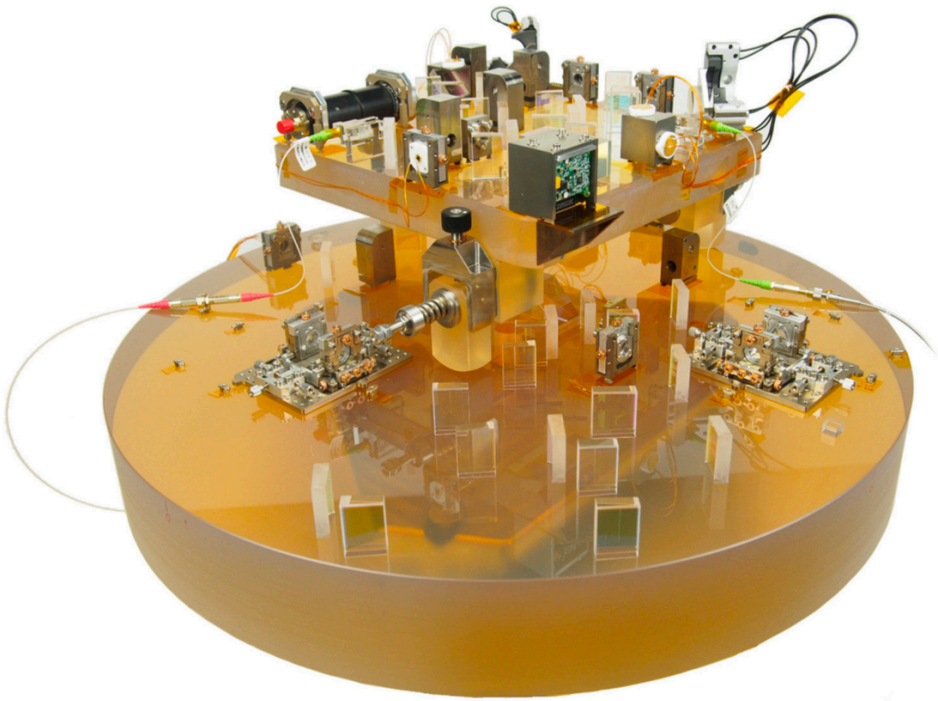
- TDI technique has been demonstrated by the LISA interferometry test bed
- The laser phase noise is suppressed by 9 order of magnitude down to the interferometer limit and LISA requireement.

Topics of my research @ AEI

- One example is the research of the Tilt-to-Length (TTL) coupling, which has great contributions to the displacement noise and often limit the sensitivity
- Another is the test of phasemeter with an unique interferometer, called Hexagon
- Main target of my research can be the estimation of the displacement noise with links and develop the LISA interferometry chain further so that it meets the requirement.

Table 3.3.: *Summary of IMS Subsystem Noise Allocations*

Effect	Total per group $\times 10^{-12}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$	Sub-allocation $\sqrt{1+(2.8\,\mathrm{mHz}/f)^4}$	Comments
Total IMS error/noise budget	18		
Total of subsystem allocations	11.7		RSS of subsystems
Subsystem allocations			
Shot noise	7.7		100 pW received power
Pathlength noise	7		
Pointing errors		5.3	
Telescope pathlength stability		1	
Optical bench pathlength stability		4.5	TTL is here
Measurement noise	5.4		RSS
Photoreceiver errors		3	
Residual laser frequency noise		2	
Residual clock frequency noise		2	
Phasemeter noise		1	
Intensity noise		1	
Phase reconstruction		1	
Stray light		2	



Appendix