Gravitational waves from binary black holes

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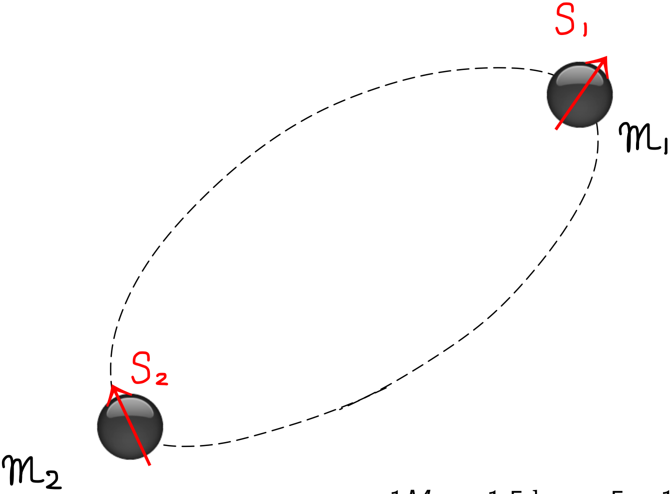
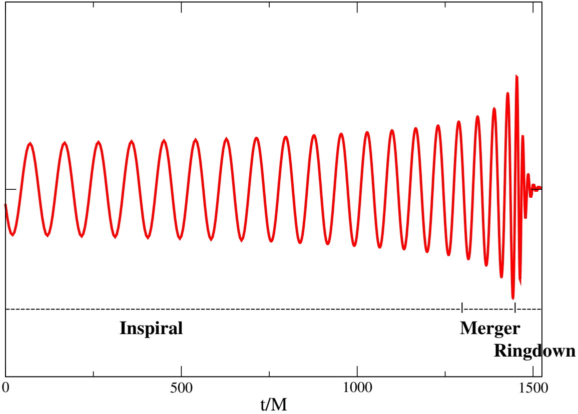
Binary black holes (BBHs)



$1 M_{\odot} \sim 1.5 \text{ km} \sim 5 \times 10^{-6} \text{ s}$

Gravitational waves from BBHs

Gravitational waveform



Inspiral: Post-Newtonian

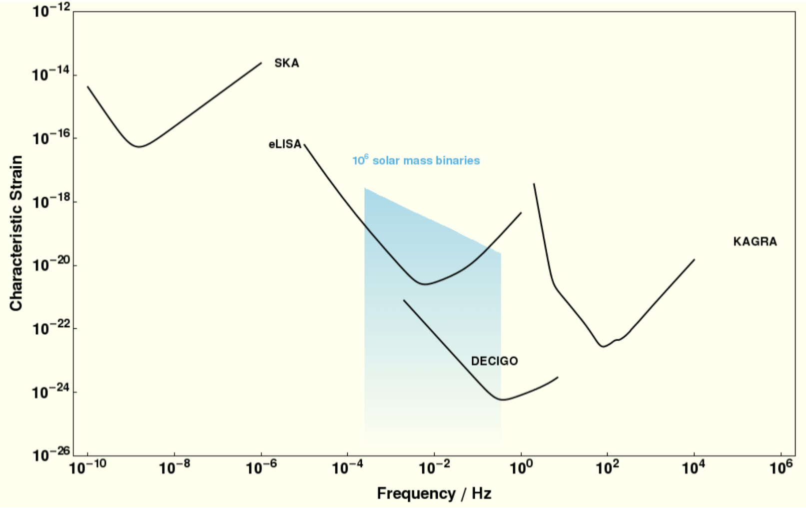
Merger: Numerical Relativity

Ringdown: Black hole perturbation

Gravitational wave observations of BBHs

Various gravitational wave detectors and $f_{\rm merge} \sim 200 \left(\frac{20 M_\odot}{M}\right)$ Hz

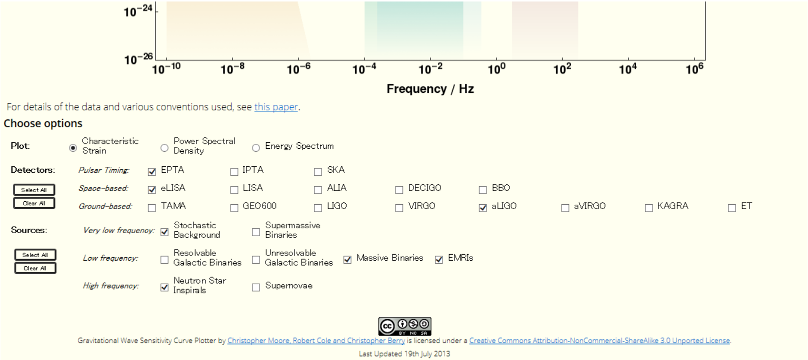
(from http://www.ast.cam.ac.uk/~rhc26/sources/)



$10^9 M_\odot$ $\quad 10^6 M_\odot$ $\quad 10^3 M_\odot$

Misc.

(from http://www.ast.cam.ac.uk/~rhc26/sources/)



By Christopher Moore, Robert Cole and Christopher Berry from the Gravitational Wave Group at the Institute of Astronomy, University of Cambridge

Numerical INJection Analysis (NINJA): 1

NINJA-1 : Numerical relativity (NR) and
Data analysis (DA) communities

- 23 numerical waveforms (10 NR groups)
- Injected into Gaussian noise colored with the frequency sensitivity of first generation detectors
- Search and parameter-estimation (9 DA groups)

Aylott *et al.*, Class. Quantum Grav. 26 (2009) 114008.
Aylott *et al.*, Class. Quantum Grav. 26 (2009) 165008.

Major limitations

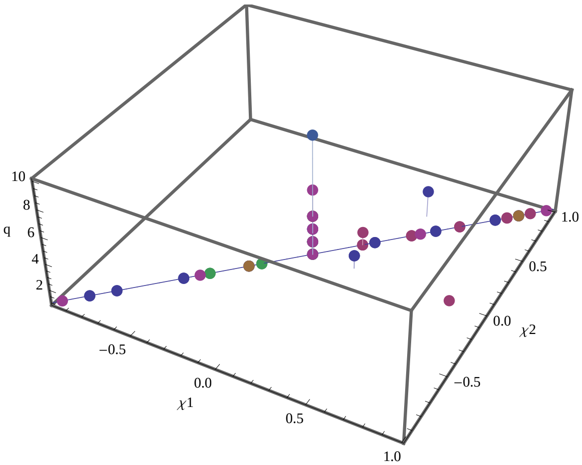
- No length or accuracy requirements for the NR waveforms
- No non-Gaussian noise transients

Numerical INJection Analysis (NINJA): 2

NINJA-2 : for real science

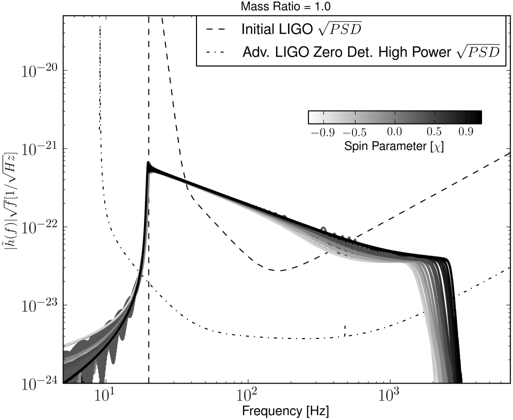
- Use real noise through LSC/Virgo MOU
- More NR waveforms (13 NR groups)
- PN-NR stitching to cover mass down to $\sim 10 M_\odot$.
- Accuracy requirements
  - 5 usable orbits before merger
  - NR (2, 2) amplitude accuracy below 5%
  - NR (2, 2) accumulated phase error ≤ 0.05 radian
  - GW stitching at $M\omega_{2,2} \leq 0.075$
  - Hybrid GWs start at $M\omega_{2,2} \leq 0.006$ ($\leftarrow 10 M_\odot$ at 20Hz)
  - Highest PN order available for phase and amplitude
- Only $\ell = |m| = 2$ required, but all harmonic modes welcome.

Aligned (anti-aligned) spinning (non-precessing) cases



q: mass ratio, χ_i: dimensionless spins

Equal-mass, equal-spin waveforms ($\chi = (\chi_1 + \chi_2)/2$)
$10M_\odot$, 100Mpc



Ajith *et al.*, Class. Quantum Grav. 29 (2012) 124001.
Ajith *et al.*, Class. Quantum Grav. 30 (2013) 199401.

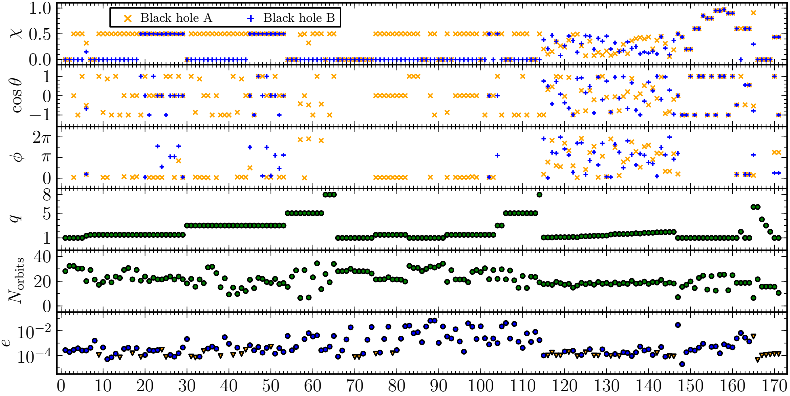
In progress

- Real data from GW detectors
- Recolored data for the sensitivities expected from aLIGO/aVirgo in 2015-16.
- "blind injections"

https://www.ninja-project.org/doku.php?id=ninja2:home

Recent progress: 1

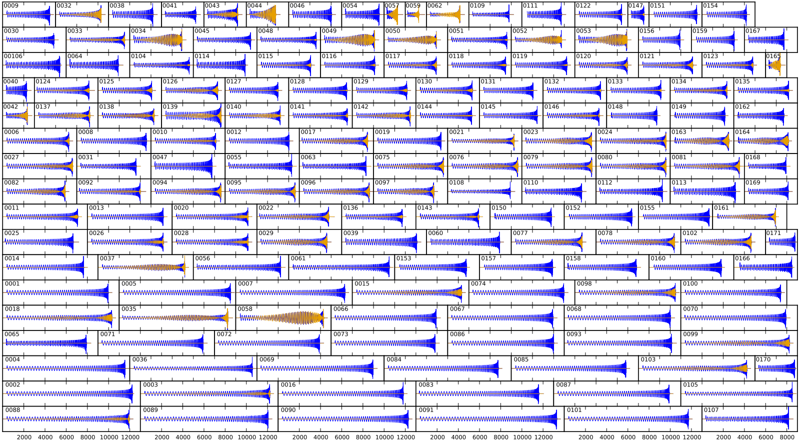
A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy, Mroue *et al.*, [arXiv:1304.6077]



χ: dimensionless spin, (θ, ϕ): initial spin direction,
$q = m_1/m_2$: mass ratio, $N_{\rm orbits}$: number of orbits before merger,
e: initial eccentricity

Recent progress: 2

A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy, Mroue *et al.*, [arXiv:1304.6077]



(See also Pekowsky *et al.*, [arXiv:1304.3176], over 220 waveforms)

Numerical Relativity/Analytical Relativity (NRAR): 1

Collaboration between Numerical and Analytical Relativity

NR groups

Abbreviation	Group name
JCP	Jena-Cardiff-Palma
FAU	Florida Atlantic University
GATech	Georgia Tech
RIT	Rochester Institute of Technology
Lean	Ulrich Sperhake
AEI	Albert Einstein Institute
PC	Palma-Caltech
SXS	Simulating eXtreme Spacetimes (Caltech, Cornell, CITA, CSU Fullerton)
UIUC	University of Illinois, Urbana-Champaign

Hinder *et al.*, [arXiv:1307.5307]

Numerical Relativity/Analytical Relativity (NRAR): 2

Plan:

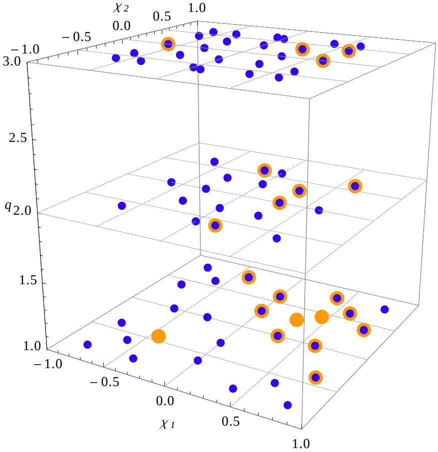
- Stage 1: Basic coverage ($q = 1$, 2, 3, mild spins)
- Stage 2: Additional precessing configurations
- Stage 3: More challenging configurations
  - Higher mass ratios ($q \gg 10$)
  - Large spins ($|\chi| > 0.95$)
  - Long ($>$ 40orbits)

Targets:

- $\sim$ 20 usable GW cycles
- Eccentricity: Aim for $e < 0.002$ as conservative target
- Phase error $\Delta\phi(t) < 0.25$radians up to $M\omega_{GW} = 0.2$ ($<$ 1orbit before merger)
- Relative amplitude error $\Delta A/A < 0.01$

[See Ian Hinder,
http://www.grg.uib.es/NRDA13/slides/Hinder_NRAR.pdf]

First stage of the NRAR collaboration
(60 simulations, for spinning non-precessing configurations)

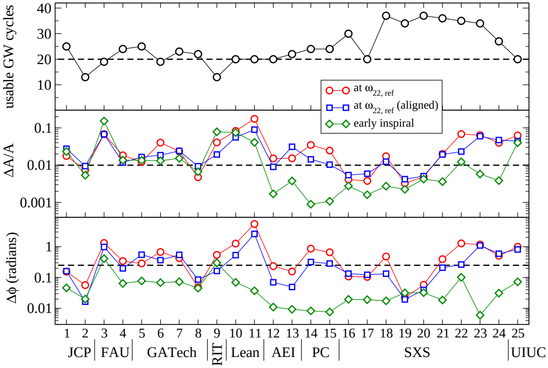


Analyzed configurations in Hinder *et al.*, [arXiv:1307.5307]

#	Group	Label	q	$\mathbf{S}_1/m_1^2$	$\mathbf{S}_2/m_2^2$	M_f/M	$\lvert\chi_f\rvert$
1	JCP	J1p+49+11	1	$(-0.128, 0.171, 0.494)$	$(0.129, -0.149, 0.106)$	0.941	0.774
2		J2−15+60	2	-0.150	0.600	0.961	0.611
3	FAU	F1p+30−30	1	$(0.000, -0.520, 0.300)$	$(0.520, 0.000, -0.300)$	0.952	0.704
4		F3+60+40	3	0.600	0.400	0.958	0.800
5	GATech	G1+60+60	1	0.603	0.603	0.927	0.858
6		G2+15−60	2	0.150	-0.607	0.962	0.635
7		G2+30+00	2	0.301	0.000	0.955	0.717
8		G2+60+60	2	0.601	0.607	0.940	0.839
9	RIT	R10	10	0.000	0.000	0.992	0.263
10	Lean	L4 *	4	0.000	0.000	0.978	0.472
11		L3+60+00	3	0.600	0.000	0.957	0.792
12	AEI	A1+30+00	1	0.300	0.000	0.947	0.732
13		A1+60+00	1	0.602	0.000	0.942	0.775
14	PC	P1+80−40	1	0.802	-0.400	0.945	0.744
15		P1+80+40	1	0.801	0.400	0.927	0.856
16	SXS	S1+44+44 *	1	0.437	0.437	0.936	0.814
17		S1−44−44 *	1	-0.438	-0.438	0.961	0.548
18		S1+30+30	1	0.300	0.300	0.942	0.775
19		S2+30+30	2	0.300	0.300	0.953	0.734
20		S3+30+30	3	0.300	0.300	0.965	0.680
21		S3p+00−15	3	0.000	$(0.260, 0.005, -0.150)$	0.972	0.536
22		S1p+30+30	1	$(0.054, -0.514, 0.305)$	$(0.054, -0.514, 0.305)$	0.937	0.804
23		S1p−30−30	1	$(0.000, 0.520, -0.300)$	$(0.000, 0.520, -0.300)$	0.958	0.638
24		S3−60+00	3	-0.599	0.000	0.978	0.271
25	UIUC	U1+30+00	1	0.300	0.000	0.947	0.732

$q = m_1/m_2$, $\mathbf{S}_i/m_i^2$: dimensionless spin, $M = m_1 + m_2$,
M_f and $\lvert\chi_f\rvert$: mass and dimensionless spin of the final BH

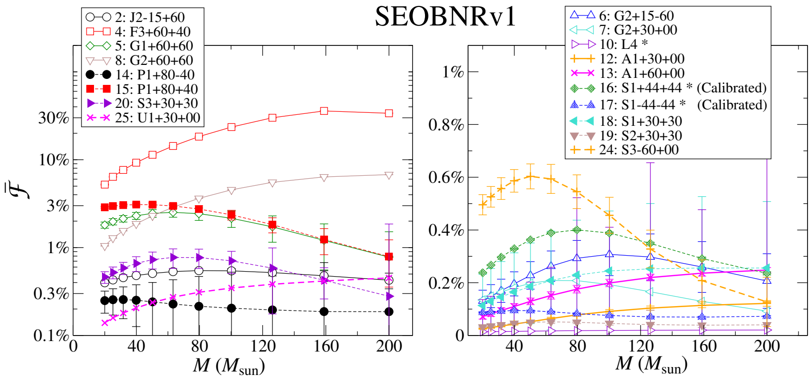
Error estimates for the NR simulations



- Finite numerical resolution
- Waveform measurement at finite distance from the source
- Computation of h from ψ_4

Spinning non-precessing template models

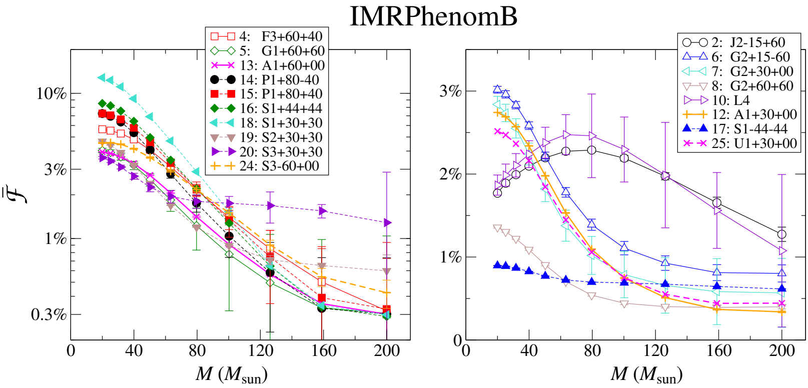
Comparison with time-domain SEOBNRv1 model



Spin-aligned EOBNR model.

Spinning non-precessing template models

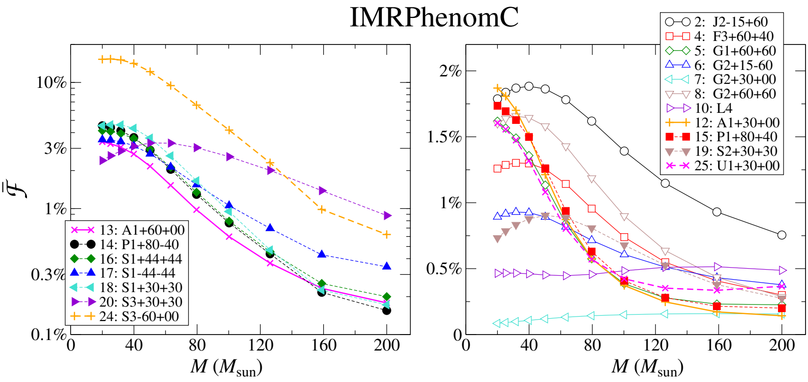
Comparison with Frequency-domain phenomenological IMRPhenomB



Frequency domain (non-precessing spins) inspiral-merger-ringdown templates of Ajith *et al.* (2011).

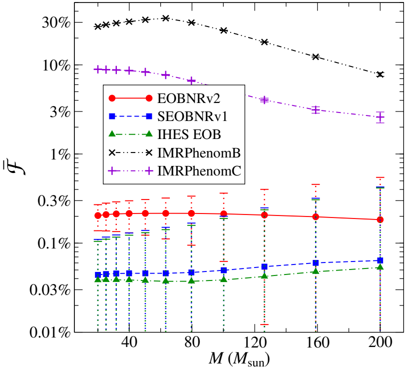
Spinning non-precessing template models

Comparison with Frequency-domain phenomenological IMRPhenomC



Frequency domain (non-precessing spins) inspiral-merger-ringdown templates of with phenomenological coefficients defined in the Table II of Santamaria *et al.* (2010).

Non-spinning $q = 10$ waveform



Unfaithfulness of analytical non-spinning $q = 10$ waveforms generated by IMRPhenomB, IMRPhenomC and three versions of EOB models with the numerical non-spinning $q = 10$ waveform (R10).

Intermediate-mass-ratio BBHs

Mass ratio: $1/10 \geq q \geq 1/100$

"Full numerical simulations" and **"Analytic treatments"**

NR: challenge in the exploration of BBH parameter space

AR: Post-Newtonian approach ($v \ll 1$)
Effective one body approach
Gravitational self-force ($q \ll 1$) and so on.

Simple as possible

- Regge-Wheeler-Zerilli formalism (BHP) + remnant BH's spin
- TaylorT4 orbital phase evolution (PN) + fitting parameters

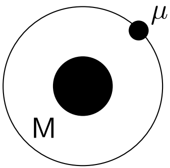
Lousto *et al.*, Phys. Rev. Lett. 104, 211101 (2010).
Lousto *et al.*, Phys. Rev. D82, 104057 (2010).
Nakano *et al.*, Phys. Rev. D84, 124006 (2011).

Final remnant of BBH mergers

Final remnant black hole's spin

	$q = 1/10$	$q = 1/15$	$q = 1/100$
Non-dim. spin	0.261 ± 0.002	0.189 ± 0.006	0.0332 ± 0.0001

- We want to introduce the spin effect into the black hole perturbation approach.
- BH ($M = m_1 + m_2$) + particle ($\mu = m_1 m_2 / M$)



Analytic, perturbative approach

Spin-Regge-Wheeler-Zerilli (SRWZ) formalism

- Extension of the RWZ for Schwarzschild perturbations.
- Include, perturbatively, a term linear in the remnant BH's spin.
- 2nd order perturbations.

$$\Psi_{\ell m}(t,r) = \Psi^{(1)}_{\ell m}(t,r) + \Psi^{(2)}_{\ell m}(t,r)\ ,$$

$$\Psi^{(\mathrm{o})}_{\ell m} = \Psi^{(\mathrm{o},1)}_{\ell m}(t,r) + 2\int dt\,\Psi^{(\mathrm{o},\mathrm{Z},2)}_{\ell m}(t,r)\ ,$$

$\Psi^{(1)}_{\ell m}$, $\Psi^{(2)}_{\ell m}$: Even parity Zerilli function

$\Psi^{(\mathrm{o},1)}_{\ell m}$: Odd parity Regge-Wheeler function

$\Psi^{(\mathrm{o},\mathrm{Z},2)}_{\ell m}$: Odd parity Zerilli function

The SRWZ formalism

Example (even parity wave equation)

$$
-\frac{\partial^2}{\partial t^2}\Psi_{\ell m}(t,r) + \frac{\partial^2}{\partial r^{*2}}\Psi_{\ell m}(t,r) - V_\ell^{(\mathrm{even})}(r)\Psi_{\ell m}(t,r) + i\,m\,\alpha\,\hat{P}_\ell^{(\mathrm{even})}\Psi_{\ell m}(t,r)
= S_{\ell m}^{(\mathrm{even})}(t,r;r_p(t),\phi_p(t))\,,
$$

$r^* = r + 2M\ln[r/(2M) - 1]$

α: Nondimensional spin parameter

$V_\ell^{(\mathrm{even/odd})}$: Potentials

$\hat{P}_\ell^{(\mathrm{even})}$: Differential operator

$S_{\ell m}$: Source term (2nd order)

- We need the particle's trajectory $(r_p(t), \phi_p(t))$.

Trajectory: Orbital frequency Ω evolution

Based on TaylorT4 evolution [Boyle *et al.* (2007)]

$$\frac{d\Omega}{dt} = \frac{96}{5}\,\Omega^{11/3} M^{5/3}\eta\,\left(1 + B\,(\Omega/\Omega_0)^{\beta/3}\right)^{-1}\Bigg[1 + \left(-\frac{743}{336} - \frac{11}{4}\,\eta\right)(M\Omega)^{2/3} + 4\,\pi\,M\Omega$$
$$+ \left(\frac{34103}{18144} + \frac{13661}{2016}\,\eta + \frac{59}{18}\,\eta^2\right)(M\Omega)^{4/3} + \left(-\frac{4159}{672}\,\pi - \frac{189}{8}\,\eta\,\pi\right)(M\Omega)^{5/3}$$
$$+ \left(\frac{16447322263}{139708800} + \frac{16}{3}\,\pi^2 - \frac{1712}{105}\,\gamma - \frac{1712}{315}\,\ln(64\,M\Omega) - \frac{56198689}{217728}\,\eta + \frac{451}{48}\,\eta\,\pi^2 + \frac{541}{896}\,\eta^2\right.$$
$$\left. - \frac{5605}{2592}\,\eta^3\right)(M\Omega)^2 + \left(-\frac{4415}{4032}\,\pi + \frac{358675}{6048}\,\eta\,\pi + \frac{91495}{1512}\,\eta^2\pi\right)(M\Omega)^{7/3} + A\,(\Omega/\Omega_0)^{\alpha/3}\Bigg],$$

$$\Omega = \frac{d\phi}{dt}\,, \quad M = m_1 + m_2\,, \quad \eta = \frac{m_1 m_2}{M^2}\,,$$

- A, α, B and β: **Fitting parameters**
- $M\Omega_0 = (1/3)^{3/2} \sim 0.19$ at $R_{\rm Sch} = 3M$ for circular orbit.
- $\alpha > 7$ and $\beta > 7$ to be consistent with the 3.5PN formula.

Trajectory: Orbital radius

Based on the ADM (Arnowitt, Deser and Misner)-TT PN
(NR $\sim$ ADM-TT/ "trumpet" stationary $1+\log$ slice of Schwarzschild)

$$R = \frac{M}{(M\Omega)^{2/3}} \left[1 + \left(-1 + \frac{1}{3}\eta \right) (M\Omega)^{2/3} + \left(-\frac{1}{4} + \frac{9}{8}\eta + \frac{1}{9}\eta^2 \right) (M\Omega)^{4/3} + \left(-\frac{1}{4} - \frac{1625}{144}\eta + \frac{167}{192}\eta\pi^2 - \frac{3}{2}\eta^2 + \frac{2}{81}\eta^3 \right) (M\Omega)^2 \right] / \left(1 + a_0 (\Omega/\Omega_0)^{a_1} \right) + C \,,$$

- R and Ω: in the NR coordinates
- a_0, a_1, C: **Fitting parameters**
- $M\Omega_0 = (1/3)^{3/2} \sim 0.19$
- $a_1 > 2$ to be consistent with the 3PN calculation.

C

Inconsistent with the ADM-TT PN formula. $\rightarrow$ But, we need!

Wave calculation in the SRWZ formalism

Radial transformation to remove the offset C between the NR and the "trumpet" coordinates by assuming $T_{\rm NR} = T_{\rm Log}$,

$$R_{\rm NR} \to R_{\rm Log} = R_{\rm NR} - C\,.$$

To the standard Schwarzschild coordinates,

$$(\,T_{\rm Log},\ R_{\rm Log}) \to (\,T_{\rm Sch},\ R_{\rm Sch})\,,$$

Final plunge trajectory:

Plunging (Schwarzschild) orbit from a matching radius $R_{\rm M} \sim 3M$ to the horizon $R = 2M$: Geodesic without the radiation reaction.

The SRWZ waveforms at a sufficiently distant location $R_{\rm Obs}$:

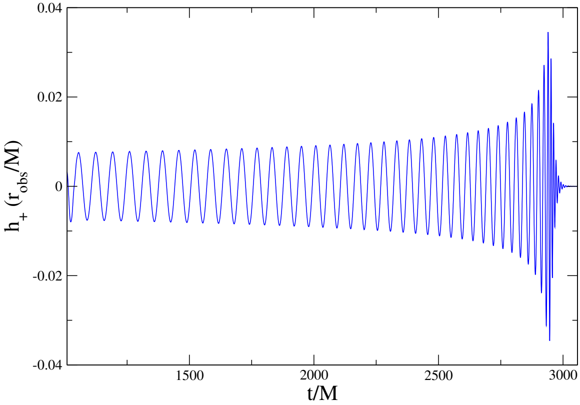
$$\frac{R_{\rm Obs}}{M}\,(h_+ - i\,h_\times) = \sum_{\ell m} \frac{\sqrt{(\ell-1)\ell(\ell+1)(\ell+2)}}{2M} \left(\Psi^{\rm (even)}_{\ell m} - i\,\Psi^{\rm (odd)}_{\ell m}\right) {}_{-2}Y_{\ell m}\,.$$

Short summary for wave calculation

Extended TaylorT4 (Trajectory)

$+$

SRWZ formalism (Wave generation)



Perturbative formula for NR waveforms

We extrapolate waveforms to $r \to \infty$,

$$\lim_{r\to\infty} [r\,\psi_4^{\ell m}(r,t)] = \left[r\,\psi_4^{\ell m}(r,t) - \frac{(\ell-1)(\ell+2)}{2} \int_0^t dt\,\psi_4^{\ell m}(r,t) \right]_{r=r_{\rm Obs}} + O(R_{\rm Obs}^{-2}),$$

$r_{\rm Obs}$: Approximate areal radius of the sphere $R_{\rm Obs}$=const.

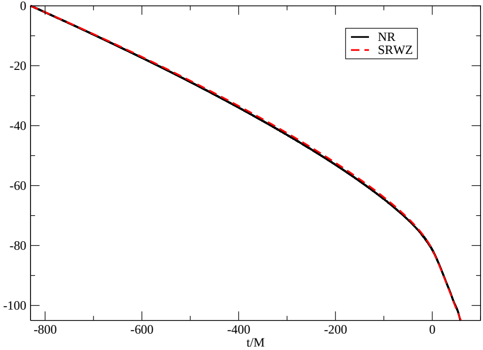
- This formula gives reliable extrapolations for $R_{\rm Obs} \gtrsim 100M$. (Numerical study by [M. C. Babiuc *et al.* (2011)])
- $\psi_4 \to h$: pyGWAnalysis code [Reisswig and Pollney (2011)] in EinsteinToolkit

Results: Gravitational wave phase ($q = 1/10$)

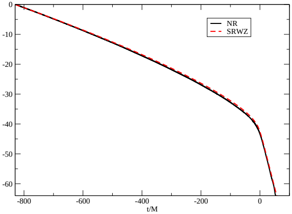
NR vs. SRWZ waveforms

$q = 1/10$, $\phi = 0$ at $t = -830M$.

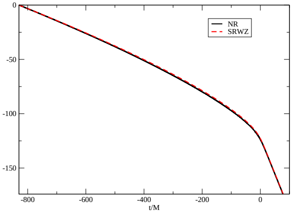
($\ell = 2,\ m = 2$)



($\ell = 2,\ m = 1$)



($\ell = 3,\ m = 3$)

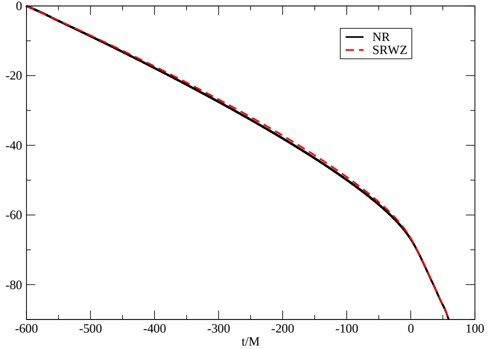


Results: Gravitational wave phase ($q = 1/15$)

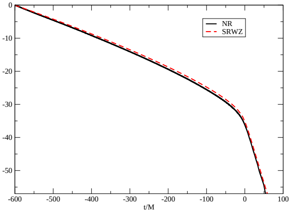
NR vs. SRWZ waveforms

$q = 1/15$, $\phi = 0$ at $t = -600M$.

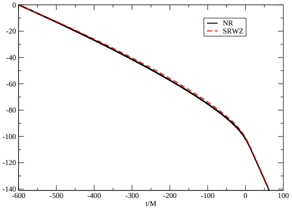
$(\ell = 2,\ m = 2)$



$(\ell = 2,\ m = 1)$



$(\ell = 3,\ m = 3)$

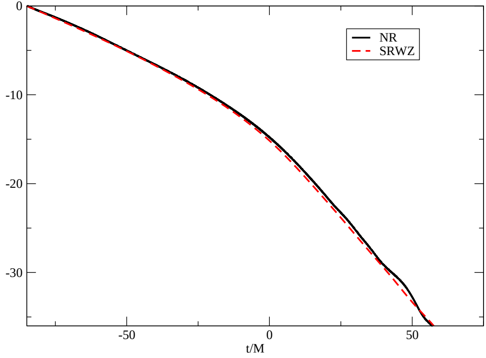


Results: Gravitational wave phase ($q = 1/100$)

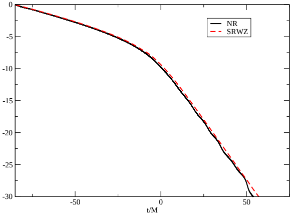
NR vs. SRWZ waveforms

$q = 1/100$, $\phi = 0$ at $t = -85M$.

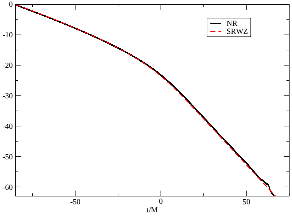
($\ell = 2,\ m = 2$)



($\ell = 2,\ m = 1$)



($\ell = 3,\ m = 3$)



Matching and next plan

Match between the NR and SRWZ ($\ell = 2$, $m = 2$) GWs in aLIGO (Zero Det, High Power). (Integration from $f_{\rm low} \sim 10$Hz.)

	$q = 1/10$	$q = 1/15$	$q = 1/100$
Range ($M\Omega_{22}$)	≥ 0.075	≥ 0.09	≥ 0.15
Total mass ($M_\odot$)	242	290	484
$\mathcal{M}_{22}$	0.994669	0.996039	0.995477

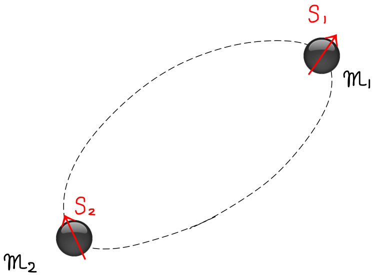
Next plan: Longer NR simulations, Various q cases

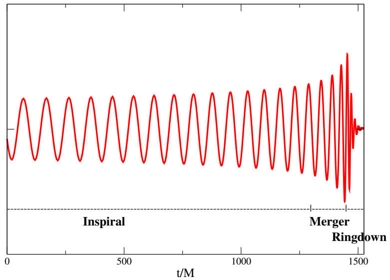
$\rightarrow$ Fitting parameters in fitting functions for the trajectory

$$(A,\, \alpha,\, B,\, \beta,\, a_0,\, a_1,\, C) = c_0 \eta^{c_1} \,.$$

Gravitational waves from binary black holes

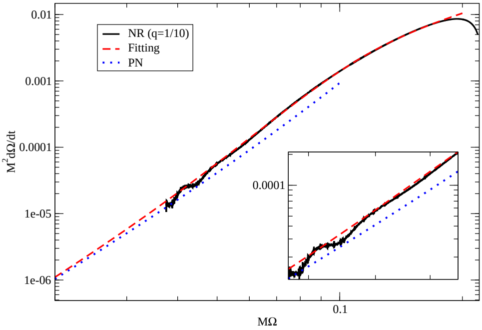
- Total mass → Frequency → Detectors
- Analytical Relativity – Numerical Relativity – Data Analysis





suppl.: Fitting for orbital frequency

- $d\Omega/dt(\Omega)$ for $q = 1/10$
- Up to $M\Omega = 0.175$:
  (NR orbital frequency around $R_{\rm Sch} = 3M$)
- End point of the NR curve:
  $R_{\rm Sch} = 2M$

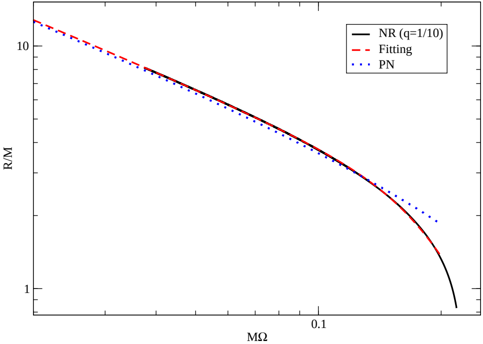


Black: NR, Red: Fitting, Blue: PN

Mass-ratio	A	α	B	β
$q = 1/10$	17.0500	7.21975 (> 7)	8.18920	12.5197 (> 7)
$q = 1/15$	26.0150	7.54047 (> 7)	8.65525	13.6168 (> 7)
$q = 1/100$	93.0650	4.32071 (< 7)	5.42457	14.9711 (> 7)

suppl.: Fitting for orbital radius

- $R(\Omega)$ for $q = 1/10$.
- Up to $M\Omega = 0.175$ ($R_{\rm Sch} = 3M$)
- End point of the NR curve: $R_{\rm Sch} = 2M$



Black: NR, Red: Fitting, Blue: PN

Mass-ratio	C	a_0	a_1
$q = 1/10$	0.216953 ($\neq 0$)	0.513214	4.68472 (> 2)
$q = 1/15$	0.237427 ($\neq 0$)	0.600321	4.57899 (> 2)
$q = 1/100$	0.198137 ($\neq 0$)	0.923360	5.29681 (> 2)